Resolution of Conflicts Involving Many Aircraft via Semidefinite Programming

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Abstract

Aircraft conflict detection and resolution is currently attracting the interest of many air transportation service providers and is concerned with the following question: Given a set of airborne aircraft and their intended trajectories, what control strategy should be followed by the pilots and the air traffic service provider to prevent the aircraft from coming too close to each other? This paper addresses this problem by presenting a distributed air-ground architecture, whereby each aircraft proposes its desired heading while a centralized air traffic control architecture resolves any conflict arising between the aircraft involved in the conflict, while minimizing the deviation between desired and conflict-free heading for each aircraft. The resolution architecture relies on a combination of convex programming and randomized searches: It is shown that a version of the planar, multi-aircraft conflict resolution problem that accounts for all possible crossing patterns among aircraft might be recast as a nonconvex, quadratically constrained quadratic program. For this type of problem, there exist efficient numerical relaxations, based on semidefinite programming, that provide lower bounds on the best achievable objective. These relaxations also lead to a random search technique to compute feasible, locally optimal and conflict-free strategies. This approach is demonstrated on numerical examples and discussed.

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1 Introduction

The air transportation system is currently the object of extensive research, following the sustained growth of air traffic over the past many years. The current enroute air traffic control system consists of the following elements:

- A geographical network whose nodes are navigation beacons (VHF Omnidirectional Range (VOR) and Distance Measuring Equipment systems (DME)), and whose links are air routes. The aircraft are allowed to fly only along these routes (with possible exceptions at those altitudes where air traffic density is very low). Flying on segments connecting two navigation beacons makes the problem of aircraft navigation and automated guidance particularly easy, although recent accidents seem to have been caused by apparent ambiguities about the identity of navigation beacons.

- Approximately 1500 enroute air traffic controllers who regulate the aircraft flow across this network and make sure no hazardous situation develops, whereby two aircraft might collide with each other (aircraft conflicts). The network structure of the aircraft routing system allows to set *a priori* guarantees on aircraft conflict geometries and their location during nominal operations: Conflicts are usually located at the nodes of the network. Knowing the conflict location *a priori* enables the decomposition of the airspace into *sectors*, managed by individual air traffic controllers, and whose boundaries are located away from the network nodes and therefore away from the most common conflict locations.

Many decades of working experience have demonstrated that this network-based architecture architecture is safe. However, it suffers from strong perceived drawbacks, such as systematic indirect routing between origin and destination, and in general a perceived lack of freedom for the aircraft pilots. The advent of a new generation of Global Navigation Satellite System (GNSS), in particular GPS, has removed in principle the limitations of the ground-based navigation infrastructure. In particular, it is now very easy to obtain precise aircraft position anywhere over the United States and not only on a pre-determined set of routes (although this idea, also named Area Navigation, has been demonstrated to be feasible for many years,[1] using the conventional navigation infrastructure, at the expense of new aircraft computational equipment). As a consequence, operational concepts such as "Free Flight"[2] have been proposed by airlines and the Federal Aviation Administration (FAA) to remove the routing constraints imposed by the conventional, fixed-route system. Under Free Flight, each aircraft would be able to optimize its trajectory according to several factors such as perceived safety, weather, direct operating costs and coordination with other flights.[3] However, the safety of a new concept of operations that sharply departs from the current, network-based architecture remains to be proved. In particular, the lack of predictability of conflict location under Free Flight seems to increase the apparent complexity of conflict detection and resolution for the human operator. This issue is currently under study. In addition, the set of standards over which operational concepts are evaluated has evolved from empirical evaluation decades ago to a sophisticated and very difficult certification process, which makes proving the safety of any new concept of operations very challenging to implement. Thus, Free Flight offers a wide array of new challenges to the research community.

This paper considers the problem of resolving conflicts arising between airborne aircraft, while accounting for aircraft preferences. Conflicts involving several (more than two) aircraft will be considered for the following reasons: First, conflicts involving a pair of aircraft have been the object of numerous studies already.[4–8] Second, conflicts involving more than two aircraft have been shown to occur in high-density sectors:[9,10] While more than two aircraft are rarely directly in conflict with each other, indirect conflict is a distinct possibility, whereby the solution to the conflict involving one pair of aircraft generates a secondary conflict with a third, neighboring aircraft. Several other approaches consider conflicts involving multiple aircraft include.[11–13] A comprehensive review has recently appeared.[14]

The current air traffic control operations are based on conflict avoidance rules and controller experience. Rule-based approaches might work for the case involving two aircraft,[5] but may require a prohibitive number of rules to handle all situations arising when more than two aircraft are involved. The present paper concentrates on optimization-based approaches, which avoid the explicit elicitation of rules. This computational approach follows the spirit of previous authors: Niedringhaus[15] proposes linear programming as a convenient modeling framework to formulate and solve efficiently conflicts arising among several aircraft. Durand, Alliot and Chansou[9] consider the same problem and propose to use genetic algorithms and linear programming to determine optimal maneuvers to solve conflicts arising among multiple aircraft. While both approaches emphasize (but are not limited to) planar conflict problems, the latter approach differs from the former in that it also optimizes the conflict resolution maneuver over possible *crossing patterns*, whereas the former approach requires *a priori* knowledge of the crossing pattern among aircraft.

In this paper, we will present an approach to the problem that is both computationally efficient and solidly rooted in recent advances in convex optimization to solve highly *nonconvex*, possibly combinatorial optimization problems:[16–19] We formulate the planar conflict resolution problem as a nonconvex, quadratically constrained quadratic program. This problem is then approximated by a convex, semidefinite program, for which very efficient solutions exist. The optimal solution to this convex program is then used to randomly generate feasible and locally optimal conflict resolution maneuvers. Based on this algorithm, we then propose a distributed conflict management architecture whereby individual aircraft are able to express their preferences at regular time intervals and are always given conflict-free, straight paths. It is shown that the ability to optimize conflict resolution over all crossing patterns not only leads to better solutions, but also allows us to identify crossing pattern rules that may be used as rules later on.

The paper is organized as follows: First, a simple model of air traffic is introduced; the basic conflict avoidance problem is then formulated using that model and an initial control architecture is proposed. Second, the combinatorial aspects of the conflict avoidance problem are discussed. The problem is formulated as a nonconvex, quadratically constrained program; an approximate method to solve this program is introduced, based on a combination of convex programming with randomization schemes. Last, numerical examples and comparisons are presented and discussed: A symmetric conflict involving eight aircraft is first introduced, and it is shown that the best solution to that problem is not symmetrical (roundabout-type conflict resolution pattern). Then a conflict involving two streams of aircraft is discussed and solved. The resulting conflict resolution strategies are compared with those proposed in the existing literature.

2 Air traffic models and problem formulation

2.1 General considerations

Like many problems of automatic control and operations research, the most challenging issue when dealing with problems in air traffic control appears during the modeling phase,[20] that is, the boundaries of the system under study are not always very well identified.

