A Model of Trading Volume with Tax-Induced Heterogeneous Valuation and Transaction Costs*

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We develop a model of trading volume when agents have different valuation and face transaction costs. In particular, the differential valuation is induced by differential tax status which generates trading around the distribution of cash dividends. Our model predicts that trading volume is negatively affected by idiosyncratic risk of dividend-paying stocks. Volume is negatively affected by the systematic risk of these stocks only in the presence of transaction costs. *Journal of Economic Literature* Classification Numbers: G11, G12, H20.   © 1996 Academic Press, Inc.

1. INTRODUCTION

Agents trade because they are different. Thus, trading can be a result of differences in endowments (e.g., Arrow, 1953), differences in preferences

(e.g., Campbell *et al.,* 1993; Dumas, 1989; Wang, 1996), differences in infor-
mation (e.g., Grossman, 1981; Tirole, 1982; Wang, 1994; Blume *et al.,* 1994),
or differences in valuation (e.g., Harris and Raviv, 1993; Biais and Bossaert,
1993; Kandel and Pearson, 1995). This study focuses on the last situation.
In particular, the source of the differences in valuation is taxes: Agents are
taxed differently on the payoffs from their security holdings. The difference
in agents' tax status induces them to trade dynamically to minimize their
tax liabilities. The extent to which they trade depends on their valuation
of the cash flow relative to the market's valuation, their risk aversion, the
transaction costs they face, and the risk in deviating from their otherwise
optimal portfolio.

   There are three reasons we consider an economy where the only motive
for trading is tax-induced differential valuation. First, differential valuation
is a significant source of trading in financial markets (e.g., Michaely and
Vila, 1996). Second, concentrating on differential taxes as the source of
differential valuation enables us to construct a model that is easy to test.
It is almost an article of faith that differential valuation is the main source
of trading around the ex-dividend day, since trading around this event
results neither from differences in information nor from differences in
preferences or opinions, but from tax-induced differential valuation of cash
flows. Thus, the predictions of the model can be tested in this (almost ideal)
environment. The third reason is related to transaction costs. While models
of trading volume based on market incompleteness or heterogeneous infor-
mation generate important insights into the time-series relation between
prices and volume, they become intractable in the presence of transaction
costs. As our model shows (as does Constantinides, 1986), the decision on
whether and how much to trade is greatly affected by transaction costs.[1]

   Several of our results can be illustrated with the following stylized exam-
ple: Imagine that the expected price drop between the cum day (the last
day the stock is traded with the dividend) and the ex day (the first day the
stock is traded without the dividend) equals the dividend paid. In this case,
agents who value dividends more than capital gains will buy the stock on
or before the cum day and sell it thereafter. Agents who have a preference
for capital gains relative to dividend income will sell a portion of their
holdings on the cum day and reverse their trade on the ex day, or shortly
thereafter. It is clear that a higher degree of differential valuation (i.e., tax
difference) across agents generates higher trading volume. If the price drop
were known for certainty, then each agent group would engage in arbitrage
trading. However, those trades are neither riskless nor costless. Both risk
and transaction costs inhibit trading.

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[1] The effect of transaction costs on prices is more ambiguous. See Constantinides (1986),
Heaton and Lucas (1996), Vayanos and Vila (1996), and Vayanos (1996).

Clearly, agents engaging in this tax-motivated trading deviate from the optimal risk sharing (i.e., the optimal portfolio choice with respect to risk considerations only). From the cum day to the ex day, they do not hold the market portfolio. An agent who prefers dividends would long the dividend-paying stocks in excess of their share in the market portfolio. He is then overexposed to the movements of these stocks' prices on the ex day. The larger the potential variation in stock prices, the greater the costs associated with deviating from optimal risk sharing. Thus, the risk of individual stocks tend to reduce the volume of trade.[2] Our results show that in the absence of transaction costs, only idiosyncratic risk restricts volume. All of the systematic risk can be costlessly hedged, and therefore does not affect volume. This implies that the two risk components (systematic and idiosyncratic) have different effects on price and volume. As in the traditional asset pricing literature, in a frictionless securities market, prices are affected by systematic risk, but not by idiosyncratic risk. Volume, on the other hand, is affected by idiosyncratic risk, but not necessarily by systematic risk.

The existence of transaction costs affects agents' trading strategies and the behavior of volume. Whatever the potential gains from trading, the amount of transaction costs reduces those gains, hence it reduces volume. Our model explores the interaction between risk and transaction costs and finds that with positive, proportional transaction costs, systematic risk affects trading volume (negatively). The higher the level of transaction costs, the bigger the effect of the systematic risk on volume. The reason is that as transaction costs increase, it becomes more costly to hedge the market risk. Consequently, agents trade less and their trades are only partially hedged against adverse market movements. Finally, since agents take smaller positions, an increase in transaction costs reduces the effect of the idiosyncratic risk (relative to the no-transaction costs case).

The paper is organized as follows. Section 2 presents the model. In Section 3 we consider the effect of differential valuation and risk on trading volume when transaction costs are zero. Section 4 analyzes the effect of transaction costs. We examine the robustness of the results in Section 5. Section 6 contains concluding remarks. Proofs are in the Appendix.

2. THE MODEL

In this section, we describe the determination of trading volume around the days of dividend payments in an equilibrium model when agents trade multiple assets and face transactions costs. The setup is similar to Michaely

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[2] Note that even in the content of our particular application, these risks may not be trivial. Assuming an overnight risk of about $1 on a $100 stock, we can see that tax-related trading is a risky investment strategy.
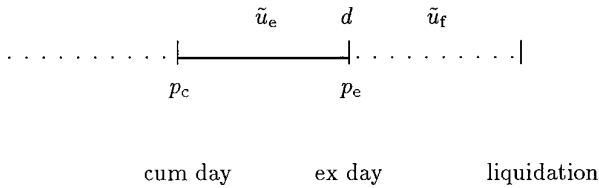
FIG 1.   Timing of events.

and Vila (1995), who consider the case of a single risky asset and no transaction costs.

2.1. *The Economy*

A. *Assets.*   There are $K + 1$ assets in the economy, a risk-free bond, and K risky dividend-paying stocks.[3] These assets are traded on two dates: the cum day, i.e., the last date where stocks are traded with the dividends, and the ex day, i.e., the first day they are traded without the dividends. After the ex day all assets are liquidated. Let $\bar{x}$ denote the vector of total number of stock shares outstanding. For simplicity in exposition, we normalized the total shares outstanding to be one for all stocks. Thus, $\bar{x} = \iota$, where ι is the vector of ones.[4]

Stock prices at liquidation are represented by an exogenous random vector. This random vector can be written as

$$\tilde{v} = \bar{v} + \tilde{u}_e + \tilde{u}_f, \tag{1}$$

where $\tilde{u}_e$ and $\tilde{u}_f$ denote the information realized at the ex day and liquidation, respectively. We assume that $\tilde{u}_e$ and $\tilde{u}_f$ are mean zero, independent, normally distributed random variables with variance–covariance matrices ω_e and ω_f, respectively. The variables p_c and p_e denote the stock price vector at the cum day and at the ex day, respectively. In addition, the stocks pay a known dividend, d, on the ex day. Without loss of generality, let $d_k > 0$ for $k = 1, \dots, K'$, where $K' \leq K$. Finally, the risk-free bond pays a constant interest rate, which for simplicity in exposition is set equal to zero. Figure 1 illustrates the timing of events.

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[3] We refer to the risky assets as "stocks." However, the structure can be generalized to incorporate other risky assets, such as indices or futures contracts.

[4] This is a non-trivial assumption since it rules out derivative instruments which have zero net supply. We make this assumption here for ease in exposition. Our results can be derived without this normalization as shown in the Appendix.

B. *Agents.* There are I agents in the economy, $i = 1, \ldots , I$. Each agent i is initially endowed with $\bar{x}^i$ shares of stocks and b^i bonds. The initial share allocation $(\bar{x}^1, \ldots , \bar{x}^I)$ is Pareto-optimal so that no trading of shares would take place were it not for the payment of dividends and the differential tax treatment of dividends and capital gains described below. Clearly, $\sum_{i=1}^{I} \bar{x}^i = \iota$. Furthermore, we assume that each agent i maximizes his expected utility of after-tax final wealth, and that his utility function exhibits constant absolute risk aversion ρ^i.

C. *Taxes and Transaction Costs.* All agents are subject to proportional taxes on both dividends and capital gains at rates τ_d^i and τ_g^i, respectively, where $i = 1, \ldots , I$. In addition, when agent i trades at the cum day in stock k, he is subject to a proportional transaction cost of c_k^i per share. The vector of transaction costs faced by agent i is denoted by c^i. Consistent with the tax code, we assume that transaction costs can be deducted from taxable capital gain income, i.e., the IRS taxes capital gains net of transaction costs. We further assume that

(1)   all taxes are paid at liquidation, which is right after the ex-dividend day;

(2)   trading at the ex day is not subject to transaction costs.

These assumptions significantly simplify the model. By assuming that all taxes are paid right after the ex-dividend day, the interest rate is zero, and the capital gains to be taxed on the liquidation day include those unrealized in previous trading, we abstract away agents' incentives in deferring taxes. In particular, agents within the same tax bracket trade the same way, independent of the different tax basis they have. In practice, taxes are paid only on realized gains. Thus, it is often preferable for agents to defer taxes by avoiding the realizations of their gains. For agents in high tax brackets, for example, those with low tax bases may be less willing to sell dividend-paying stocks on the cum day than those with high tax bases. This effect of differential tax bases can lead to complex dynamic effects on agents' trading strategies, equilibrium prices and trading volume. In the current setting, we avoid these problems by assuming a zero interest rate and a finite liquidation day. Zero interest rate diminishes the incentive to defer taxes. More discussions on relaxing this assumption are given in Section 5.B.

