

Determining Muscle Forces in the Leg During Normal Human Walking—An Application and Evaluation of Optimization Methods

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The individual muscle forces in the leg during human walking are unknown, because of a greater number of muscles when compared to degrees of freedom at the joints. The muscle force-joint torque equations can be solved, however, using optimization techniques. A linear programming solution of these equations applied at discrete, time-independent steps in the walking cycle using dynamic joint torque data is presented. The use of this technique, although capable of providing unique solutions, gives questionable muscle force histories when compared to electromyographic data. The reasons for the lack of confidence in the solution are found in the inherent limitations imposed by the linear programming algorithm and in the simplistic treatment of the muscles as tensile force sources rather than complex mechanochemical transducers. The definition of a physiologically rationalized optimal criterion requires both a global optimization approach and more complete modelling of the system.

Introduction

Classical mechanical analysis of human gait has been primarily concerned with those quantities most readily measured on human subjects. Consequently, the oldest area of study is the photographic measurement of human motion begun by Muybridge [1]¹ in the 1880's and now exemplified in the elegant classification studies of Murray [2]. More complete characterization of the system was provided by the work of Bresler and Frankel [3] who combined motion data with the mass properties of the human body and the measured foot-floor reaction to permit modelling of the lower limb dynamics. These techniques have since been applied many times [4, 5] to determine the torques at the skeletal joints caused by gravitational and inertial forces during movements of the lower limbs.

These calculated torques represent the requirements for dynamic equilibrium placed upon the muscles and ligaments of the musculoskeletal system. Since the lower limbs possess a mechanically redundant set of muscles, direct solution of muscle force—moment arm—joint torque equations and consequently the muscle force-joint reaction equations—is not possible. In addition, noninvasive techniques of measuring muscle force such as electromyography (EMG) do not provide the quantita-

tive accuracy required nor do they permit access to all muscles of interest. It is for these reasons that gait research has not been able to complete the mechanical characterization of the lower limb system and has not yet found wide application to clinical problems.

The importance of considering muscle action cannot be overstated. Besides being the actuators of the musculoskeletal system, muscular forces often exceed body weight and hence can dominate the loading on bones both at the joints and at the muscle attachment areas. It is clear that a complete mechanical description of human gait would have many applications including bone and joint mechanics, (osteoarthritis research, bone pathologies, prosthetic design) and muscle function research (*in-vivo* muscle activity or muscle as the final control element in overall movement strategies) as well as clinical studies of pathological gait arising from muscular activation abnormalities.

The indeterminant nature of the muscle force equations has been approached in several ways and for differing purposes. In relation to the hip joint reaction forces during gait and other activities, Paul [6] circumvented the intrinsic redundancy by grouping functionally similar muscles and their attachment points until the system was soluble. Morrison [7] followed the same concept for the knee and both authors produced data which gives valuable insight into the total joint reaction, that is the muscular component as well as the externally calculable gravitational and inertial contribution, however, detail as to individual muscle activity was lost. In areas other than human gait the concept of mathematical optimization has been used to determine muscle forces for eye movements [8], kicking movements [9] and static postures [10].

¹Numbers in brackets designate References at end of paper.

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This paper addresses the problem of obtaining individual force histories of all important muscles of the lower limb during normal level walking. The solution technique is static optimization and the full three dimensional dynamic equilibrium of the system is considered. While such an approach has been applied to quasi-static walking [11] and even considered as an accepted technique [12, 13] it has never been fully demonstrated. Indeed it will be shown that this technique applied only to the mechanical aspects of the problem leads to physiologically unreasonable results. In this sense the following is a feasibility study and can serve as a foundation for more refined approaches to this complex and multi-disciplinary problem.

The System

Fig. 1 is an idealized representation of a pelvis and lower limb system. The hip and ankle joints are represented by perfect ball joints (3 orthogonal rotatory degrees of freedom) while the knee is shown as a simple hinge (flexion-extension only). The muscle geometry is represented by point attachments with muscle force vectors directed along a line connecting these two points. The muscle is considered solely as a tension source with no time-varying qualities.