In this paper, we are interested in solving conflicts arising among several aircraft. For that purpose, we assume that a finite set of aircraft has been isolated from the rest of the air transportation system. Several criteria can be used to detect those aircraft simultaneously involved in the same conflict.[4,7,9]

Although designing and analyzing systems for aircraft conflict detection and resolution needs to account for the three dimensions, this paper will investigate air traffic evolving in two dimensions (planar conflict resolution): All aircraft are assumed to evolve in the plane. This paper can be extended to three dimensions, at the expense of more notations. However, while vertical maneuvers appear to be most efficient for tactical conflict resolution (such as in the case of TCAS (Traffic Alert and Collision Avoidance System)), horizontal maneuvers might be more adapted for the "strategic" conflict resolution context considered in this paper, because they induce less passenger discomfort, do not require flight level changes and thus may not perturb the vertically stratified traffic structure as it exists today in the enroute airspace.

Following a model first introduced and justified by Andrews,[21] we assume the state of each aircraft to be described by its position and its speed. For a given conflictual situation, conflict resolution maneuvers consist of simultaneous and instantaneous speed and bearing changes for all aircraft involved in the conflict. In the proposed architecture, the conflict resolution is centralized. However, the pilots are free to indicate their desired headings. Thus the overall conflict resolution is a mix of centralized decision making structure for safety and decentralized preferences for efficiency. Roughly speaking, this architecture is the one currently adopted in Collaborative Decision Making structures in air transportation.

2.2 Notations

Let n be the number of aircraft involved in one conflict and let each aircraft be identified by its index $i \in \{1, \dots, n\}$. Denote the initial position of aircraft i by $p_{i,0}$ and its initial velocity by $v_{i,0}$. Denote its position at any time t by $p_i(t)$ (or the shorthand p_i). Denote the commanded velocity changes by u_i. We will use a double-index notation for aircraft relative positions and velocities. Thus, the relative position p_{ij} is defined by $p_{ij} = p_i - p_j$; the relative speed v_{ij} is defined as $v_{ij} = v_i - v_j$ and the relative velocity changes will be noted $u_{ij} = u_i - u_j$.

2.3 Collision avoidance constraints

Conflict resolution constraints can be expressed in many ways. While expressing collision avoidance constraints in terms of a given *minimum miss distance* appears to be the most attractive option from a geometrical standpoint, it may be better substituted for a time-based separation criterion, especially when considering tactical conflict resolution. The present context is concerned with strategic conflict resolution. In this case, a distance-based criterion is acceptable, because the main factor for this separation requirement is radar resolution. Assume then (i) a minimum safety distance d_s, (ii) no initial conflict between aircraft, and (iii) that aircraft follow straight trajectories at constant speed. The conflict avoidance constraint is then shown graphically in Fig. 1 for a given aircraft pair (i, j) and can be written as

$$p_{0ij}^T(v_{0ij} + u_{ij}) + ||v_{0ij} + u_{ij}|| \sqrt{||p_{0ij}||^2 - d_s^2} \geq 0, \tag{1}$$

for each aircraft pair (i, j), where $|| \bullet ||$ is the Euclidean norm. As may be seen from Fig. 1, the conflict avoidance condition may be seen as the *union* of the half planes defined by the two linear constraints

$$(v_{0ij} + u_{ij})^T n_{1ij} \geq 0 \tag{2}$$

and

$$(v_{0ij} + u_{ij})^T n_{2ij} \geq 0, \tag{3}$$

where n_{1ij} and n_{2ij} are shown in Fig. 1.

2.4 Maneuvering constraints

The maneuvering constraints of an enroute aircraft are significant. In particular, while enroute at high altitude, the speed range of an aircraft is narrow. Aircraft bearing is usually not limited over the time scales under consideration. However, passenger comfort as well as trajectory smoothness preference might result in constraints on the bearing changes as well. In this paper, we will follow the formulation proposed by Niedringhaus,[15] where speed changes are constrained to stay within a given set around the current aircraft speed: The set of possible changes is the convex set of possible speed commands shown in Fig. 1 (right), and is mathematically described by the following quadratic and linear constraints:

$$||v_{0i} + u_i|| \leq v_{\max}, \quad (v_{0i} + u_i)^T v_{0i}/||v_{0i}|| \geq v_{\min}. \tag{4}$$

Usually, $(v_{\max} - v_{\min})/v_{\max} \leq 0.1$ for most commercial jet aircraft at high altitudes. At lower speeds, the aircraft encounters stall buffeting. At higher speeds, the aircraft encounters Mach buffeting. At lower altitudes, the speed range can increase considerably.

2.5 Cost function

The cost function is chosen so as to minimize the speed deviations from the desired speed expressed by each aircraft. It is chosen to be a quadratic function of the speed deviations

$$J = \sum_{i=1}^{n} ||u_i - u_{i,d}||^2, \tag{5}$$

which is a measure of the total "energy" necessary for conflict avoidance. In this context, $u_{i,d}$ is the desired speed deviation. Choosing quadratic objective functions is a relatively standard practice,[22] but may be replaced by other convex objective functions.[15] The cost function (5) may incorporate weighting terms to emphasize lateral speed changes over longitudinal speed changes, for example.

2.6 Target control architecture for conflict avoidance

The proposed control architecture will comprise two loops, as shown in Fig. 2. The first loop manages the conflict detection and resolution and provides control commands so that straight aircraft trajectories are conflict-free over a given time horizon (in the case of the aircraft clusters considered in this paper, this horizon is ∞). In general, it is considered that 20 minutes is an "optimal" horizon that trades off between deviation cost and uncertainty of conflict prediction.[7,9] The second guidance loop provides speed vector preferences at a higher rate than the chosen conflict-free horizon (for example five minutes). While the operation of the first loop is done by a centralized algorithm, the second loop may be centralized or decentralized. In the latter case, each aircraft chooses its preferred course, such as in the case of Free Flight;[3] the pilot may then express route preference according to other safety or economic criteria, such as weather or expected arrival time (ETA) constraints.

Compared with other strategies, this strategy offers the following characteristics: First, the centralized, conflict avoidance loop only computes conflict-free, straight trajectories. While this approach is obviously motivated first by computational requirements (as described later), it also offers an attractive option to pilots and controllers alike. Indeed, segmented trajectories require significant attention on the part of the pilot and the controller if they are not totally automated, and are therefore subject to pilot lack of attention and possibly pilot maneuvering delays.[23] Thus, while many existing approaches propose segmented, conflict free trajectories with little or no guarantees about what happens if waypoints are missed, the proposed approach always generates straight, conflict-free trajectories over a time horizon longer than that necessary, and generates segmented trajectories only through updates from individual preferences.