We also assume that trading on the ex day is not subject to transaction costs. This assumption is not as restrictive as it sounds. Indeed, it amounts to assuming that when doing a round-trip transaction (for example, buying a share at the cum day and selling it at the ex day), transaction costs are paid up front. In other words, the tax-motivated trades are assumed to take the form of repo-arrangements. Combined with the assumption that agents arrive at the cum day with optimal risk-sharing portfolios, this assumption

implies that agents return to their optimal risk-sharing portfolios on the ex day. This simplification allows us to obtain several results in closed form. However, it should be recognized that in a truly dynamic model with transaction costs, agents in general do not hold optimal risk-sharing portfolios, either before the cum day or on the ex day. We provide some further discussions on this issue in Section 5.A.

2.2. *Equilibrium*

We first describe agent i's intertemporal budget constraint. Agent i's before-tax wealth is given by

$$W_{\mathrm{B}}^{i} = b^{i} + p_{\mathrm{c}}'(\overline{x}^{i} - x_{\mathrm{c}}^{i}) - (c^{i})'|x_{\mathrm{c}}^{i} - \overline{x}^{i}| + d'x_{\mathrm{c}}^{i} + p_{\mathrm{e}}'(x_{\mathrm{c}}^{i} - x_{\mathrm{e}}^{i}) + \tilde{v}'x_{\mathrm{e}}^{i}, \quad (2)$$

where b^{i} denotes agent i's bond holding and x_{e}^{i} and x_{c}^{i} his stock holding at the ex day and the cum day, respectively. Agent i's taxes are given by

$$T^{i} = \tau_{\mathrm{g}}^{i}[(p_{\mathrm{c}} - \overline{p}^{i})'\overline{x}^{i} + (p_{\mathrm{e}} - p_{\mathrm{c}})'x_{\mathrm{c}}^{i} + (\tilde{v} - p_{\mathrm{e}})'x_{\mathrm{e}}^{i} - (c^{i})'|x_{\mathrm{c}}^{i} - \overline{x}^{i}|] + \tau_{\mathrm{d}}^{i}d'x_{\mathrm{c}}^{i}, \tag{3}$$

where $\overline{p}_{k}^{i}$ is agent i's tax basis for stock k (i.e., the price initially paid for the $\overline{x}_{k}^{i}$ shares). Combining Eqs. (2) and (3), we write agent i's final wealth after taxes W_{A}^{i} as

$$
\begin{aligned}
W_{\mathrm{A}}^{i} = {} & \{b^{i} + [p_{\mathrm{c}} - \tau_{\mathrm{g}}^{i}(p_{\mathrm{c}} - \overline{p}^{i})]'\overline{x}^{i}\} + (1 - \tau_{\mathrm{d}}^{i})d'x_{\mathrm{c}}^{i} \\
& + (1 - \tau_{\mathrm{g}}^{i})\{(p_{\mathrm{e}} - p_{\mathrm{c}})'x_{\mathrm{c}}^{i} - (c^{i})'|x_{\mathrm{c}}^{i} - \overline{x}^{i}| + (\tilde{v} - p_{\mathrm{e}})'x_{\mathrm{e}}^{i}\}.
\end{aligned}
\tag{4}
$$

Equation (4) says that after-tax final wealth equals after-tax initial wealth plus after-tax capital gains net of transaction costs plus after-tax dividend income. We denote the after-tax initial wealth by

$$W_{0}^{i} = b^{i} + p_{\mathrm{c}}'\overline{x}^{i} - \tau_{\mathrm{g}}^{i}(p_{\mathrm{c}} - \overline{p}^{i})'\overline{x}^{i}$$

and the tax-induced preference for dividends versus capital gains by α^{i}, that is,

$$\alpha^{i} = \frac{1 - \tau_{\mathrm{d}}^{i}}{1 - \tau_{\mathrm{g}}^{i}}.$$

With these notations, agent i's final wealth after taxes can be written as

$$W_{\mathrm{A}}^i = W_0^i + (1 - \tau_{\mathrm{g}}^i)\{[(p_{\mathrm{e}} - p_{\mathrm{c}}) + \alpha^i d]' x_{\mathrm{c}}^i - (c^i)' |x_{\mathrm{c}}^i - \bar{x}^i| + (\tilde{v} - p_{\mathrm{e}})' x_{\mathrm{e}}^i\}. \tag{5}$$

We now describe the equilibrium prices and quantities. This is done in two steps, first to derive the equilibrium at the ex day and then to specify the equilibrium at the cum day. For convenience, we define by θ^i the tax-adjusted risk tolerance of agent i,

$$\theta^i = \frac{1}{(1 - \tau_{\mathrm{g}}^i)\rho^i},$$

and θ the aggregate tax-adjusted risk tolerance,

$$\theta = \sum_{i=1}^I \theta^i.$$

Given the normality and constant absolute risk aversion assumptions, standard deviations yield

$$p_{\mathrm{e}} = \bar{v} + u_{\mathrm{e}} - \theta^{-1}\omega_{\mathrm{f}}\iota$$

and

$$x_{\mathrm{e}}^i = \bar{x}^i = (\theta^i/\theta)\iota.$$

As expected, because transaction costs are assumed away on the ex day, the economy reverts to the initial Pareto-optimal risk sharing.

Finally, we derive the demand for stocks on the cum day and the equilibrium prices and volume. We denote by $\bar{p}_{\mathrm{e}}$ the expected ex day price, i.e.,

$$\bar{p}_{\mathrm{e}} = \bar{v} - \theta^{-1}\omega_{\mathrm{f}}\iota.$$

Given the assumptions of normality and constant absolute risk aversion and Eq. (5), agent i's problem is to maximize

$$(\bar{p}_{\mathrm{e}} - p_{\mathrm{c}})' x_{\mathrm{c}}^i + \alpha^i d' x_{\mathrm{c}}^i - (c^i)' |x_{\mathrm{c}}^i - \bar{x}^i| - \frac{1}{2\theta^i}(x_{\mathrm{c}}^i)' \omega_{\mathrm{e}} x_{\mathrm{c}}^i. \tag{6}$$

The first order condition is

$$\overline{p}_{\mathrm{e}} - p_{\mathrm{c}} + \alpha^i d - c^i \otimes \varepsilon^i - (\theta^i)^{-1} \omega_{\mathrm{e}} x_{\mathrm{c}}^i = 0,$$

where the vector ε^i is defined as

$$\begin{cases} \varepsilon_k^i = 1 & \text{if } x_{\mathrm{c},k}^i > \overline{x}_k^i \\ \varepsilon_k^i = -1 & \text{if } x_{\mathrm{c},k}^i < \overline{x}_k^i \\ -1 \le \varepsilon_k^i \le 1 & \text{if } x_{\mathrm{c},k}^i = \overline{x}_k^i, \end{cases}$$

the vector $c^i \otimes \varepsilon^i$ defined as

$$(c^i \otimes \varepsilon^i)_k = c_k^i \varepsilon_k^i,$$

and $k = 1, \ldots, K$. We define by δ^i the share of agent i in the economy, as measured by his risk tolerance. That is,

$$\delta^i = \frac{\theta^i}{\theta}.$$

The δ^i can be thought of as probabilities. Let E_δ be the expectation operator defined over δ^i, and let $\overline{\alpha}$ be the weighted average preference for dividends versus capital gains, i.e.,

$$\overline{\alpha} = \sum_{i=1}^{I} \delta^i \alpha^i \equiv \mathrm{E}_\delta \alpha.$$

From the first-order condition, the equilibrium price satisfies

$$p_{\mathrm{c}} = (\overline{p}_{\mathrm{e}} + \overline{\alpha} d - \theta^{-1} \omega_{\mathrm{e}} \iota) - \sum_{i=1}^{I} \delta^i c^i \otimes \varepsilon^i. \tag{7}$$

Equation (7) decomposes the cum day price into two components. The first component (within brackets) is the cum day price in the absence of transaction costs. It is equal to the expected (risk-adjusted) ex day price plus the after-tax market value of the dividend vector, $\overline{\alpha} d$. The second component measures the impact of transaction costs on the ex day price. The sign of the second component is ambiguous: If buyers face larger

(lower) transaction costs than sellers, then the cum day price is lower (higher) than in the absence of transaction costs.

Substituting (7) into the first-order condition yields the equilibrium holdings

$$x_c^i = [\overline{x}^i + \delta^i(\alpha^i - \overline{\alpha})\omega_e^{-1}d] + \delta^i\omega_e^{-1}\left(\sum_{i'=1}^{I} \delta^{i'}c^{i'} \otimes \varepsilon^{i'} - c^i \otimes \varepsilon^i\right). \qquad (8)$$

The first term in Eq. (8) represents agent i's optimal trading strategy in the absence of transaction costs. The interpretation is very simple. If the tax differential between dividend income and capital gain income of agent i is greater than the market average, he holds more of the dividend-paying stocks than he usually does during non-dividend-paying periods. The second term represents the adjustment of his strategy due to the fact that (1) the equilibrium price may be different because other agents face transaction costs and (2) he himself faces transaction costs. The difficulty with Eqs. (7) and (8) is that the variables ε_k^i are endogenous, since they depend upon the sign of individual trades. Thus, in general, it is difficult to solve ε_k^i in closed form.

In what follows, we analyze the properties of the equilibrium. To simplify our analysis, we assume that the uncertainty in stocks' payoffs have a one-factor structure. (The extension to the multi-factor case is discussed later.) In particular, the information shock on the ex day can be written as

$$\tilde{u}_e = \beta\tilde{q} + \tilde{\eta}, \qquad (9)$$

where $\tilde{q}$, $\tilde{\eta} = (\tilde{\eta}_1, \ldots, \tilde{\eta}_K)'$ are one-dimensional and K-dimensional independent normal random variables with covariance matrices σ_q^2 and $\omega_\eta = \mathrm{diag}\{\sigma_1^2, \ldots, \sigma_K^2\}$, respectively, and $\sigma_k \leq \sigma$, $k = 1, \ldots, K$, i.e., the variances of idiosyncratic risk have a uniform bound. Without loss of generality (except degenerate cases), we normalize the factor so that the average beta is one, i.e.,

$$\frac{1}{K}\sum_{k=1}^{K} \beta_{k,q} = 1.$$

Furthermore, we assume that there is a large number of stocks, i.e., $K \gg 1$. (Effectively, we will take the limit $K \to \infty$ in our future analysis.) Given that K is large, $\tilde{q}$ is (almost) perfectly correlated with the market portfolio. Also the stocks' β's with respect to the market portfolio (almost) equal

the stocks' β's with respect to $\tilde{q}$. For the sake of the presentation, we refer to $\tilde{q}$ as the market portfolio (see the Appendix for details).