A model of the dynamics of this system could take the form

$$\ddot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \dot{\mathbf{x}}) + \mathbf{g}(\mathbf{u}) \quad (1)$$

\mathbf{x} = three-dimensional joint angles

$\dot{\mathbf{x}}$ = joint angular velocities

\mathbf{u} = vector of all muscle forces

where the function $\mathbf{f}(\mathbf{x}, \dot{\mathbf{x}})$ contains mass and geometric properties of the system while $\mathbf{g}(\mathbf{u})$ effectively transforms muscle forces to joint torques. The problem is, therefore, to find a unique con-

trol vector $\mathbf{u}(t)$ that will give the required trajectories. However, since the joint angles ($\mathbf{x}(t)$) are prescribed, i.e., can be measured, they can be differentiated with respect to time to provide $\dot{\mathbf{x}}(t)$ and differentiated again to provide $\ddot{\mathbf{x}}(t)$. Equation (1) is now reduced to:

$$\mathbf{m}(t) = \mathbf{g}(\mathbf{u}(t)) \quad (2)$$

where

$$\mathbf{m}(t) = \ddot{\mathbf{x}} - \mathbf{f}(\mathbf{x}, \dot{\mathbf{x}}) \quad (3)$$

With appropriate adjustment of the units involved, (2) can be interpreted as a vector of dynamic moment trajectories equated to a vector of muscle torques. All explicit dynamics have vanished and are contained implicitly in $\mathbf{m}(t)$ (hence the label "dynamic" moments). It is now clear that the general function \mathbf{g} is a matrix of orthogonal moment arms which transforms the muscle forces into joint torques. These moment arms are in general limb position dependent, therefore the complete system equation is

$$\mathbf{R}(\mathbf{x}(t))\mathbf{u}(t) = \mathbf{m}(t) \quad (4)$$

where \mathbf{R} = matrix of position dependent moment arms. Now the problem of muscle-joint redundancy becomes evident, since the dimension of \mathbf{u} , (31 muscles in the present model), is greater than three times that of \mathbf{m} , (7 degrees of freedom) precluding any unique solution of (4) as it stands.

The solution approach followed here is to provide an optimal criterion for selecting one member from the infinite set of possible $\mathbf{u}(t)$ trajectories that will satisfy (4). Further simplification is made by:

- 1 treating equation (4) as a time sequence of time independent states,
- 2 defining a penalty function (J) that is linear in \mathbf{u} .

These conditions prepare the problem for solution by a linear programming algorithm, (a static optimization method), which takes the general form:

$$\min J \quad ; \quad J = \mathbf{c}^T \mathbf{y}$$

\mathbf{y}
[\mathbf{c}^T = transpose of \mathbf{c} , a vector of constants] subject to the constraints

$$\mathbf{A}\mathbf{y} = \mathbf{b} \quad [\mathbf{A} \text{ and } \mathbf{b} \text{ are matrices of constants}]$$

$$\mathbf{C}\mathbf{y} \leq \mathbf{d} \quad [\mathbf{b} \text{ and } \mathbf{d} \text{ are vectors of constants}]$$

$$\mathbf{y} \geq 0$$

If we define $\mathbf{y} = \mathbf{u}$, then $\mathbf{A} = \mathbf{R}$, $\mathbf{b} = \mathbf{m}$ and $\mathbf{C} = 0$ (no inequality constraints). The restriction $\mathbf{y} \geq 0$ is fortuitous since muscle force is in fact unidirectional. For the present application, the cycle of walking is defined from heel contact to heel contact of one foot and is quantized into 50 equidistant intervals [$T = 1/50 \tau$; τ = period of cycle]. The final form of the optimization now becomes

$$\text{for } n = 1 \dots 50 \quad (5)$$

$$\min (\mathbf{c}^T \mathbf{u}(nT))$$

$$\mathbf{u}(nT)$$

subject to the constraints: $\mathbf{R}(nT)\mathbf{u}(nT) = \mathbf{m}(nT)$ and: $\mathbf{u}(nT) \geq 0$