3 Conflict Resolution Loop: Problem properties and formulation as a quadratically constrained, quadratic program

3.1 Problem properties

The main feature of the conflict resolution problem presented in the previous paragraphs is its inherent combinatoriality. The complexity of this problem seems to grow exponentially with the number of aircraft involved in the conflict. This may be seen using the following intuitive argument: The number of aircraft pairs involved in the solution to one conflict involving n aircraft is $n(n-1)/2$. For each aircraft pair the conflict resolution algorithm needs to decide whether each crossing pattern corresponding to each aircraft pair should be "clockwise" (the vector p_{ij} rotates clockwise) or "counter-clockwise" (the vector p_{ij} rotates counter-clockwise). Once the crossing pattern is chosen, then the conflict resolution problem becomes a convex, quadratic optimization problem (other problem formulations have led to alternative convex optimization problems, such as linear programs[9,15]). Solving convex, quadratic programs is particularly simple and their theoretical computational complexity has recently been shown to be polynomial.[16]

Thus, much of the complexity in the proposed conflict resolution formulation is to find an "optimum" crossing pattern. In this section, we propose to investigate and demonstrate via numerical examples that quadratically constrained quadratic programming and its semidefinite relaxation can be used to achieve that goal.

3.2 Nonconvex, quadratically constrained, quadratic programs

The general format for a non-convex, quadratically constrained quadratic optimization problem is

$$\begin{array}{ll} \text{Minimize} & z^T P_0 z + 2q_0^T z + r_0 \\ \text{subject to} & z^T P_i z + 2q_i^T z + r_i \leq 0, \quad i \in \mathcal{I}, \end{array} \tag{6}$$

where $\mathcal{I}$ is a given index set. In this problem, the objective function and the constraints are quadratic forms. The signature of these quadratic forms is a priori *arbitrary*. Although this problem can be shown to be very difficult to solve in general (it includes all binary integer problems as special cases[24]), it has been the focus of recent research attention, because there exist powerful methods, based on convex optimization, to obtain approximate (but often very good) solutions, along with very good lower bounds to it. These relaxations to the Problem (6) can be given a number of interpretations,[17,19,24,25] including the following:[18] Instead of looking for a specific decision variable z that solves Problem (6) optimally, consider instead the problem of looking for a random variable z with given first order moment (denoted $\hat{z}$) and second-order moment denoted Z (that is, $\mathbf{E}\, zz^T = Z$), such that the optimization problem (6) is solved *on average* over that distribution. It is easy to see that the relaxed problem can then be formulated as

$$\begin{array}{ll} \text{Minimize} & \mathbf{Tr}\, P_0 Z + 2q_0^T \hat{z} + r_0 \\ \text{subject to} & \mathbf{Tr}\, P_i Z + 2q_i^T \hat{z} + r_i \leq 0, \quad i \in \mathcal{I}, \\ \left[\begin{array}{cc} Z & \hat{z} \\ \hat{z}^T & 1 \end{array}\right] \geq 0. & \end{array} \tag{7}$$

The last constraint is added to ensure that the covariance matrix $Z - \hat{z}\hat{z}^T$ is positive semidefinite. In addition to providing lower bounds (which may be interpreted as *limits of performance* to the original, non-convex problem), the random distribution defined by $\hat{z}$ and Z may also be used to obtain good feasible solutions to the original problem by searching randomly across such a distribution, thus proposing one form of efficient *randomized algorithm*.[26] If $Z = \hat{z}\hat{z}^T$, then the distribution in fact consists of a unique point, and in this case this point is then the optimal solution to Problem (6) as well. Cases where this is known to occur systematically include the case when $\mathcal{I}$ contains only one element (presence of a single quadratic constraint[27]). Nontrivial cases where this relaxation has been known to work very well include the work by Goemans and Williamson[17] and the work by Karger and Motwani.[28] In both cases, the semidefinite relaxation was followed by an algorithm examining several random draws from the distribution defined by $(\hat{z}, Z)$. An interesting feature of the proposed relaxation is that, while it appears to approximate the original, non-convex problem very well, it also may be solved in polynomial time, thus yielding interesting perspectives for real-time applications.[19,29]

Previous applications of this approach include the solution to a variety of problems appearing in robust control systems analysis,[25,30] analysis via Linear Matrix Inequalities,[19] actuator placement problems,[31] network optimization problems, semiconductor manufacturing and quantum physics, as well as communications.[32] It includes in particular all linear integer programming problems as special cases.

The problem that remains is then to find what strategies should be chosen to eventually find good, feasible solutions to the problem. The latter issue has been dealt with in many different fashions in the past: It often happens that no randomized solution is feasible, yet custom-designed algorithms have been able to retrieve very good solutions[17] from these initial random solutions. One strategy is the following: Considering the Problem (6), one can build a conservative approximation of it by keeping all convex constraints unchanged and linearizing the non-convex constraints in the vicinity of the random sample, a standard practice in nonlinear optimization theory and practice.[16,24]

3.3 Formulation of the conflict avoidance problem as a quadratically constrained, quadratic program and solution procedure

Much of the research involved in the solution to the planar conflict resolution problem hinges on the ability to formulate usable quadratic constraints (there is a thorough treatment of this issue in the case of linear integer programming[33]). For the current problem, all constraints are already expressed directly as (convex) linear or quadratic constraints on the aircraft's speed vectors, except for the conflict avoidance constraint (1). This is done most easily by the following proposition:

Proposition 3.1 *The constraint (1) is equivalent to the set of quadratic constraints*

$$p_{0ij}^T(v_{0ij} + u_{ij}) + w_{ij}\sqrt{||p_{0ij}||^2 - d_s^2} \geq 0, \quad ||v_{0ij}||^2 \geq w_{ij}^2, \tag{8}$$

where $w_{ij} \geq 0$ are new slack variables.

This proposition is trivial to prove.

It is worthy to note that constraints like (1) are traditionally transformed into mixed integer linear constraints using standard methods.[34] The proposed method is an attractive and efficient alternative to these traditional approaches.

It is readily seen that the above constraints and cost function form a non-convex quadratic program of the form (6), with $z := [u_1,\, u_2\, \ldots,\, u_n,\, w_{12},\, \ldots,\, w_{(n-1),n}]^T$.

If the optimal solution Z to the semi-definite relaxation has unit rank, then $\hat{z}$ is the solution to the original problem. Otherwise, the following randomization procedure is applied: Considering the Gaussian distribution with mean $\hat{z}$ and covariance $Z - \hat{z}\hat{z}^T$, pick samples $\tilde{z}$ according to that distribution. The "linearization" procedure is then to pick the crossing pattern for each aircraft pair by computing

$$C = \mathrm{sign}(p_{0ij} \times (v_{0ij} + \tilde{u}_{ij})),$$

where $\tilde{u}_{ij} = \tilde{u}_i - \tilde{u}_j$ is computed from $\tilde{z}$ and $\times$ is the usual outer product between two planar vectors. The crossing pattern is then chosen to be counter-clockwise if $C = 1$ (the linear constraint (2) is chosen) and clockwise if $C = -1$ (the linear constraint (3) is chosen). By convention, we will assume that the crossing pattern is clockwise in the very unlikely case when $C = 0$.

Once the crossing pattern is chosen, the corresponding convex optimization problem may be solved using using recently introduced optimization methods[16] or otherwise.