Finally, we assume that the number of stocks that pay dividends on a given day is small compared to the total number of stocks in the economy, i.e., $K'/K \ll 1$. In particular, we maintain K' to be finite even in the limit that K goes to infinity. This assumption is a reasonable approximation to reality. On the New York Stock Exchange, for example, where more than 2000 stocks are listed, an average of 20 stocks go ex each day (Michaely and Vila, 1996).

The above assumptions allow us to characterize the agents' trading strategies in a simple way. In general, the agents' trading strategies consists of two parts: the ''tax-arbitrage'' part and the hedging part. When agents engage in ''tax-arbitrage'' by buying or selling dividend-paying stocks, they pick up additional risk in deviating from optimal risk-sharing portfolios. Part of this risk—that is common to all stocks—can be hedged by trading in non-dividend-paying stocks. In general, the optimal hedging strategy can be complicated, depending on both the common risks and idiosyncratic risks of all stocks.[5] When $K \gg 1$ and $K'/K \ll 1$ (i.e., there is a large number of non-dividend-paying stocks and a small number of dividend-paying stocks), the strategy is significantly simplified. Hedging portfolios can be formed using the non-dividend-paying stocks which carry only common risks. Agents then use these hedging portfolios to hedge away the common risk of their ''tax-arbitrage'' positions. When the common risk has a one-factor structure, the agents only need one hedging portfolio, i.e., a portfolio that carries only the market risk (which is approximated by $\tilde{q}$).

3. THE CASE OF ZERO TRANSACTION COSTS

We first consider the case in which all transaction costs are zero, i.e., $c^i = 0$. In this case, the prices are given by

$$p_c^* = \overline{p} + \overline{\alpha}d - \frac{1}{\theta}\beta\sigma_q^2, \tag{10}$$

where β is the column vector of β_k's. Here, we have taken the limit $K \gg 1$ (see the Appendix for details). Given the normality assumption, the tax-

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[5] This is probably best seen by considering the extreme case in which all stocks pay dividends. In this case, there is no simple separation of an agent's trading strategy into a tax-arbitrage part and a hedging part.

adjusted CAPM holds, so that stock prices depend only on the market risk term $\beta\sigma_q^2$ and not on idiosyncratic risk σ_k^2's.[6]

If transaction costs are zero, agent i's trading strategy can be easily described. Let y_k^i be the number of shares of stock k he buys. Then,

$$y_k^i = \theta^i(\alpha^i - \overline{\alpha})\left[\frac{d_k}{\sigma_k^2} - \frac{\sigma_q^2}{D}\frac{\beta_k}{\sigma_k^2}\sum_{k'=1}^{K}(\beta_{k'}d_{k'}/\sigma_{k'}^2)\right], \qquad k = 1,\ldots,K \quad (11)$$

where $D = 1 + \sigma_q^2 \iota' \omega_\eta^{-1}\beta$ (see the Appendix for details). Let

$$a^i = \theta^i(\alpha^i - \overline{\alpha})\omega_\eta^{-1}d \qquad \text{and} \qquad h^i = -(\beta'a^i)D^{-1}\sigma_q^2\omega_\eta^{-1}\beta.$$

We have

$$y^i = a^i + h^i.$$

The first term of his trading strategy a^i can be interpreted as the tax-arbitrage part, and the second term h^i the hedging part. His tax-arbitrage trade gives rise to the total exposure of common risk $(\beta'a^i)$. In the absence of common risk (i.e., $\beta = 0$), there is no hedging and the second term is zero. Given the tax-arbitrage position a^i, the hedging part of the trading strategy in general depends on both the common risk and idiosyncratic risk of all stocks. Since non-dividend-paying stocks carry both the common risk and idiosyncratic risks, they do not provide perfect hedges against the common risk of tax-arbitrage positions. Their idiosyncratic risks give rise to the ''basis risk'' in using them as hedging vehicles. Consequently, agents reduce their tax-arbitrage positions and only partially hedge the common risk.

Under the assumption that $K \gg 1$ and $K'/K \ll 1$, the trading strategies

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[6] Equation (10) could have been derived without the normality assumption as a one-factor APT equation. But, the normality assumption is needed for the determination of the market volume. Equation (10) is similar to what has been derived by Brennan (1970) and Litzenberger and Ramaswamy (1979). In the current setting, we start with the primitives of the economy (preferences, payoff distributions, etc.) and derive the equilibrium stock prices. The risk premium is expressed in terms of the discount on price itself, not in terms of discount rate (on the cash flow). This is somewhat different from the formulation in Brennan (1970) and Litzenberger and Ramaswamy (1979), which derives only the equilibrium conditions on expected returns in a mean-variance setting. However, it is straightforward (but tedious) to show that our formulation also lead to similar results on the returns. In particular, given the stock prices, we can show that $E[r_k] - r_f = \alpha_k + \beta_k(E[r_M] - r_f) + \gamma\delta_k$ where β_k is stock k's market beta and δ_k its dividend yield ($k = 1, \ldots, K$). (The derivation is available from the authors.) This is very similar to, for example, Eq. (15) in Litzenberger and Ramaswamy (1979), except that the constant term α_k is stock specific here but not in their case.

are greatly simplified. In this case, a perfect hedging vehicle can be formed from the non-dividend-paying stocks. In particular, one can form a portfolio of non-dividend-paying stocks with total number of shares normalized to one that carries only the common risk (assuming that it has a non-trivial β). We then replace the $(K'+1)$th stock with this portfolio and other stocks while keeping the other stocks unchanged, and use this as the new asset base.[7] This implies that $\sigma_{K'+1} \approx 0$. It immediately follows that

$$y_k^i = \begin{cases} a_k^i & k = 1, \ldots, K' \\ \beta_{K'+1}^{-1}(\beta' a^i) & k = K' + 1 \\ 0 & k = K' + 2, \ldots. \end{cases} \tag{12}$$

In this case, agent i's trading strategy has a simple characterization: If he likes dividends more than the average agent, i.e., if $\alpha^i > \overline{\alpha}$, then he buys the dividend-paying stocks on the cum day and hedges by selling a small number of shares of each non-dividend-paying stock (i.e., the hedging portfolio). The opposite is true if he likes dividends less than the average agent.

From (12), it follows that the intensity of trade in stock k is an increasing function of (1) the size of the dividend d_k; (2) the difference between the agent's valuation and the market's valuation, $(\alpha^i - \overline{\alpha})$; and (3) the agent's risk tolerance, θ^i. The intensity of trade is a decreasing function of the idiosyncratic risk, σ_k^2. Since the market component of the stock's risk can be hedged costlessly, the trade y_k^i is independent of the market risk, σ_q^2, and of the stock's beta, β_k.

The trading volume on a dividend-paying stock is given by

$$v_k^* = \left(\sum_{i=1}^I \delta^i |\alpha^i - \overline{\alpha}| \right) \frac{\theta d_k}{2\sigma_k^2} = (\mathrm{E}_\delta |\alpha - \overline{\alpha}|) \frac{\theta d_k}{2\sigma_k^2} \tag{13}$$

which does not depend upon σ_q^2 and β_k. In addition, the volume on each individual stock going ex does not depend upon how many stocks are going ex.

Let us now examine the relation between trading volume, heterogeneity, and risk in the absence of transaction costs. Although there is also volume

[7] Note that in the limit $K \to \infty$, the newly formed portfolio carries only common risk $\tilde{q}$. In the case when K is finite, the portfolio also carries non-trivial idiosyncratic risk. In order to maintain the structure that the idiosyncratic risks are independent across assets, we also need to replace the remaining non-dividend-paying stocks by appropriate portfolios. The new asset base can be obtained by simply rotating the original asset base.

in non-dividend-paying stocks since they are used as hedging instruments, we focus on the volume of dividend-paying stocks. (See the Appendix for the volume pattern in non-dividend-paying stocks.)

First, note that $E_\delta|\alpha - \overline{\alpha}|$ measures the tax heterogeneity in the economy. As the tax heterogeneity increases, the difference in valuation of cash dividends increases, and so do gains from trading. Consequently, trading volume increases as (13) shows. This result is quite intuitive and is formally stated in the following proposition.

PROPOSITION 1. *The trading volume is an increasing function of the degree of tax heterogeneity in the economy.*

Keeping all other variables constant, as the dividend yield increases, the gains from transferring the dividend from high-tax agents to low-tax agents also increases. This leads to the following proposition.

PROPOSITION 2. *The trading volume is an increasing function of the dividend yield.*

The dependence of volume on various risk components is quite different from the dependence of prices on these risk components. Note that agents buying (selling) dividend-paying stocks find themselves over/under invested in these stocks. As a result, they are exposed to the idiosyncratic risk of these stocks. The higher the idiosyncratic risk of a dividend-paying stock, the more costly it is to buy or to sell it for tax purposes. We have the following proportion.

PROPOSITION 3. *The trading volume is a decreasing function of the idiosyncratic risk*, σ_k^2.

It is interesting to note that unlike prices, idiosyncratic risk affects volume even when transaction costs are not present.

Since there are no transaction costs, all of the market risk component can be costlessly hedged (when $K \gg 1$ and $K'/K \ll 1$), thus it does not inhibit trading as (13) indicates.

PROPOSITION 4. *The trading volume is independent of the market risk*, σ_q^2, *and of the stock's beta*, β_k.

Propositions 3 and 4 combined with Eq. (10) show that risk affects prices and volume differently. Market risk affects prices but not trading volume, while idiosyncratic risk affects trading volume but not prices.

