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TECHNICAL NOTE

A FIVE BODY – THREE DIMENSIONAL DYNAMIC ANALYSIS OF WALKING*

This note describes a technique for the dynamic analysis of articulated linkages (e.g. the lower limbs) employing a full three dimensional kinematic description of the segments and producing dynamic joint–force components without the use of a foot–floor force measuring device. The method is an extension of that described by McGhee *et al.* (1976) to three dimensions, and the results have been used with success as input data to a muscle force optimization procedure (Hardt, 1978).

For this development, it is assumed that the position of the center of gravity of each segment is known as a function of time as well as the angular displacement history of the segment. This data was provided here by a system developed at M.I.T. (called TRACK, *T*elemetered *R*eal-*T*ime *A*cquisition and *C*omputation of *K*inematics) that employs the Selspot (a trademark of SELCOM AB, Sweden) system combined with a PDP11/40 minicomputer to measure the 3-D position of points in space and then, by associating these points with body fixed coordinate frames, calculates the complete kinematic description of each segment without any assumption as to its connection with other segments. Details of this system can be found in Conati (1977). In addition, techniques are presently being developed to automatically locate the center of gravity given the assumption that it lies on a line connecting the instant centers of the segment articulations (see Antonsson, 1978).

The model is based on a Newtonian formulation (which might not immediately suggest itself for such a complex linkage) because this is an "inverse dynamics" problem, that is the forces and moments are the desired outputs rather than inputs and a full kinematic description of each free body in the system is assumed. It is soon obvious that a Lagranian method would be inappropriate here since the system model can be developed by simple repetition of the free body model of each segment.

DYNAMIC MODEL OF THE LOWER LIMBS

The human body can be approximately modelled as a system of articulated, rigid, massy links, and, given information about the motion of these links, the forces and moments acting on a system can be calculated. If each link is represented by a free body with a general set of forces and moments acting on its end points, as shown in Fig. 1, the equation of motion for each body can be written:

$$\Sigma F = m\ddot{\vec{x}}_{c.g.} = \vec{F}_1 + \vec{F}_2 + m\vec{g}, \qquad (1)$$

$$\Sigma \vec{M}_{c.g.} = \frac{d}{dt}(I_{c.g.}\vec{\omega}) = \vec{M}_1 + \vec{M}_2 + \vec{r}_1 x \vec{F}_1 + \vec{r}_2 \times \vec{F}_2, \qquad (2)$$

$\vec{F}_1, \vec{F}_2 \equiv$ end forces applied at joint centers

$\vec{M}_1, \vec{M}_2 \equiv$ joint moments

$m \equiv$ mass

$I_{c.g.} \equiv$ inertia tensor about the center of gravity

$\vec{r}_1, \vec{r}_2 \equiv$ moment arms of force vectors about the center of gravity,

where all vector quantities are referred to a common inertial reference frame. (Notice that the effect of muscle forces have been condensed into pure moment generators on the limb. This makes the problem solvable, but it also leaves the joint forces, F_1 and F_2 incomplete since the muscular component of this force must also be included.)

From complete kinematic information the accelerations $\ddot{\vec{x}}$ and $d/dt\,\vec{\omega}$ can be derived using numerical differentiation and the mass distribution information of Braune and Fischer (1889), for example, can be used to estimate m and I_{cg} given characteristic lengths and diameters for each subject. With this information, equation 1 can be solved for the forces

$$\vec{F}_1 + \vec{F}_2 = m\ddot{\vec{x}}_{c.g.} - m\vec{g}. \qquad (3)$$

If a force plate is available to measure the foot–floor force, equation (3) can be solved completely for the foot segment with the foot forces as the input and the ankle force as the output. Since no force attenuation occurs across the joints, subsequent proximal segment equations can now be solved provided their motion is known. Similarly, knowing all forces and measuring the foot–floor moment, equation (2) can be used to solve for all the joint moments. This is essentially the method pioneered by Bresler and Frankel (1950) and used by many contemporary investigators.

If a force plate is not available, as was the case here, the most distal force (the foot force) cannot be measured and a slightly different approach must be taken. If the links are assembled into a complete lower limb system, such as shown in Fig. 2, there will be five force and five moment vector equations, one for each link. However, there are six unknown force and moment vectors. This indeterminateness can be resolved by considering the normal walking cycle. During

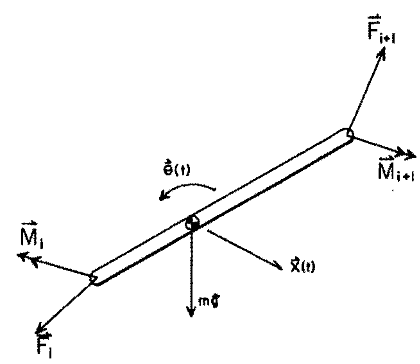

Fig. 1.