$\mathbf{u}(nT)$ = muscle forces at time nT

$\mathbf{m}(nT)$ = dynamic joint torques at time nT

$\mathbf{R}(nT)$ = position and therefore time-dependent moment arm matrix at time nT

A revised simplex algorithm, implemented on a PDP11-40, was used to solve the equation system (5).

As discussed in the foregoing, data for joint angle and torque trajectories, $\mathbf{x}(nT)$ and $\mathbf{m}(nT)$, plus the matrix of muscle moment

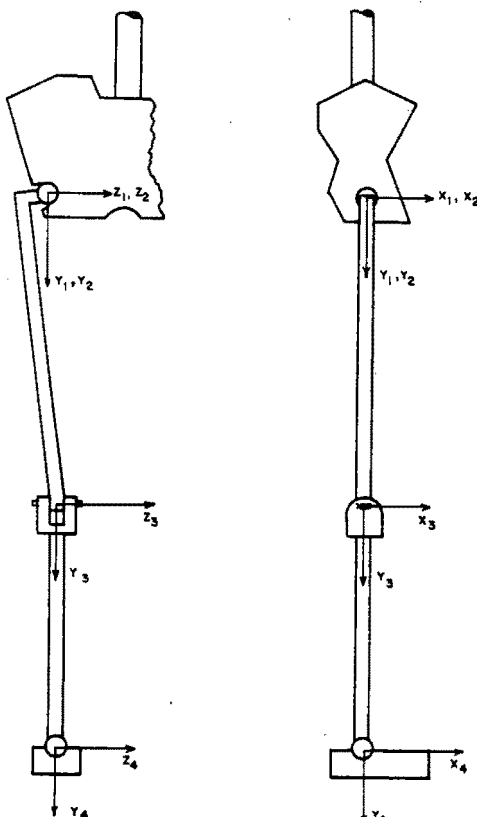


Fig. 1 Skeletal model and coordinate frames used. (Frame 1 is fixed to the pelvis, 2 to the femur, 3 to the tibia and 4 to the talus or foot.) Both the ankle and foot are ball joints while knee is a simple hinge.

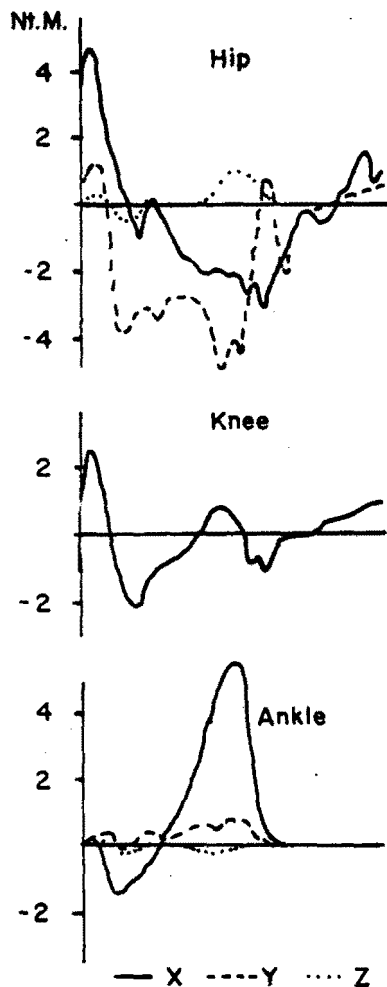


Fig. 2 Joint torque data from Bresler and Frankel [3] for one heel strike to heel strike cycle. (Z is in direction of progression, Y is vertical and X is out from the sagittal plane.)

