# University of Pennsylvania

## ESE 605 Modern Convex Optimization
## Homework 8
Due on Friday April 6

1. *Robust least-squares with interval coefficient matrix:* An interval matrix in $R^{m \times n}$ is a matrix whose entries are intervals:

$$\mathcal{A} = \{A \in \mathbb{R}^{m \times n} | A_{ij} - \bar{A}_{ij}| \leq R_{ij}, \quad i = 1, \cdots, m, \quad j = 1, \cdots, n\}.$$

   The matrix $\bar{A} \in \mathbb{R}^{m \times n}$ is called the nominal value or center value, and $R \subset \mathbb{R}^{m \times n}$, which is element-wise nonnegative, is called the radius. The robust least-squares problem, with interval matrix, is

$$\text{minimize } sup_{A \in \mathcal{A}} \|Ax - b\|_2,$$

   with optimization variable $x \in \mathbb{R}^n$. The problem data are $\mathcal{A}$ (i.e., $\bar{A}$ and $R$) and $b \in \mathbb{R}^m$. The objective, as a function of $x$, is called the worst-case residual norm. The robust least-squares problem is evidently a convex optimization problem.

   (a) Formulate the interval matrix robust least-squares problem as a standard optimization problem, e.g., a QP, SOCP, or SDP. You can introduce new variables if needed. Your reformulation should have a number of variables and constraints that grows linearly with $m$ and $n$, and not exponentially.

   (b) Consider an over-determined specific problem instance with $m = 10$, $n = 5$, with $A$ generated in Matlab as $\bar{A}$=round(10*rand(10,5)), b=round(5*rand(10,1)) . The matrix $R$ should be chosen in such a way that the uncertainty in each entry of $A$ is $\pm 0.1$. Find the solution $x_{ls}$ of the nominal problem (i.e., minimize $\|\bar{A}x - b\|_2$), and robust least-squares solution $x_{rls}$. For each of these, find the nominal residual norm, and also the worst-case residual norm. You need to use $CVX$ for this problem.

2. Assume the set $\mathcal{A}$ in problem 1 is given by $\bar{A} + U$, where $\|U\|_2 \leq 0.1$. Find the robust least squares solution with the same problem data as before. Find the worst case residual norm.

3. Problem 8.9 and 8.20 from the text

4. State and solve the optimality conditions for the problem

$$\textbf{minimize} \quad \log \det \begin{bmatrix} X_1 & X_2 \\ X_2^T & X_3 \end{bmatrix}^{-1}$$

$$\textbf{subject to}$$

$$\textbf{tr } X_1 = \alpha$$
$$\textbf{tr } X_2 = \beta$$
$$\textbf{tr } X_3 = \gamma$$

The optimization variable is $X = \begin{bmatrix} X_1 & X_2 \\ X_2^T & X_3 \end{bmatrix}$

with $X_1 \in \mathbb{S}^n$, $X_2 \in \mathbb{R}^{n \times n}$, $X_3 \in \mathbb{S}^n$. The domain of the objective function is $\mathbb{S}_{++}^{2n}$. We assume $\alpha > 0$, and $\alpha\gamma > \beta^2$.