4 Numerical examples

The proposed approach is now illustrated on a number of numerical experiments. First, a "Free-flight"-like scenario is presented and solved using the proposed numerical approach. Then, an irrealistic but geometrically elegant symmetric conflict involving eight aircraft is considered. It is shown that the optimal solution to this problem is not a symmetric "turn around". Then, an example where two aircraft streams fly "miles-in-trail" is considered. It is shown that the proposed approach works better than approaches that do not optimize over crossing patterns. Branch-and-bound tests reveal the proposed procedure produces excellent solutions (close to optimal) for these two examples. In addition, it is shown how the optimization algorithm automatically generates conflict avoidance maneuvers by "platooning" aircraft together. The optimization software SDPPACK[35] was used for the numerical experiments.

4.1 Random encounter pattern

A set of 10 aircraft flying at a nominal, desired speed of 200 knots was positioned and oriented at random, as shown in Figure 3. This initial configuration generates 8 conflicts among those aircraft. The conflicts need to be solved simultaneously due to the generally convergent nature of the aircraft flow. In this example, only one maneuver is issued to all aircraft. The maximum speed for all aircraft is $v_{\max} = 220$ knots and the minimum speed is $v_{\min} = 180$ knots. The minimum miss distance between aircraft was chosen to be $d_s = 5$ Nm in this case. In Figure 3 and later, the circles surrounding the aircraft have a diameter the size of the minimum miss distance between aircraft.

The combination of convex programming and randomized search led to a set of solutions whose best element scored a cost of 663 knt^2. The semidefinite programming relaxation yielded a lower bound on the best possible cost of 603 knt^2. Thus, the gap between the best obtained cost and the best possible cost is less than 10% in this case. To get an idea of the performance of the proposed algorithm in this case, we ran a simulation in which 500 random solutions were generated using the proposed approach and examined: The randomized algorithm found a feasible solution for 100% of all the generated samples. A normalized histogram of the performance obtained for each random trial is shown in Fig. 3. It shows that one out of three random trials yields the best found performance.

4.2 Symmetric encounter pattern

A set of 8 aircraft is shown in Fig. 4. These aircraft converge to the same point at the same speed (200 knots), and it is desired to find optimal aircraft deviations so that conflicts are avoided. In this example as in the previous one, only one resolution command is issued to examine the performance of the proposed optimization algorithm. The maximum speed $v_{\max}$ is 220 knots and the minimum speed $v_{\min}$ is 180 knots. An intuitive optimal solution would follow a "roundabout" pattern,[13] whereby every aircraft deviates its course by the same angle. The randomization algorithm found that a better solution exists, which is not symmetric and rather counter-intuitive: This solution requires two airplanes to fly straight through the center (one accelerates, the other decelerates), and the others to deviate from their original course following a roundabout pattern, as shown in Fig. 4. In this figure, the circles surrounding the aircraft have a diameter the size of the minimum miss distance between aircraft (5 Nm).

The best cost found is 3801.7 knt^2. The optimal cost provided by the semidefinite relaxation (and thus a lower bound on the best achievable cost) is 1100 knt^2. Thus there is a significant difference between upper and lower bounds in this case. However, the best roundabout solution corresponds to a cost of 5486 knt^2 and thus the best found solution represents a 40% improvement compared with the roundabout solution. Any rotation or flip to that solution remains valid as well. Again, we ran a simulation in which a large number of trial solutions were generated and examined: The randomized algorithm found a feasible solution for about 68% of all the generated samples, and the distribution of solutions according to performance is very skewed towards the best cost found, as may be seen in Fig. 4. Thus, it usually takes only a few random trials to generate a good solution. We also compared the performance of the proposed algorithm with that of a purely random scheme to generate crossing patterns : Less than 0.5% of the generated crossing patterns yielded feasible solutions and the generated solutions were always considerably worse in terms of performance than those generated via the proposed approach.

This algorithm did not provide a convincing proof that the solutions are indeed optimal or close to optimal, because the gap between upper and lower bounds is large. To obtain more information about the optimality of the proposed solution, a straightforward branch-and-bound algorithm was implemented, whereby branching consists of choosing crossing patterns for each aircraft pair in ascending order (the aircraft pairs are assumed to have been organized in an ordered list) and bounding consists of applying the proposed semidefinite relaxation algorithm (which provides lower bounds) and the randomization procedure (which provides upper bounds) to the non-branched crossing patterns for the remaining aircraft pairs. Details about branch-and-bound algorithms may be found in most optimization textbooks.[34] While considerably more time consuming, branch-and-bound procedures are global optimization methods. In this case the branch-and-bound procedure showed that the global optimum value for this problem is 3673 knt^2, thus within 4% of the value found with the initial relaxation approach.

4.3 Crossing aircraft streams

This example illustrates the possible efficiency of the proposed algorithm to handle the intersection between two aircraft streams. Indeed, a "classical" solution to that problem would rely on fixed aircraft routing and aircraft staging at the intersection, which would result in larger than necessary separation between aircraft in the same stream. This example was inspired from the article by Niedringhaus.[15] In that article, the author used a linear programming approach to solve this problem, and used a fixed and pre-determined crossing pattern. The example below shows that some improvements and insight may be obtained by considering the option to also optimize over the crossing patterns as well. Considering first the case when the aircraft are allowed to perform one and only one simultaneous and instantaneous turn, this section then considers the case when the aircraft are allowed to perform many turns, to recover their initial course.

4.3.1 Single turn pattern

Two strings of four aircraft spaced 25 miles-in-trail are converging towards each other as shown in Fig. 5. The aircraft's speeds are 200 knots, $v_{\min} = 180$ knots and $v_{\max} = 220$ knots. The minimum miss distance was chosen arbitrarily to be 20 Nm. As a result, it is impossible to resolve conflicts arising between these two aircraft streams by using staging without path deviations: The distance between two consecutive aircraft in one stream does not allow the controller to insert an aircraft from the other stream between them without creating a conflict. The results of the proposed approach are illustrated in Fig 5. Again, the outcome of the semidefinite relaxation generates a probability distribution which is skewed to the left. About 80% of the randomly generated solutions are feasible, whereas a pure randomization algorithm did generate only 1% feasible crossing patterns. The best feasible solution found generated a cost of 4968.3 knt^2, whereas the semidefinite relaxation provided a lower bound value of 3888.6 knt^2. A subsequent, costlier branch-and-bound optimization revealed that the optimum value for this problem is 4959 knt^2. Thus the best solution value found with the proposed procedure is within less than 0.2% of optimality. Compared with the published solutions,[15] the proposed optimization procedure results in smaller trajectory deviations, because it optimizes over the crossing patterns as well.

