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UNCERTAINTY, INVESTMENT, AND INDUSTRY EVOLUTION*

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We study the effects of industry-wide and idiosyncratic uncertainty on the entry of firms, total investment, and prices in a competitive industry with irreversible investment. We determine entry decisions and the resulting industry equilibrium and its characteristics, emphasizing effects of different sources of uncertainty. We stress how irreversibility affects the equilibrium distribution of prices, which in turn affects entry. Finally, we use four-digit U.S. manufacturing data to measure the extent of uncertainty and gauge its importance for investment. We find that a doubling of industry-wide uncertainty raises the required rate of return on new capital by about 20 percent.

1. INTRODUCTION

Most investment expenditures are at least in part irreversible, that is, are sunk costs that cannot be recovered should market conditions change adversely. As a result, the cost of investing includes an opportunity cost of commiting resources rather than waiting for new information. A growing literature has shown how this opportunity cost can be evaluated, and demonstrated that it is very sensitive to uncertainty over future project values, so that changing market conditions that affect the riskiness of future cash flows can have a large impact on investment spending. These results emphasize the role of uncertainty as a determinant of investment spending, and suggest that policies that reduce volatility (over, say, exchange rates, prices, or interest rates) may lower the required cost of capital.[2]

In most of the recent literature, the emphasis is on the investment decisions of an individual firm, rather than industry-wide investment and growth, and uncertainty is modelled by introducing an exogenous state variable (e.g., a demand or cost shift parameter, the price of the firm's output, or the interest rate) that follows some

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[2] McDonald and Siegel (1986) were among the first to demonstrate the implications of irreversibility for investment decisions. Other examples of this literature include Bertola and Caballero (1994), Dixit (1989b), Majd and Pindyck (1987), and Pindyck (1988). For an overview, see Dixit (1992), Pindyck (1991), and Dixit and Pindyck (1994). The earlier literature on investment under uncertainty, e.g., Hartman (1972) and Abel (1983), demonstrates how uncertainty will increase the expected value of a marginal unit of capital if the marginal revenue product of capital is a convex function of the stochastic variable (an implication of Jensen's inequality), and thereby increase investment.

stochastic process. However, similar effects of uncertainty on investment can be found at the industry level. The reasons for these effects, however, may not be the same.

What always matters for investment are the distributions of future values of the marginal profitability of capital—if these distributions are symmetric (and the firm is risk-neutral), increasing uncertainty will not affect investment. For a monopolist, irreversibility causes the distributions to be asymmetric because the firm cannot disinvest in the future if negative shocks arrive; hence the firm invests less today to reduce the frequency of bad outcomes in the future (i.e., the frequency of situations in which the firm has more capital than desired). On the other hand, in a competitive industry with constant returns to scale, the distribution of the future marginal profitability of capital is independent of the firm's current investment. But this distribution is not independent of *industry-wide* investment if the elasticity of demand faced by the industry is less than infinite.

As a result, when studying irreversible investment in an industry context, it is important to distinguish between aggregate (industry-wide) and idiosyncratic (firm-level) shocks. To see this, consider idiosyncratic and aggregate shocks to productivity that are both symmetrically distributed. Although either type of shock might affect the expected future market price and hence the expected marginal profitability of capital, idiosyncratic shocks lead to a symmetric probability distribution for the marginal profitability.[3] Aggregate shocks, however, do not; although negative shocks can reduce the market price, positive shocks are accompanied by the entry of new firms and/or expansion of existing firms, which limits any increases in price. Hence the distribution of outcomes for individual firms is truncated; negative shocks to productivity reduce profits more than positive shocks increase them, and irreversible investment is reduced accordingly.[4] Thus an important objective of this paper is to clarify the different mechanisms through which aggregate and idiosyncratic shocks interact with irreversibility in a competitive industry.

Uncertainty affects irreversible investment in two ways: first, through the effect of the firm's current investment on the expected path of its marginal profitability of capital; and second, through the effects of *competitors' investment* on the path of this marginal profitability. Caballero (1991) has shown that with constant returns to scale, the importance of the first effect decreases as the demand curve facing the firm becomes more elastic, as long as the uncertainty is firm-specific. But this does not mean that industry-level uncertainty will not affect industry investment and output in a competitive equilibrium. As shown by Pindyck (1993), irreversibility has the same type of effect on industry investment as it does for a monopolist once one allows for entry of new firms or the expansion of existing ones. The reason is that irreversibility combined with the possibility of entry affects the distribution of the marginal profitability of capital seen by each individual firm. Hence another objective of this paper is to characterize the distribution of the marginal profitability of

[3] For simplicity, we are ignoring the effect of uncertainty through the convexity of the marginal revenue product of capital, as stressed by Hartman (1972).

[4] This is an example of the *bad news principle* discussed by Bernanke (1983).

capital and its evolution, and show how it affects entry, investment, and the price level itself.

This paper extends and complements recent work by Dixit (1989b), Leahy (1993), and others. Dixit characterizes industry evolution in the presence of aggregate uncertainty by using dynamic programming methods to determine the entry and exit decisions of individual firms of discrete size. Leahy models an industry equilibrium in which price is endogenous, and shows that under reasonable assumptions, it is optimal for individual firms to make their investment decisions under the myopic assumption that price follows an exogenous lognormal random walk. Other related work includes the discrete-time models of Hopenhayn (1992a, b) and Lambson (1991). Hopenhayn examines industry equilibrium allowing for endogenous exit and firm heterogeneity, but restricts uncertainty to be firm-specific. Lambson develops a model similar to ours in that it allows for industry-wide as well as firm-specific shocks, but he focuses on the effects of evolving market conditions on technology selection.[5] In the first part of this paper we use a continuous-time approach similar to that used by Dixit (1989b) and Leahy (1993), but we emphasize the effects of different sources of uncertainty on entry. We then go on to show how the distribution of prices, conditional on the time elapsed since entry, can be used as an alternative way to characterize firms' behavior and industry equilibrium. This helps to clarify how investment is affected by the interaction of irreversibility with different forms of uncertainty in an equilibrium setting.

We examine the effects of idiosyncratic and aggregate uncertainty using a simple model of a competitive market in which firms have constant returns to scale and there is a sunk cost of entry. In the next section, we cast the model as a dynamic programming problem, and we obtain a solution and examine its properties. In Section 3 we re-cast the problem in terms of the conditional distribution of the marginal profitability of capital. We calculate the time path for this distribution, and show how it provides additional insight into the effects of uncertainty on investment and industry evolution. In Section 4 we use four-digit U.S. manufacturing data to measure the extent of uncertainty and gauge its importance for industry investment. Section 5 concludes, and discusses possible extensions of our work.