arms $R(x(nT))$, for the hip, knee and ankle must be available for the force optimization. If limb angles are measured and mass properties of the body segments are known (or approximated), the joint torques can be found by solving equation (3). For this study, the measurement of joint angles and the derived torques, of Bresler and Frankel [3] (using a lumped mass assumption), were used, (see Fig. 2). $R(x(nT))$, the matrix of muscle-joint moment arms, was found by representing each muscle by position vectors from the assumed center of rotation of the appropriate joint to both the origin and insertion of each muscle with the limbs in a reference position. The data of Arvikar [14], who treated the muscles as straight line actuators with point attachments, was used for this purpose with the exception of the tensor fascia lata muscle which was treated as a uniaxial hip muscle as suggested by Hollingshead [15].

The attachment point vectors were referred to a coordinate frame fixed on the skeletal member of their attachment and relative rotations at the joints were represented by a 3×3 rotation matrix. Matrices for each joint were derived from the motion data by the use of modified Euler angles updated at the discrete time intervals. By applying the appropriate rotation matrix the origin and insertion vectors were put into a common coordinate frame and the individual muscle moment arm vectors were calculated by forming the cross product of the muscle origin vectors with their respective muscle length unit vectors. The right hand side of the constraint equation was also made consistent by referring the joint torque vectors (calculated in an

absolute or inertial frame) to the frames appropriate to the moment arm vectors at each joint.

The above information completes the constraint equations of (5) and attention can turn to definition of the cost of coefficients in the penalty function $J = c^T u$. Ideally this function should reflect a physiologically relevant quantity which is in fact minimized by the natural control system. However, the choice of such a function can be quite difficult, as illustrated by the penalty function "shopping list" given by Chao and An [16].

The simplest function, from the point of view of interpreting the results, sets all elements of c , the cost coefficients, equal to one thereby making J the sum of all muscle forces, which are then to be minimized. The first optimization discussed in the next section used this minimum force criterion.

Results

The muscle forces resulting from the minimum force optimization are shown in Fig. 4. Before discussing individual muscle force histories, however, the convenient integration of these forces at the joints will be exploited for comparison with other studies. In Fig. 3, the resultant joint force magnitudes are shown by the solid line. This represents the sum of the muscular forces predicted by the optimization and the gravitational and inertial forces due to the mass of the limbs. The latter two quantities were again borrowed from the study of Bresler and Frankel [3]. For the hip, (Fig. 3(a)), the result from Paul [6] and the *in-vivo* data from the instrumented prosthesis of Rydell [17] are cross-plotted. There is good agreement with Paul, in both magnitude and temporal aspects, with variations probably due to individual subject differences. The high force at 50 percent of the cycle (5.7 body weights), is primarily due to the large force in the tensor fascia lata muscle (see Fig. 4). The need for such a high force is possibly a result of imposing full three-dimensional moment equilibrium and the exclusion of the intrinsic hip rotator muscles from the Arvikar [14] model. In the three-dimensional treatment of dynamic equilibrium at the joints, the classical definition of agonist-antagonist muscle pairs breaks down since in general each muscle will have finite moment arms about all three axes and therefore antagonistic pairs cannot always be defined. This situation could lead to higher joint forces than if the muscles were arranged orthogonally.

At the knee (Fig. 3(b)), the calculated resultant when compared to that of Morrison [7] again shows good magnitude agreement, but does not contain the initial peak just after heel strike. This can be traced directly to differences between the moment data of Bresler and Frankel used here and that used by Morrison, since the general shape of the joint force curve mimics that of the rectified joint torque curve. The presence of antagonistic muscle activity (here well defined with only one degree of freedom allowed), is indicated by the simultaneous activity of the quadriceps muscles, (rectus femoris, and vastus medialis, intermedius and lateralis), and the gastrocnemius, and it has reduced the valleys in the 30 to 50 percent region of the curve. Morrison predicted that this would occur and that his solution represented a minimum joint force since he specifically excluded antagonist activity.

