The Yowie Factor

a simple estimate of load rate during climber fall arrest

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Center for Sports Innovation
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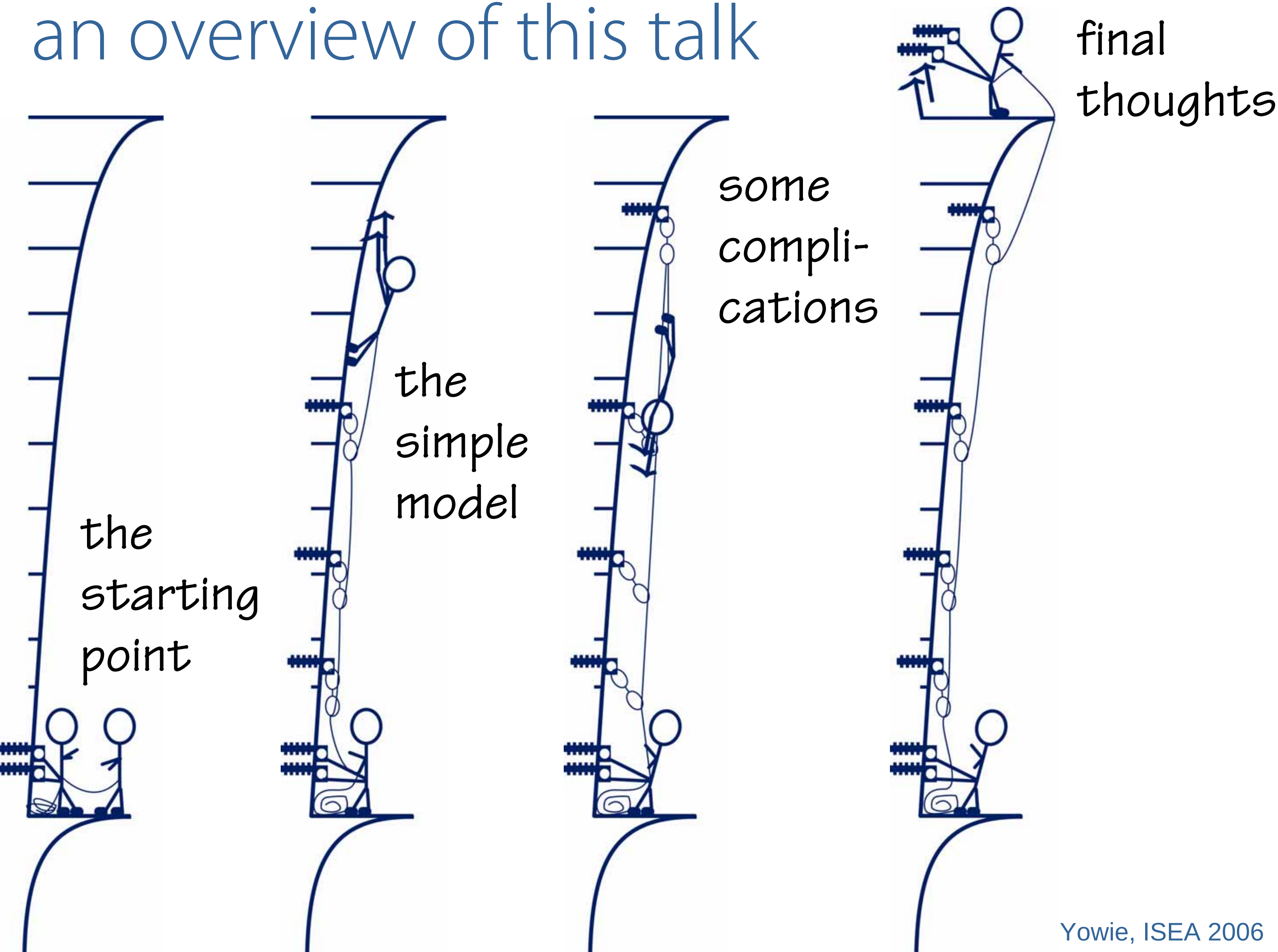
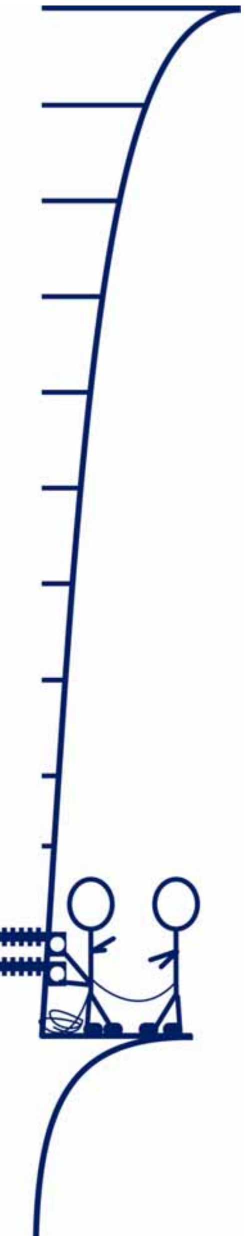
an overview of this talk



motivation

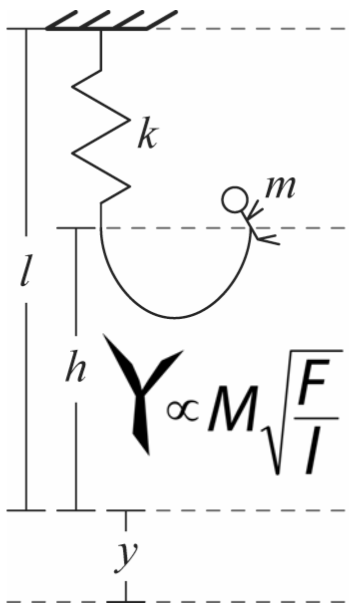


both ice and ice screws exhibit reduced strength with increased load rate. a 100 X increase in rate halves the strength.

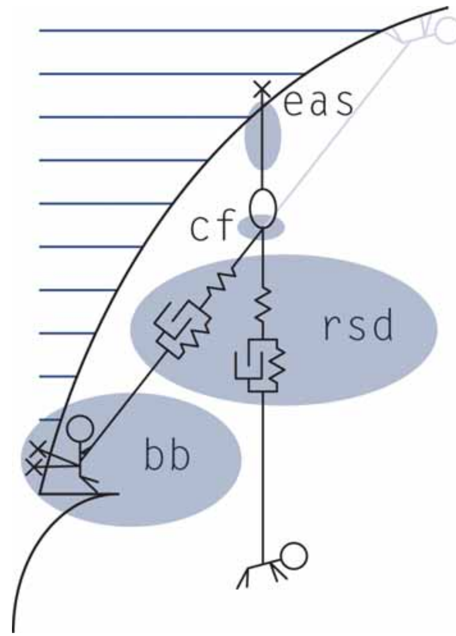
objectives

- determine degree of "impact"
- make impact accessible
- guide safety system design