2. A STYLIZED MODEL

We begin with a highly stylized model in which the value of a marginal unit of capital is stochastic and exogenous. For simplicity, our formulation eliminates the conventional positive Jensen's inequality effect of uncertainty on the value of a marginal unit of capital that arises from the endogenous response of variable factors to exogenous shocks. This lets us focus on the way in which the effects of uncertainty are mediated through the equilibrium behavior of all firms.

[5] In related studies, Lippman and Rumelt (1985) model a competitive industry equilibrium with free entry and exit, Dixit (1991) characterizes the equilibrium for a competitive industry with irreversible investment and a price ceiling, and Lambson (1992) examines the long-run determinants of average profit rates in a model of industry equilibrium with sunk costs.

We consider a market with a large number of productive units. Each productive unit might be a single firm, or individual firms might each own several units. These units are industry specific, so that their installation involves a sunk cost. Entry occurs when new productive units are added, either because new firms invest and enter the market, or existing firms invest in new capacity. What matters is that idiosyncratic shocks apply to these units individually, that is the units all have the same expected productivity, but will have randomly differing realized productivities. To clarify the ways in which uncertainty affects investment, we assume that the owners and managers of these units are risk-neutral. (Hence the investment rules we derive maximize firms' values in a competitive financial market, whether or not idiosyncratic or aggregate shocks are spanned by the set of traded assets in the economy.)

We assume that these productive units are small enough and the number of them is large enough so that we can represent them as a continuum whose mass at time t is $N(t)$. Total industry output, $Q(t)$, is given by:

$$Q(t) = \int_0^{N(t)} A_i(t)\, di \tag{1}$$

where $A_i(t)$ is the output of productive unit i at time t. The A_i's are assumed to follow arbitrary and possibly correlated exogenous stochastic processes. We decompose these individual productivity variables into two parts, their average (the aggregate) and the remainders:

$$A_i(t) = A(t)a_i(t), \quad \text{such that } \int_0^{N(t)} a_i(t)\, di = N(t).$$

Here $A(t)$ is the average productivity of the industry, so that $Q(t) = A(t)N(t)$, and $a_i(t)$ is the productivity of unit i relative to that of the industry as a whole.

We allow for one idiosyncratic and two aggregate sources of uncertainty. First, we let $a_i(t)$ and $A(t)$ follow separate stochastic processes, so that productivity has both an idiosyncratic and an aggregate component. Second, we introduce another source of aggregate uncertainty through the industry demand curve. Industry demand is taken to be isoelastic:

$$P(t) = M(t)Q(t)^{-1/\eta}, \tag{2}$$

where $M(t)$ is an exogenous stochastic process that captures aggregate shocks. We will assume that $M(t)$ follows a diffusion.

The measure of industry size, $N(t)$, increases with entry and decreases with *failures*, the (involuntary) removal of productive units. We assume that the latter occurs at an exogenous proportional rate γ. At the level of an individual unit, a *failure* is a Poisson arrival, and the intensity of the Poisson process is γ.[6] Alternatively, we could have assumed a deterministic depreciation rate γ that applies to all units; our results (from (4) below onwards) would be the same.

[6] It would be more realistic, of course, to make the Poisson arrival rate depend on the age of the specific unit. However, that complicates the model but adds little additional insight.

To introduce irreversibility, we assume that entry of a productive unit requires a sunk cost F. Free entry determines that there are no profits to be made by adding another productive unit to the industry, so that:

$$F \geq \mathrm{E}_i\left\{\mathrm{E}_0\left[\int_0^\infty P(t)A_i(t)e^{-\delta t}\,dt\right]\right\}, \tag{3}$$

which holds with equality at all times in which there is entry. The parameter δ is the discount rate. Note that the expectation E_i is over all unit-specific uncertainty, which includes the stochastic productivity process $a_i(t)$ as well as the Poisson failure process for each unit. The expectation E_0 is over the distribution of the future marginal revenue product of capital, $P(t)A_i(t)$, and therefore accounts for the possible (irreversible) entry of new productive units. As will become evident, the ability to enter the industry reduces the probability of good outcomes by truncating the upper part of the distribution for the aggregate component of $P(t)A_i(t)$, namely $P(t)A(t)$.

By Fubini's theorem and the construction of $A_i(t)$ we can pass the expectation operator E_i inside the integral in eqn. (3), so that it reduces to:

$$F \geq \mathrm{E}_0\left[\int_0^\infty P(t)A(t)e^{-(\delta+\gamma)t}\,dt\right]. \tag{4}$$

Note that the only idiosyncratic effect that remains in (4) is the failure rate γ, and this is now indistinguishable from an industry-wide depreciation rate. Because the value of the output of each unit is linear in the output-specific stochastic state variable, we can eliminate all other idiosyncratic elements from the right-hand side of (3). This is an extreme result that will help to focus and clarify our analysis. In the concluding section we discuss natural modifications of the model that give an additional role to idiosyncratic shocks. However, these modifications do not affect our basic conclusions.

Since $Q(t) = A(t)N(t)$, we can use the market demand equation to construct a measure of the value of output for an average productive unit. Letting $B(t)$ denote the average value of output:

$$B(t) \equiv P(t)A(t) = M(t)A(t)^{(\eta-1)/\eta}N(t)^{-1/\eta}. \tag{5}$$

Because the industry size $N(t)$ is endogenous, $B(t)$ will follow a regulated stochastic process, where $N(t)$ regulates $B(t)$. Letting lower case letters represent the logarithm of the corresponding variable, we can write:

$$d\log B(t) \equiv db(t) = dm(t) + \left(\frac{\eta-1}{\eta}\right)da(t) - \frac{1}{\eta}dn(t). \tag{6}$$

In order to obtain analytical results that can be used to illustrate the implications of different sources of uncertainty, we make the simplifying assumption that the aggregate stochastic state variables follow geometric Brownian motions. Thus, we

write the dynamics of $m(t)$ and $a(t)$ as:

$$dm(t) = \left(\alpha_m - \tfrac{1}{2}\sigma_m^2\right) dt + \sigma_m\, dz_m(t) \tag{7}$$

$$da(t) = \left(\alpha_a - \tfrac{1}{2}\sigma_a^2\right) dt + \sigma_a\, dz_a(t). \tag{8}$$

