Predicting Rope Impact Forces Using a Non-linear Force Deflection

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Abstract

Existing theories for predicting impact loads do not match well with data from rope system drop tests. The existing formulas are based on a constant rope modulus. Recent testing has suggested that this linear approximation for the force-deflection relation could result in underestimating the impact forces. This paper presents new formulas for predicting the maximum impact force in a rope based on a second order polynomial force-deflection relation.

Introduction

Technical rescue requires a good understanding of the mechanics of rope behavior. It is important for rope rescue technicians to be able to predict the maximum impact force from a potential fall. The force in a rope generated when a falling weight is arrested depends on how fast the weight is stopped. An understanding of the limitations of predictive methods for rope impact forces can be gained by comparing these methods against test data.

Weber (2001) measured the load deflection curves for ropes and also measured the impact force for real rope systems. He has shown that existing formulas for predicting impact forces, based on a constant rope modulus, are not predictive. They may be mis-leading and non-conservative for certain ropes. These existing formulas are based on a linear approximation of the rope stiffness. By assuming that the rope modulus does not change with load, a closed-form solution for the impact force can be computed.

The goal of this paper is to provide better predictions by using a second order fit for the force in a rope as a function of strain.

Introduction

Typical force deflection curves for assorted ropes are shown in Figure 1.

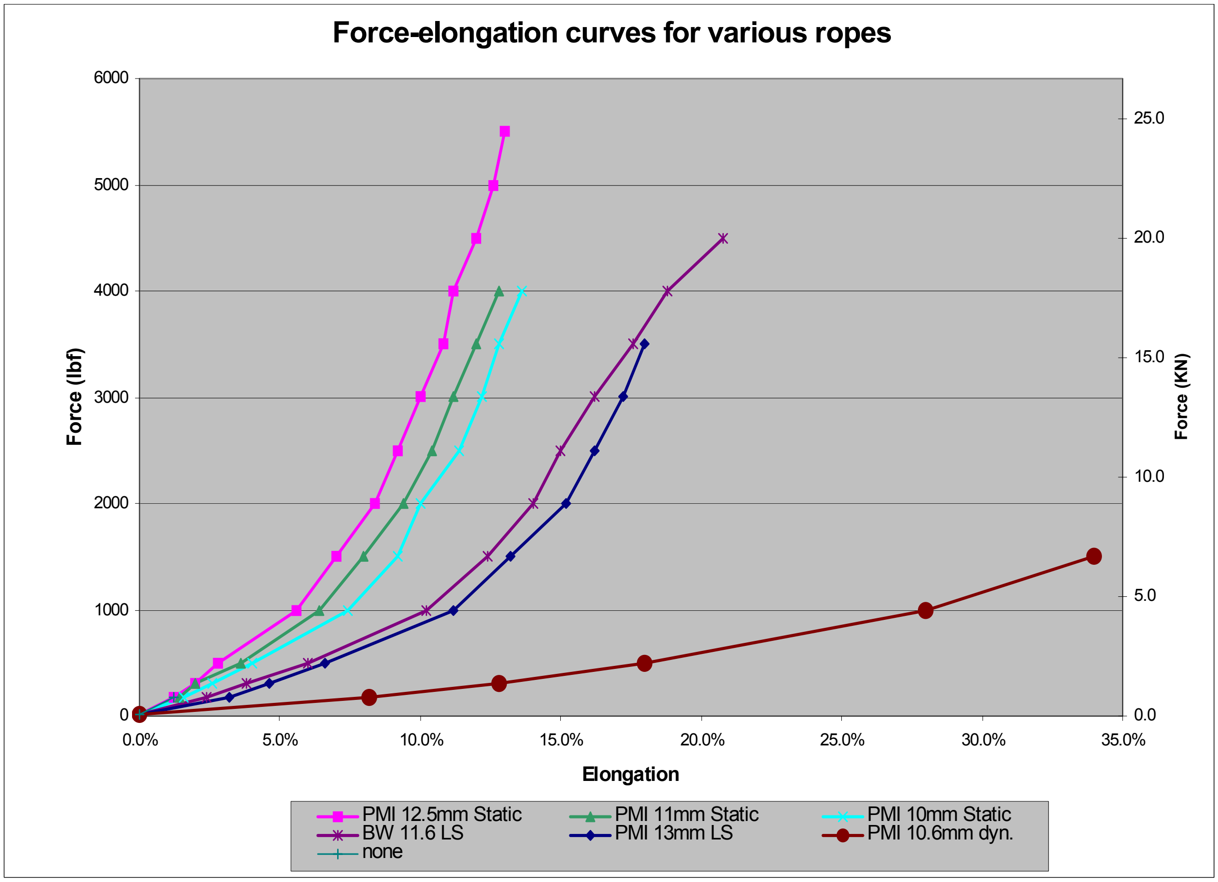


Figure 1Load deflection curves for different ropes as measured by Weber 2001.