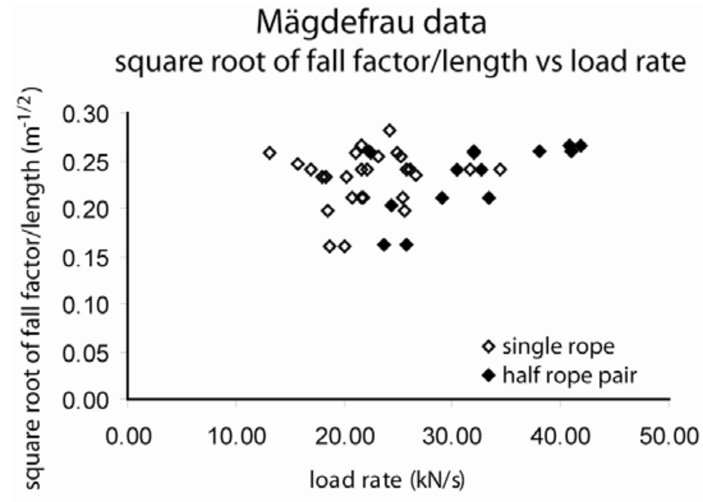
methods



determine the load rate based on a (very) simple rope model

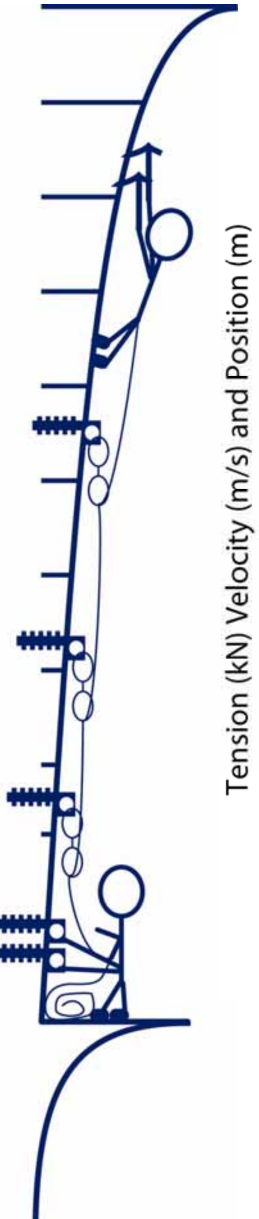


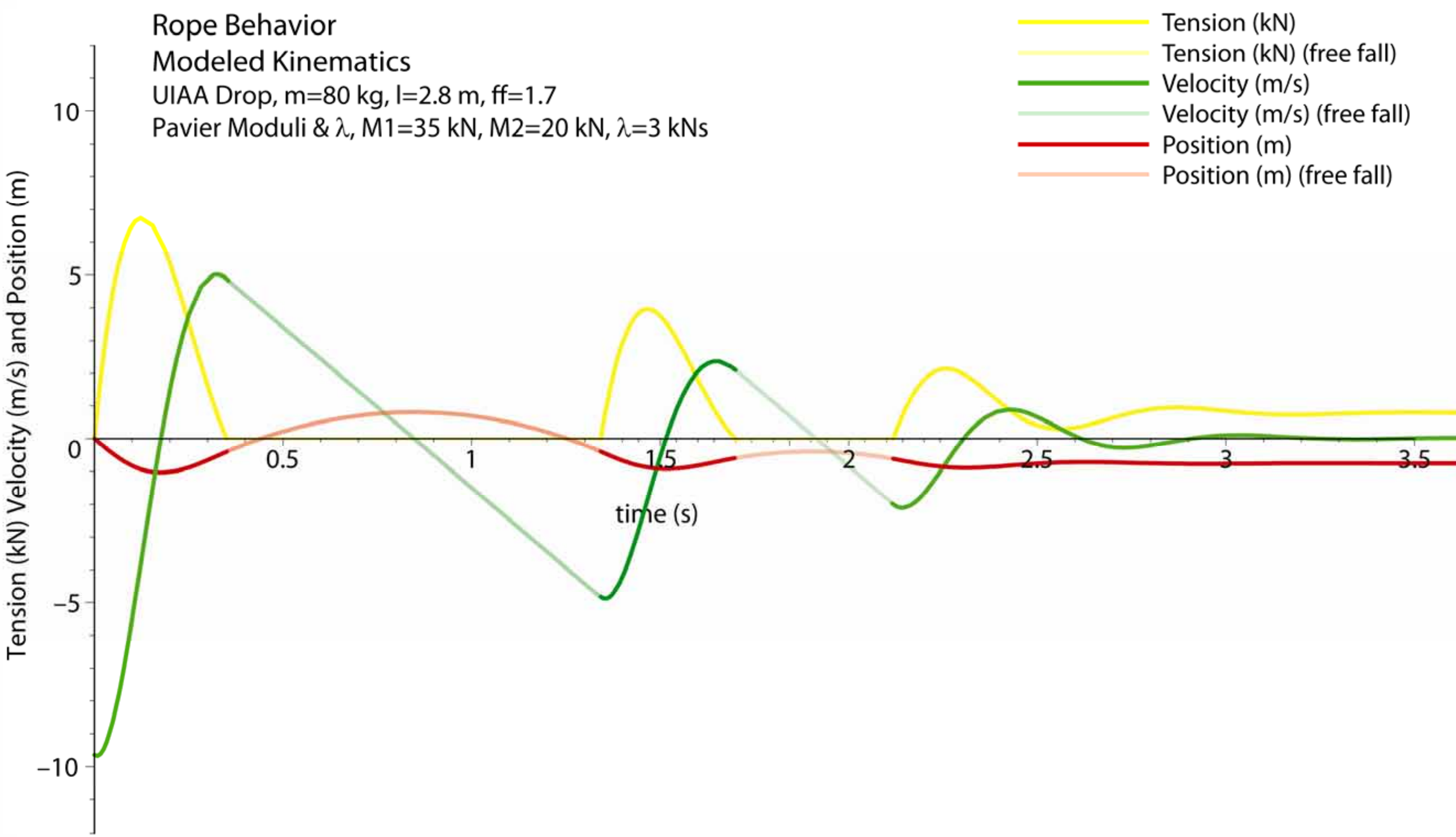
compare the simple model to complicated models (Wexler and Pavier)



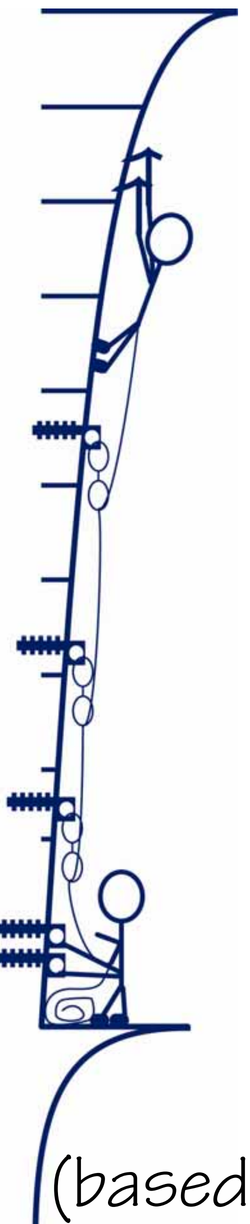
compare the simple model to data (Mägdefrau)

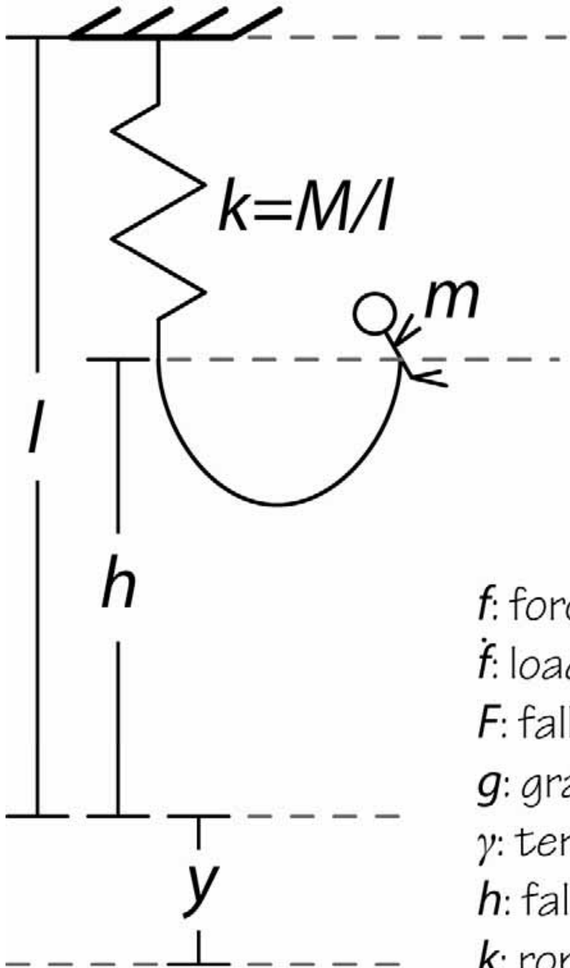
expected kinematics behavior





the simple model





the potential energy of fall height is exchanged for energy stored in the stretched rope spring:

$$mgh = \tfrac{1}{2} m\dot{y}^2_{max} = \tfrac{1}{2} k y^2_{max}$$

simple mass/spring oscillation:

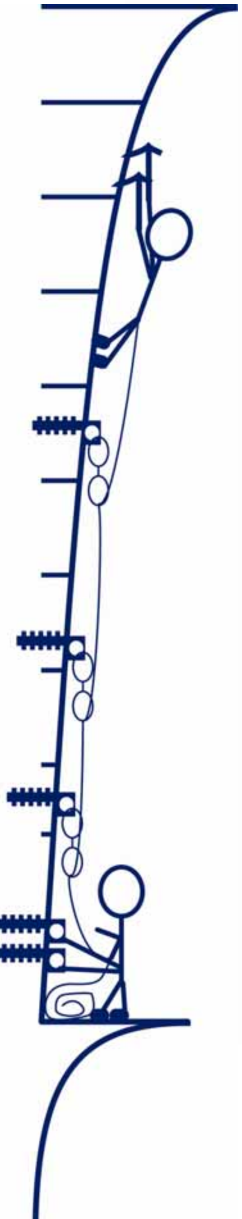
$$f = -ky = m\ddot{y}$$

f: force on climber
$\dot{f}$: load rate
F: fall factor (h/l)
g: gravity acceleration
γ: tension ratio
h: fall height
k: rope's spring constant
l: rope length
λ: damping coefficient

m: climber's mass
M: rope modulus
$\dot{y}$: rope stretch
$\ddot{y}$: climber velocity
$\ddot{y}$: climber acceleration
y: climber jerk
ω: oscillation frequency
Y: yowie factor