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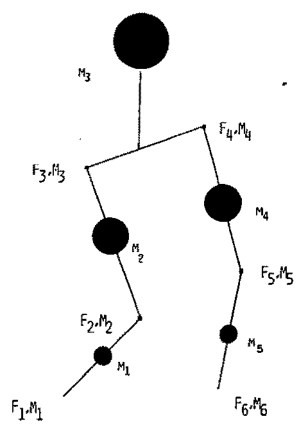

Fig. 2.

most of the cycle one of the legs is swinging above the ground, thus the most distal force and moment on that leg is zero and the system of equations is solvable. However, a problem arises when both feet are on the ground, which occurs during double leg stance. During this time the distribution of forces and moments between the feet is unknown. McGhee *et al.* (1976) approximated this force and moment transfer between the present stance foot and the present swing foot as a linear function of double leg stance time. To apply this to the present model consider the right leg in contact with the ground with the left leg completing its swing phase. The dependence of the forces and moments would be expressed:

$$\vec{F}_0 = \vec{F}_1 + \vec{F}_6, \quad \vec{M}_0 = \vec{M}_1 + \vec{M}_6$$

$$\vec{F}_6 = \frac{t - t_1}{t_2 - t_1}\vec{F}_0. \tag{4}$$

$$\vec{M}_6 = \frac{t_2 - t}{t_2 - t_1}\vec{M}_0, \tag{5}$$

where

t_1 = beginning of double leg stance

t_2 = end of double leg stance

$\vec{F}_1, \vec{M}_1$ = forces and moments on right ankle

$\vec{F}_6, \vec{M}_6$ = forces and moments on left ankle.

This dependence between the force and moment vectors of the ankle again yields a solvable set of equations and all forces and moments for the model can be found for the entire walking cycle provided the timing of the double leg stance portion is measured. (The latter was accomplished by the use of foot switches on the heel and toe of the subjects shoes.)

This solution procedure was implemented in the following manner. Combining link equations (1) and (2) into a system describing the 5 link model of Fig. 2, the matrix equations

$$M\vec{\ddot{x}} = \Gamma\vec{f} + \overrightarrow{mg} \tag{6}$$

$$\frac{d}{dt}(I\vec{\omega}) = \Gamma\vec{m} + \overrightarrow{rxF}, \tag{7}$$

can be written, where:

M = mass matrix (15 × 15 diagonal)
$\vec{\ddot{x}}$ = acceleration vector (15 element)
$\overrightarrow{mg}$ = gravity vector
$\vec{f}$ = force vector (15 elements)
I = inertia matrix (15 × 15)
$\vec{\omega}$ = angular velocity vector (15 elements)
$\vec{m}$ = moment vector (15 elements)
$\overrightarrow{rxF}$ = cross product vector
Γ = matrix of constants relating forces across the joints and the ankle dependence during double leg stance (15 × 15).

Solving these for the force and moment vectors yields:

$$\vec{f} = \Gamma^{-1}\,(M\vec{\ddot{x}} - \overrightarrow{mg}) \tag{8}$$

$$\vec{m} = \Gamma^{-1}\left(\frac{d}{dt}(I\vec{\omega} - \overrightarrow{rxF}\right). \tag{9}$$

Implementation of the single leg stance–double leg stance algorithm is accomplished by using the appropriate set of force and moment vectors (i.e. for left leg stance, the right leg ankle forces are dropped from the vectors) and by using the proper Γ matrix for that set. There will be three Γ matrices, one for each single leg stance phase and one for the double leg stance phase. In single leg stance, the Γ matrix reflects the coupling of equal and opposite forces across the joints, but for double leg stance the Γ matrix also incorporates equations (4) and (5) into the elements relating ankle forces and moments. The calculation routine must switch between these three matrices depending upon which phase of gait is occurring. During double leg stance the dependent ankle forces and moments must be calculated separately by application of equations (4) and (5) to the force set found above, which yields all six force and moment vectors.

The use of a five segment human body model necessitates several assumptions. By not including the feet their inertial contribution to the ankle moments is neglected. This does not seem severe since the feet represent only 5.7% of the total leg weight (Drillis and Contini, 1964). A greater loss is the lack of information about the foot position and the ankle angle. (As a result the assumption must be made that the ankle remains locked at zero degrees of flexion.)

By lumping the head, arms, trunk, and pelvis into one body (the HAT) the independent motion of these four elements is neglected. Probably the most significant of these motions is rotation of the trunk with respect to the pelvis and head about the vertical axis, which is normally accompanied by the characteristic swinging of the arms. The swing of the arms and rotation of the trunk tend to counterbalance an opposite rotation of the pelvis as the legs swing forward. By neglecting the arm swing, the calculated moments at the HAT–femur joints (the hips) will be higher than if arm swing were included. Although the HAT is rather massive, representing 63% of total body weight, this trunk rotation is of small magnitude and the concomitant error is not expected to be significant.