Since hedging the market risk is costless, the additional risk in the ex day "arbitrage" is independent across stocks. Thus, the trading activity on one dividend-paying stock is not affected by the trading activity on another dividend-paying stock. This is stated in the following proposition.

PROPOSITION 5. *The trading volume on any stock going ex does not depend upon the number of stocks going ex.*

The independence between volume and the number of stocks going ex is a direct outcome of the constant absolute risk aversion assumption. In general, the relation between the trading volume in each stock and the number of stocks going ex depends on the form of utility function.

It should be emphasized that the results stated in Propositions 1 through 5 are obtained under the assumption that $K \gg 1$ and $K'/K \ll 1$. As discussed earlier, the motivation for these two assumptions is quite transparent. In particular, these assumptions allow agents to perfectly hedge the common risk of their "tax-arbitrage" positions by trading in a diversified portfolio of non-dividend-paying stocks. The result in Proposition 4, for example, depends on the existence of such a hedging portfolio. If the number of non-dividend-paying stocks is small, any possible hedging portfolios still bear the idiosyncratic risks that are not diversified away. Thus, hedging becomes "costly" due to this basis risk and agents optimally choose not to hedge all the common risk of their tax-arbitrage positions. Consequently, their total trade would be affected by the common risk of dividend-paying stocks. Proposition 5 holds only when $K \to \infty$ and K' remains finite (or more generally $K'/K \to 0$). When K is finite (or K'/K is finite), the volume does depend on the number of stocks going ex as (11) shows.

4. THE CASE OF NON-ZERO TRANSACTION COSTS

We now consider the effect of transaction costs on agents' trading behavior. Transaction costs limit agents' tax-related trading. To see that the effect of transaction costs is likely to be significant, consider a \$100 stock that pays a \$1 quarterly dividend, and assume that $\overline{\alpha} = 0.8$ [this value is consistent with Elton and Gruber (1970) and Michaely (1991)]. With these numbers, a corporate agent ($\alpha^i = 1.72$) makes \$0.92 per share by selling the stock on the cum day and repurchasing it on the ex day. For a corporate investor, the return on this trade is about 0.92%, which is above a plausible cost of transacting. A tax-exempt agent ($\alpha^i = 1$) makes 0.2%, which is probably below the range of transaction costs for most agents. Given the remarks above, we expect transaction costs to be a significant factor in the ex day trading activity.

With transaction costs, the equilibrium prices and quantities are much harder to calculate. Transaction costs limit the number of agents who participate in the ex day trading as well as the trading activity of those who do participate. The effect on volume seems clear: The volume is reduced. On the other hand, since transaction costs limit the trading of both buyers

$(\alpha^i > \overline{\alpha})$ and sellers $(\alpha^i < \overline{\alpha})$, the resulting effect on prices is somewhat ambiguous.[8]

To analyze the effect of transaction costs on trading volume, we consider a special case within the general setting presented in Section 2.2. In addition to the assumptions in Section 2.2, we assume that: (1) all the stocks that are not going ex have the same beta, β_N, and idiosyncratic risk, σ_N^2; (2) all the stocks that are going ex pay the same dividend, d, have the same beta, β_E, and idiosyncratic risk, σ_E^2; (3) all agents face the same transaction costs, $c_k^i = c$, on every stock; and (4) the distribution of agents' tax preference parameter α^i (with probability δ^i) is symmetric around $\overline{\alpha}$.

Assumption (1), (2), and (3) effectively reduce the model to the case of two stocks: one is dividend-paying with both market risk and idiosyncratic risk and the other is non-dividend-paying with only market risk. These assumptions make the model tractable. (Another tractable case is where one of the non-dividend-paying ''stock'' has no idiosyncratic risk. This stock can be interpreted as an index fund or a stock index futures.) Assumption (4) further implies that prices are not affected by transaction costs, and the tax-adjusted CAPM holds (see the Appendix). This allows for a simple description of trading strategies and, therefore, market volume. We discuss in Section 5 what happens when some of these assumptions are relaxed.

Under these assumptions, agents can be divided into five endogenous groups characterized by two constants, $0 < s < t$, which depend (among other things) on the level of transaction costs and the dividend amount:

1. $|\alpha^i - \overline{\alpha}| \leq s$: The agent's valuation of a dollar of dividend is close to the market price of a dollar of dividend. Trading is too costly compared to the benefits.

2. $s < \alpha^i - \overline{\alpha} \leq t$: The agent buys dividend-paying stocks, but does not hedge because hedging is too costly. In this case, dividend arbitrage subjects the agent to the market risk of the portfolio of stocks going ex. Since transaction costs are proportional, diversification is not costly: An agent prefers to buy all dividend-paying assets rather than just a subset. (In the presence of fixed transaction costs, the agent would trade only a subset of the dividend-paying stocks.)

$-t \leq \alpha^i - \overline{\alpha} \leq -s$: Similarly, the agent sells dividend-paying stocks and does not hedge.

3. $t < \alpha^i - \overline{\alpha}$: The agent's private valuation of dividends is quite different from the market's. Therefore, he takes a large position in dividend-paying stocks and hedges a portion of the market risk that he undertakes. The optimal hedging strategy (derived in the Appendix) equates the mar-

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[8] See Vayanos and Vila (1996) and Vayanos (1996) for a discussion of the relation between transaction costs and prices.
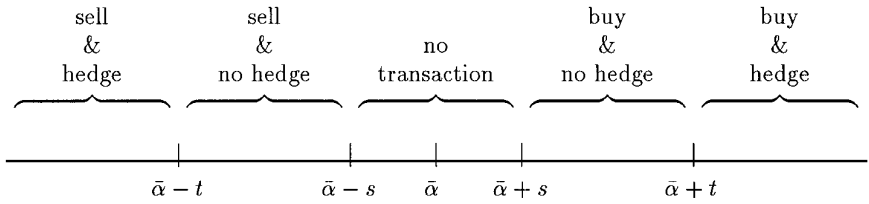
Fɪɢ 2.   Equilibrium trading with transaction costs.

ginal cost of hedging, i.e., the transaction cost, to the marginal benefit, i.e., the marginal cost of the market risk. In addition, the optimal arbitrage decision equates the marginal profit net of transaction costs to the marginal cost of additional risk. Since the marginal cost of the additional risk equals the marginal cost of the idiosyncratic risk plus the marginal cost of market risk, we obtain the first-order condition: Marginal profit of arbitrage equals marginal cost of idiosyncratic risk plus marginal cost of hedging. It follows that the agent's trade is independent of the level of market risk, and a decreasing function of idiosyncratic risk.

$\alpha^i - \bar{\alpha} < -t$: The agent sells dividend-paying stocks and hedges the market risk.

Figure 2 illustrates this classification of agents in terms of their equilibrium trading strategy, which is determined by their risk-adjusted tax status α. Given this classification of agents and their trading strategies, we can examine in detail the behavior of trading volume (see the Appendix for details).

In the presence of transaction costs such as brokerage commissions and bid–ask spreads, agents trade less and hedge less. The following proposition simply confirms this intuition.

PROPOSITION 6.   *The trading volume is lowered by the presence of transaction costs.*

Transaction costs make "tax-arbitrage" not only more costly, but also more risky. In saving the transaction costs, agents have to bear some of the market risk they pick up in "tax-arbitrage" trading. Consequently, in addition to the idiosyncratic risk, the market risk of dividend-paying stocks also inhibits agents from trading. The effect of the idiosyncratic risk is, however, stronger. Clearly, agents have to bear all the idiosyncratic risk while they do not have to bear the market risk since it is hedgable. While some active agents do not hedge the market risk to save transacting costs, some agents do. In other words, the trading of all active agents is affected by the idiosyncratic risk, while the market risk may only affect a subset of

active agents. This result is summarized in Proposition 4′, in parallel to Proposition 4.

PROPOSITION 4′. *The trading volume is a decreasing function of both the market risk component of the stock risk, $\beta_k^2 \sigma_q^2$, and the idiosyncratic risk, σ_k^2. The effect of the idiosyncratic risk is stronger.*

Furthermore, we have Proposition 5′, in parallel to Proposition 5.

PROPOSITION 5′. *The trading volume on each dividend-paying stock is a decreasing function of the number of stocks going ex.*

In the presence of transaction costs, agents do not fully hedge the market risk of the dividend-paying portfolio. Thus, the unhedged risk (including the idiosyncratic and market component) becomes correlated across the stocks going ex. Since agents' willingness to assume risk is limited, the trading volume per stock is lower.[9]

Finally, in the presence of transaction costs, agents take smaller positions. This is particularly true for agents who do not hedge. As a result, the idiosyncratic risk is less important in the determination of trading volume.

PROPOSITION 7. *As transaction costs increase, the trading volume is less sensitive to the idiosyncratic risk, σ_k^2.*

5. ROBUSTNESS

The model presented in the previous section relies on several specific assumptions that were useful (if not necessary) to obtain analytical results. In this section, we discuss the robustness of the results with respect to the relaxation of some of these assumptions.