(based on Wexler work)

the simple estimate of load rate



kinematics solutions:

$$\omega = \sqrt{\frac{k}{m}}$$

$$y = -\sqrt{\frac{2mgl^2F}{M}}\sin(\omega t)$$

$$\dot{y} = -\sqrt{2gFl}\cos(\omega t)$$

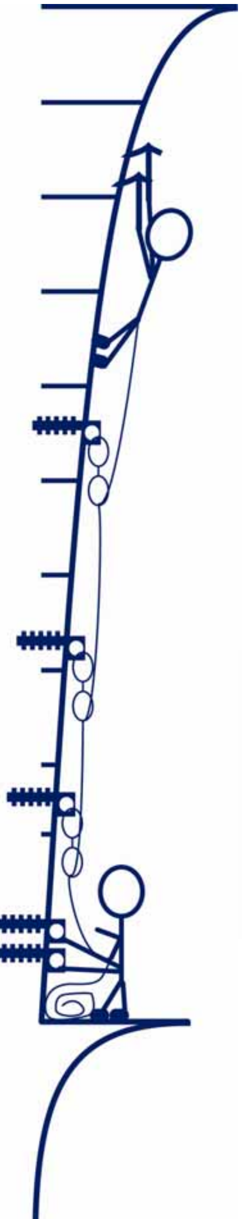
$$\ddot{y} = \sqrt{\frac{2gFM}{m}}\sin(\omega t)$$

$$\dddot{y} = \frac{M}{m}\sqrt{\frac{2gF}{l}}\cos(\omega t)$$

$$\dot{f} = \frac{df_{(t)}}{dt} = M\sqrt{\frac{2gF}{l}}\cos(\omega t)$$

$$Y \propto M\sqrt{\frac{F}{l}}$$

inclusion of potential energy of stretch



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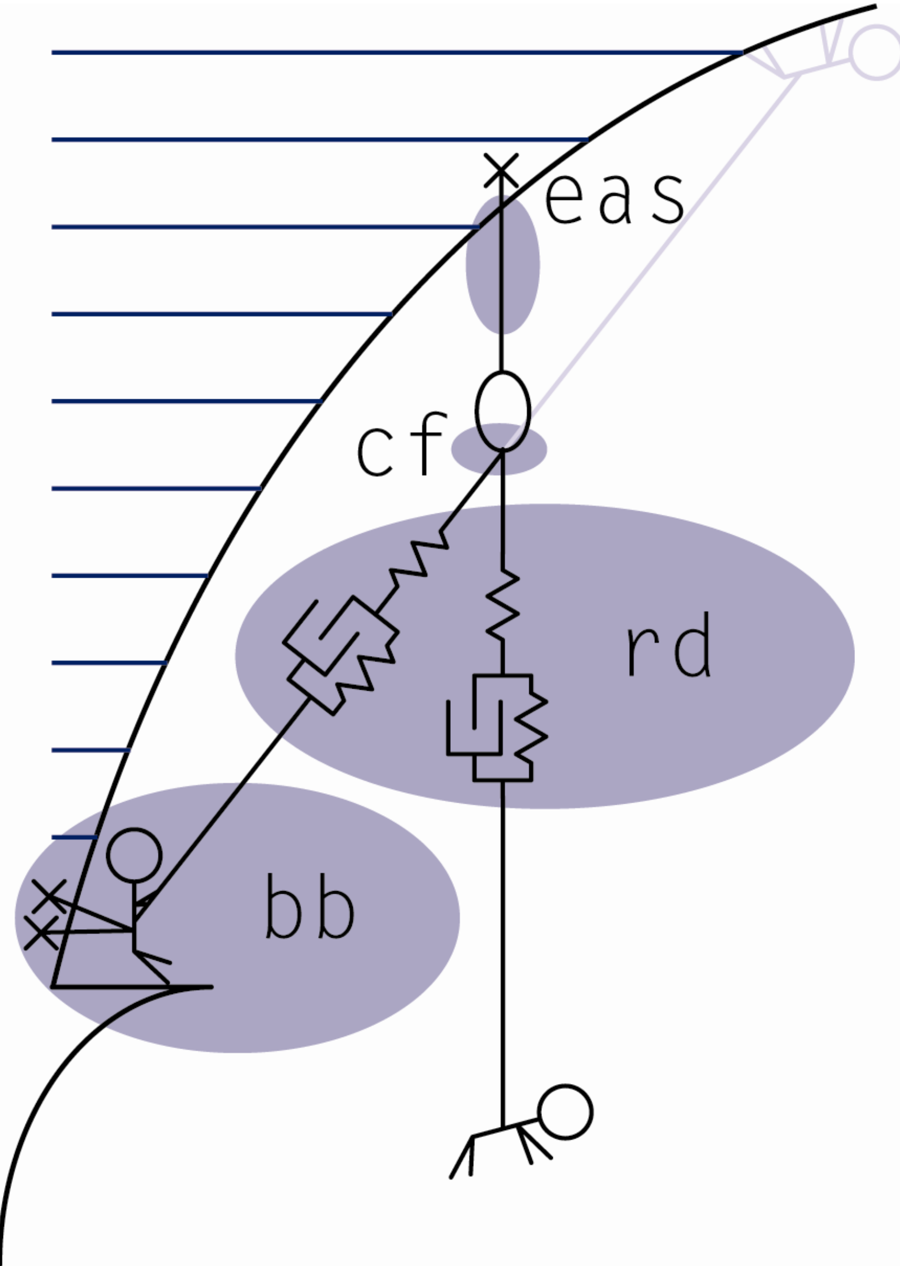
a slight adjustment of boundary conditions leads to a 15%-20% increase in force, but no change in frequency:

$$mgh + mgy_{max} = \tfrac{1}{2}ky_{max}^2$$

$$\ddot{y} = g\left(1 + \sqrt{1 + \frac{2MF}{mg}}\right)$$

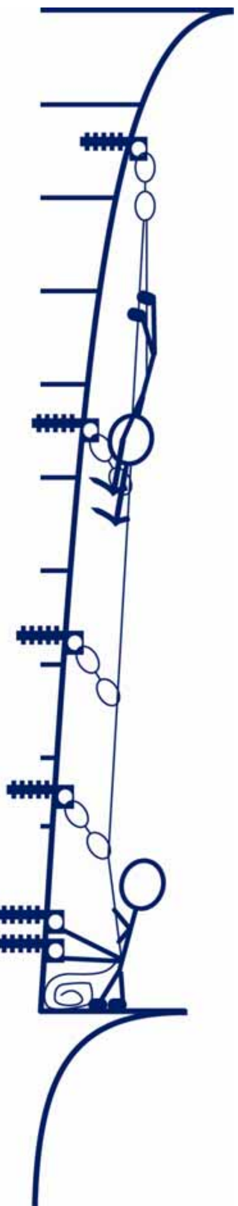
$$\dot{f} = mg\sqrt{\frac{M}{lm}}\left(1 + \sqrt{1 + \frac{2MF}{mg}}\right)\cos(\omega t)$$

complications

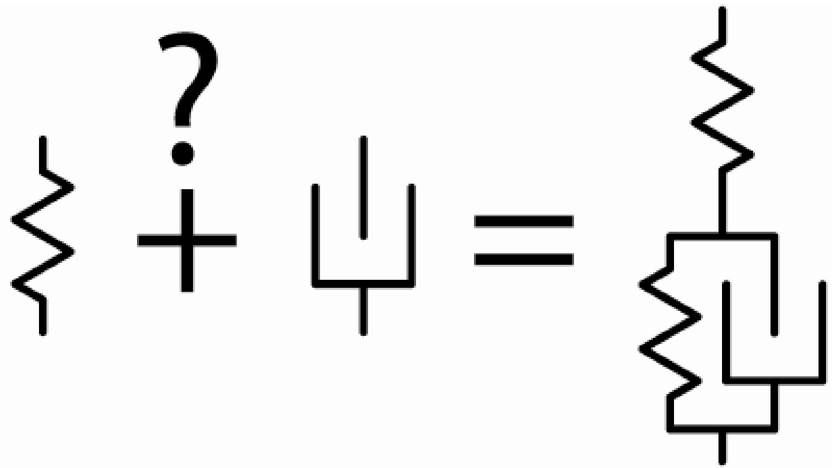


- rope damping
- carabiner friction
- belayer behavior
- energy absorbing systems