We also assume that the Wiener processes $dz_m(t)$ and $dz_a(t)$ are uncorrelated. (It is easy to relax this assumption.) Then $B(t)$ follows a particularly simple regulated geometric Brownian motion. Specifically, $B(t)$ will remain at or below a fixed upper boundary. This boundary, which we denote by U, is yet to be determined as part of an industry equilibrium. Regulation is due to entry; when this is not occurring, $n(t) \equiv \log(N(t))$ will follow:

$$dn(t) = -\gamma\, dt,$$

and $b(t)$ is given by:

$$db(t) = \beta\, dt + \sigma_b\, dz(t), \tag{9}$$

where

$$\beta = \alpha_m - \frac{1}{2}\sigma_m^2 + \frac{\gamma}{\eta} + \frac{\eta - 1}{\eta}\alpha_a - \frac{\eta - 1}{2\eta}\sigma_a^2,$$

and

$$\sigma_b = \sqrt{\sigma_m^2 + \left(\frac{\eta - 1}{\eta}\right)^2 \sigma_a^2}.$$

This model is simple enough so that we can find a closed form solution for the optimal investment rule, i.e., for the upper boundary U. (Later we will see how the entire problem can be recast in terms of the conditional distribution of marginal revenue product.) Let $W(x)$ denote the value of entering the industry at $t = 0$ when $b(0) = x$, so that $B(0) = e^x$:

$$W(x) = \int_0^\infty e^{-(\delta+\gamma)t}\, \mathrm{E}_0[B(t) | B(0) = e^x]\, dt. \tag{10}$$

By arbitrage, over an interval dt, the total expected return from being in the industry must be equal to $(\delta + \gamma)W dt$. This expected return has two components, an expected capital gain, $\mathrm{E}_0\, dW$, and a flow of revenue $B(0)\, dt = e^x\, dt$. By Ito's Lemma, $\mathrm{E}_0\, dW = \beta W'(x)\, dt + (1/2)\sigma_b^2 W''(x)\, dt$, so $W(x)$ must satisfy the following differential equation:

$$\tfrac{1}{2}\sigma_b^2 W''(x) + \beta W'(x) - (\delta + \gamma)W(x) + e^x = 0. \tag{11}$$

In addition, $W(x)$ must satisfy the following boundary conditions:

$$\lim_{x \to -\infty} W(x) = 0, \tag{12}$$

and

$$W'(u) = 0, \tag{13}$$

where $u = \log U$. Boundary condition (12) follows from the fact that 0 is an absorbing boundary for B. Condition (13) follows from the (left) continuity of the value function at the trigger point u.

The reader can check that (11) has the following simple solution that satisfies the associated boundary conditions:

$$W(x) = \frac{e^x}{\delta + \gamma - \beta - \sigma_b^2/2} - \frac{e^u/\lambda}{\delta + \gamma - \beta - \sigma_b^2/2} e^{\lambda(x-u)}, \tag{14}$$

where

$$\lambda = \frac{-\beta + \sqrt{\beta^2 + 2(\delta + \gamma)\sigma_b^2}}{\sigma_b^2}. \tag{15}$$

A sufficient condition for the existence of this solution is that the discount rate be large enough so that the value of a unit remains bounded even if entry into the industry were prohibited throughout the future. Specifically, we require that $\delta + \gamma - \beta - \sigma_b^2/2 > 0$, that is $\delta > \alpha_m + [(\eta - 1)/\eta](\alpha_a - \gamma) - [(\eta - 1)/2\eta^2]\sigma_a^2$. This ensures that $\lambda > 1$, and simply implies that the neoclassical cost of capital is positive.

We can now determine U, the upper boundary of $B(t)$. If we had solved this as a central planning problem, we would determine U from the first-order condition that $W''(U) = 0$. Instead, we follow Leahy (1993) and use the free entry condition, which in this case is $F = W(u)$. Hence:

$$\frac{U}{F} = \frac{\lambda}{\lambda - 1}\left(\delta + \gamma - \beta - \frac{1}{2}\sigma_b^2\right). \tag{16}$$

Because of free entry, $\mathrm{E}_0 \int_0^\infty B(t) e^{-(\delta+\gamma)t}\, dt = F$, where $t = 0$ is the time of entry. Since $U \geq \mathrm{E}_0[B(t)]$ for all t and $U > \mathrm{E}_0[B(t)]$ for all $t > 0$, we know that $\mathrm{E}_0 \int_0^\infty U e^{-(\delta+\gamma)t}\, dt > F$. This is a result of irreversibility; there is an opportunity cost of investing now rather than waiting for new information. If firms could *uninvest* and recoup the cost F, we would instead have the standard Marshallian result that $\mathrm{E}_0 \int_0^\infty U e^{-(\delta+\gamma)t}\, dt = F$.

For simplicity, in what follows we assume that aggregate productivity is constant, so that $\alpha_a = \sigma_a = 0$ (and hence $\sigma_b = \sigma_m$). Recall that α_m and σ_b represent the mean and the standard deviation of the rate of growth of revenue per productive unit averaged over the industry when there is no entry. With tedious calculation, one