A. *Multiperiod Extensions under Transaction Costs*

We first consider how to embed our model into an infinite horizon setting. The following condition is needed: agents start their trades on the cum day from positions that are Pareto-optimal with respect to risk-sharing and revert to these positions on the ex day (after the dividend payments). In the absence of transaction costs, this condition can be satisfied: The best policy would be to deviate from optimal risk-sharing just before the ex day, and revert to the original portfolio just after the dividend is paid. In other words, the heterogeneity in valuation only causes a temporal deviation

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[9] An analogous situation occurs in portfolio theory: A constant absolute risk aversion agent having access to two positively correlated assets invests less in each of them than he would if he had access to only one of them. However the total amount invested is higher.

from optimal risk-sharing. There is no benefit from tilting a portfolio away from the optimal position before the cum day, or keeping it away from the optimal position after the ex day. Differential taxes result in dynamic trading, but they do not have any permanent effect on asset allocations.[10]

In the presence of transaction costs, however, a permanent deviation from optimal risk sharing may occur. Agents may partially shift their portfolio allocation toward or away from the dividend-paying stocks, depending on their after-tax valuation of dividends and capital gains relative to the market. The possible formation of a ''holding'' clientele (in addition to a trading clientele) implies that agents arrive at the cum-dividend day with portfolios that are not optimal risk-sharing portfolios, because they now face the trade-off between deviation from optimal risk-sharing and paying transaction costs. In general, it is difficult to solve such a model in a truly dynamic setting (see, e.g., Heaton and Lucas, 1996; Vayanos, 1996). However, we can illustrate the effect of transaction costs on agents' portfolios and their trading around the ex day using the simple setting described in Section 2, but with an additional trading day, day 0, say, before the cum day. Trading on day 0 allows agents to achieve their optimal portfolios in the presence of transaction costs, which are different from the optimal risk-sharing portfolios. The trading on the cum day then represents temporary deviations from their optimal portfolios, instead of the optimal risk-sharing portfolios. As shown in Appendix B, in this case, higher transaction costs tend to push agents toward holding portfolios that reduce future trading. In other words, agents are more likely to arrive at the cum day with portfolios that are optimal with respect to risk, transaction costs, and their valuation of the dividend payment. Consequently, the trading activity on the cum day is lowered. If the risk over the periods between dividend payments is significantly large, the risk-sharing motive becomes dominant over these periods. Agents would always want to hold optimal risk-sharing portfolios between dividend payments, and they only deviate from these portfolios on cum days. In this case, we effectively return to the simple situation in Section 4.

B. *Differential Tax Basis*

In the basic model, the interest rate is assumed to be zero and agents liquidate their positions at the terminal date. This is equivalent to assuming that capital gains taxes are paid on stock investments independent of whether or not these gains are realized. With non-trivial time discounting (i.e., $r \neq 0$), agents may want to defer the realizations of their taxes on capital gains. Differences in tax bases give agents different incentives for trading. For example, agents with low tax bases tend to be less willing to

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[10] A derivation about embedding the basic model in an infinite horizon setting in the absence of transaction costs is available from the authors upon request.

sell their holdings than those with high tax bases. If agents can "short against the box," they can circumvent the effect of differential tax bases. Thus, without transaction costs, our (implicit) assumption that investors disregard their tax basis is justified. However, if short selling is costly or impossible [for example, the Clinton administration is considering legislative changes that would "discourage" investors from shorting against the box as a tax maneuver (Wall Street Journal, December 15, 1995)], it is possible that not all shareholders are willing to trade to the extent the model predicts.

In order to capture the effect of different tax bases within the framework of our basic model, we consider the following variation of the model. Continue to let the interest rate be zero, but assume that agent i's final wealth after taxes has the form

$$
\begin{aligned}
W^i_{\mathrm{A}} = W^i_0 + (1 - \tau^i_{\mathrm{g}})\{ & (p_{\mathrm{e}} - p_{\mathrm{c}})x^i_{\mathrm{c}} + (\tilde{v} - p_{\mathrm{e}})'x^i_{\mathrm{e}} \\
& + \alpha^i d' x^i_{\mathrm{c}} + (b^i)'(x^i_{\mathrm{c}} - \bar{x}^i)^- - (c^i)'|x^i_{\mathrm{c}} - \bar{x}^i|\},
\end{aligned}
\tag{14}
$$

where $i = 1, \ldots, I$. Equation (14) differs from (5) by only one additional term $(b^i)'(x^i_{\mathrm{c}} - \bar{x}^i)^-$, where $(x^i_{\mathrm{c}} - \bar{x}^i)^- = \min[0, x^i_{\mathrm{c}} - \bar{x}^i]$. This term simply says that if $b^i_k > 0$ (<0), agent i can decrease (increase) his final wealth by selling stock k from his original position $\bar{x}^i_k$. For an agent i with a low-tax basis in stock k, selling stock k on the cum day is costly since it implies early realization of capital gains and more capital gains taxes, and the cost is linear in the number of shares sold. Thus, $b^i_k > 0$. It can be verified that an extension of the basic model, in which $r > 0$ and agents want to realize capital gains (losses) late (early) due to different tax basis, can be reduced to the basic model with the above modification. Two effects are omitted in (14): the risk of deferred tax liabilities and the capital gains from the cum to ex day (which can affect the ex day trading). If the incentive to revert to the Pareto-optimal holding on the ex day is strong, the second effect would be small. We can further reexpress (14) as

$$
\begin{aligned}
W^i_{\mathrm{A}} = W^i_0 + (1 - \tau^i_{\mathrm{g}})\{ & (p_{\mathrm{e}} - p_{\mathrm{c}})'x^i_{\mathrm{c}} + (\tilde{v} - p_{\mathrm{e}})'x^i_{\mathrm{e}} \\
& + \alpha^i d' x^i_{\mathrm{c}} - (c^i_+)'(x^i_{\mathrm{c}} - \bar{x}^i)^+ - (c^i_-)'(x^i_{\mathrm{c}} - \bar{x}^i)^-\},
\end{aligned}
\tag{15}
$$

where $c^i_+ = c^i$ and $c^i_- = c^i - b^i$. The formal analysis in Section 2 still holds true. In particular, the equilibrium holdings on the cum day have the same form as in (8) except that now

$$
(c^i \otimes \varepsilon^i)_k = \begin{cases} c^i_{k,+} & \text{if } x^i_{\mathrm{c},k} > \bar{x}^i_k \\ -c^i_{k,-} & \text{if } x^i_{\mathrm{c},k} < \bar{x}^i_k . \\ -c^i_{k,-} \leq \varepsilon^i_k \leq c^i_{k,+} & \text{if } x^i_{\mathrm{c},k} = \bar{x}^i_k \end{cases}
$$

The equilibrium stock prices and trading volume on the cum day can then be characterized in a manner similar to that in the basic model. The above discussion shows that the tax basis affects transaction costs (mainly for sellers). The effect of tax basis on equilibrium trading volume is ambiguous, depending on the relative distribution of tax bases among agents with respect to their risk-adjusted tax preferences and transaction costs. We do not attempt a detailed analysis of the effect of differential tax bases.

The above discussion takes agents' tax bases as given. In a truly dynamic setting, tax bases are actually endogenous. They are determined by agents' optimal trading strategies. Consequently, the heterogeneity among agents is also determined endogenously, which can manifest itself in dynamic clienteles. Solving such a dynamic model is difficult and we leave it for future research.

C. *Multi-factor Extensions of Uncertainty Distribution*

In the basic model, we have assumed a one-factor structure about the uncertainty distribution. In other words, the uncertainty in the stocks' pay-offs has one common factor and idiosyncratic factors that are mutually independent across stocks. This assumption is not crucial for the results in the paper. It only helps to simplify the agents' trading strategies. When agents take positions in the dividend-paying portfolio on the cum day, they also want to use other stocks to hedge the risk they pick up. Their hedging strategy depends on how the risk of dividend-paying portfolio is correlated with the risks of other stocks. In the one-factor case, only the factor risk can be hedged. The hedging strategy is simply to use a hedging portfolio that carries only the factor risk (i.e., the market risk). In the more general case with multiple factors, the situation is similar, namely, to use a set of hedging portfolios that carry only factor risks. With a small number of factors, the number of hedging portfolios is small. In addition, these hedging portfolios have little "basis risk," i.e., they carry only factor risks. Our discussions in the paper can be easily extended to this case. In the most general case in which the number of factors equals the number of stocks, the number of hedging portfolios becomes big and they also carry non-trivial basis risk. The basis risk gives rise to an additional cost for hedging (in addition to transaction costs).

D. *Discussions on the Simplifying Assumptions*

In deriving the analytic results in Section 4, the following simplifying assumptions are made: (1) stocks not going ex all have the same beta, β_N, and idiosyncratic risk, σ_N^2; (2) stocks going ex all pay the same dividend, d, and have the same beta, β_E, and idiosyncratic risk, σ_E^2; (3) the transaction costs are the same for all stocks and all agents, i.e., $c_k^i = c$; and (4) the

distribution of agents' tax preference parameter α^i (with probability δ^i) is symmetric around $\bar{\alpha}$. We now discuss how these results may change if some of the assumptions are relaxed. We do this by considering several simple examples.

Assumption (1) is a mild one in its effect on our results. In order to see this, let us consider the special case when there is a stock not going ex that has no idiosyncratic risk. (We are still maintaining the other assumptions.) In this case, agents only use this single stock (or index), instead of a portfolio of stocks, to hedge the market risk of their tax-arbitrage strategies. This reduces the transaction costs of hedging. Furthermore, the transaction costs decrease as this stock's absolute value of β increases. This is because in our model the transaction costs are specified as linear in the number of shares traded (not the dollar amount). With a large absolute value of β, few shares are needed to hedge a given exposure to the market risk. In the more general case when the market risk and idiosyncratic risk of non-dividend-paying stocks differ, agents would rely more heavily on those stocks with small idiosyncratic risk but large market risk to do the hedge. However, some agents still may not hedge, and even those who do may not hedge completely, due to transaction costs. By the same intuition discussed in Section 4, we still expect Propositions 4′ and 5′ to hold when Assumption (1) is relaxed.

Assumption (2) is non-trivial. This can be shown by considering the following example: Suppose that there are only two stocks going ex, 1 and 2. They have the same dividend and idiosyncratic risks but offsetting market risk, i.e., $\beta_1 = -\beta_2$. It is clear that agents optimal tax-arbitrage trading bears no market risk. Consequently, the volume of tax-arbitrage trading is independent of market risk, in contrast with Propositions 4′ and 5′. This counter example relies on the fact that the market risk of dividend-paying stocks are offsetting each other. In a more realistic situation where the dividend-paying stocks all have positive but different β, we would expect Propositions 4′ and 5′ to still be true. When dividends, idiosyncratic risks, and market risk are all different across dividend-paying stocks, the situation becomes more complicated.

Let us now consider what may happen when we relax Assumption (3) to allow different transaction costs for different stocks. When the non-dividend-paying stocks have different transaction costs, agents would use those with smaller transaction costs to do the hedge. As long as there is a large number of them, they form a well-diversified portfolio. The results in Section 4 should still hold. When the transaction costs vary across dividend-paying stocks, it can be shown (as the derivation in the Appendix shows) that similar results can be obtained as in Section 4.