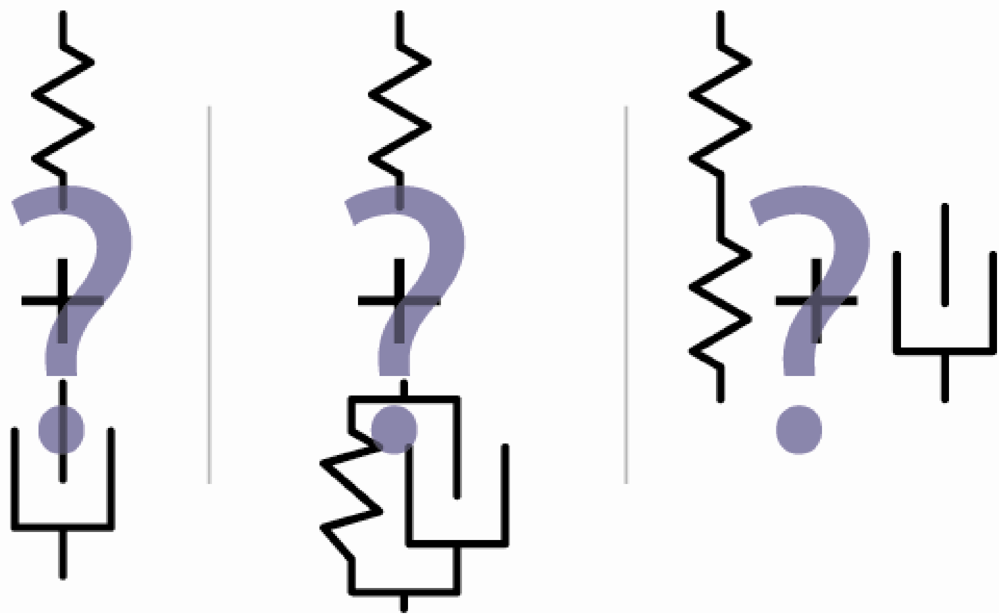
how does one compare a spring to a damped spring system?



Wexler spring model

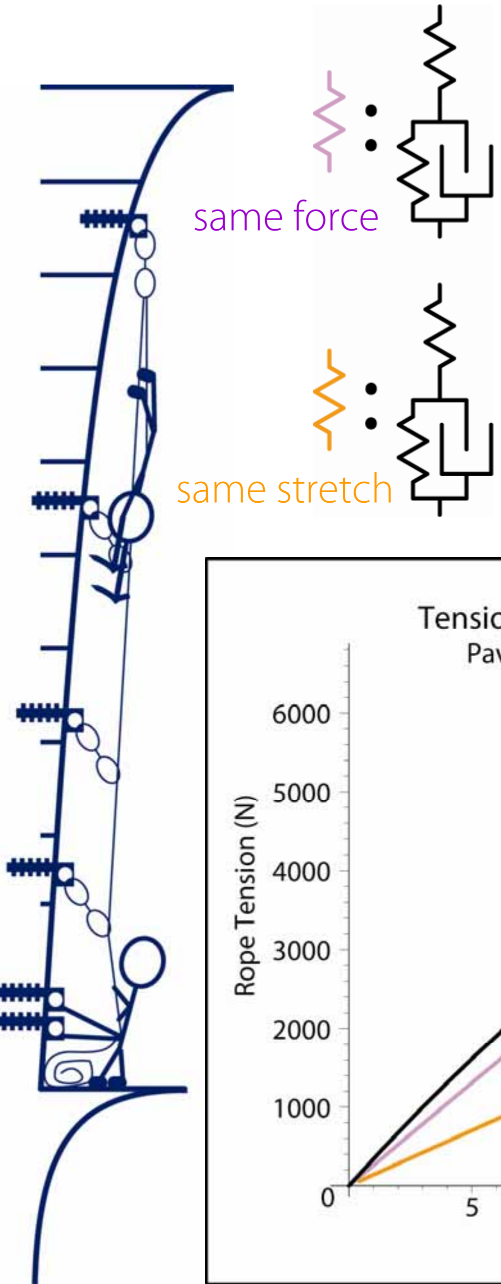


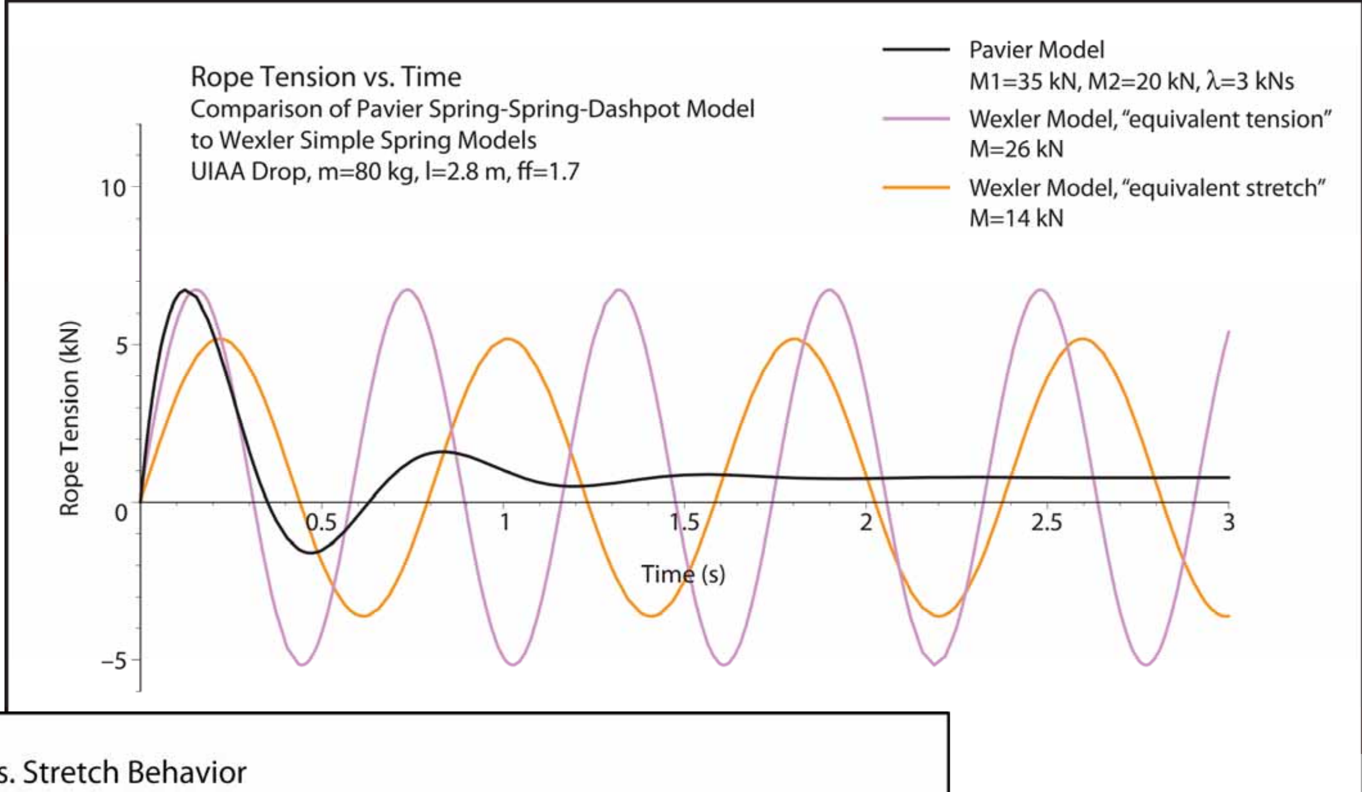
Pavier spring spring/ dashpot model

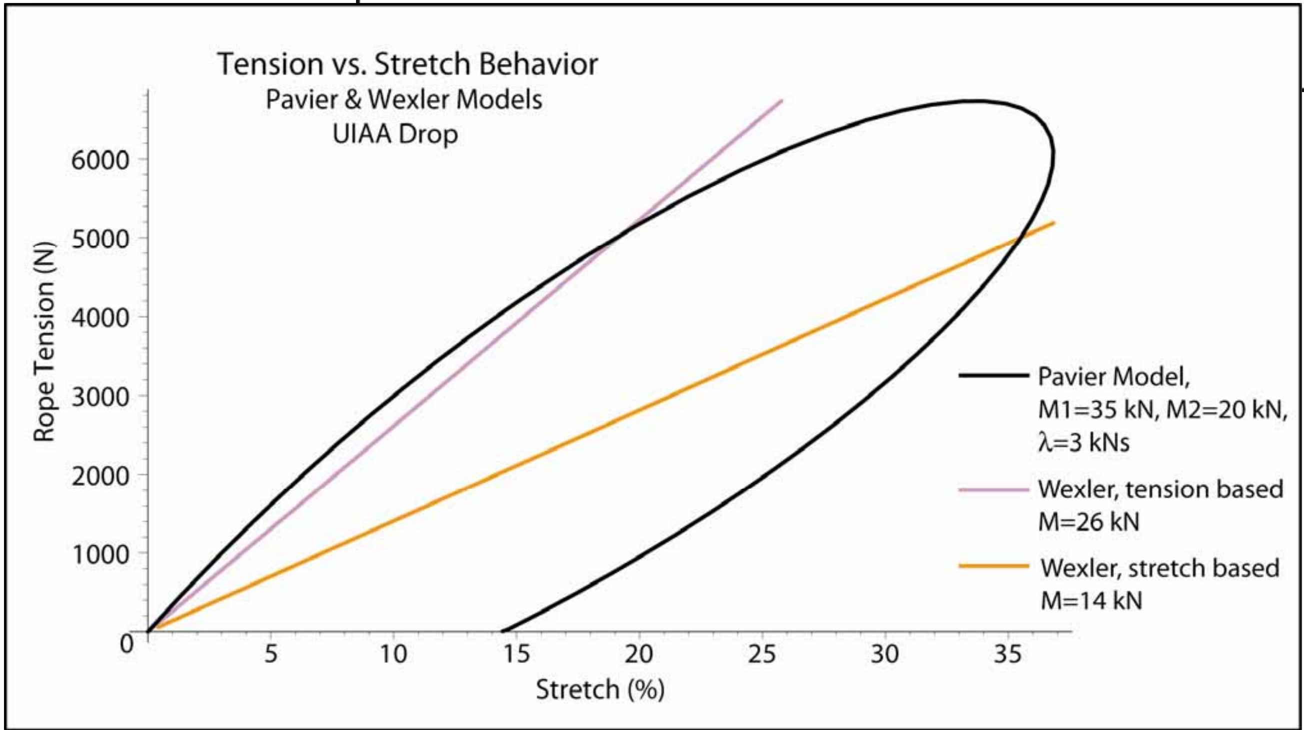


I have no simple answer; rather, 4 complicated ones.