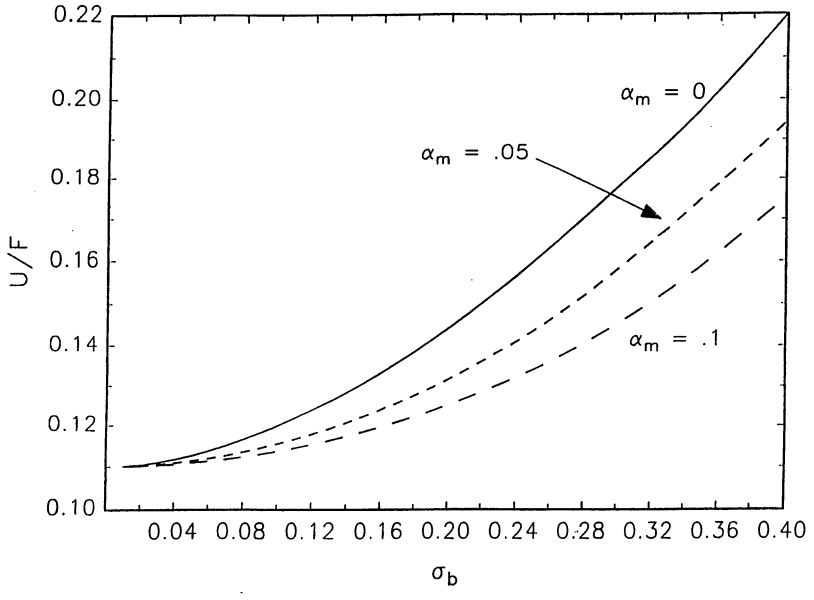

FIGURE 1

DEPENDENCE OF U/F ON σ_b AND α_m

can show that $\partial(U/F)/\partial\sigma_b > 0$ and $\partial(U/F)/\partial\alpha_m < 0$. A smaller value of α_m raises U/F because given any value of U/F, it implies a lower expected price so that less entry is needed to satisfy the zero profit condition. (This is discussed further below.) A higher value of σ_b raises U/F by increasing the opportunity cost of investing, and thereby raising the threshold required for a firm to pay the sunk cost F. But note that only *aggregate* uncertainty matters; U/F is unaffected by idiosyncratic shocks. Figure 1 shows this dependence of U/F on α_m and σ_b.

One can also show that $\partial(U/F)/\partial\eta > 0$, and $\partial(U/F)/\partial\delta > 0$. An increase in the elasticity of demand, η, implies that the potentially positive effect of the failing units on the price is reduced. This lowers expected revenue flow and hence raises the threshold required for investment. An increase in δ likewise raises the threshold by directly lowering the expected present value of returns and by increasing the opportunity cost of investing in the unit now, rather than waiting and discounting the expenditure F. As for $\partial(U/F)/\partial\gamma$, the discounting effect described above again holds (capital depreciates faster when γ is larger). However, there is an offsetting effect from the increased depreciation of the capital of other firms, which tends to raise the expected industry price as seen from the time of entry. The first effect dominates for most reasonable parameter values.

We can now describe the behavior of industry investment, output, and price in equilibrium. Suppose, for example, that aggregate demand increases. Then entry of new productive units will occur, so that price will rise only to the point that $P(t)A(t) = U$. Figures 2A, 2B, and 2C illustrate this by showing a particular sample

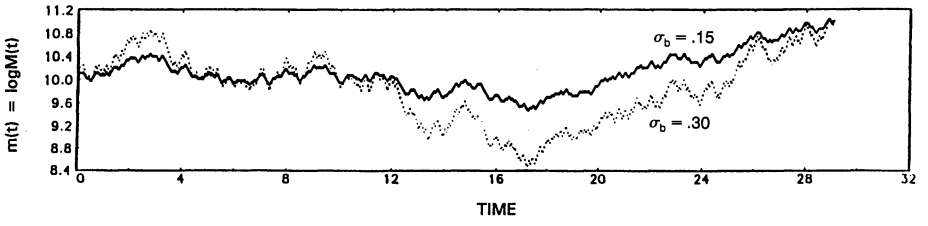

FIGURE 2A

LOG DRIVING FORCE

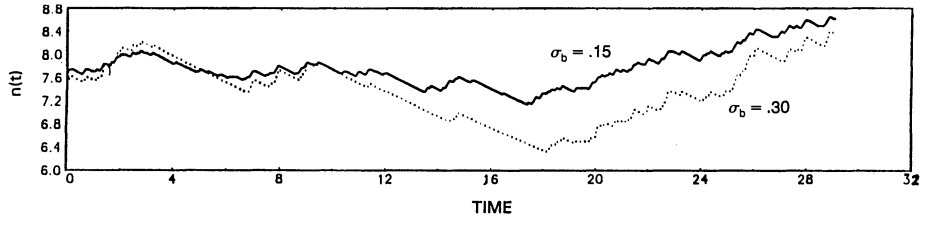

FIGURE 2B

LOG OF NUMBER OF PRODUCTIVE UNITS

path for industry evolution for two values of σ_m, 0.15 and 0.30. (In this simulation, the other parameters are $\eta = 2$, $\alpha_m = 0.02$, $\alpha_a = \sigma_a = 0$, $\gamma = 0.03$, $\delta = 0.06$, and $F = 100$.) The top graph shows the log of the stochastic driving force, $m(t)$. (The realization for $z(t)$ is the same for the two lines, but the values of σ_b are different.) Figure 2B shows the log of the number of productive units, $n(t)$. Note that when $m(t)$ is falling (e.g., between $t = 12$ and 18), there is little or no investment, so $n(t)$ falls due to failures (or depreciation). For $t > 18$, $m(t)$ is generally rising, and so entry occurs and $n(t)$ rises.

Figure 2C shows the realization for the log of price, $p(t)$. Note that $p(t)$ appears stationary; that is because we have set $\alpha_a = \sigma_a = 0$ for all t, so that $A(t) = 1$ always.

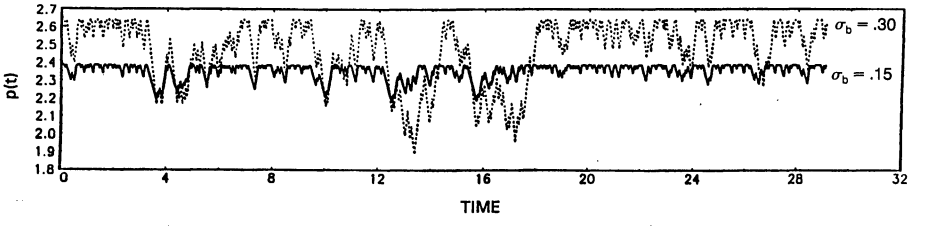

FIGURE 2C

LOG OF PRICE

(Hence price is equal to the average revenue per productive unit, which is the relevant state variable for the decision-making unit. In the more general case, $b(t)$ would follow the same pattern as in Figure 2C, and $p(t)$ would be the sum of $b(t)$ and a Brownian motion.) As the figure illustrates, during *recessions* (when $m(t)$ is falling) price will also fall, and will fall farther when σ_b is larger. But during *good times*, $p(t)$ is generally higher when σ_b is larger. The reason is that with a larger σ_b, there is a greater chance of deeper recessions, so during good times firms wait longer before entering, n is smaller, and p is higher.

Underlying these results is a forecast of future revenues by firms that are considering entry. In fact, this forecast (which must take into account entry by other firms) completely determines the decision to enter. Hence, looking directly at the expected value of future revenues, and their dependence on the underlying parameters, helps to understand industry evolution. We turn to this next.