The ankle resultant (Fig. 3(c)) appears reasonable and the double peak at about 3.5 body weights (1 due to gravitational and inertial forces and 2.5 arising from muscular forces) is closely correlated with the flexion-extension moments required at that joint.

When individual muscle forces are examined (Fig. 4) the masking effect of considering only the resultant is apparent. In addition to the impossible force requirements placed on the tensor fascia lata muscle, the lack of activity in seven of the muscles, despite electromyographic evidence of their use, raises questions as to the overall fidelity of the solution. Although several muscles, notably the gluteus maximus, gluteus medius and the

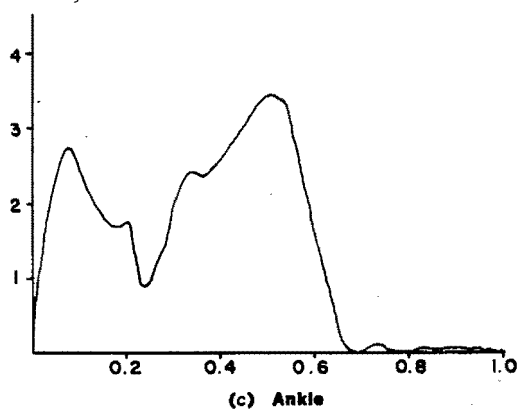
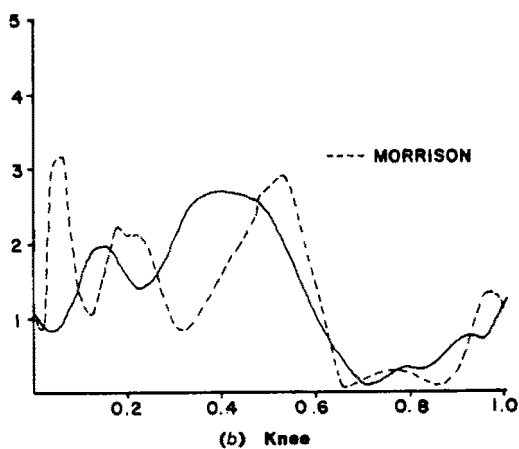
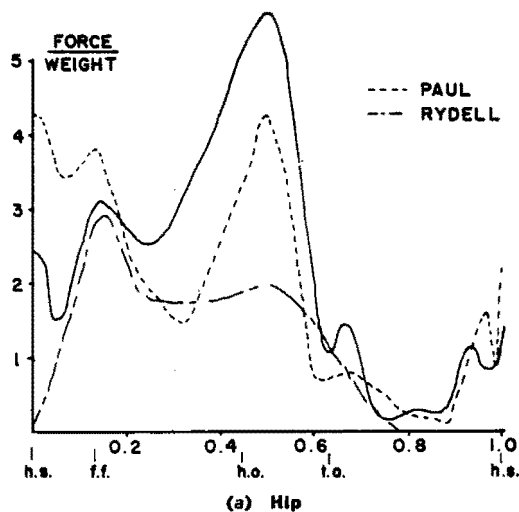


Fig. 3 Magnitude of the resultant force at each joint. The solid plot is the sum of the muscular forces from optimization plus inertial and gravitational forces from Bresler and Frankel [3]. Abscissa notations: h.s.—heel strike, f.f.—foot flat, h.o.— heel off, t.o.—toe off.

biceps femoris, show good temporal agreement with the EMG envelopes, most other active muscles show less than acceptable timing. The good agreement found is best explained in geometric terms: those muscles and the requirements placed upon them during level walking correspond closely to their geometric adaptation to these functions, this is, there is little choice but to use these particular muscles. However, when a true mechani-

cal redundancy exists, as in the case of the quadriceps for example, the distinction based on the information included in this model is not so clear. Indeed the placing of only moment constraints on the system is clearly responsible for the apparent paradox of reasonable joint forces with unreasonable muscle use.