damping I

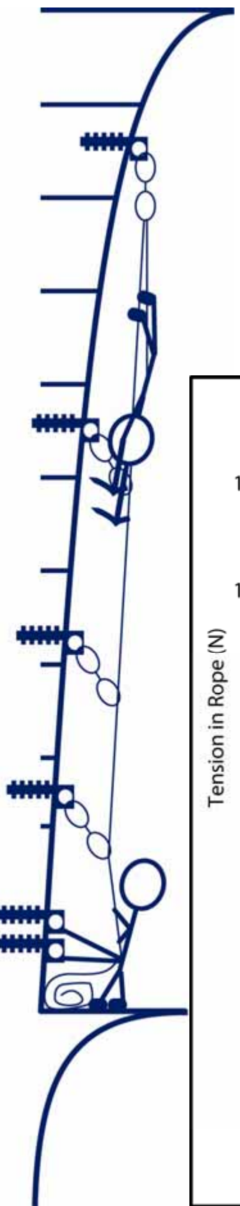


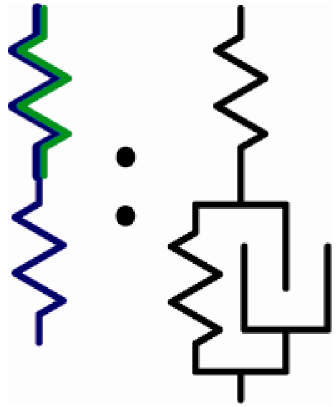


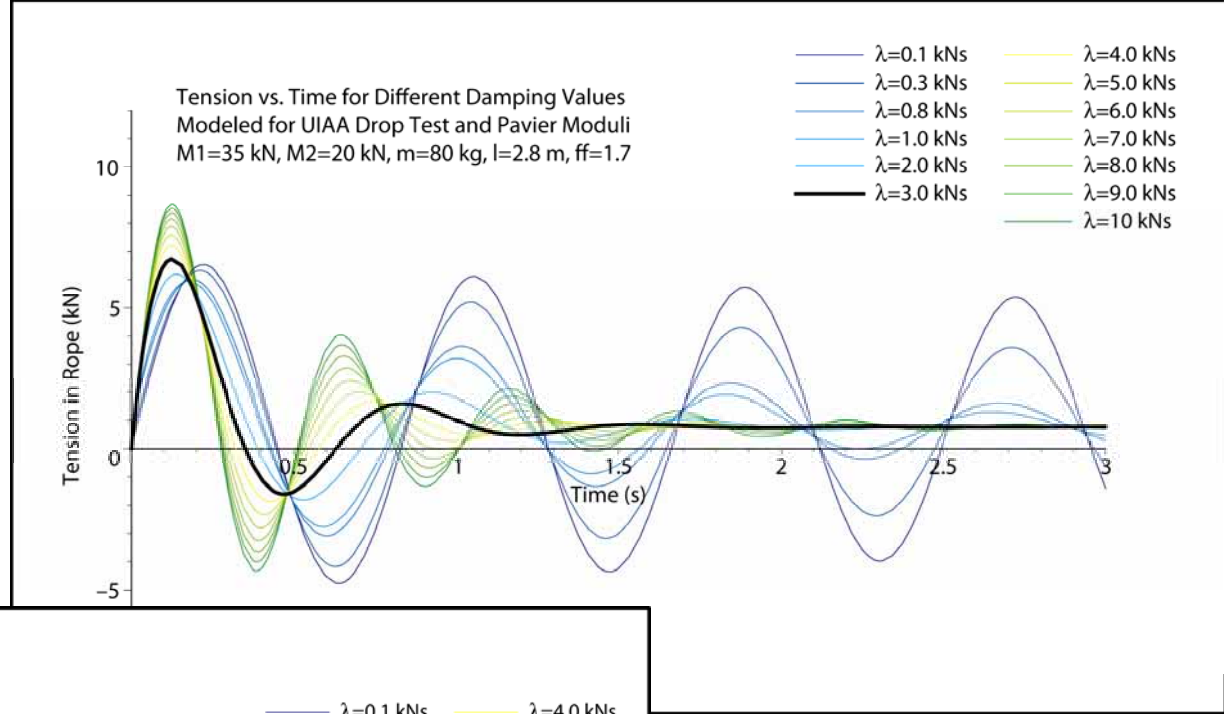


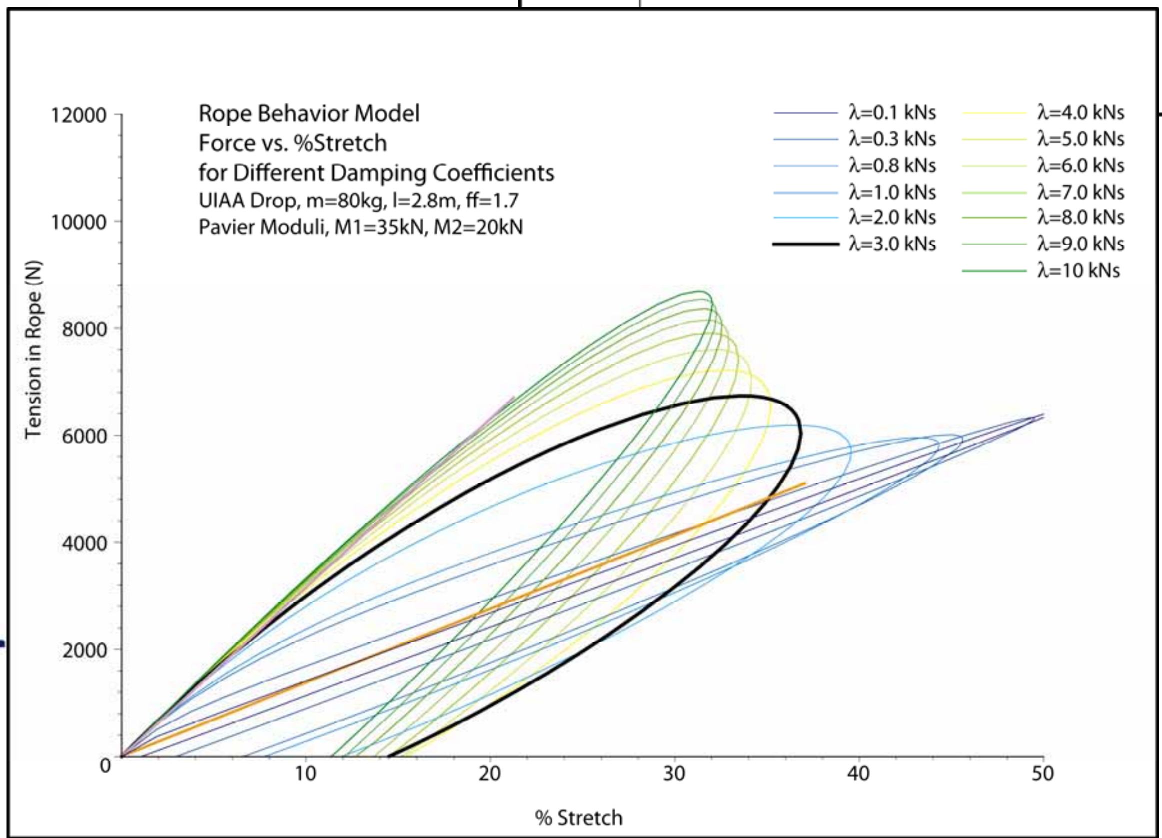
here, the addition of damping increases load rate