4.3.2 Multiple turn simulation

Two strings of five aircraft each are converging towards each other. In this section, the proposed resolution procedure was used every five minutes to update the aircraft trajectories and possibly allow the aircraft to recover their nominal flight paths. Thus, every five minutes, a new, conflict-free rectilinear trajectory is generated according to the scheme shown in Figure 2, and this trajectory is conflict free for the aircraft set under consideration. The guidance law that drives the preference of each aircraft is a simple proportional guidance law, whereby the desired speed vector is proportional to the lateral deviation of the aircraft from its intended (rectilinear) course. In this case, the guidance law is chosen so that, if an aircraft is granted its desired speed vector, it returns to its desired course within one time step (5 minutes). The characteristics and maneuverability of all aircraft are the same as in the previous examples.

Fig. 6 shows the trajectories followed by the ten aircraft, along with four snapshots taken at $t = 0$ min, 27 min, 54 min, and 80 min. In this case the chosen strategy is *platooning*: Because the spacing between aircraft does not allow the two aircraft streams to cross by staging airrcaft without generating conflicts, the proposed algorithm groups aircraft in pairs. Interestingly enough, platooning has been proposed as a viable, although heuristic option in many intelligent, hierarchical transportation systems.[36] After the conflict is resolved, the aircraft recover their positions and trail each other again. This example shows another interesting aspect of the proposed optimization-based approach to solve conflict arising among aircraft: It allows the engineer to find and/or justify specific hierarchical structures meant to reduce the complexity of systems involving many interacting vehicles.[36]

4.3.3 Multi-segmented simulation with aircraft passing

Next, the following scenario was simulated: Considering again two intersecting aircraft streams, it is assumed that one aircraft wants to fly faster than the other aircraft. Initially, all aircraft fly at the same speed (200 knots); however, the accelerating aircraft progressively indicates a faster desired speed (up to 300 knots). Fig. 7 shows a simulation of the aircraft flow at $t = 0$ min, $t = 24$ min, $t = 48$ min, $t = 72$ min, $t = 96$ min and $t = 120$ min. Again it may be seen in Fig. 7 that all conflicts are avoided, while the faster aircaft is allowed to pass the preceding aircraft.

5 Conclusion

In this paper, the problem of resolving conflicts arising among several aircraft has been considered. A conflict resolution architecture combining decentralized aircraft preferences with centralized conflict resolution while minimizing path deviations from the desired paths is proposed. The centralized conflict resolution system is based on the formulation of a related nonconvex quadratic programming problem and its solution via semidefinite programming combined with a randomization scheme. Numerical simulations have indicated that the proposed approach may be used to find nontrivial solutions to complex conflicts involving multiple aircraft. They also indicate specific patterns (such as aircraft platooning) that may be used in future, rule-based conflict resolution systems.

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References

[1] Bobick J. (1972) *Improved Navigation by Combining VOR/DME Information with Air or Inertial Data* PhD thesis, Stanford University.

[2] RTCA (1995) RTCA Task Force 3: Free Flight Implementation, Draft Final Report Technical report, RTCA.

[3] Pujet N. and Feron E. Flight Plan Optimization in Flexible Air Traffic Environments (1996) In *AIAA Conference on Guidance, Navigation, and Control*, San Diego.

[4] Kuchar J.K. and Yang L.C. Incorporation of Uncertain Intent Information in Conflict Detection and Resolution (1997) In *Proc. IEEE Conf. on Decision and Control*, pages 1810–1815, San Diego, CA.

[5] Schild R. (1998) *Rule Optimization for Airborne Aircraft Separation* PhD thesis, T. U. Wien.

[6] Kuchar J. (1996) A Methodology for Alerting System Performance Evaluation. *AIAA Journal of Guidance, Control and Dynamics.* **19**(2), 438–444.

[7] Paielli R.A. and Erzberger H. (1997) Conflict Probability Estimation for Free Flight. *AIAA Journal of Guidance Control and Dynamics.* **20**(3), 588–596.

[8] Krozel J. and Peters M. Strategic Conflict Detection and Resolution for Free Flight (1997) In *Proc. IEEE Conf. on Decision and Control*, pages 1822–1828, San Diego, CA.

[9] Durand N. , Alliot J.M. , and Chansou O. (1995) An Optimizing Conflict Solver for Air Traffic Control. *Air Traffic Control Quarterly.*

[10] Alliot J.-M. , Durand N. , and Granger G. FACES: a Free Flight Autonomous and Coordinate Embarked Solver (1998) In *FAA-Eurocontrol Air Traffic Management Seminar*, Orlando, Fla.

[11] Eby M.S. (1994) A self-organizational approach for resolving air traffic conflicts. *The Lincoln Laboratory Journal.* **2**(7), 239–253.

[12] Zeghal K. Towards the logic of an airborne collision avoidance system which ensures coordination with multiple cooperative intruders (1994) In *Proc. ICAS*, pages 2208–2218, 1994.

[13] Tomlin C. , Pappas G. , and Sastry S. (1998) Conflict Resolution for Air Traffic Management: A Study in Multiagent Hybrid Systems. *IEEE Trans. Aut. Control.* **43**(4), 509–521.

[14] Kuchar J.K. and Yang L.Y. Survey of Conflict detection and resolution modeling method (1997) In *Proc. AIAA Guidance, Navigation and Control Conf.*, New Orleans, LA.

[15] Niedringhaus W. (1995) Stream Option Manager: Automated Integration of Aircraft Separation, Merging, Stream Management, and Other Air Traffic Control Functions. *IEEE trans. on Man, Systems and Cybernetics.* **4**, 140–147.

[16] Nesterov Yu. and Nemirovsky A. (1994) *Interior-point polynomial methods in convex programming*, volume **13** of *Studies in Applied Mathematics* SIAM, Philadelphia, PA.

[17] Goemans M. and Williamson D. (1995) Improved Approximation Algorithms for Maximum Cut and Satisfiability Problems Using Semidefinite Programming. *J. ACM.* **42**, 1115–1145.

[18] Bertsimas D. and Ye Y. (1997) Semidefinite Relaxations, Multivariate Normal Distributions, and Order Statistics.

[19] Boyd S. , El Ghaoui L. , Feron E. , and Balakrishnan V. (1994) *Linear Matrix Inequalities in System and Control Theory*, volume **15** of *SIAM Studies in Applied Mathematics* SIAM.

[20] Hansman R. J. The National Airspace System (NAS), how well do we know the plant? SAE Control Systems Meeting, October 1998.

[21] Andrews John (1978) A Relative Motion Analysis of Horizontal Collision Avoidance Technical report, Massachusetts Institute of Technology, Lincoln Laboratory.

[22] Menon P.K. , Sweriduk G.D. , and Sridhar B. (1999) Optimal Strategies for Free-Flight Air Traffic Conflict Resolution. *AIAA Journal of Guidance Control and Dynamics*. **22**(2), 202–211.

[23] Yang L. C. and Kuchar J. K. A Prototype Conflict Alerting System for Free Flight (1997) In *35th AIAA Aerospace Sciences Meeting and Exhibit*, Reno, NV.

[24] Vandenberghe L. and Boyd S. (1996) Semidefinite Programming. *SIAM Review*. **38**(1), 49–95.