A closer look at the implications of the minimum force criterion reveals that it yields a purely geometric optimization, whereby the set of muscle moment arm vectors which produce the lowest muscle forces will be chosen over all other possibilities. Consequently, the only representation of the muscles in the mathematics is in the form of their moment arms, ignoring the physiology of the system and the dynamic properties of the muscles. It is imperative, therefore, that muscles be considered more than just ideal unidirectional force actuators, since the central nervous system must surely consider their dynamic and static properties when devising the control strategy which this solution addresses.

As force sources, muscles have the following properties relevant to this study:

- nonlinear maximal force-velocity and maximal force-length relationships
- power producing and absorbing capability but with "impure" dissipation in the latter (i.e., input power still required)
- nonconstant mechanochemical efficiency
- different modes and therefore dynamics of energy storage (PCr, aerobic and anaerobic energy sources)
- functional adaptation to various types of activity
 - low power-low fatigue ("slow" muscles)
 - high power-high fatigue ("fast" muscles)
- passive force contribution due to elastic structural elements.

To incorporate some of these properties into the present formulation, it is proposed to define a cost function that will minimize the instantaneous energy requirements of the muscles. This involves representing the muscle thermodynamics, a description not presently available. Therefore, to approximate muscle energetics while conforming to the linear programming format, the following assumptions are made:

1 When actively contracting, the energy flux into a muscle is proportional to the mechanical power developed. (This implies a constant mechanochemical efficiency which is known *not* to be true [18].)

2 The energy requirement of the muscle when actively lengthening (absorbing power) is nonzero and is a small constant value which is independent of force or velocity [19].

3 The internal work during isometric contraction provides a bias for the entire positive power mode.

The elements of c (the cost coefficients) are shown as a function of shortening velocity in Fig. 5.

Therefore

$$\dot{E}_i = f_i v_i + 1.0 \text{ when } v_i \geq 0 \text{ (} i\text{th muscle)}$$

$$\dot{E}_i = 0.1 \text{ when } v_i < 0$$

$$v_i = \text{shortening velocity of muscle } i$$

and over the entire sequence

$$E_{\text{total}} = \sum_{n=1}^{50} c^f(nT) = \sum_{n=1}^{50} \sum_{i=1}^m c_i(nT) f_i(nT)$$

for m muscles ($m = 31$)