damping II



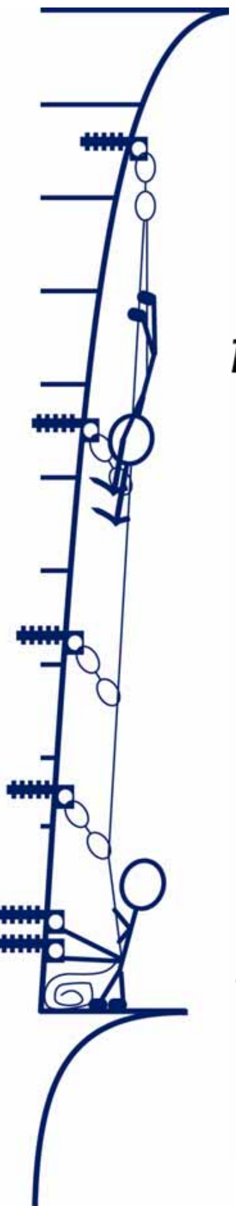


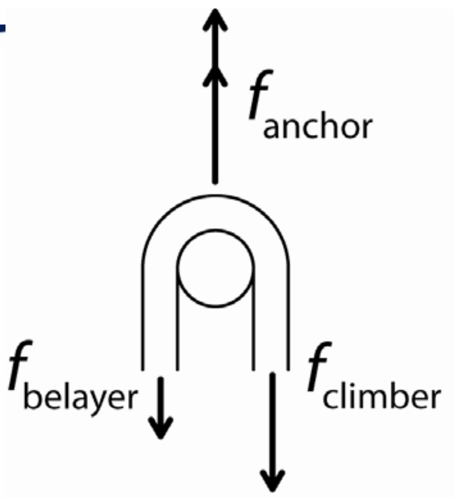




“critical” damping produces a load rate midway between the top spring and the two springs in series.

carabiner friction





$$\gamma f_{belayer} = f_{climber}$$

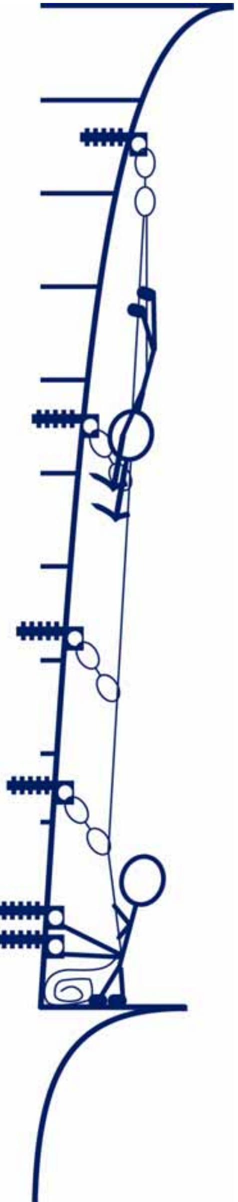
$$\gamma \approx 2$$

$$l_{effective} = l\left(\frac{\left(1 - \frac{1}{\gamma}\right)}{2}F + \frac{1}{\gamma}\right)$$

1) increases the force on the top anchor but does not change the proportionality of the yowie factor

2) decreases the effective rope length and thus also increases the fall factor; expect a 20% increase in load rate & reduced proportionality

belayer behavior



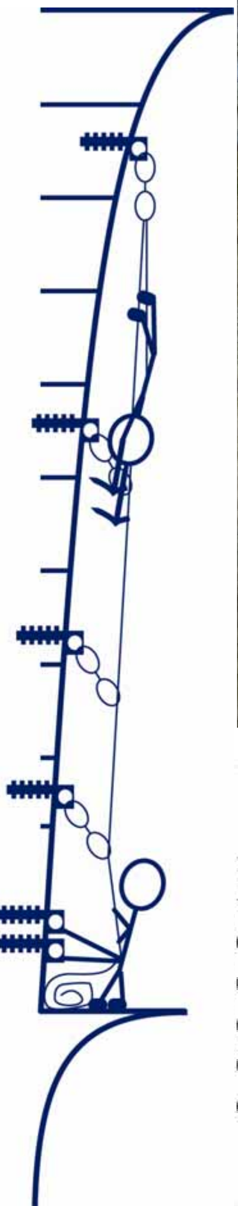
the belayer can reduce the energy absorbed by the rope by allowing rope to slip through the belay device and by being lifted up. the reduced energy results in reduced force, increased time, and thus reduced load rate.

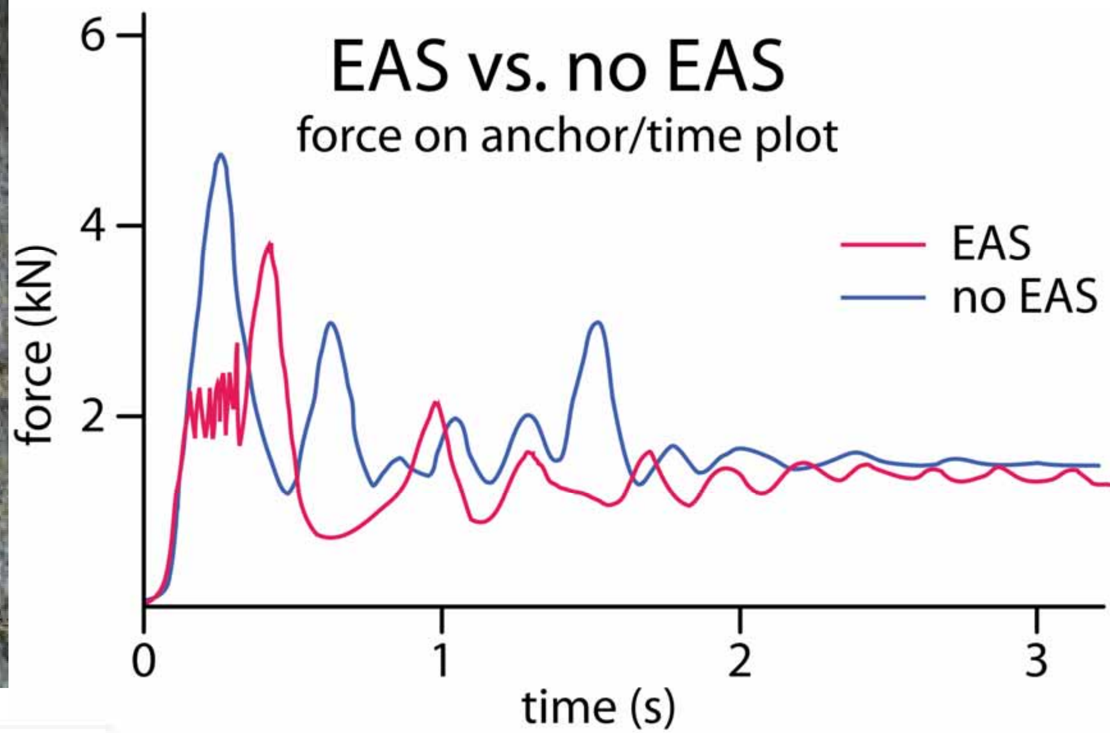
$$mgh + mgy_{max} = \tfrac{1}{2} k y_{max}^2$$

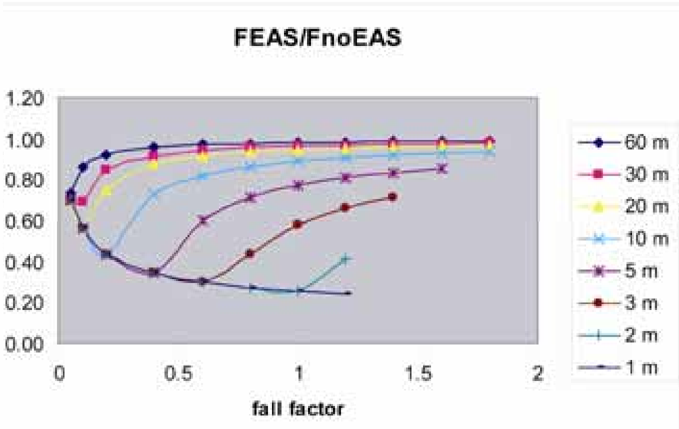
$$\ddot{y} = g\left(1 + \sqrt{1 + \frac{2MF}{mg}}\right)$$

$$\dot{f} = mg\sqrt{\frac{M}{lm}}\left(1 + \sqrt{1 + \frac{2MF}{mg}}\right)\cos(\omega t)$$