Attaway (1996) derived an equation for the maximum force a climber could experience in a fall using an energy balance approach, where the potential energy of the climber was balanced against the strain energy in the rope. At the time, a simple linear relation was used to model the force in the rope as a function of the rope stretch. It was assumed that the force for any deflection of the rope would be of the form:

$$F = K\delta \qquad \text{Eq(1)}$$

This linear forced-displacement relationship was used to derive the following expression for impact force as a function of fall height:

$$F = W + W\sqrt{1 + \frac{2hM}{WL}}, \qquad \text{Eq(2)}$$

where W is the weight of the climber, h is the fall height, M is the modulus of the rope, $K = \frac{M}{L}$, and L is the length of the rope. The impact force depends on the weight of the climber, the rope stiffness, and the ratio of the rope length to the fall height. This ratio,

$$FF = \frac{h}{L}, \qquad \text{Eq(3)}$$

has become known as a "fall factor". Based on the above formulation, a fall of 20 ft. on 100 ft. of rope should generate the same force as a fall of 2 ft. on 10 ft. of rope. In a normal lead climbing or rescue situation, the fall factor can range from 0.0 to 2.0.

During recent testing, Weber (2001) has shown that Eq. 2 does an inadequate job of predicting the forces in static ropes. Weber's results suggest that the force predictions based on Eq 2 becomes noticeably less accurate as the test weights and fall factors increase - as much as 30% low in certain cases. Weber found that no single value of the modulus, M, would produce accurate results in all cases.

In the section below, new equations for predicting rope impact forces will be derived based on a better analytical representation of rope force as a function of rope strain. A better fit to the load deflection curves for rope can be found by using a least squares minimization for a second order polynomial. The force, F, as a function of strain, ε is assumed to have the following form:

$$F = a\varepsilon + b\varepsilon^2. \qquad \text{Eq(4)}$$

Here, a and b are constants determined from a least squares fit. Many tools are currently available for fitting equations to data. For example, the curve fitting options in Microsoft Excel can be used to obtain the second order polynomial fits shown in Figure 2. The force deflection curves were reflected about the y-axis in order to improve the data fit. The equations were also constrained so that the force would be zero for zero deflection.

The strain is related to the displacement of the rope by:

$$\delta = \varepsilon L. \qquad \text{Eq(5)}$$

Which gives the force as a function of displacement as:

$$F = \frac{a}{L}\delta + \frac{b}{L^2}\delta^2 \qquad \text{Eq(6)}$$

Even though the force displacement relation is no longer linear, the strain energy method can still be used to compute the maximum force in a fall. As a climber falls, the potential energy in the fall will be converted to strain energy in the rope. At point of maximum stretch, the strain energy will equal the potential energy:

$$PE = SE \qquad \text{Eq(7)}$$

The fall energy is given by:

$$PE = mgh + mg\delta \qquad \text{Eq(8)}$$

Strain Energy will be:

$$SE = \int_0^{\delta} F(x)dx \qquad \text{Eq(9)}$$

Assuming a second order fit for the force as a function of deflection as shown in Figure 2, the strain energy will become:

$$SE = \frac{1}{2}\frac{a}{L}\delta^2 + \frac{1}{3}\frac{b}{L^2}\delta^3 \qquad \text{Eq(10)}$$