where

$$E = \text{free energy input to the muscles}$$

$$\dot{E} = \text{rate of free energy input to the muscles}$$

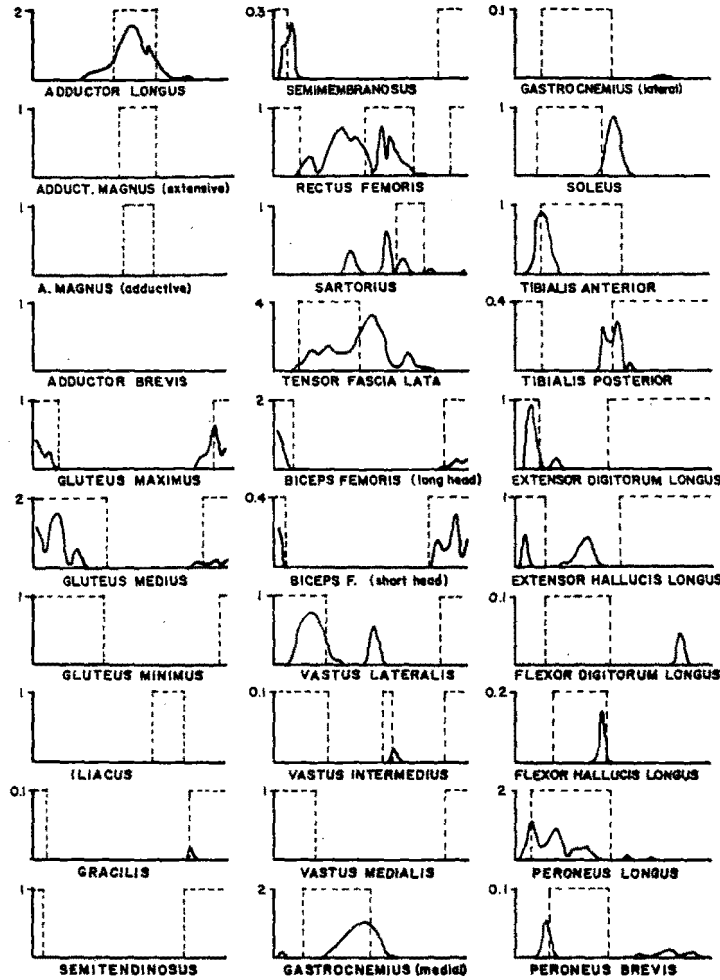


Fig. 4 Force trajectories from the minimum force optimization. The cycle is from heel strike to heel strike of one leg. Dashed envelopes indicate average presence of EMG activity as reported by UCLA Berkeley [24].

The results of such a cost function are shown in Fig. 6. The shortening velocities were determined by numerical differentiation of the muscle length trajectories precomputed according to the methods described previously. This overly simplified cost function again has a less elegant interpretation and in effect treats those muscles that are lengthening as the least costly. Next those with shorter moment arms and therefore lower shortening velocities are favored over those with longer moment arms. The general pattern has changed in that more muscles are participating in the movement, however, the foregoing comments still apply. The increased input as to the dynamic state of the muscle has distributed the duty but the problem of mechanical redundancy remains unresolved.

Conclusions

The mechanical aspects of the muscle force-limb movement problem can be suitably expressed mathematically and the indeterminacy of the solution can be resolved by linear programming. However, to accept this technique solely on this basis would be hazardous since the foregoing application illustrates the lack of agreement with those measures available. On the other hand, it would be just as perilous to assert that optimization solutions in general are inadequate or inappropriate for this problem. On the contrary, when motor control is viewed in a

context larger than just the mechanical requirements the concept of optimization arises [19] not because of its property of resolving indeterminacy, but in its proper role of providing a unifying criterion for decision making. Indeed, motor control can be analytically viewed within the formalism of adaptive control theory, where the basic elements—feedback of the outputs, monitoring the control signal and modification of the controller

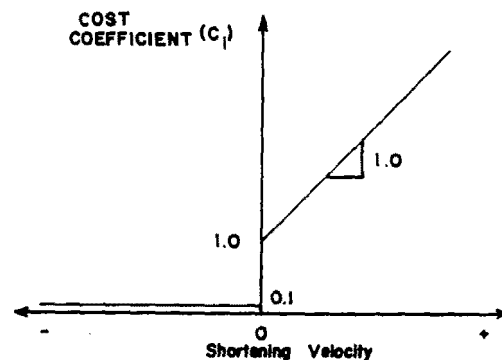


Fig. 5 Minimum muscle energy cost coefficient dependence on muscle shortening velocity

according to a performance index—, all have neurophysiological parallels in the rich kinesiological feedback from the muscles and joints [21], the direct feedback of the control signal (sometimes called corollary discharge [22]), and the gradual refinement of motor coordination with repeated trials or practice. This suggests that a natural process of optimization is at work, thus a purely biomechanical approach such as presented here is inadequate since the intrinsic qualities of the system being controlled, (primarily the muscles), are not sufficiently represented in the model.