energy absorbing systems

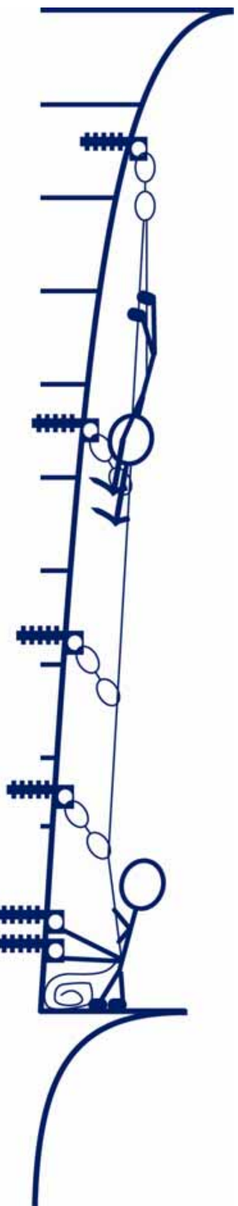


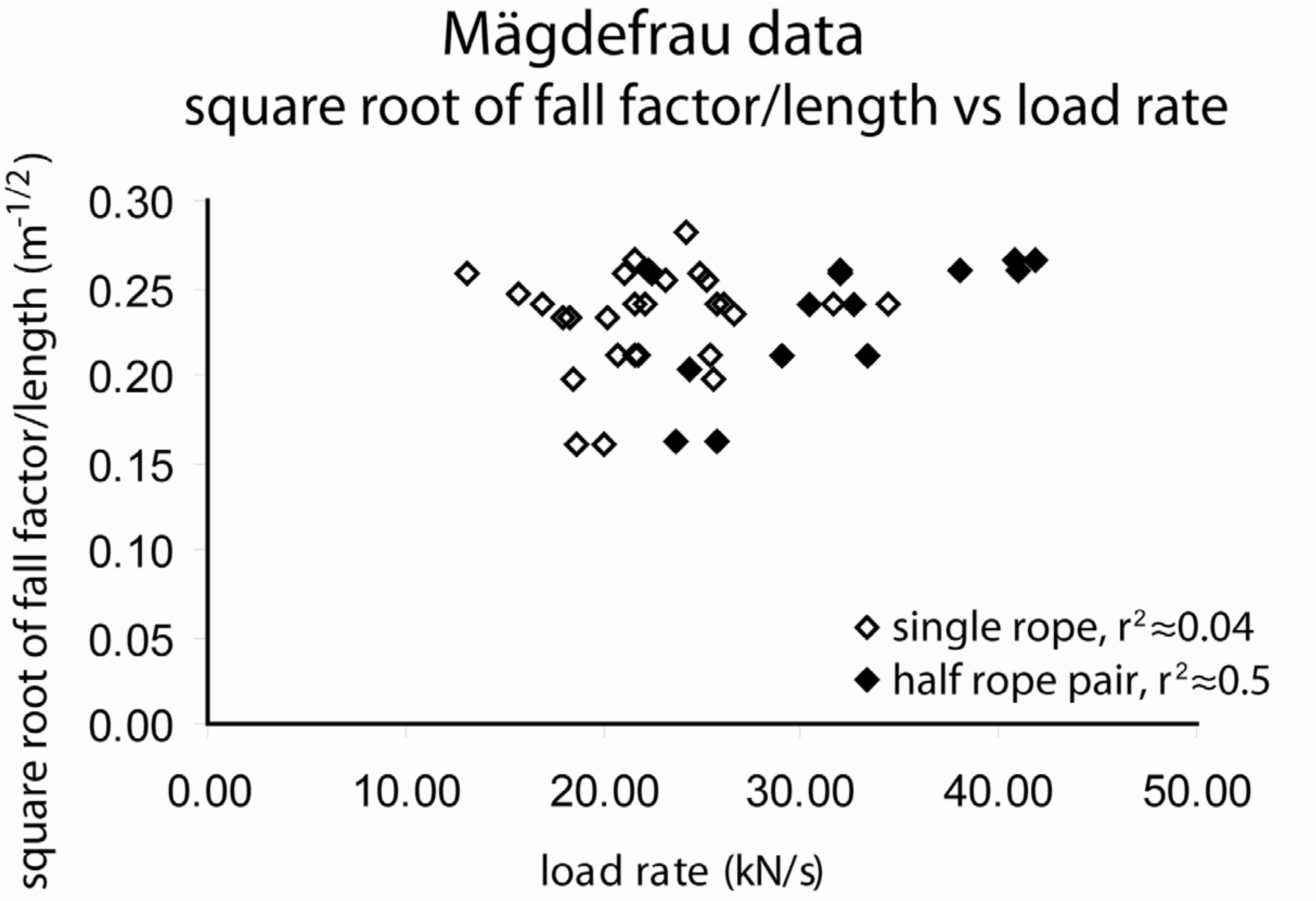




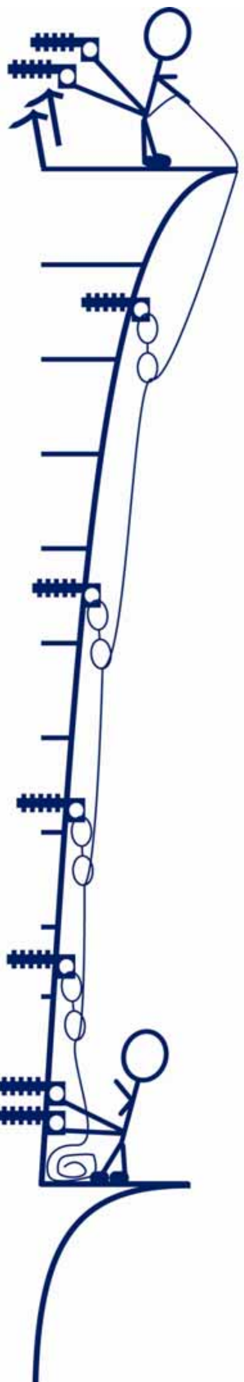
EAS reduce load rate

model compared to data





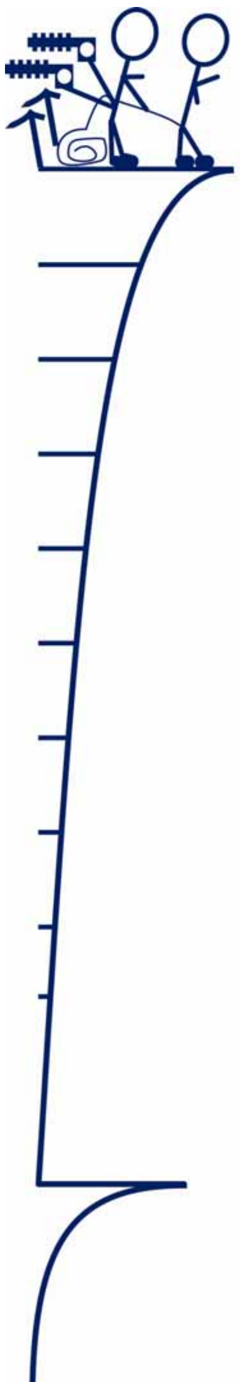
model and data correlate only loosely



conclusions

as a rule of thumb: $Y \propto M\sqrt{\frac{F}{I}}$

- climbers can apply the rule by protecting the belay and using low "modulus" ropes.
- use EAS and a dynamic belay.
- in the future, the rule might be used to guide the design of better ice screws and perhaps the use of plastic anchor components.



acknowledgements

- title slide photo by Luca Marinelli
- many thanks to the folks at MIT's aero/astro department and center for sports innovation
- kudos to Susan Ruff for patient editorial comment

abridged bibliography

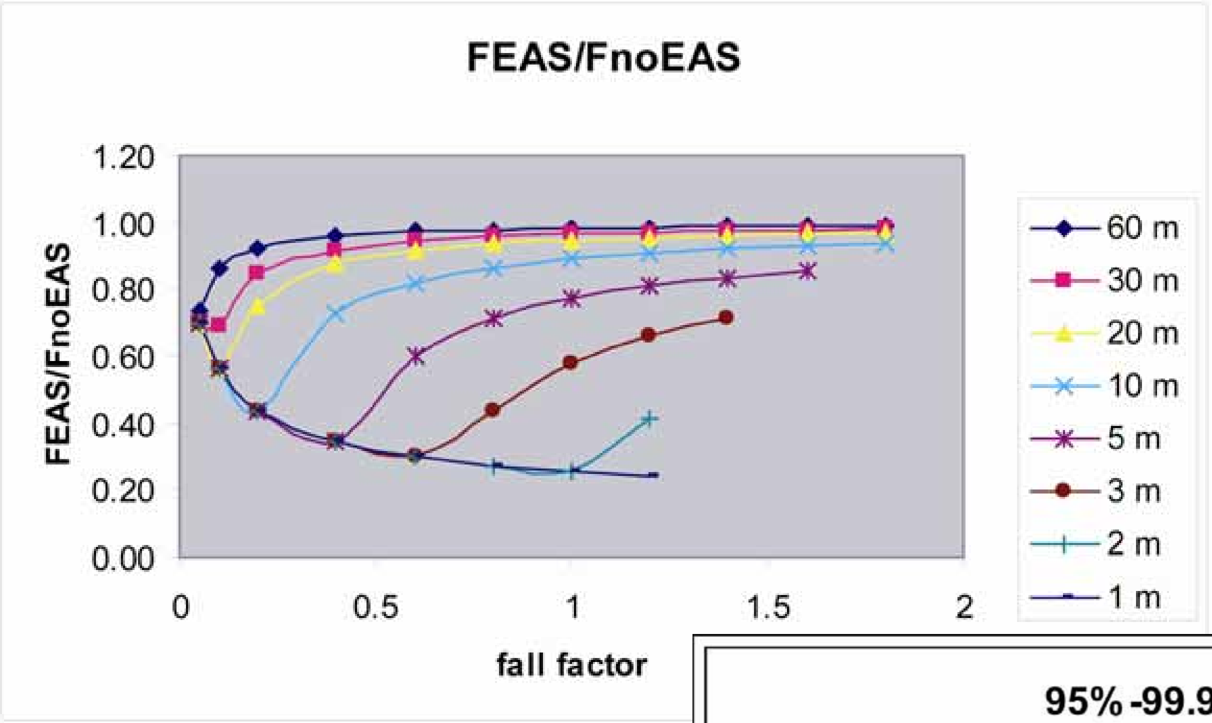
Mägdefrau, H., (1989) Die Belastung des menschlichen Körpers beim Sturz ins Seil und deren Folgen, dissertation, Ludwig-Maximilians University, translated by David LiaBraaten, 1994.