[25] Doyle J. (1982) Analysis of Feedback Systems with Structured Uncertainties. *IEE Proc.* **129-D**(6), 242–250.

[26] Motwani R. and Raghavan P. (1997) *Randomized Algorithms* Stanford-Cambridge Program. Cambridge University Press, Cambridge, UK.

[27] Uhlig F. (1979) A Recurring Theorem About Pairs of Quadratic Forms and Extensions: A Survey. *Linear Algebra and Appl.* **25**, 219–237.

[28] Karger D. , Motwani R. , and Sudan M. Approximate Graph Coloring by Semidefinite Programming (1994) In *Proceedings of the 35th Annual Symposium on Foundations of Computer Science*, pages 2–13, 1994.

[29] McGovern L. and Feron E. Requirements and Hard Computational Bounds for Real-Time Optimization Problems (1998) In *IEEE Conf. on Decision and Control*, San Diego.

[30] Safonov M. G. (1982) Stability margins of diagonally perturbed multivariable feedback systems. *IEE Proc.* **129-D**, 251–256.

[31] Jamoom M. , Feron E. , and McConley M. Optimal Distributed Actuator Control Grouping Schemes (1998) In *IEEE Conf. on Decision and Control*, December 1998.

[32] Lovasz L. (1979) On the Shannon Capacity of a Graph. *IEEE Trans. on Info. Theory.* **IT-25**(1), 1–7.

[33] Lovasz L. and Schriver A. (1991) Cones of Matrices and Set-Functions and 0-1 Optimization. *SIAM J. on Optimization.* **1**(2), 166–190.

[34] Hillier F.S. and Lieberman G.J. (1990) *Introduction to Operations Research* Computer Science Workbench. McGraw Hill, New York.

[35] Alizadeh F. , Haeberly J.-P. , Nayakkankuppam M.V. , Overton M.L. , and Schmieta S. *SDPPPACK Ver. 0.9: User's guide* New-York University, 1997.

[36] Varaiya P. (1993) Smart Cars on Smart Roads: Problems of Control. *IEEE Trans. Aut. Control.* **38**(2), 195–207.

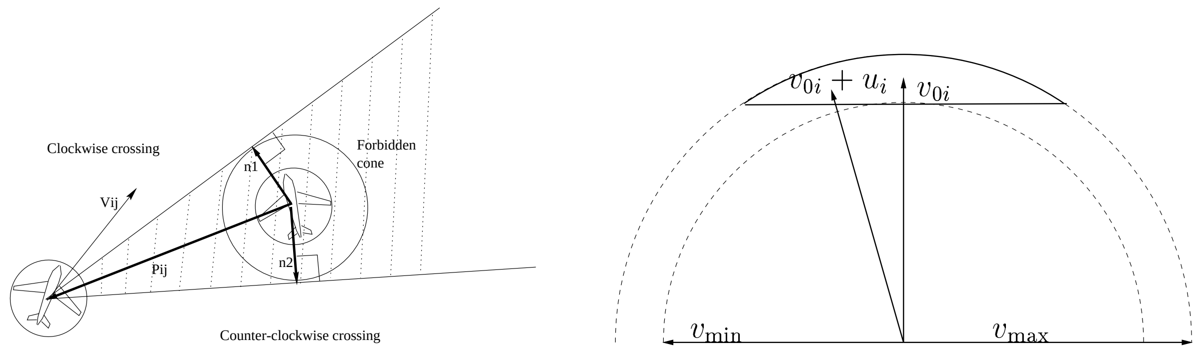

Figure 1: Constraints on aircraft maneuvers. Left: Conflict avoidance constraints. Right: Maneuvering constraints

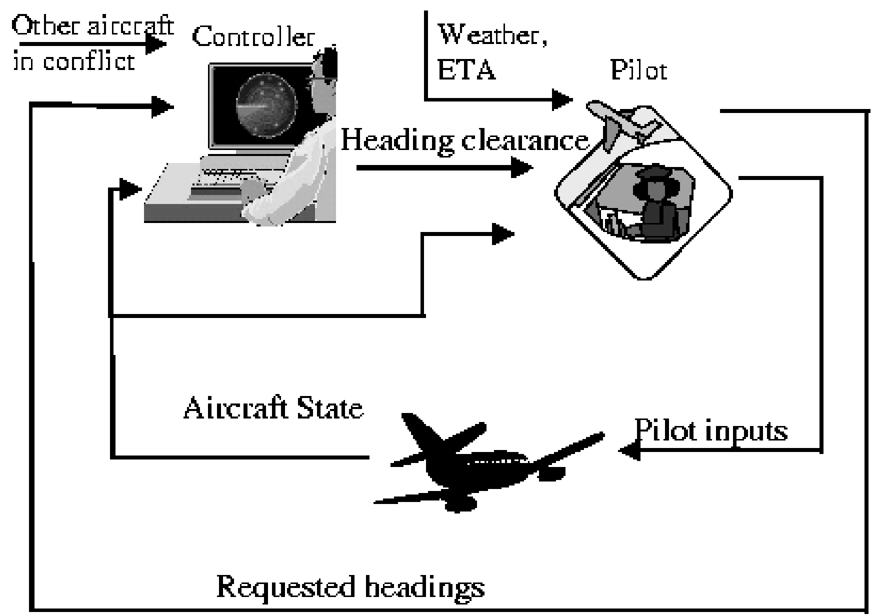

Figure 2: Mixed centralized/decentralized air traffic control scheme

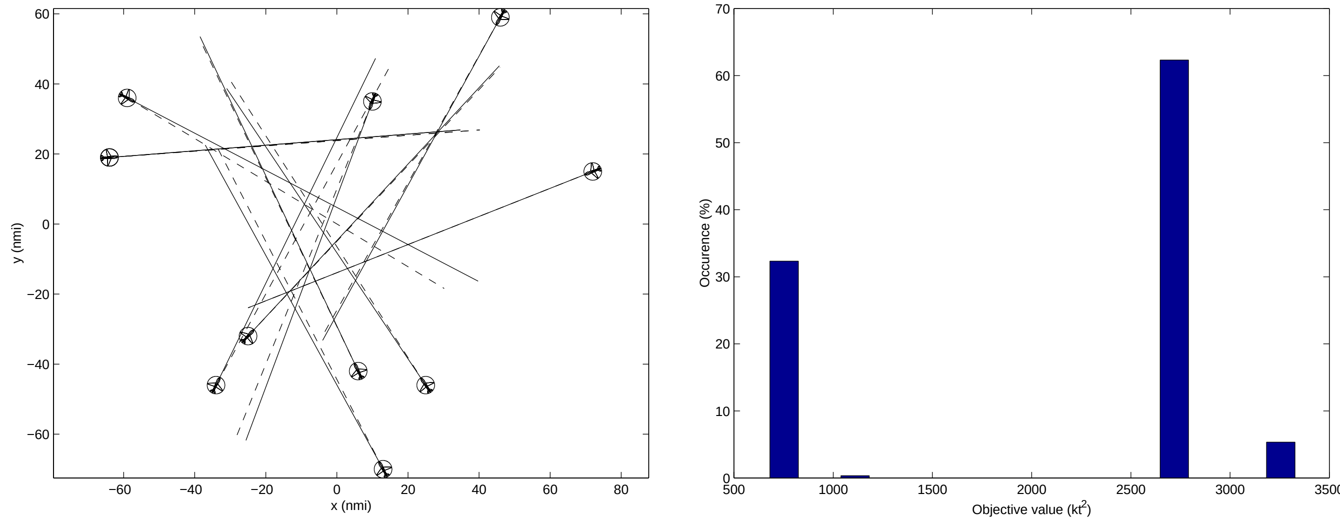

Figure 3: Test case for multi-aircraft conflict resolution algorithm. Left: Ten converging aircraft. Right: Distribution of results from randomized algorithm. Dashed lines: Initial configuration. Continuous: Configuration after conflict resolution.