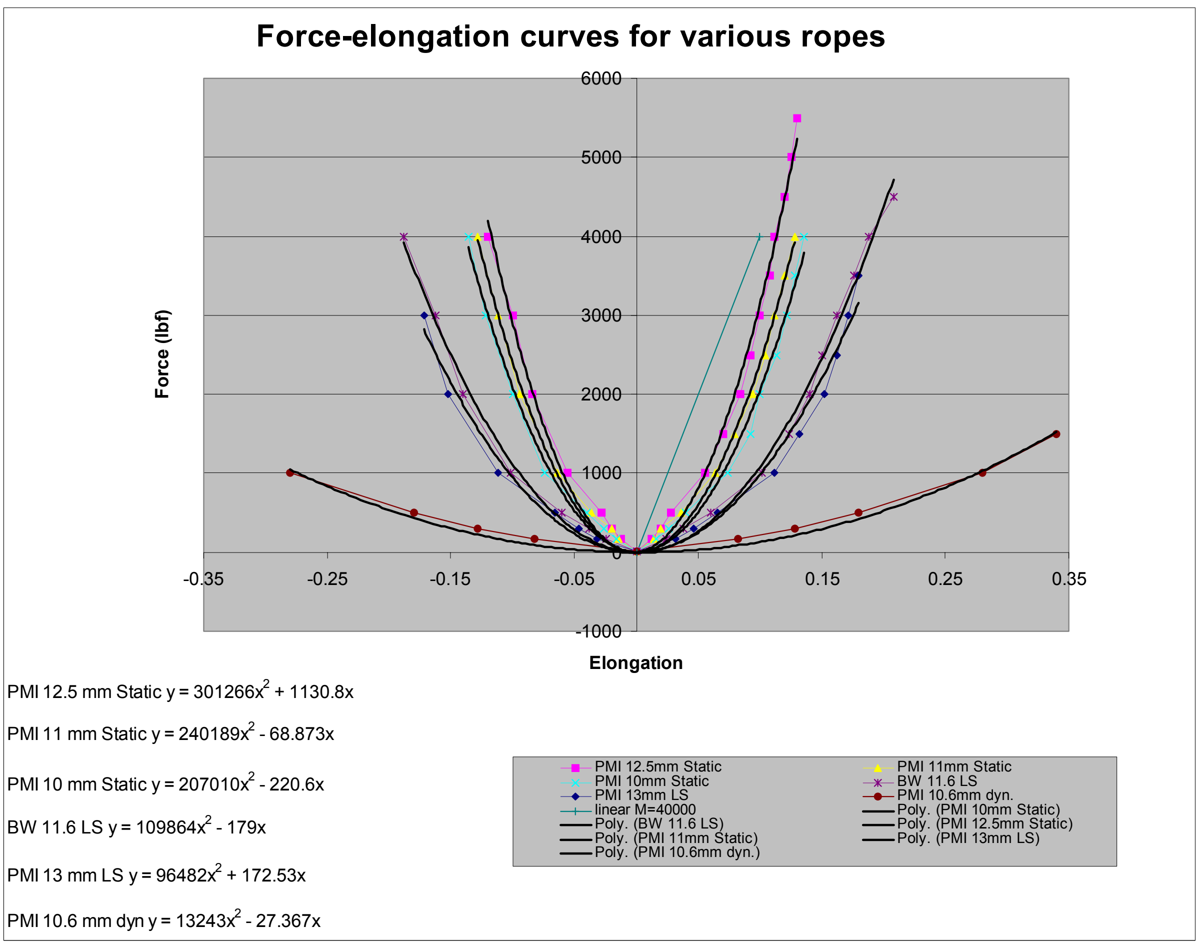


Figure 2 Second order polynomial fits for rope deflection.

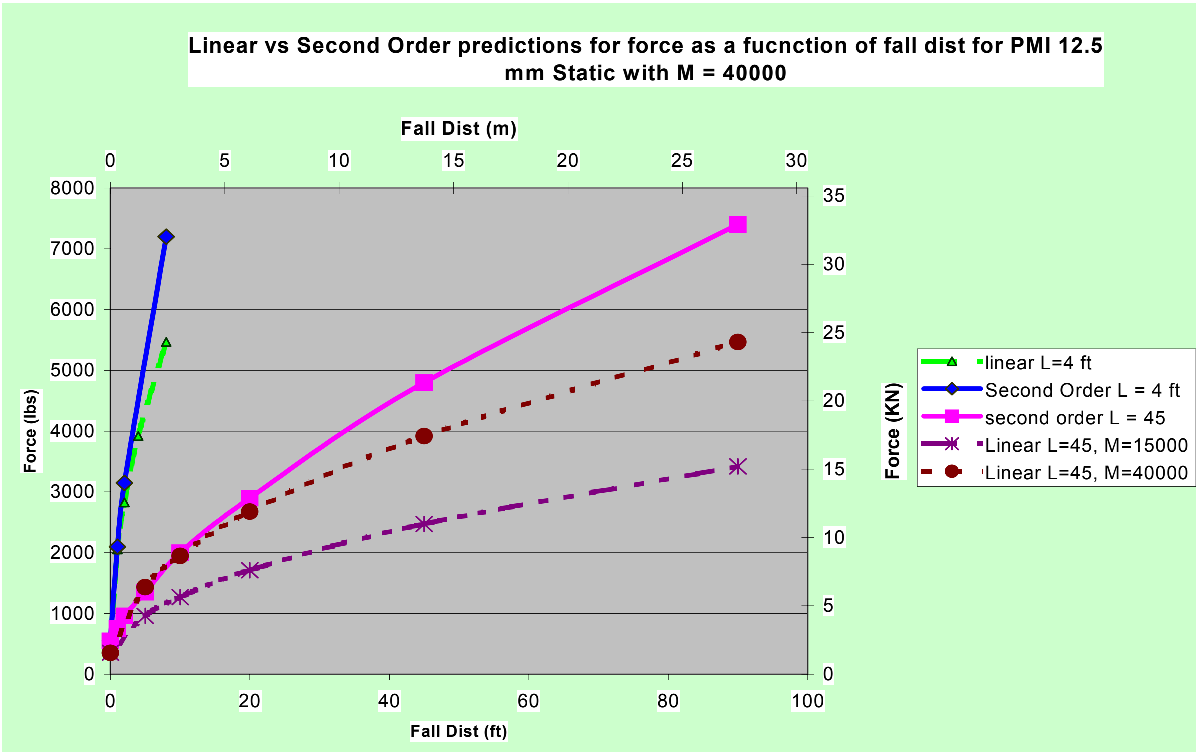


Figure 3 Impact force predicted using linear and second order rope models.

Expanding the balance of the fall energy with the strain energy will give:

$$mgh + mg\delta - \left(\frac{1}{2}\frac{a}{L}\delta^2 + \frac{1}{3}\frac{b}{L^2}\delta^3\right) = 0 \qquad \text{Eq(11)}$$

The techniques for solving the above cubic equation for δ in closed form are well known (Hudson, 1944). The results are kind of messy, complete with real and imaginary numbers and will not be presented here. Once a solution for δ has been found, the impact force can be computed using Eq. 10.