The use of linear programming can be extended well beyond that demonstrated here, particularly with respect to additional constraints such as maximum force capabilities, intrinsic muscle response time or by direct incorporation of EMG temporal data. However, several severe limitations are imposed by this algorithm. A property of a linear constraint space with a linear performance index is that it limits the number of nonzero variables in a particular solution to a range between the number of equality constraints and the total number of constraints (equality + inequality), the so-called basic feasible solution. Since the demonstrated solution involved seven equality and no inequality constraints, only seven of the 31 muscles were active at any time step. This is an artificial restriction imposed on muscle use before the optimization is performed. The linear penalty function is also unnecessarily restrictive and as illustrated here and by others [10, 11, 14, 16] leads to function definitions that are em-

pirical in nature and may not, therefore, have any physiological analog. This is particularly true in the muscle force problem since no direct measure of muscle force exists.

Finally the fact that a static optimization is performed at discrete times implies that each of these events is independent of the others, (except for the connectivity of the moment data mentioned earlier). This is most severely reflected in the objective function where the local optimality implies that a global optimum has been found. This is not consistent with known muscle dynamics and muscle energy requirements which are clearly state dependent with a finite bandwidth of operation [23].

Summary

Optimization techniques are not only applicable but desirable in the determination of muscle forces during walking. However, the proper solution will require more input as to the physiology of the system since the optimization process itself must be viewed and therefore formulated as an analog to the real system rather than as solution convenience. Attention can then be focused more complete models of the system, such as thermodynamic descriptions of muscle function, and on defining optimal criteria which parallel possible natural criteria. It is only in this way that confidence can be placed in a solution for which there is no direct means of experimental verification.

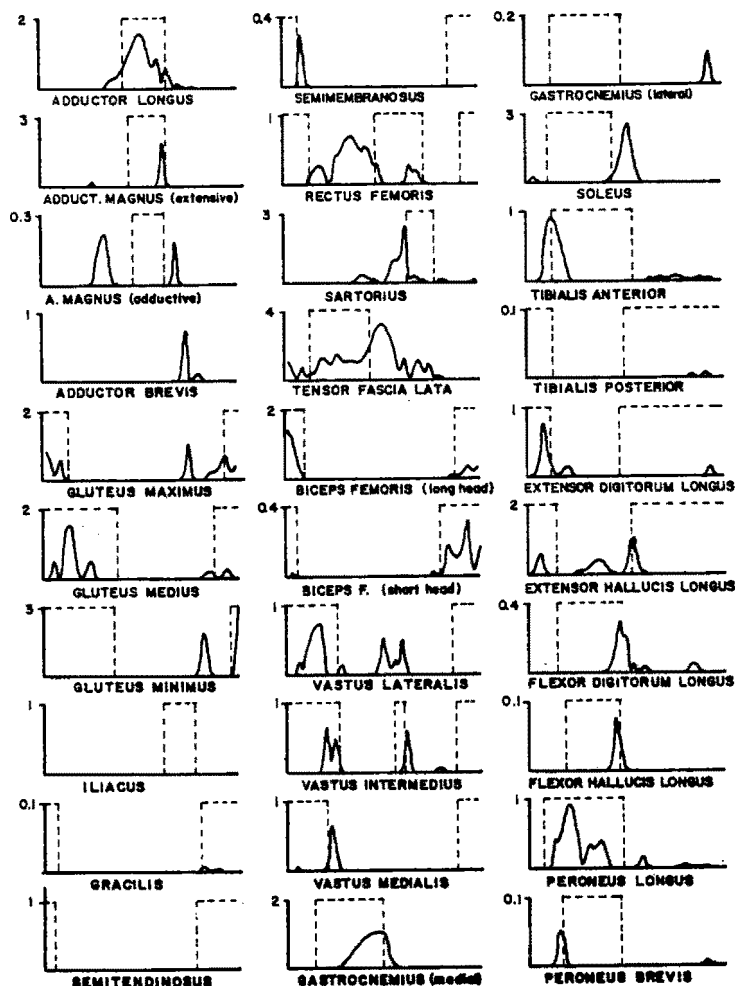


Fig. 6 Force trajectories from the minimum muscle energy optimization—EMG envelopes are the same as Fig. 2

Acknowledgments

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