Assumption (4) is very important in deriving the results. The symmetric distribution in agents' tax preference parameter α^i (with probability δ^i)

around its mean $\bar{\alpha}$ (together with the other assumptions) gives the result that stock prices on the cum day are independent of the transaction costs. Without this assumption, the stock prices on the cum day also depend on the transactions costs and the solution becomes more complicated (see the general solution in Section 2).

We have only considered the possible impact of relaxing the four assumptions individually. Of course, things would be even more complicated if some of these assumptions are relaxed simultaneously. Then we must return to the general solution in Section 2. Given the difficulty in expressing the solution in closed form, it is likely that we will have to rely on numerical solutions.

6.   DISCUSSIONS AND CONCLUSIONS

We have analyzed the behavior of volume when agents trade because of differential valuation. In particular, we have considered a situation in which taxes are the sole reason for the differential valuation among agents, and investigate how risk and transaction costs affect their trading behavior and the resulting trading volume. The model indicates that transaction costs reduce volume and heterogeneity increases volume. Systematic risk affects (negatively) volume only in the presence of transaction costs, while idiosyncratic risk always reduces volume. An increase in the level of transaction costs diminishes the effect of the idiosyncratic risk on volume.

Despite the model's simplifying assumptions (such as CARA preferences, no effect of tax basis, etc.), it gives several clear predictions that are consistent with the data. In particular, Michaely and Vila (1996) use data on dividend paying stocks to test several of the model's implications. They find that abnormal volume around the ex-dividend day is negatively related to the level of both market risk and idiosyncratic risk, consistent with the model's prediction. The level of transaction costs is also found to have a significant effect on the volume of trade: (1) Stocks with higher transaction costs (proxied by the bid-ask spread and the market value of equity) exhibit lower trading volume; and (2) trading volume is lower in periods associated with lower transaction costs. Consistent with Proposition 4′, they also find that market risk has more effect on trading volume in periods of higher transaction costs. Thus, when hedging systematic risk is more costly, agents take smaller positions. Idiosyncratic risk has the opposite effect: As transaction costs decrease, as they did in 1975, the effect of idiosyncratic risk is increased. Finally, as the degree of tax heterogeneity increases, the volume of trade increases significantly.

The model's predictions are also consistent with the evidence presented by Grundy (1985), Lakonishok and Vermaelen (1986), and Koski (1992).

Grundy shows that trading volume increases with yield (consistent with Proposition 1). Consistent with Proposition 6, Lakonishok and Vermaelen show that the increase in volume is more pronounced after 1975, when the cost of transacting was lower. Koski shows that when traders are able to reduce their risk exposure and transaction costs by using nonstandard settlement trades, the abnormal volume increases more than tenfold.

In the current model, differential taxes give rise to differential valuation which generates trading. It appears that when differential valuation is the dominant trading motive, the model's predictions about the relation between volume, risk, and transaction costs are consistent with the data.

The interaction between differential valuation, risk, and transaction costs suggests a potential advantage of changing the trading structure on the ex day.[11] Imagine the effect of having a market structure in which stocks go ex dividend not at the end of the day but in the middle of the day. Then agents who are trading for tax reasons can buy and sell in a matter of seconds or minutes. (This change does not affect the corporate traders who have to hold the stock for at least 45 days if they want to exclude 70% of the dividends from taxes.) The immediate impact of such a change would be to substantially reduce the risk involved in tax-arbitrage trading: Instead of having to hold the stock overnight, agents only have to hold it for a very short period of time. This reduction in risk can greatly increase the amount of trading. In fact, Koski (1992) documents that Japanese insurance companies prearranged ex day trading (by changing the settlement day) to ''synthetically'' achieve exactly such a market structure. Moreover, her evidence shows that trading volume of shares involved in such trade increases dramatically. Because the exchanges' profits are driven by volume, they may have the incentive to consider such a proposal.

Another possible change to trading structure is related to the effect of transaction costs. A significant portion of the transaction costs is the bid–ask spread. It is well known that two factors contributing to the spread are asymmetric information (see, for example, Kyle, 1985; Glosten and Milgrom, 1985; O'Hara, 1995) and inventory costs (see, for example, Amihud and Mendelson, 1980 and Ho and Stoll, 1983). If, however, investors and the specialist know that trades are tax motivated rather than information motivated, the required spread would be narrower for these trades.[12] In the current market structure, a trader cannot know for sure if a trade is tax motivated, even when it occurs around the ex day. If agents could prearrange ex day trades such that they commit to buy (sell) cum and sell

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[11] We thank a referee for leading us to consider the effect of changing the trading process.
[12] Koski and Michaely (1996) present evidence that even with the current market structure, the bid–ask spread around the ex day is lower than on other days. This is consistent with the assertion that the proportion of non-informational trade is low on ex days.

(buy) ex, they would be able to credibly signal that their trades are not informational, hence reduce the transactions costs. Thus, a sunshine trading mechanism, in the spirit of Admati and Pfleiderer (1991), could be implemented in order to reduce transaction costs and increase trading volume. This can be done without changing either the settlement day or the cum day ex day timing.

As discussed in the paper, in addition to trades in dividend-paying stocks (tax-arbitrage trades), agents also trade in non-dividend-paying stocks to hedge the common risk incurred from tax-arbitrage trades. These hedging trades can be quite costly given the transaction costs in trading individual stocks. An attractive vehicle for agents to hedge the common risk at possibly lower costs is index futures contracts. [For a discussion on why it may be less costly to trade market baskets, see, for example, Subrahmanyam (1991).] Thus, in a model like ours with multiple assets and transaction costs, there is a natural demand for the introduction of new securities based on market baskets. It also should be pointed out that our model is capable of including securities like index futures. Note that an index futures contract is equivalent to a ''stock'' (with zero net supply) that only carries the common risk (and possibly has low transaction costs). The existence of such a security can further simplify the model as mentioned earlier.

In addition to differential valuation, there are other motives of trading such as information or risk allocation (see, e.g., Wang, 1994). When these motives of trading are dominant, the relation between volume, risk, and transaction costs can be different from what this model predicts. It is therefore important to extend the framework presented here to situations where trading motives are based on reasons other than differential valuation. Modeling the interaction between trading volume, private information, risk, and transaction costs in this more general setting will not be an easy task, but we hope that the insights developed here will prove useful.

APPENDIX

A. *Proofs*

In this appendix we provide a proof of the results stated in Sections 3 and 4. We first analyze the one-factor model. Next, we consider the no transaction costs case and derive Propositions 1 through 5. We then derive the trading strategies in the presence of transaction costs. Finally, we derive Propositions 6, 4′, 5′, and 7.

Step 1: *The One-Factor Model.* Here, we consider the general situation that the risky securities traded in the market can have zero net supply. Let

$\bar{x}$ be the vector of shares outstanding for the stock (without assuming that it can be normalized to ι). Furthermore, define

$$w_k = \frac{\bar{x}_k}{\bar{x}_M}, \qquad \text{where } \bar{x}_M = \sum_{k=1}^{K} \bar{x}_k.$$

Thus, w_k gives the weight of the stock k in the market portfolio (in terms of shares). We normalize the factor so that

$$\sum_{k=1}^{K} w_k \beta_{k,q} = 1,$$

i.e., the "average beta" is one. We further assume that w_k is small compared with one for all $k = 1, \ldots, K$. (This is apparently true if $\bar{x} = \iota$.)

Let $\tilde{u}_{e,M}$ be the information shock on the market portfolio, i.e.,

$$\tilde{u}_{e,M} = \sum_{k=1}^{K} w_k \tilde{u}_{e,k} = \tilde{q} + \sum_{k=1}^{K} w_k \tilde{\eta}_k.$$

Let σ_M^2 be the variance of the information shock on the market portfolio, i.e.,

$$\sigma_M^2 = \sigma_q^2 + \sigma_\eta^2, \qquad \text{where } \sigma_\eta^2 = \sum_{k=1}^{K} w_k^2 \sigma_k^2. \tag{A.1}$$

Let $\beta_{k,M}$ be the stock's β with respect to the market portfolio, i.e.,

$$\beta_{k,M} = \frac{\text{cov}(\tilde{u}_{e,k}, \tilde{u}_{e,M})}{\text{var}(\tilde{u}_{e,M})} = \beta_k + \frac{w_k \sigma_k^2 - \beta_k \sigma_\eta^2}{\sigma_q^2 + \sigma_\eta^2}. \tag{A.2}$$

Let $\sigma_{k|M}^2$ be the variance of $\tilde{u}_{e,k}$ conditional on $\tilde{u}_{e,M}$, i.e.,

$$\sigma_{k|M}^2 = \text{var}(\tilde{u}_{e,k} \,|\, \tilde{u}_{e,M}) = \sigma_k^2 + \beta_k^2 \sigma_q^2 - \frac{(w_k \sigma_k^2 + \beta_k \sigma_q^2)^2}{\sigma_q^2 + \sigma_\eta^2}. \tag{A.3}$$

From (A.1), (A.2), and (A.3) it follows that if K is large and all the w_k's are small with respect to one, σ_M^2, $\beta_{k,M}$, and $\sigma_{k|M}^2$ are equal to σ_q^2, β_k, and σ_k^2, respectively.[13] It is therefore legitimate to interpret σ_q^2 as the risk on the market portfolio, β_k as the stock's beta, and σ_k^2 as its idiosyncratic risk.

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[13] Formally, we consider a sequence of economies. In the Kth economy, the risky assets are $1, \ldots, K$ with weights w_k^K. To take limits, we assume that $\sup_{k=1,\ldots,K} w_k^K \to 0$ as $K \to \infty$, and that $\sup_{K=1,\ldots,\infty} \sigma_k^2 < \infty$.