The impact force can still be expressed as a function of the fall factor, FF=h/L. Even when a second order fit is used for the rope deflection curve, a fall of 20 ft. on 100 ft. of rope will produce the same maximum force as a fall of 2 ft. on 10 ft. of rope.

Rather than solving the cubic equation in Eq. 11, a spread sheet was to used to solve for δ. A zero function can be computed using the following function:

$$Z(\delta) = mgh + mg\delta - \left(\frac{1}{2}\frac{a}{L}\delta^2 + \frac{1}{3}\frac{b}{L^2}\delta^3\right) \qquad \text{Eq(12)}$$

A plot of Z as a function of δ will show where the strain energy and fall energy are in balance, the point where Z crosses the y axis.

The solution for $Z(\delta) = 0$ will give the displacement that sets the strain energy equal to the fall energy. If the Z function does not go to zero within the bounds of $\delta = 0$ and $\delta = \delta_{max}$ then no solution for the strain energy exist. The rope would be expected to fail. After finding $\delta|_{z = 0}$, use Eq. 6 to compute the load.

Figure 3 shows a plot of the force predicted using both a linear and a second order fit for a 4 ft. and 45 ft. PMI 12.5 mm static rope. The results are compared with the predictions based on the linear relation from Eq. 2. **Note that the linear equations UNDER PREDICT the force by more than 30% when compared with the more accurate second order fit.**

In addition to predicting different forces for large fall factors, the force predicted by a fall factor or zero is much different for a second order fit. A fall factor zero is a fall where the climber is at the end of a non-tensioned rope and falls the stretch distance. Even though h is zero, the weight will still displace downward as the stretch is removed from the rope. A linear spring will never produce more than 2X for a zero length fall. The non-linear system predicts a force of more than 3 times the weight.

Real Systems Have Knots

The second order polynomial force deflection formula does an excellent job of matching the static load deflection curves. As discussed above, it shows that the linear force-deflection formula can under predict the maximum impact loads. The next question is how well does the second order fit match real data.

Shown in Figure 4 is the measured and predicted values for W=176 lbs. Comparing the computed forces with the measured forces for the 20 ft. rope length shows that the current predictions over estimate the force in the rope: 3300 lbs vs. 4700 lbs for a FF = 1.0.

Based on this result, it appears we are not predictive for a “real” system. Real systems use knots. Knots can absorb energy. Unless their stiffness is included in the energy balance equation, the force estimates will be inacurate. Knots are complicated, twisted masses of rope that slide past each other as they tighten. The mechanics for a knot is quite different from the mechanics for a rope. However, it appears that for the purposes of modeling, a knot can be treated as an equivalent length of rope. The idea is that the knots add to the “real” length of the rope. If this were so, then the measured rope test with knots should overlap with an “effective” length rope that does not have knots. One would hope that a constant amount of rope could be added to each measured rope length to account for the knots. For example, an equivalent rope length of 14 ft. compares very well with the result from the 4 ft. test. This would mean that each knot would add the equivalent of 5 ft. of rope.

Unfortunately, it looks like more rope length is needed for the longer ropes. An effective length of 45 ft. must be used to match the 20 ft. test. This requires each knot to behave as though it were 12.5 ft. of rope.

Instead of simply adding rope, it may be better to compute the strain energy of the rope and knot combination. We can do this by finding a second order fit to the knot and including its strain energy in the energy balance equation. One advantage this method has over an assumed knot length is that knot pre-tensioning can be modeled using a fit with a constant term.

$$F = c + \frac{d}{L_k}\delta_k + \frac{e}{L_k^2}\delta_k^2 \qquad \text{Eq(13)}$$

In the above fit, c would be the value of the force for zero displacement. Note that this does not really mean that the force would be 176 lbs with zero load, but it means that no displacement would occur until the load reaches 176 lbs.