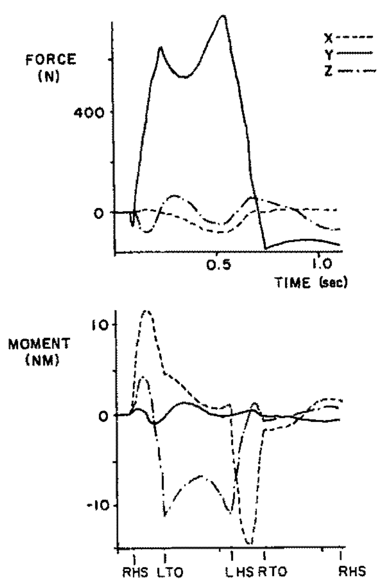

Fig. 3.

A final note on the actual calculations: the kinematic data used here was essentially position and orientation data referenced to the center of gravity of each segment in the absolute or fixed reference frame, therefore equation 8 can be solved by direct use of this data for the forces. The moment equation 9, on the other hand, is considerably more difficult to solve in the fixed reference frame because the inertia matrix as seen in this frame will change with time. However, if the equation is written instead in a body fixed reference frame the inertia matrix will be constant, and if the body fixed frame is aligned with the principal axes of the segment the inertia matrix becomes diagonal. To properly evaluate the term

$$\frac{\mathrm{d}}{\mathrm{d}t}(I\vec{\omega}),$$

with I expressed in a body fixed or relative frame, the angular momentum vector $(I\omega)$ is treated exactly like any other general vector, and the formula for the time rate of change of a vector in a rotating frame (Crandall *et al.*, 1968) is used:

$$\frac{\mathrm{d}}{\mathrm{d}t}(I\vec{\omega}) = I_{rel}\vec{\dot{\omega}} + \vec{\omega} \times I_{rel}\vec{\omega}.$$

This relationship is then applied to each segment in its respective body fixed frame. Before pre-multiplication by Γ^{-1}, as dictated by equation (9), each segment in the system must be referred to a common frame, the best choice being the fixed reference frame. With this modification, the moment equation of equation (9) becomes

$$\vec{m} = \Gamma^{-1}\ (C^{-1}\ (I\vec{\dot{\omega}} + \vec{\omega x I \omega} - \overline{rxF})_{rel}), \qquad (10)$$

where the elements of I and $\overline{rxF}$ are all referred to their respective body fixed frames and C^{-1} represents a 15×15 matrix that collects the individual segment rotation matrices along its diagonal. The resulting joint moments are thus referred to the fixed frame consistent with the joint force calculation.

The result of these calculations are moment–time histories for all of the leg joints and those components of the joint reaction forces arising from inertial and gravitational influences only. (As stated earlier the total joint force will also have a large contribution due to the muscular forces that can only be determined after the joint moments are known.)

RESULTS AND DISCUSSION

The calculation procedure detailed above was applied to data from three subjects. The resulting joint forces and moments for one of these trials is shown in Fig. 3. Considering the overall shape of the curves, they compare favorably with data from other investigators (e.g. Bresler and Frankel, 1950; Paul, 1965) or Crowninshield *et al.*, 1978). Clearly evident in the moment results, for example, is the abductive component (M_x) during stance required to support the trunk, and the strong extensive and flexive moments (M_z) at each double leg stance period. However, closer examination reveals several problems all of which can be traced to the assumed force dependence during double leg stance.

Points of greatest concern are: the sometimes abrupt transition at left toe-off and left heel-strike (see F_y) and the dependence of peak magnitudes (e.g. M_x and F_y) on the duration of double leg stance. This latter point places a great importance on the accurate determination of foot contact times during the walking cycle.

With only these reservations, however, this method appears quite powerful. Perhaps the greatest virtue is not the simultaneous calculation of bilateral dynamic joint forces and moments, but the illustration of the simplicity of the dynamic equations when the input kinematic data is presented as absolute motion of the segment centers of gravity. This eliminates relative rotations and the resulting Coriolis formulation, and yields a completely general set of equations from which any mechanical quantity of interest can be derived. Thus there is not "built-in" data structure that would restrict the application of this tool.

The double leg stance problems with this method can be resolved with force plate data and the resulting system would provide accurate limb dynamics for the entire lower limb system. This approach, which places fewer restrictions on the detail with which data parameters can be calculated, holds the promise of greater clinical use where, for example, lateral symmetry arguments are often unfounded.

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