3. THE PRICE DISTRIBUTION AND ENTRY

In the previous section we found the optimal investment rule in the standard way —by using dynamic programming to calculate the firm's value function. In general, this approach is useful in that studying the local (in time) behavior of the value function allows one to fully characterize complex dynamic problems. Problems in which the optimal or competitive outcome consists of regulating a Brownian motion, as in the model developed in the previous section, are good examples of this. Value matching, smooth pasting and the Bellman equation are all intuitive properties arising from this local analysis.

Although dynamic programming is a powerful tool, it sometimes conceals the economic intuition as to how changes in parameters affect optimal policies. As we explained in the Introduction, the combination of irreversibility and industry-wide uncertainty causes the threshold that triggers investment to rise because of the asymmetry in the distribution of the future marginal profitability of capital that the irreversibility constraint brings about. This is hidden in the dynamic programming formulation.

For example, (10) defined the value function, $W(x)$, as the expected present value of the flow of marginal revenue product. Thus, any effects of changes in the variance or drift parameters on the value function, and hence on the optimal investment rule, must come through their effects on either the path of the expected marginal profitability of capital or the discount rate. In this section we illustrate this mechanism by looking at the path of expected marginal profitability directly and showing how it is affected by the underlying parameters. Although this approach is more cumbersome than that used in the previous section, it makes the nature of the irreversible investment problem more apparent.

To proceed, we need to derive the conditional probability density for b, which we denote by $f(b,t)$. Since we know that a firm will enter only when $b(t)=u$, we can replace x by u in (10). Hence $f(b,t)$ is the probability density of b after a time t has elapsed from the moment of entry, conditional on $b(0)=u$. As mentioned above, any effects of parameters such as β and σ_b on the entry point u and hence on price will occur through their effects on the path of the density $f(b,t)$, and in particular

on the function:

$$\mathrm{E}[B(t)|B(0)=U]=\int_{-\infty}^{\log U}e^{b}f(b,t)\,db.$$

This expected value begins at the moment of entry at U, and then converges over time to the ergodic mean. In the Appendix we derive the entire path of the density $f(b,t)$ and its conditional moments. In particular, its ergodic mean, which we denote by $\bar{B}_{\infty}$, is given by:

$$\bar{B}_{\infty}\equiv\lim_{t\to\infty}E[B(t)|B(0)=U]=\frac{2\beta}{\sigma_b^2+2\beta}U. \tag{17}$$

If the discount and depreciation rates are small, this ergodic mean, as opposed to the transition path to this mean, has a relatively large weight in determining the response of the equilibrium entry point, U, to changes in the drift and uncertainty parameters. It is straightforward to see that $\bar{B}_{\infty}$ rises with β and falls with σ_b; thus, by the free entry condition, U must fall with β and rise with σ_b.

If the discount and/or depreciation rates are large, the transition path to the ergodic mean carries more weight, so the problem is more complicated. In this case we need to account for the entire path of $f(b,t)$. (Intuitively, we know that $f(b,t)$ must start as a spike at u when $t=0$, and as t increases it must converge smoothly to the ergodic density.) Because $b(t)$ follows the diffusion equation (9), $f(b,t)$ must satisfy the Kolmogorov forward equation:

$$f_t(b,t)=\tfrac{1}{2}\sigma_b^2 f_{bb}(b,t)-\beta f_b(b,t). \tag{18}$$

(See Karlin and Taylor 1981.) Since $b(t)$ is regulated at u, the solution to this equation must satisfy the following boundary conditions for $t>0$:

$$f(u,t)=\frac{\sigma_b^2}{2\beta}f_b(u,t), \tag{19}$$

$$\lim_{b\to-\infty}f(b,t)=0, \tag{20}$$

as well as the initial condition:

$$\int_{-\infty}^{x}f(b,0)\,db=\begin{cases}0 & x<u\\ 1 & x=u\end{cases}. \tag{21}$$

In Appendix A we derive the solution for $f(b,t)$. Using this, we find an expression for the trajectory of the expected marginal profitability of capital, conditional on its value at the time of entry, U. From the free entry condition, the present value of the flow of this expected marginal profitability must equal the cost of entry, and this determines U.

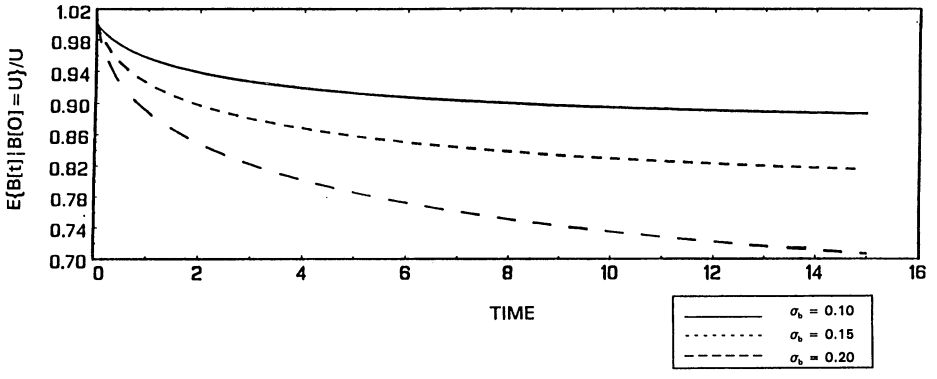

FIGURE 3A

$E\{B(t)|B(0) = U\}/U$

Figure 3 illustrates how the mechanisms underlying industry-wide investment are revealed by the conditional expectation of the marginal profitability of capital. The figure divides the investment problem into two steps. The first step, shown in panel (a), removes the effect of individual firms' optimal entry decisions by normalizing the path of the conditional expectation of $B(t)$ by its value at the time of entry. Thus it isolates the impact on firms' expected marginal revenue of the interaction between industry-wide entry (optimal or otherwise) and the stochastic environment. The second step, shown in (b), adds back in the effect of firms' individual entry decisions.