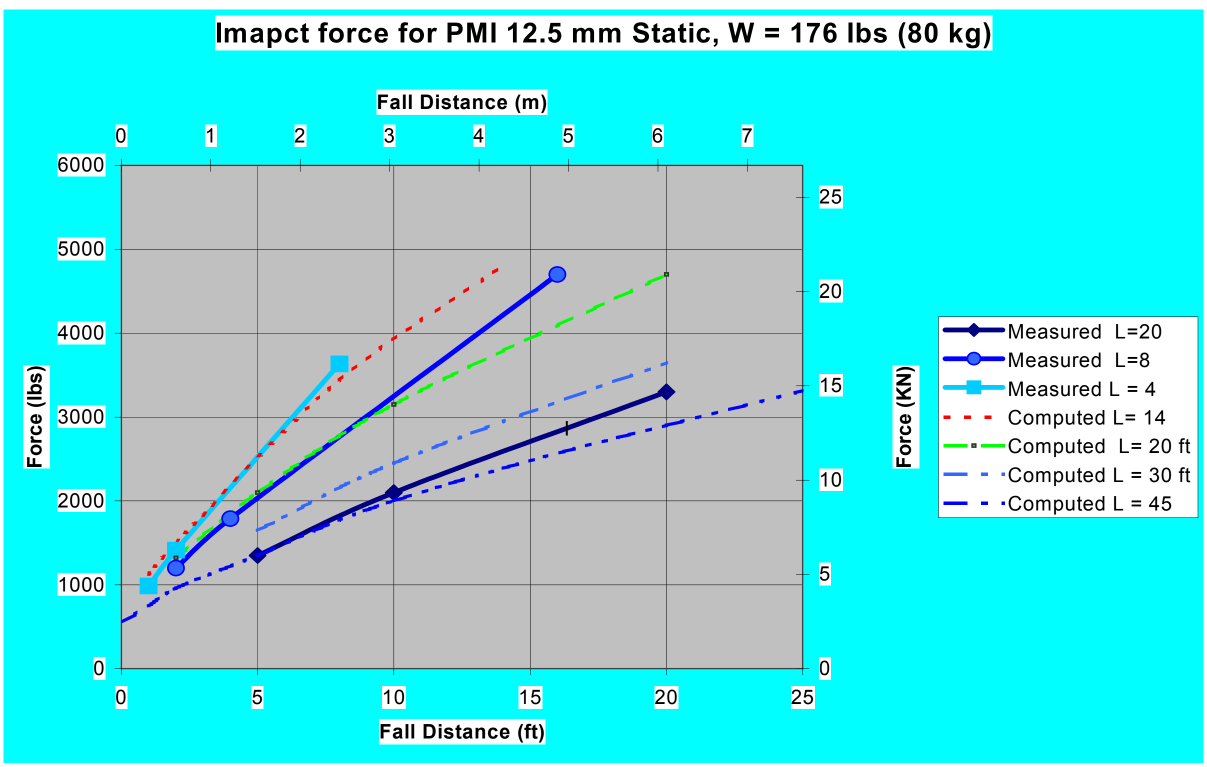


Figure 4 Predicted vs. measured for PMI 12.5 mm static.

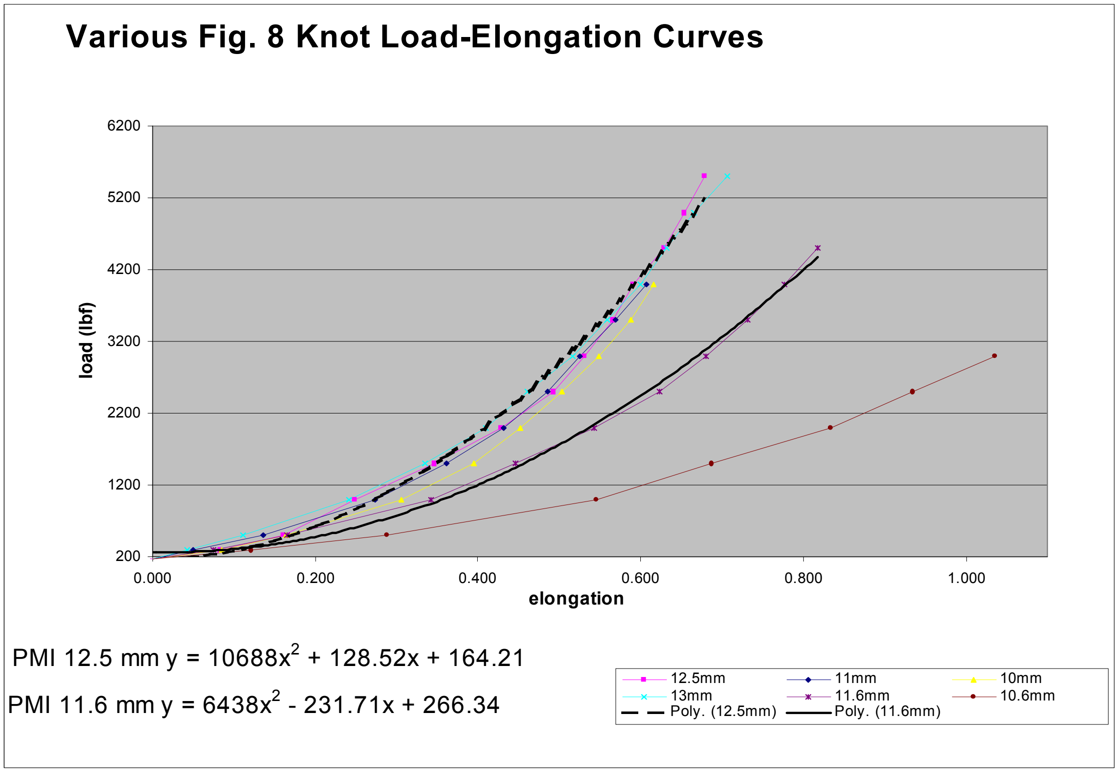


Figure 5 Knot stretch vs. rope stretch.

Figure 5 shows a typical plot of a force-strain curve for knots. A second order fit for the force displacement of the knot can be found from these plots.