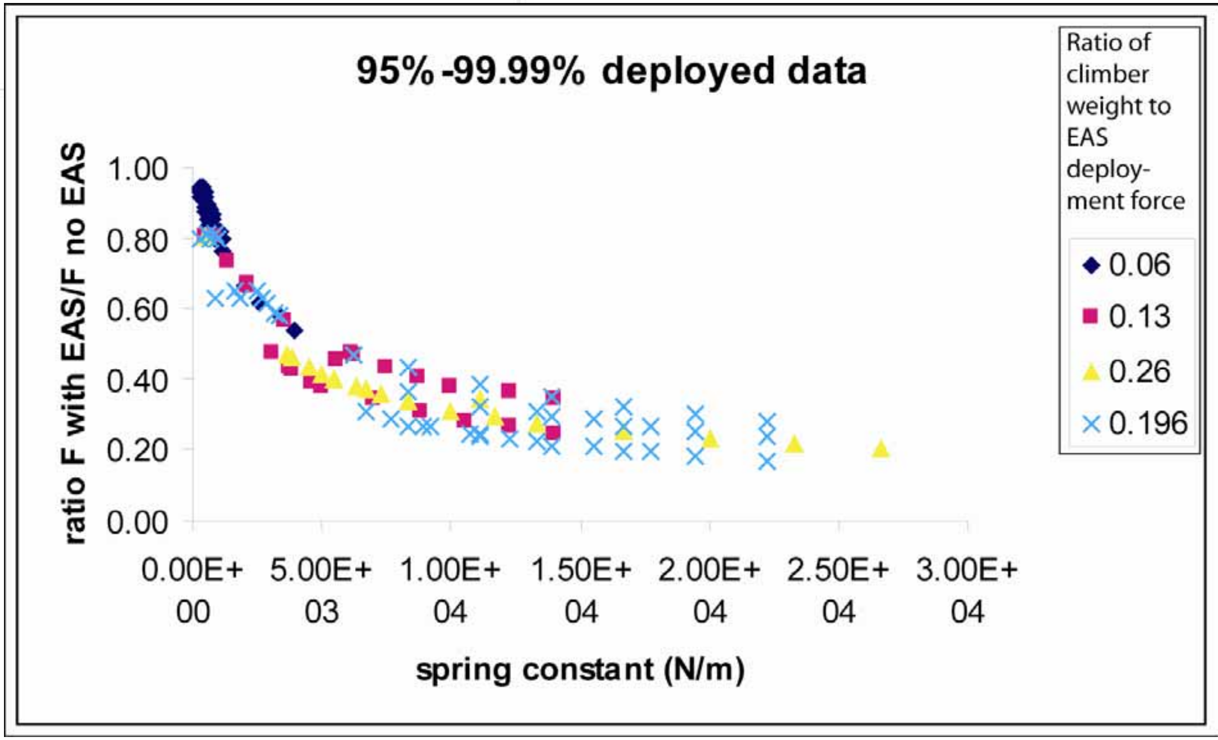
Pavier, M. (1998) Experimental and theoretical simulations of climbing falls, *Sports Engineering*, **1**, 79–91.

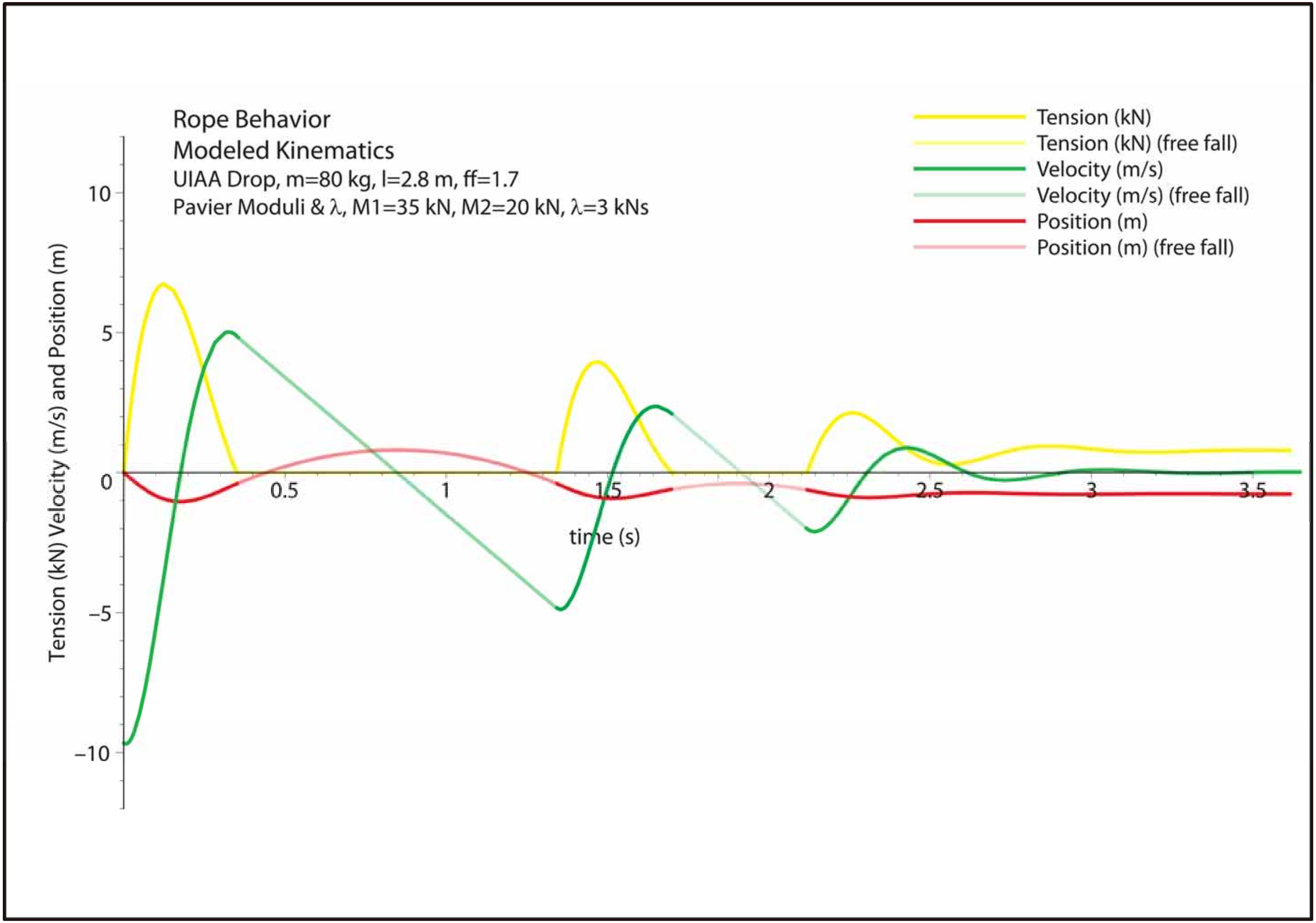
Wexler, A. (1950) The theory of belaying, *American Alpine Journal*, **7**, 379-405.


questions?
$Y \propto M\sqrt{\frac{F}{I}}$

stray EAS graphs






Rope Behavior
Modeled Kinematics
UIAA Drop, m=80 kg, l=2.8 m, ff=1.7
Pavier Moduli & λ, M1=35 kN, M2=20 kN, λ=3 kNs
Tension (kN)
Tension (kN) (free fall)
Velocity (m/s)
Velocity (m/s) (free fall)
Position (m)
Position (m) (free fall)
Tension (kN) Velocity (m/s) and Position (m)
10
5
0
−5
−10
0.5
1
1.5
2
2.5
3
3.5
time (s)

alternate expression

$$Y \propto \frac{M}{I}\sqrt{h}$$