Step 2: *The No-Transaction-Cost Case.* In the absence of transaction costs, Eq. (7) reduces to the tax-adjusted CAPM so that the price of asset k is given by

$$p_{c,k}^* = \overline{p}_{c,k} + \overline{\alpha}d_k - \frac{1}{\theta}\beta_{k,M}\sigma_M^2 = \overline{p}_{e,k} + \overline{\alpha}d_k - \frac{1}{\theta}\beta_k\sigma_q^2. \qquad (A.4)$$

Let $y^i = (y_k^i) = x_c^i - \overline{x}^i$ be agent i's trading strategy at the cum day. In the absence of transaction costs, y^i can be derived from Eq. (8). It is useful, however, to consider the problem faced by agent i at the cum. If p_c is given by (7) (with $c^i = 0$), the optimization problem (6) reduces to maximizing

$$(\alpha^i - \overline{\alpha})d'y^i - \frac{1}{2\theta^i}(y^i)'\omega_e y^i \qquad (A.5)$$

with respect to y^i. In the one-factor case, this is equivalent to maximizing

$$(\alpha^i - \overline{\alpha})\sum_{k=1}^{K} d_k y_k^i - \frac{1}{2\theta^i}\left[\sum_{k=1}^{K}(y_k^i)^2\sigma_k^2 + \left(\sum_{k=1}^{K}\beta_k y_k^i\right)^2\sigma_q^2\right]. \qquad (A.6)$$

The solution to this problem is given by

$$y_k^i = \theta^i(\alpha^i - \overline{\alpha})\left[\frac{d_k}{\sigma_k^2} - \frac{\beta_k\sigma_q^2}{\sigma_k^2}\frac{\sum_{k'=1}^{K}(\beta_{k'}d_{k'}/\sigma_{k'}^2)}{1 + \sum_{k'=1}^{K}(\beta_{k'}\sigma_q^2/\sigma_{k'}^2)}\right]. \qquad (A.7)$$

Under the assumptions that K is large and only a small number of stocks pay dividends, the expression in (A.7) reduces to Eq. (12). Therefore the trading volume in the absence of transaction costs is given by (13) and Propositions 1 through 5 follow easily.

Step 3: *Trading Strategies in the Presence of Transaction Costs.* To solve for the equilibrium trading strategies, we show that at the zero transaction costs prices given by (A.4), supply equals demand. We then calculate trading volume at these prices.

Consider an agent i with $\alpha^i > \overline{\alpha}$. In the presence of transaction costs the agent maximizes

$$(\alpha^i - \overline{\alpha})\sum_{k=1}^{K} d_k y_k^i - \sum_{k=1}^{K} c_k^i|y_k^i| - \frac{1}{2\theta^i}\left[\sum_{k=1}^{K}(y_k^i)^2\sigma_k^2 + \left(\sum_{k=1}^{K}\beta_k y_k^i\right)^2\sigma_q^2\right]. \qquad (A.8)$$

We have assumed that: (1) the distribution of agents' tax preference parame-

ter α^i (with probability δ^i) is symmetric around $\overline{\alpha}$; (2) all the stocks that are going ex pay the same dividend d, have the same beta β_E, and idiosyncratic risk σ_E^2; (3) all the stocks that are not going ex have the same beta β_N, and idiosyncratic risk σ_N^2, and (4) all agents face the same transaction costs, $c_k^i = c$, on every stock. Given these assumptions, the agent's problem can be described as follows: Choose how many shares, a, of the dividend-paying assets to buy and how many shares, h, of the non-dividend-paying assets to sell as hedges. Indeed, since transaction costs are proportional, diversification is not costly. Hence, the agent prefers to buy all dividend-paying assets rather than just a subset. (Note that in the presence of fixed transaction costs, the agent trades only a subset of the dividend-paying assets.) From (A.8) above, and with the number of stocks going ex denoted by S, a and h are chosen so as to maximize

$$(\alpha^i - \overline{\alpha})Sad - c[Sa + (K - S)h]$$
$$- \frac{1}{2\theta^i}\{S\sigma_E^2 a^2 + (K - S)\sigma_N^2 h^2 + [S\beta_E a - (K - S)\beta_E h]^2 \sigma_q^2\} \qquad (A.9)$$

with respect to $a > 0$ and $h > 0$. Define

$$s = \frac{c}{d}, \qquad t = \frac{c}{d}\frac{S(1 + \beta_E)\beta_E\sigma_q^2 + \sigma_E^2}{S\beta_E\beta_N\sigma_q^2}. \qquad (A.10)$$

The solution to (A.9) is described as follows:

$$\text{if } \alpha^i \le \overline{\alpha} + s, \qquad \begin{cases} a = 0 \\ h = 0 \end{cases}$$

$$\text{if } \overline{\alpha} + s < \alpha^i \le \overline{\alpha} + t, \qquad \begin{cases} a = \theta^i \dfrac{(\alpha^i - \overline{\alpha})d - c}{S\beta_E^2\sigma_q^2 + \sigma_E^2} \\ h = 0 \end{cases} \qquad (A.11)$$

$$\text{if } \alpha^i > \overline{\alpha} + t \qquad \begin{cases} a = \theta^i \dfrac{(\alpha^i - \overline{\alpha})(\gamma\beta_N\sigma_q^2 + \sigma_N^2)d - [\gamma(\beta_N + \beta_E)\sigma_q^2 + \sigma_N^2]c}{S\beta_E^2\sigma_q^2\sigma_N^2 + \gamma\beta_N\sigma_E^2\sigma_q^2 + \sigma_E^2\sigma_N^2} \\ h = \theta^i \dfrac{(\alpha^i - \overline{\alpha})S\beta_E\beta_N\sigma_q^2 d - [S(\beta_N + \beta_E)\beta_N\sigma_q^2 + \sigma_N^2]c}{S\beta_E^2\sigma_q^2\sigma_N^2 + \gamma\beta_N\sigma_E^2\sigma_q^2 + \sigma_E^2\sigma_N^2} \end{cases}$$

where $\gamma = (K - S)\beta_E$.

Since the distribution of α^i is symmetrical, for every $\alpha^i < \overline{\alpha}$ there exists

$\alpha^{i'} > \overline{\alpha}$, such that $\theta^i = \theta^{i'}$ and $\alpha^i - \overline{\alpha} = \overline{\alpha} - \alpha^{i'}$. From Eq. (A.11) it can be seen that $y^i + y^{i'} = 0$, and, hence, supply equals demand.

Step 4: *Trading Volume in the Presence of Transaction Costs.* Having derived a and h, we calculate the trading volume on any dividend-paying stock under the assumption that K is large (which implies that the β of non-dividend-paying stocks is equal to the average β, i.e., one).

We first define $\mu^i = \alpha^i - \overline{\alpha}$. When $K \to \infty$, Eq. (A.11) yields the following expression for the trading volume on dividend-paying stocks:

$$v_c = \theta \left[\sum_{s \leq \mu^i < t} \delta^i \frac{\mu^i d - c}{S\beta_E^2 \sigma_q^2 + \sigma_E^2} + \sum_{\mu^i > t} \delta^i \frac{\mu^i d - c(1 + \beta_E)}{\sigma_E^2} \right]. \quad (A.12)$$

From Eq. (A.12) we now derive Propositions 6, 4′, 5′, and 7. We note that when taking the derivative of v_c with respect to any variable, s and t can be treated as constant. Indeed, by continuity of the trading strategy y_k^i,

$$\frac{\partial v_c}{\partial s} = \frac{\partial v_c}{\partial t} = 0$$

for s and t given by (A.11). We now calculate the partial derivatives of v_c with respect to c, β_E, and S:

$$\frac{\partial v_c}{\partial c} = -\theta \left[\sum_{s \leq \mu^i < t} \delta^i \frac{1}{S\beta_E^2 \sigma_q^2 + \sigma_E^2} + \sum_{\mu^i > t} \delta^i \frac{1 + \beta_E}{\sigma_E^2} \right] < 0 \quad (A.13a)$$

$$\frac{\partial v_c}{\partial \beta_E} = -\theta \left[\sum_{s \leq \mu^i < t} \delta^i \frac{2(\mu^i d - c)S\beta_E^2 \sigma_q^2}{(S\beta_E^2 \sigma_q^2 + \sigma_E^2)^2} + \sum_{\mu^i > t} \delta^i \frac{c}{\sigma_E^2} \right] < 0 \quad (A.13b)$$

$$\frac{\partial v_c}{\partial \sigma_q^2} = -\theta \left[\sum_{s \leq \mu^i < t} \delta^i \frac{(\mu^i d - c)S\beta_E^2}{(S\beta_E^2 \sigma_q^2 + \sigma_E^2)^2} \right] < 0 \quad (A.13c)$$

$$\frac{\partial v_c}{\partial \sigma_E^2} = -\theta \left[\sum_{s \leq \mu^i < t} \delta^i \frac{\mu^i d - c}{(S\beta_E^2 \sigma_q^2 + \sigma_E^2)^2} + \sum_{\mu^i > t} \delta^i \frac{\mu_i d - c(1 + \beta_E)}{\sigma_E^4} \right] \quad (A.13d)$$

$$\frac{\partial v_c}{\partial S} = -\theta \left[\sum_{s \leq \mu^i < t} \delta^i \frac{(\mu^i d - c)\beta_E^2 \sigma_q^2}{(S\beta_E^2 \sigma_q^2 + \sigma_E^2)^2} \right] < 0. \quad (A.13e)$$

Propositions 6 and 5' follow immediately from Eqs. (A.13). The first part of Proposition 4' is also a direct consequence of (A.13). We now compare the relative impact of $\beta_E^2 \sigma_q^2$ and σ_k^2 (second part of Proposition 4').