In order to account for the knots, the strain energy will become:

$$SE = SE_{rope} + SE_{knot} \qquad \text{Eq(14)}$$

Zero function for strain energy balance becomes:

$$mgh + mg(\delta_r + \delta_k) - \left(\frac{1}{2}\frac{a}{L_r}\delta_r^{\,2} + \frac{1}{3}\frac{b}{L_r^{\,2}}\ \delta_r^{\,3}\right) - \left(c\delta_k + \frac{1}{2}\frac{d}{L_k}\delta_k^{\,2} + \frac{1}{3}\frac{e}{L_k^{\,2}}\ \delta_k^{\,3}\right) = 0 \qquad \text{Eq(15)}$$

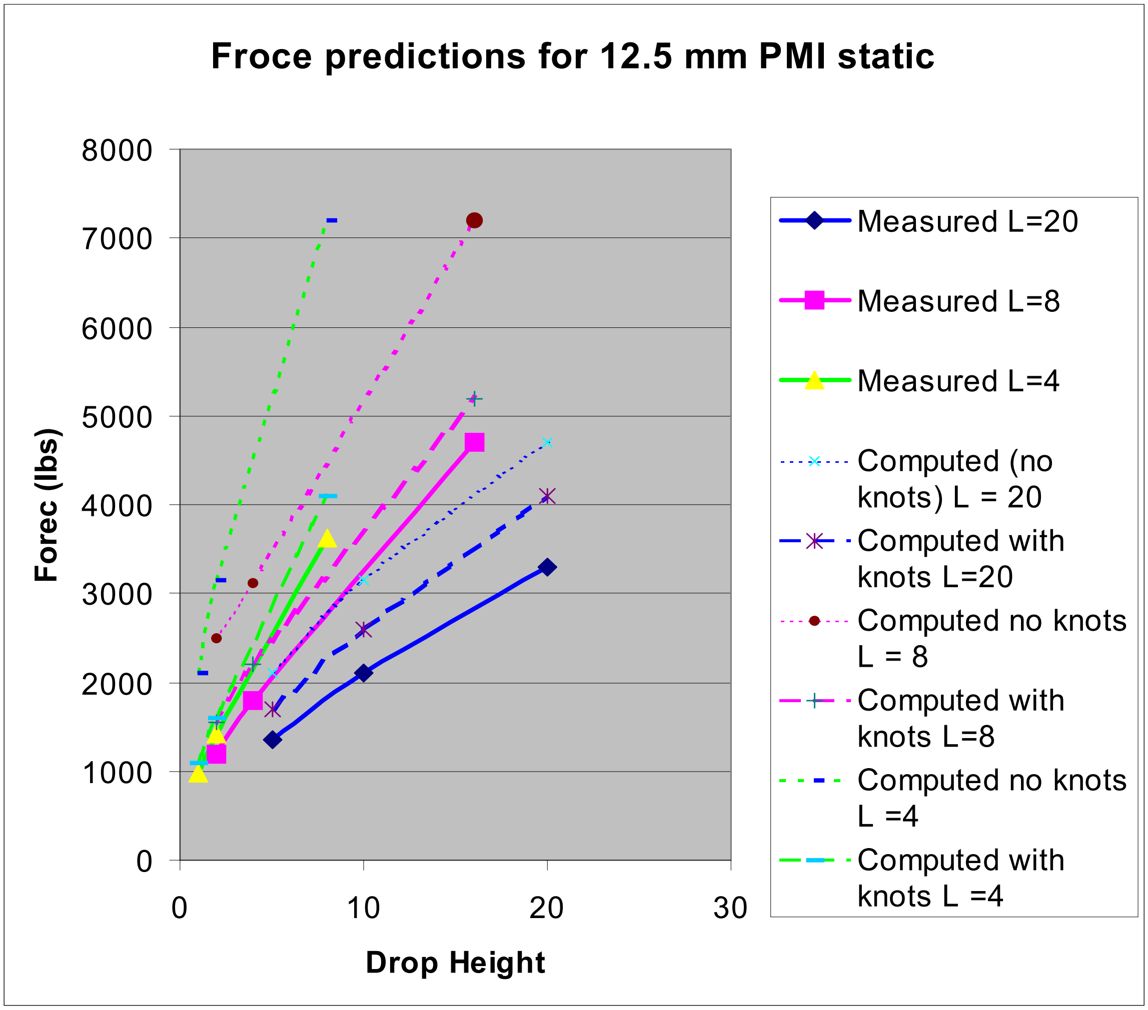


Figure 6 Computed vs. measured impact for PMI 12.5 mm static for different rope lengths and drop heights. The computed results are based on 2nd order fits with and without knots.