Panel (a) shows $E[B(t)|B(0) = U]/U$ as a function of time for $\sigma_b = 0.10$, 0.15, and 0.20. (Other parameter values are $\gamma = 0.03$, $\alpha_m = 0.02$, $\alpha_a = 0$, $\sigma_a = 0$, $\eta = 2$, $\delta = 0.06$, and $F = 1$.) Two points should be noted. First, the asymmetry of the irreversibility constraint (i.e., there is free entry but no exit) implies that the expected marginal

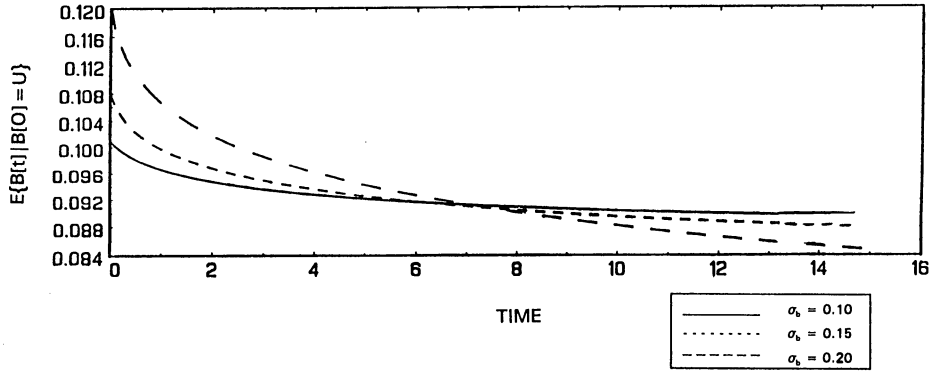

FIGURE 3B

$E\{B(t)|B(0) = U\}$

profitability is largest at the time of entry, and declines monotonically thereafter. Second, the more uncertainty there is the faster and deeper is this decline. The reason is that free entry truncates the distribution from above, while larger negative shocks imply larger reductions in marginal profitability.

Panel (b) shows $E[B(t)|B(0) = U]$ for the same three values of σ_b. This incorporates the industry-wide determination of U in response to the post-entry pattern of the expected marginal profitability. Note first that the entry threshold U (the intersections of these curves with the vertical axis) is always greater than the frictionless neoclassical cost of capital, which is equal to $\delta + \gamma - \beta - (1/2)\sigma_b^2$. In order to recoup the initial investment with a declining path for expected marginal profitability, expected returns in early periods must exceed the neoclassical cost of capital. Second, U increases with uncertainty. This is the case because (as we saw above) greater uncertainty implies a steeper decline in expected marginal profitability.[7]

4. SOME EVIDENCE FROM U.S. MANUFACTURING

In this section we use data for two- and four-digit U.S. manufacturing industries to obtain measures of uncertainty over the marginal profitability of capital. We then use these measures to gauge the importance of uncertainty for investment.

Given assumptions about the production technology and market structure, we estimate a times series for our marginal profitability variable, $B(t)$, up to a scaling factor. In particular, we assume that the industry is competitive and the production function is Cobb-Douglas with constant returns to scale. We can thus express the output of a productive unit as:

$$Y(t) = S_t K_t^{\alpha} L_t^{\phi(1-\alpha)} M_t^{(1-\phi)(1-\alpha)}, \tag{22}$$

where S_t is an index of profitability, α is the share of capital, and ϕ is the share of labor in a labor-materials composite which we will denote by H. (This might appear different from the model in Section 2, but it is not. Note that if the firm chooses the flexible factors L and M optimally, given CRTS, Y_t will be proportional to K_t.) Given this expression for output, the marginal profitability of capital is given by:

$$\Pi_K(t) = \alpha(1-\alpha)^{(1-\alpha)/\alpha} (P_t S_t)^{1/\alpha} P_{H,t}^{-(1-\alpha)/\alpha}, \tag{23}$$

where P_t is the price of output, and $P_{H,t}$ is the price of the labor-materials composite. Letting $A_t \equiv \alpha(1-\alpha)^{(1-\alpha)/\alpha} S_t^{1/\alpha} P_{H,t}^{-(1-\alpha)/\alpha}$, we can write the marginal

[7] If the discount rate δ and depreciation rate γ were zero, these curves would all converge to the same value, and would not cross each other. The reason is that with no discounting, only the long-run steady-state matters, and not the transition to that steady-state. Different values of σ_b would result in different values of U such that the resulting ergodic means for the marginal profitability of capital would be the same. With discounting, however, the transition matters, so that the curves cross.

profitability of capital as:

$$\Pi_K(t) = A_t P_t^{1/\alpha}.$$

Note that this is equivalent to our expression for $B(t)$ in Section 2, except for the exponent on P_t. (In Section 2 we eliminated this standard convexity exponent for purely expositional reasons.) We will work with $b(t) = \log B(t) = \log \Pi_K(t)$. This is given by:

$$b(t) = a_t + \frac{1}{\alpha} p_t, \tag{24}$$

where again, lower case letters represent logs of the corresponding variables.

We cannot measure $b(t)$ at the individual firm or plant level. Instead we focus on investment and the marginal profitability of capital at two different levels of aggregation: 20 two-digit manufacturing industries, and 443 four-digit subsectors that make up these two-digit industries. While it is not clear which level of aggregation is more representative of what we have called an industry, shocks at the four-digit level are likely to have a larger idiosyncratic component. Hence it is useful to compare the volatility of $b(t)$ and its implications for investment across these levels of aggregation. For each industry, we use data on the real value of output, real inputs of capital, materials, and labor, and the corresponding price deflators to obtain a time series for $b(t)$ over the 29-year-period 1958 through 1986. We denote these series by $b_2(t)$ and $b_4(t)$ for the two-digit and four-digit industries respectively. The data and the calculation of the $b(t)$'s are discussed in the Appendix.[8]

We calculated the sample standard deviations of $\Delta b(t)$ for each of the 20 two-digit industries, which we denote by SDB2, and for each of the 443 four-digit industries, which we denote by SDB4. If we view shocks at the four-digit level as idiosyncratic, then SDB4 would measure *total* (aggregate and idiosyncratic) uncertainty.[9] Table 1 shows SDB2 and the average of the SDB4s for each of the two-digit industries. Observe that the average four-digit standard deviation is typically two or three times as large as the corresponding two-digit standard deviation. The two-digit standard deviations are on the order of 10 percent per year (consistent with an annual standard deviation of real returns on the New York Stock Exchange Index of 20 percent per year and an average debt/equity ratio of one).

Table 1 also shows the premia over the neoclassical cost of capital implied by our model, for the two- and four-digit standard deviations, assuming for both that all uncertainty is aggregate (the premium is $U/F - (\delta + \gamma - \beta - \frac{1}{2}\sigma_b^2)$, with U/F given by (16)). Note that for the two-digit level of aggregation, the implied premia are on

[8] We used a database assembled by Brian K. Sliker, who graciously made it available for our use. We included only 443 of the 450 four-digit SIC industries because of missing data in seven of the industries.