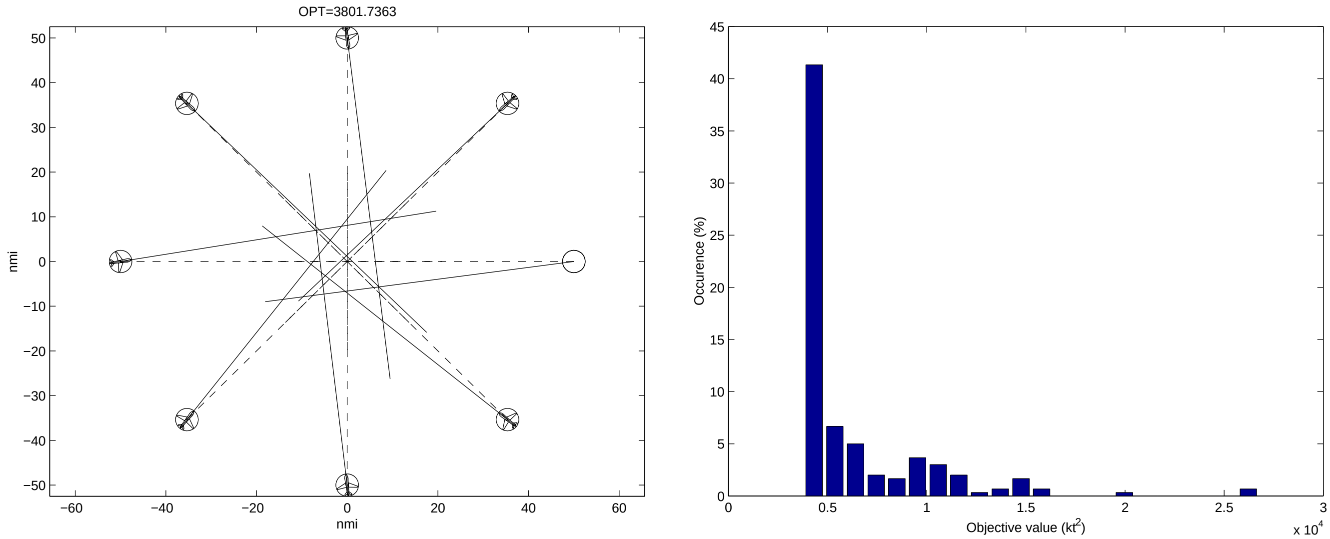

Figure 4: Test case for multi-aircraft conflict resolution algorithm. Left: Eight converging aircraft. Right: Distribution of results from randomized algorithm. Dashed lines: Initial configuration. Continuous: Configuration after conflict resolution.

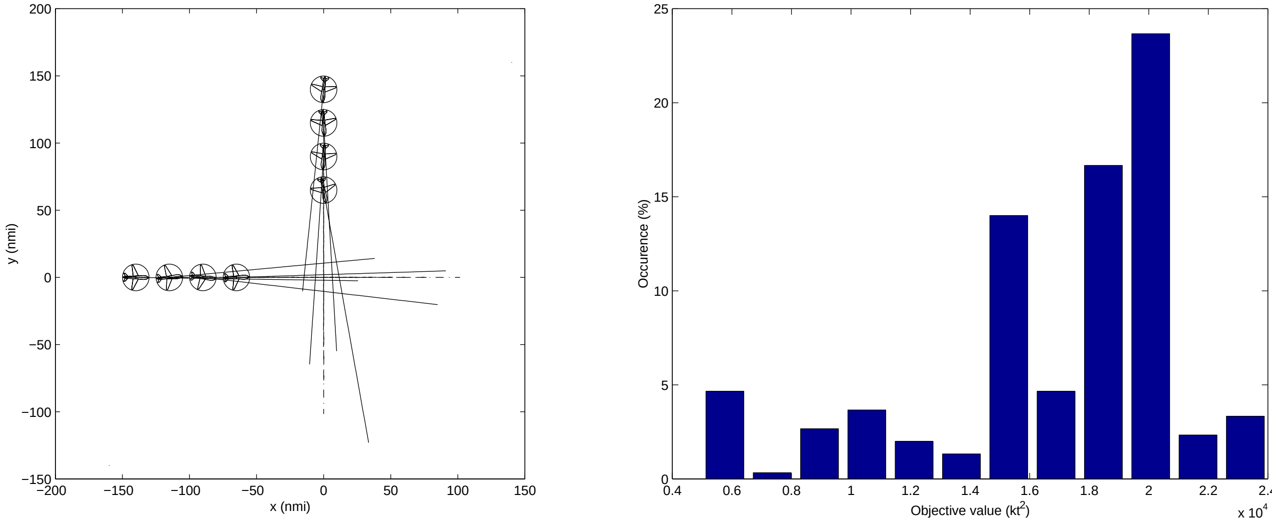

Figure 5: Test case for multi-aircraft conflict resolution algorithm. Left: Two converging lines of four aircraft. Right: Distribution of results from randomized algorithm. Dashed lines: Initial configuration. Continuous: Configuration after conflict resolution