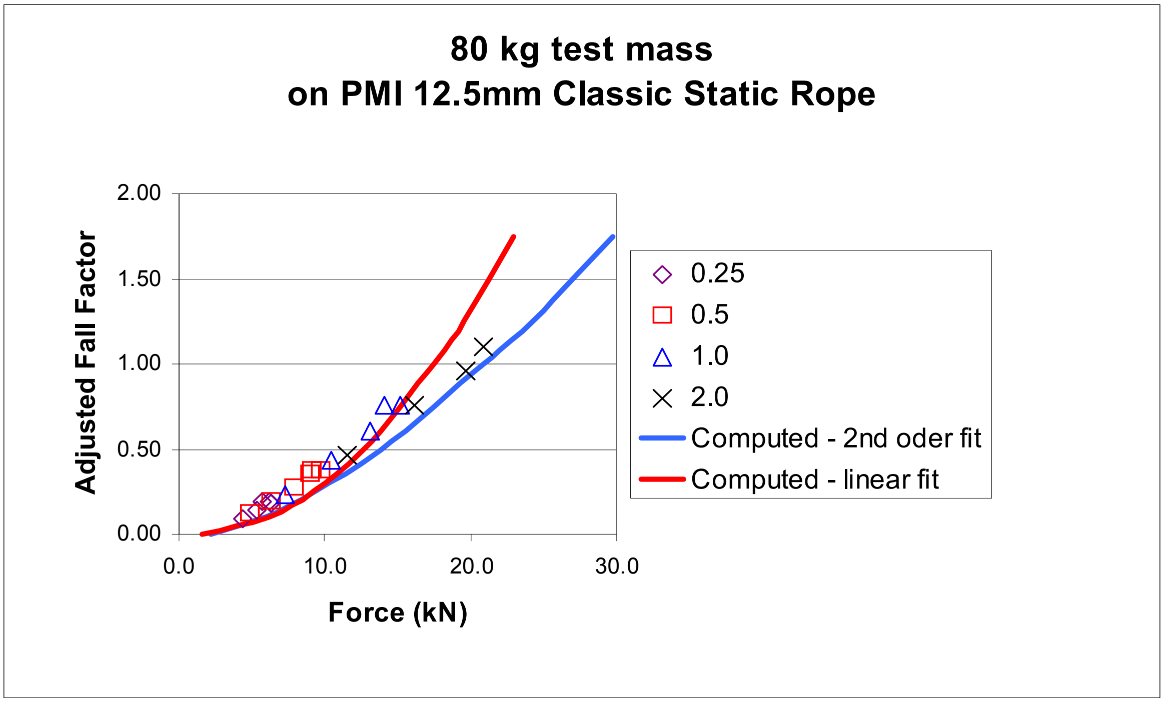
The force in the rope will be

$$F_r = \frac{a}{L_r}\delta_r + \frac{b}{L_r^{\ 2}}\ \delta_r^2 \qquad \text{Eq(16)}$$

The force in the knot will be equal to the force in the rope, therefore:

$$c + \frac{d}{L_k}\delta_k + \frac{e}{L_k^{\ 2}}\ \delta_k^2 \angle F_r = 0 \qquad \text{Eq(17)}$$

where the constants c, d, and e are from the curve fit of the rope, and we require $F_r > c$ for $\delta_k \neq 0$.