[9] Our estimators of SDB2 and, possibly, SDB4 are biased downwards from the true standard deviations because $b(t)$ is a regulated process. Equation (9) applies when it is not regulated, but our sample includes periods of regulation.

TABLE 1
BASIC STATISTICS FOR TWO- AND FOUR-DIGIT INDUSTRIES

SIC	NOB	SDB2	Implied Premium	Mean of SDB4	Implied Premium
20	47	0.058	0.039	0.246	0.085
21	4	0.104	0.047	0.451	0.170
22	30	0.118	0.050	0.366	0.130
23	33	0.076	0.042	0.304	0.105
24	17	0.168	0.062	0.327	0.114
25	13	0.125	0.051	0.258	0.089
26	17	0.113	0.049	0.217	0.076
27	17	0.061	0.039	0.192	0.068
28	28	0.088	0.044	0.224	0.078
29	5	0.201	0.071	0.256	0.088
30	6	0.127	0.052	0.242	0.084
31	11	0.093	0.045	0.231	0.080
32	27	0.099	0.046	0.236	0.082
33	26	0.250	0.086	0.506	0.198
34	32	0.123	0.051	0.277	0.095
35	44	0.160	0.060	0.301	0.104
36	39	0.147	0.056	0.264	0.091
37	15	0.184	0.066	0.403	0.147
38	12	0.105	0.047	0.220	0.077
39	20	0.109	0.048	0.255	0.088

NOB, number of 4-digit industries in each 2-digit industry; Mean of SDB4, cross-sectional sample mean of the 4-digit sample standard deviations of $\Delta b(t)$ corresponding to the 2-digit industry; implied premium, premium over the neoclassical cost of capital resulting from uncertainty and irreversibility, i.e., it is $U/F - (\delta + \gamma - \beta - (1/2)\sigma_b^2)$, where U/F is given by (16), with $\gamma = 0.03$, $\alpha_m = 0.02$, $\alpha_a = \sigma_a = 0$, $\eta = 2$, and $\delta = 0.06$.

the order of 5 or 6 percent, while for the four-digit level they are on the order of 12 percent. These premia are substantial, and are distinct from any premia associated with systematic risk, for instance, in the context of the CAPM. Finally, we also gauge the importance of uncertainty for investment by computing the semi-elasticity $\Delta \log(U/F)/\Delta \sigma_b$. For the industries in Table 1, this semi-elasticity is about 2 for either level of aggregation.

Ideally we would like to estimate the semi-elasticity $\Delta \log(U/F)/\Delta \sigma_b$ using a direct measure of the required return U/F, rather than the one implied by the model. However, the threshold U is not directly observable. Instead, we compute proxies for this threshold, and then use them to estimate the semi-elasticity assuming that the model is correct. We proxy $u = \log U$ by extreme values of $b(t)$; since u is the upper barrier for $b(t)$, $b(t)$ should be close to u when it is large relative to its average value. We use three variables, all computed relative to the industry mean of $b(t)$, to proxy u at both the two- and four-digit levels: (i) the maximum of $b(t)$ over the 29 years of data, denoted by DBMAXn, where n = 2 or 4 for the two- and four-digit industries; (ii) the average of the top decile (three observations) of the 29 annual values of $b(t)$, denoted by DBDECn; and (iii) the average of the top quintile (six observations), denoted by DBQUINTn. We average over several extreme values and use DBDEC and DBQUINT rather than just DBMAX because in practice $b(t)$ may rise above u temporarily if there are lags in investment, if there are predictable temporary increases in $b(t)$, or if firms do not

TABLE 2
CROSS-SECTION REGRESSION RESULTS

Dependent Variable	Const.	SDBn	NOB	R^2
DBMAX2	0.0651	2.3347	20	0.246
	(0.1290)	(0.9641)		
DBDEC2	0.0072	2.3634	20	0.296
	(0.1151)	(0.8598)		
DBQUINT2	0.0070	1.8928	20	0.359
	(0.0799)	(0.5966)		
DBMAX4	0.2136	1.5274	443	0.580
	(0.0208)	(0.0619)		
DBDEC4	0.1886	1.2038	443	0.550
	(0.0161)	(0.0480)		
DBQUINT4	−0.1677	0.9361	443	0.588
	(0.0135)	(0.0403)		

SDB2, sample standard deviation of $\Delta b(t) = \Delta \log B(t)$ for each 2-digit industry; SDB4, average sample standard deviation of $\Delta b(t)$ for the 4-digit industries that comprise the 2-digit industry. Standard errors corrected for heteroscedasticity are shown in parentheses.

always optimize. We compute these variables relative to the mean because $b(t)$ is identified only up to a constant, which may differ across sectors.[10]

Table 2 shows cross-section regressions of DBMAXn, DBDECn, and DBQUINTn against SDBn and a constant, for $n = 2$ and 4. These regressions provide alternative estimates of the semi-elasticity $\Delta \log(U/F)/\Delta \sigma_b$.[11] For the two-digit industries, we again find that this semi-elasticity is about 2, close to what we obtained by computing implied premia directly from the model (see Table 1). This implies that an increase in the annual standard deviation of the marginal profitability of capital from, say, 0.1 to 0.2 should increase the required return on investment by 20 percent (so that if the required return was 30 percent, it should rise to about 36 percent). This is a sizable effect, consistent with the simulated elasticities illustrated in Figure 1, but less than predictions based on analyses of individual projects, such as those by McDonald and Siegel (1986), Majd and Pindyck (1987), and others. For the four-digit industries, the estimates of $\Delta \log(U/F)/\Delta \sigma_b$ are about half as large. One interpretation of this is that the four-digit standard deviations have a much larger idiosyncratic component (as we would expect), which, as the model predicts, does not affect the required return.

5. CONCLUSIONS

In a competitive equilibrium, uncertainty over market demand or average productivity affects irreversible investment through the feedback of industry-wide capacity

[10] Note from (17) that u minus the mean of b is affected by uncertainty in the same qualitative way as is u itself, because the mean is much less sensitive to uncertainty than is u. When the discount rate is zero, the mean of b is unaffected by uncertainty.