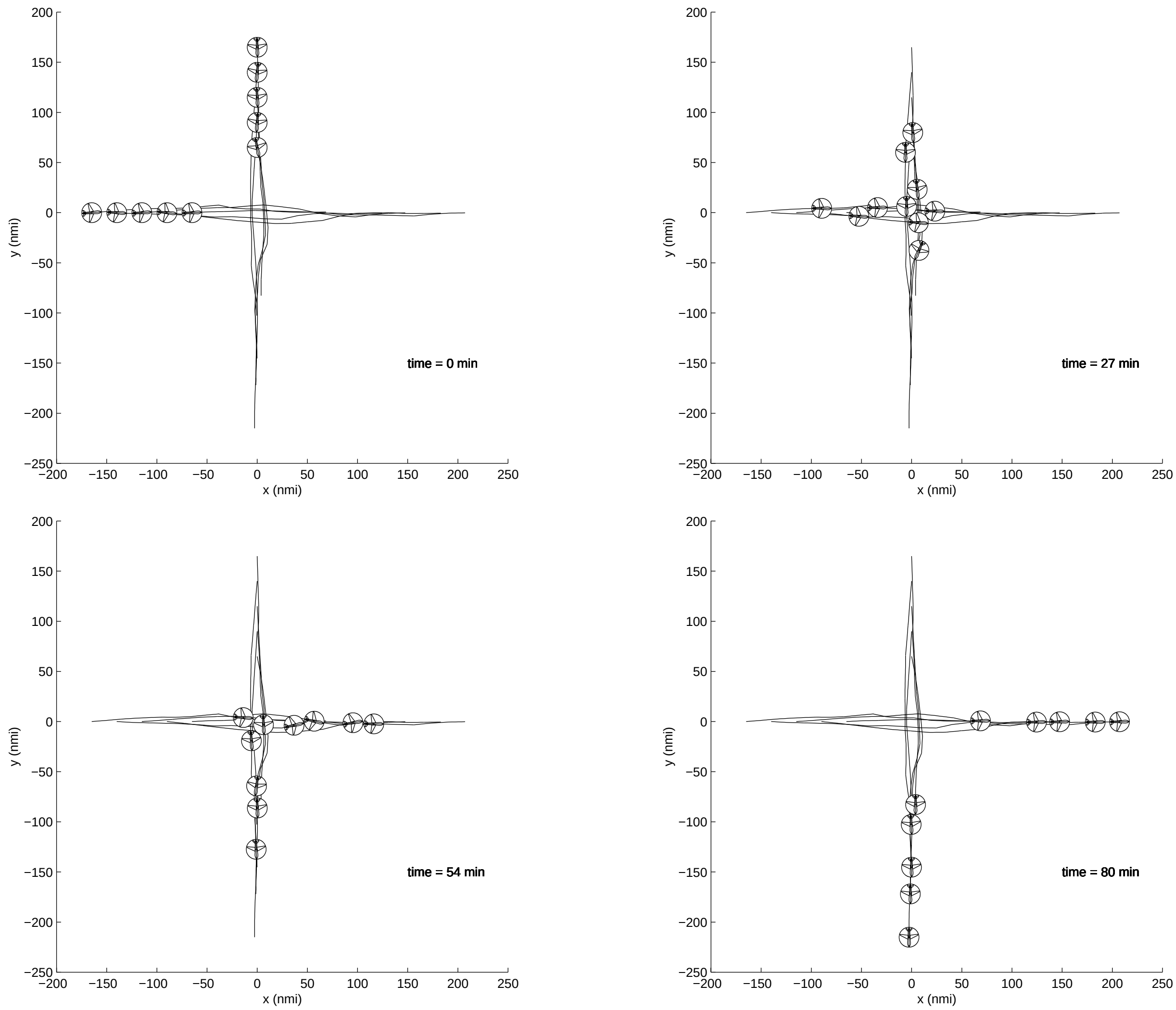

Figure 6: Two crossing aircraft strings; platooning is the resulting behavior to solve the conflicts arising between the two aircraft strings.

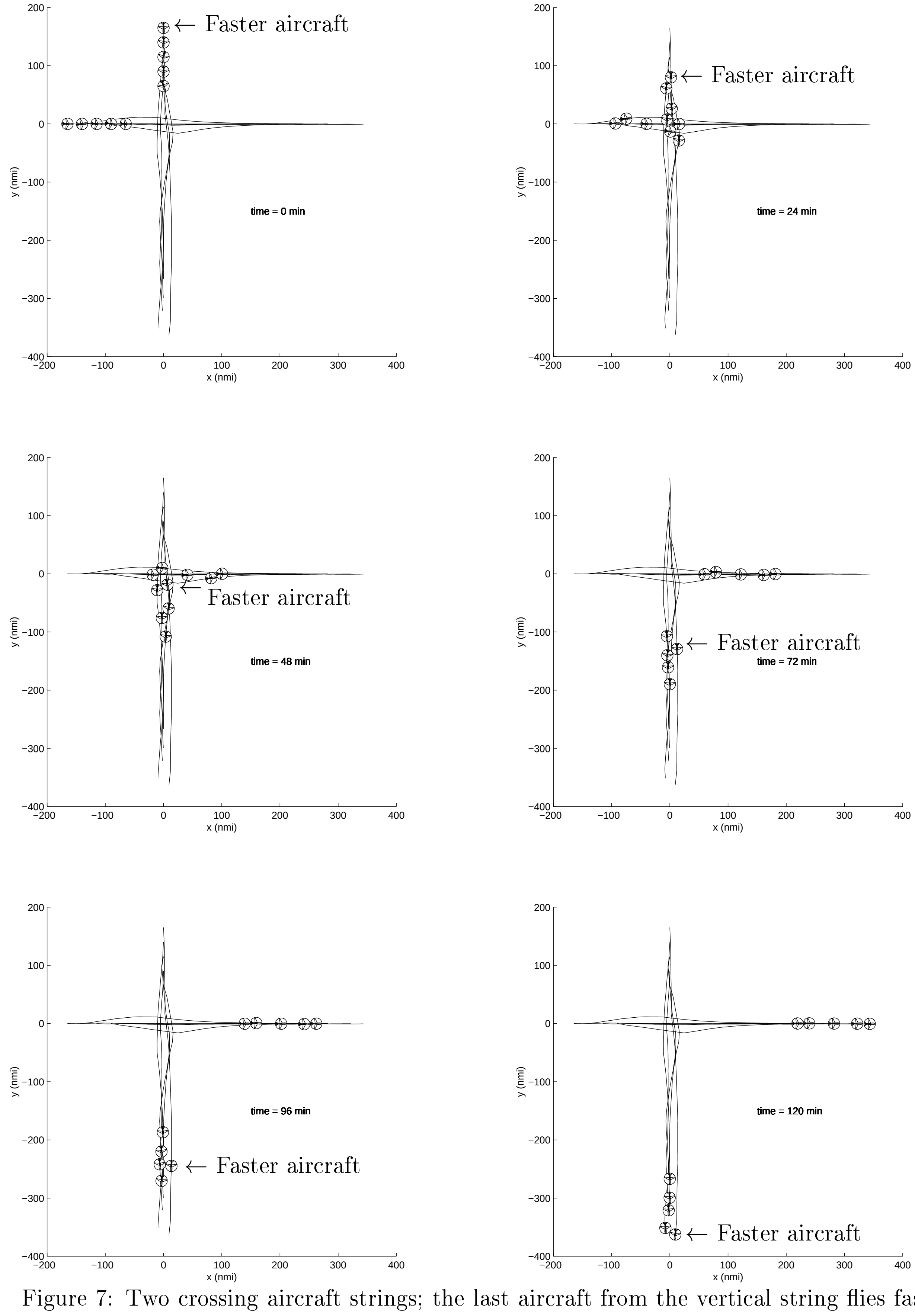

Figure 7: Two crossing aircraft strings; the last aircraft from the vertical string flies faster and passes its predecessors with simultaneous conflict resolution.