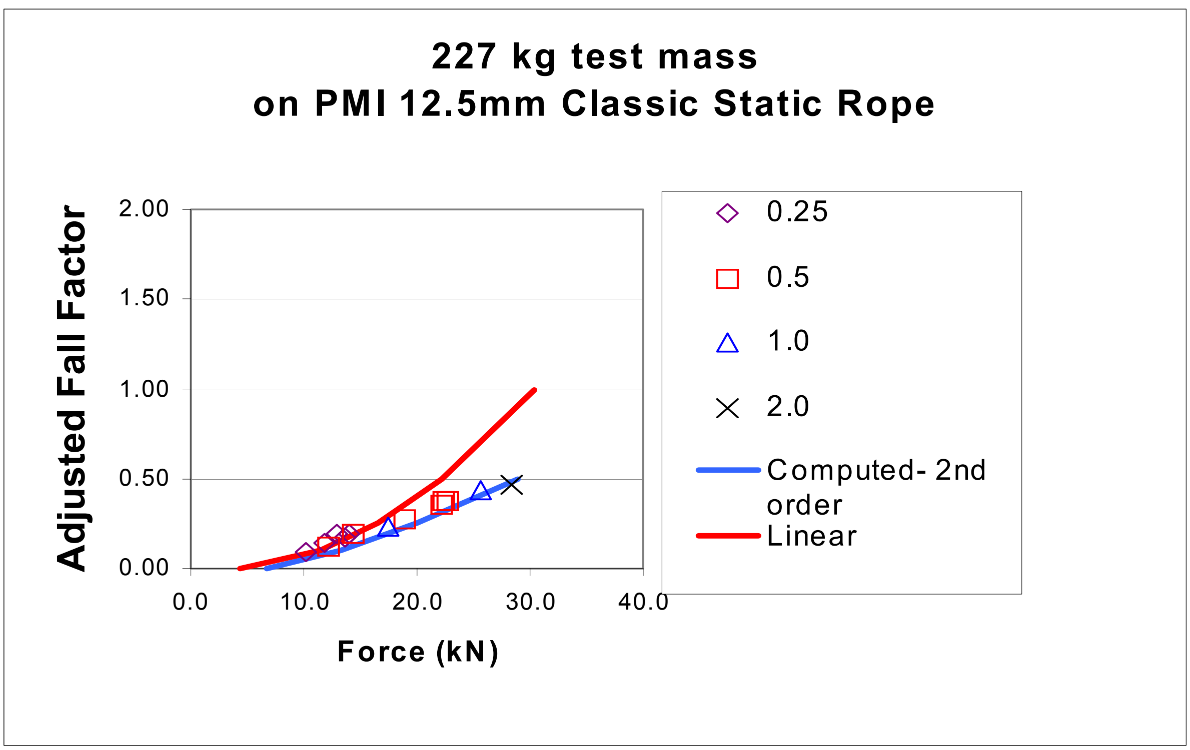


Figure 7Adjusted fall factor for 12.5 mm PMI rope compared with computed fall factors based on strain energy balance for a linear and second order fit for force-deflection.

The solution procedure remains the same. We step from $\delta_r = 0$ to $\delta_r = max$. From δ_r we can compute the force in the rope. Once the force in the rope is known, the knot displacement can be computed. Given both the rope displacement (prescribed) and the knot displacement (computed), the zero function strain energy can be computed from Eq. 15.

A total knot length of 1.3 ft. was used based on the measured length of the two knots in the system. In addition, the knot length must be subtracted from the total rope length. The results for the computed strain energy for the case with and without knots is shown in Figure 6. Adding the effects of the knots really improves rope force prediction. The predicted forces follow the correct trends, but the values are a little high. It is uncertain why the predicted values are high.

As a second measure of how well the existing method predicts rope impact forces, the experimental data from Weber (1999) was recast and plotted in terms of a computed effective rope length. The data was recast by assuming each knot to have an equivalent length of 5.8 ft. of rope. The results from the series of drop tests for 12.5 mm PMI are replotted using the adjusted lengths in Figure 7. Also shown in this figure are the computed values from the strain energy balance equations using both the linear rope force-deflection curve and the second order fit for 12.5 mm PMI. Note that the impact forces computed here are for ropes without knots. In order to compare the results, the test rope lengths were adjusted. As can be seen in this plot, the forces based on the linear predictions under predict the measured impact loads. The second order fit matches the data quite well.

For the 500 lb test series, the linear equations force estimates for a fall factor of 1.0 was 6840 lb. The maximum rope force from the test was measured at 6382 for the knot-adjusted fall factor of 0.47.The second order prediction estimates a 6500 lb load for a fall factor 0.47. Even when the knot-adjusted lengths are used, the linear estimate for maximum fall factor is off by almost a factor of 2.0, while the second order fit produces results that are with 2% of the test data.

It appears that using a second order polynomial for the force deflection curves allows us to more accurately compute the balance between the strain energy and the potential energy. To confirm or improve the existing predictions, more research may be needed. Duplications of Weber's measurements could confirm or improve the understanding of uncertainty in the experimental set up.

Real world systems are complex, and rope behavior is still not fully understood. Before we can fully resolve the question "How well do fall factors work?", we should:
1) remove the knots from the system and measure impact forces (i.e. only test one thing at a time).
2) determine if the loading rate affects the dynamics of the load-displacement curve.

Measuring a knotless system will help us separate rate effects of the rope from rate effects of the knot. To understand rate effects, we need to know if a rope is stiffer or softer for high versus low speed loading. Rate effects for nylon have been observed by other researchers. The design of nylon airbags for a car crash require predictive models that will not work until rate effects are included. A second example of rate effect is in parachute design. High-speed parachute designers must include rate effects to get good predictions (Gwinn). Parachute designers have seen strengths of nylon ribbon drop by as much as 30 percent for high loading rates of 12 (1/sec). The loading rate for impacts on ropes should not be that high, but we could expect a 5 to 10% reduction in strength. A great test would be to measure the load-deflection curve during a series of drop tests.

Summary

- Linear approximations to rope stiffness is a bad idea.
- Errors in the linear approximation can result in an underestimation of the maximum impact force by as much as 30%. The impact force for a zero fall factor will be more than three times the static load. (should be easy to test)
- A second order fit provides a more accurate estimate of impact loads for nylon ropes.
- In theory, the fall factor should still control the load in a rope even when non-linear rope behavior is present.
- Knots in real systems absorb energy and can increase the “effective” rope length.
- Knots are much softer than rope and make it difficult to estimate an effective rope length.
- Including separate terms for the strain energy of knots improves the accuracy of the impact force prediction.
- Better tools are need to illustrate the concept of a fall factor and how it works for real systems.
- The ratio of the fall height to the rope length (a.k.a. fall factor), can be used to predict the maximum force in a rope provided that the effects from knots are included either as an effective length of rope or through separate terms in the strain energy equations.

More testing is needed before these findings can really be accepted and put into widespread use. Perhaps, the ultimate goal could be for rope manufacturers to one day provide a "look-up" graph for all ropes that includes curves to predict forces for different weights at different fall factors. As far as overall system safety goes, these new formulas are definitely a step in the right direction because they "over predict" slightly versus "under predict."

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