We first consider an increase in the market component of the stock's risk due to an increase of β_E. Note that increasing β_E means increasing the market risk in all the S stocks that are going ex. Ideally, we would like to consider an increase in β for only one stock. This, however, would require solving an intractable model with different β's. Within the framework of our model, the correct approach is either to take $S = 1$ or to scale by S. Straightforward calculations show that

$$\frac{1}{S}\frac{\partial v_c}{\partial(\beta_E^2 \sigma_q^2)}\bigg|_{\sigma_q^2 = \text{constant}} - \frac{\partial v_c}{\partial \sigma_E^2} = \theta \sum_{\mu^i \geq t} \delta^i \frac{(\mu^i - t)d + c\sigma_E^2/(2\beta_E \sigma_q^2)}{S\sigma_E^4} > 0$$

and therefore

$$\text{abs}\left\{\frac{1}{S}\frac{\partial v_c}{\partial(\beta_E^2 \sigma_q^2)}\bigg|_{\sigma_q^2 = \text{constant}}\right\} < \text{abs}\left\{\frac{\partial v_c}{\partial \sigma_E^2}\right\}, \qquad (A.14)$$

where abs$\{\cdot\}$ denotes the absolute value. Equation (A.14) shows that an increase of the market component of risk due to an increase in β has a lower impact on volume than an increase in the idiosyncratic risk has.

We next consider an increase in the market component of risk due to an increase in the market risk. Straightforward calculations show that

$$\frac{1}{S}\frac{\partial v_c}{\partial(\beta_E^2 \sigma_q^2)}\bigg|_{\beta_E = \text{constant}} - \frac{\partial v_c}{\partial \sigma_E^2} = \theta \sum_{\mu^i \geq t} \delta^i \frac{\mu^i d - c(1 + \beta_E)}{S\sigma_E^4} > 0$$

and therefore

$$\text{abs}\left\{\frac{1}{S}\frac{\partial v_c}{\partial(\beta_E^2 \sigma_q^2)}\bigg|_{\beta_E = \text{constant}}\right\} < \text{abs}\left\{\frac{\partial v_c}{\partial \sigma_E^2}\right\}. \qquad (A.15)$$

This concludes our proof of Proposition 4'.

Finally, we show that the effect of the idiosyncratic risk, σ_E^2, is smaller in the presence of transaction costs (Proposition 7). In the non-transaction-costs case volume is given by

$$v_c^* = \frac{\theta d}{\sigma_E^2} \sum_{\mu^i > 0} \delta^i \mu^i$$

so that

$$\left| \frac{\partial v_{\mathrm{c}}^{*}}{\partial \sigma_{\mathrm{E}}^{2}} \right| = \frac{v_{\mathrm{c}}^{*}}{\sigma_{\mathrm{E}}^{2}} > \frac{v_{\mathrm{c}}}{\sigma_{\mathrm{E}}^{2}} > \left| \frac{\partial v_{\mathrm{c}}}{\partial \sigma_{\mathrm{E}}^{2}} \right|. \qquad (A.16)$$

B. The Effect of Transaction Costs on Optimal Portfolios and Trading Behavior

To illustrate the effect of transaction costs on portfolio allocation (static) and on trading volume around the ex day (dynamic) in a multiperiod setting, we describe the optimal behavior of an agent (with CARA utility function) in an economy with one risky asset that pays dividends. We can capture the idea of more trading periods by introducing another trading day, day 0, say, one year before the dividend payment. As before, agents may trade on both the cum and the ex days. The initial day 0 represents the trade that may result in a long-run deviation (static) from optimal risk-sharing, and the potential trade on the cum day may result is a temporary deviation from optimal holdings via a dynamic strategy. For simplicity, we assume that the liquidation and ex are the same day.

Let z^{i} denote initial purchase at day 0, x^{i} the Pareto-optimal holdings, y_{i} cum day trading, c transaction cost (proportional), σ_{1}^{2} the variance between day 0 and the cum day, σ_{2}^{2} the variance between the cum day and the ex day, d the dollar amount of the dividend, θ^{i} tax-adjusted risk tolerance, θ the market's tax-adjusted risk tolerance, α^{i} relative valuation of dividend to capital gains, and $\overline{\alpha}$ the market's relative valuation of dividends to capital gain. (Using the same distributional assumptions as before, the longer time horizon between day 0 and the cum day than between the cum and the ex day implies that $\sigma_{1}^{2} \gg \sigma_{2}^{2}$.)

The agent has two decision variables: ξ and y. Setting the budget constraint and given CARA utility function, a solution to this optimization problem yields

$$y^{i} = (x^{i} - z^{i}) + \frac{\theta^{i}(\alpha^{i} - \overline{\alpha})}{\sigma_{1}^{2}} d - c \qquad (B.1)$$

$$z^{i} = x^{i} + \frac{\theta^{i}}{\sigma_{2}^{2}} c. \qquad (B.2)$$

Expressing the cum day trading in terms of the exogenous variables yields

$$y^{i} = \frac{\theta^{i}(\alpha^{i} - \overline{\alpha})}{\sigma_{1}^{2}} d - c - \frac{\theta^{i}}{\sigma_{2}^{2}} c. \qquad (B.3)$$

Thus, a higher level of transaction costs pushes agents toward higher trading activity at day 0 (i.e., greater permanent deviation from optimal risk-sharing), and lower trading activity on the cum day. In other words, the higher the level of transaction costs, the more attractive are buy-and-hold strategies and the less attractive are dynamic strategies. Thus, instead of starting from a portfolio that is Pareto-optimal from a risk-sharing perspective only, agents come to the cum day with holdings of z^i [see Eq. (B.2)] which is the optimal holding with respect to both risk and transaction costs.

It is important to notice, however, that the basic relation between transaction costs and volume has not changed: The higher the level of transaction costs, the lower the level of trading around the ex day, and day 0 trading is higher.

REFERENCES

ADMATI, A., AND PFLEIDERER, P. (1991). Sunshine trading and financial market equilibrium, *Rev. Finan. Stud.* **4,** 443–482.

AMIHUD, Y., AND MENDELSON, H. (1980). Dealership market: Market making with inventory, *J. Finan. Econ.* **8,** 31–53.

ARROW, K. (1953). Le rôle des valeurs boursières pour la repartition la meilleure des risques, *Econométrie* **40,** 41–47.

BIAIS, B., AND BOSSAERT, P. (1993). Asset prices and trading volume in a beauty contest, Working paper, California Institute of Technology, CA.

BLUME, L., EASLEY, D., AND O'HARA, M. (1994). Market statistics and technical analysis: The role of volume, *J. Finance* **49,** 153–181.

BRENNAN, M. (1970). Taxes, market valuation and financial policy, *Nat. Tax J.* **23,** 417–429.

CAMPBELL, J., GROSSMAN, S., AND WANG, J. (1993). Trading volume and serial correlation in stock returns, *Quart. J. Econ.* **108,** 905–939.

CONSTANTINIDES, G. (1986). Capital market equilibrium with transaction costs, *J. Polit. Econ.* **94,** 842–862.

DUMAS, B. (1989). Two-person dynamic equilibrium in the capital market, *Rev. Finan. Stud.* **2,** 157–188.

ELTON, E., AND GRUBER, M. (1970). Marginal stockholders' tax rates and the clientele effect, *Rev. Econ. Statist.* **52,** 68–74.

GLOSTEN, L., AND MILGROM, P. (1985). Bid, ask, and transaction prices in a specialist market with heterogeneously informed traders, *J. Financ. Econ.* **13,** 71–100.

GROSSMAN, S. (1981). An introduction to the theory of rational expectations under asymmetric information, *Rev. Econ. Stud.* **48,** 541–559.

GRUNDY, B. (1985). Trading volume and stock returns around ex-dividend dates, Working paper, University of Chicago, Chicago, IL.

HARRIS, M., AND RAVIV, M. (1993). Differences in opinion make a horse race, *Rev. Finan. Stud.* **6,** 473–506.

HEATON, J., AND LUCAS, D. (1996). Evaluating the effects of incomplete markets on risk-sharing and asset pricing, *J. Polit. Econ.* **104,** 443–487.

Ho, T., and Stoll, H. (1983). The dynamics of dealer markets under competition, *J. Finance* **38,** 1053–1074.

Kandel, E., and Pearson, N. (1995). Differential interpretation of public signals and trade in Speculative markets, *J. Polit. Econ.* **103,** 831–872.

Koski, J. (1992). Nonstandard settlement trading on the ex-dividend day, Working paper, University of Washington, Seattle, Washington.

Koski, J., and Michaely, R. (1996). Trade size, information intensity and price effects, Working paper, Cornell University, Ithaca, NY.

Kyle, A. (1985). Continuous auctions and inside trading, *Econometrica* **53,** 1315–1336.

Lakonishok, J., and Vermaelen, T. (1986). Tax induced trading around the ex-day, *J. Finan. Econ.* **3,** 287–320.

Litzenberger, R., and Ramaswamy, K. (1979). The effects of personal taxes and dividends on capital asset prices, *J. Financ. Econ.* **7,** 163–195.

Michaely, R. (1991). Ex-dividend day stock price behavior: The case of the 1986 Tax Reform Act, *J. Finance* **46,** 845–859.

Michaely, R., and Vila, J.-L. (1995). Investors' heterogeneity, prices and volume around the ex-dividend day, *J. Finan. Quant. Anal.* **30,** 171–198.

Michaely, R., and Vila, J.-L. (1996). Trading volume with private valuation: Evidence from the ex-dividend day, *Rev. Finan. Stud.* **9,** 471–510.

O'Hara, M. (1995). ''Market Microstructure Theory,'' Blackwell, Cambridge, MA.

Subrahmanyam, A. (1991). A theory of trading in stock index futures, *Rev. Finan. Stud.* **4,** 17–51.

Tirole, J. (1982). On the possibility of speculation under rational expectations, *Econometrica* **50,** 1163–1181.

Vayanos, D. (1996). Transaction costs and asset prices: A dynamic equilibrium model, Working paper, Stanford University, Stanford, CA.

Vayanos, D., and Vila, J.-L. (1996). Equilibrium interest rate and liquidity premium under proportional transactions costs, Working paper, Stanford University, Stanford, CA.

Wang, J. (1994). A model of competitive stock trading volume, *J. Polit. Econ.* **102,** 127–168.

Wang, J. (1996). The term structure of interest rates in a pure exchange economy with heterogeneous investors, *J. Finan. Econ.* **41,** 75–110.