[11] These estimates should be taken with caution. Even if the model were not true, generally there will be a positive association between the variance of the increments of a random variable and the maximum of the random variable. We do not purport these results as a test of the model; they are at best *suggestive* numbers, especially when compared with the theoretical values reported in Table 1.

expansion and new entry on the distribution of prices. If demand increases, existing firms will expand or new firms will enter until the market clears. From the point of view of an individual firm, this limits the amount that price can rise under good industry outcomes. But if investment is irreversible, there is no similar mechanism to prevent price from falling under bad outcomes. Each firm takes price as given, but knows that the distribution of future prices is affected by the irreversibility of investment industry-wide, which leads it to raise the trigger point at which it is willing to invest. Idiosyncratic shocks, which affect only an individual firm, do not induce entry and thus should have less impact on the firm's willingness to invest.

We have tried to clarify these channels through which aggregate and idiosyncratic uncertainty affect investment and industry evolution. Our model is simple enough so that it can be solved using standard dynamic programming methods, but we have emphasized the effects of uncertainty on the conditional distribution of the marginal profitability of capital, and shown how this distribution can be derived and used as an alternative means of determining and understanding the behavior of firms and the resulting industry equilibrium.

It is useful to compare our model with standard NPV models of investment based on the CAPM. In those models, it is systematic (economy-wide) uncertainty that affects the discount rate. In our model, aggregate uncertainty, which is related to but not the same as systematic uncertainty, increases the trigger point, which corresponds to a higher required rate of return. Thus the mechanisms are very different, but the effects of different sources of uncertainty are similar as in NPV-CAPM models. We have ignored CAPM effects; they may magnify the effects of aggregate uncertainty that we have derived. However, a recent study by Leahy and Whited (1993) using firm-level data finds that CAPM effects are negligible, whereas the effects of irreversibility seem substantial.

The model we present is highly stylized and makes a number of simplifying assumptions. Some are important and should be kept in mind when interpreting our results. First, as we noted in the empirical section, if there is a flexible factor, or if the firm can costlessly and temporarily shut down when price falls below variable cost, the marginal profit function will be convex in price and in exogenous productivity. Then for an industry of fixed size, an increase in idiosyncratic uncertainty will raise the present value of an additional unit of capital, and so to preserve the zero-profit condition, the trigger point at which entry occurs must decline.

Second, we have ignored abandonment. Suppose a productive unit can be scrapped at any time for some positive value. This puts a floor on the value of the unit. (The unit will be scrapped once the combination of price and its productivity reach the point where its value equals the scrap value.) This possibility raises the value of the unit for any combination of price and expected productivity, which lowers the entry point u, and hence reduces the effect of aggregate uncertainty described by our model. Also, an increase in idiosyncratic uncertainty will raise the value of the unit. The reason is that potential entrants cannot know what their relative productivity will be until they enter. However, exit is done selectively, when idiosyncratic productivity is low *ex post*. Selective exit raises the value of a unit, lowering the critical cutoff point for entry. Hence a scrap value reduces the negative

effect of aggregate uncertainty and creates a positive effect of idiosyncratic uncertainty. Also, the combination of faster entry and the incentive to exit when conditions are bad tends to reduce the variability of price.[12]

A price floor will have an effect similar to that of a scrap value, but only for aggregate uncertainty. It also reduces the negative effect of aggregate uncertainty on entry by limiting one of the two possible reasons for bad aggregate outcomes. (Bad aggregate outcomes that are due to a decline in average industry productivity are still possible.) However, a price floor will not alter the effect (or lack thereof) of idiosyncratic uncertainty.

Obviously such extensions of the model may alter the *absolute* effects of aggregate and idiosyncratic uncertainty. However, these extensions will not alter the basic mechanism that generates the asymmetry in the roles of these two types of uncertainty.

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APPENDIX

A. *The Density Function and Conditional Expectation of $B(t)$.* Let $y \equiv b - u$, and $g(y,t)$ be the density of y at time t, so that $f(b,t) = g(b-u,t)$. In this setup, finding the path of the conditional density of $b(t)$ amounts to solving the problem defined below by (A.1) to (A.6):

$$g_t(y,t) = \frac{1}{2}\sigma_b^2 g_{yy}(y,t) - \beta g_y(y,t), \tag{A.1}$$

$$g(0,t) = \frac{\sigma_b^2}{2\beta} g_y(0,t), \tag{A.2}$$

$$\lim_{y \to -\infty} g(y,t) = 0, \tag{A.3}$$

$$g(y,t) \geq 0 \quad \forall y \leq 0 \text{ and } t \geq 0, \tag{A.4}$$

$$\int_{-\infty}^{0} g(y,t)\, dy = 1 \quad \forall t \geq 0, \tag{A.5}$$

$$\int_{-\infty}^{x} g(y,0)\, dy = \begin{cases} 0 & x < 0 \\ 1 & x = 0 \end{cases}. \tag{A.6}$$

A solution to a similar problem, although with a different initial condition, can be found in Bertola and Caballero (1994). Here we only outline the basic steps of the solution, which is obtained by the method of Separation of Variables.

[12] This is strictly correct only when $A(t)$ is stationary (possibly around a deterministic trend), since otherwise the variance of price becomes infinite. But even if $A(t)$ had a stochastic trend component, the statement would hold for finite intervals.

Writing the solution of the homogeneous problem as $g(y,t) = T(t)Y(y)$, we can decompose the problem into two ordinary differential equations:

$$T'(t) + \lambda T(t) = 0, \tag{A.7}$$

$$Y''(y) - \frac{2\beta}{\sigma_b^2}Y'(y) + \frac{2\lambda}{\sigma_b^2}Y(y) = 0, \tag{A.8}$$

subject to the boundary conditions above, with λ a constant. The solution method has the following steps: First, find the values of λ for which the homogenous problem has a solution. Second, characterize each of these solutions. And third, combine these solutions to satisfy the inhomogenous initial condition.

The characteristic equation of (A.8) has real solutions for $\lambda \leq \theta\beta/4$, where $\theta \equiv 2\beta/\sigma_b^2$. It is easy to verify that the only real solution that satisfies the homogenous boundary conditions occurs when $\lambda = 0$, which yields the particular solution:

$$Y(y; \lambda = 0) = \theta e^{\theta y}. \tag{A.9}$$

However, there is a continuum of solutions for values of $\lambda > \theta\beta/4$, which have the form:

$$Y(y;\psi) = B(\psi)e^{(\theta/2)y}\left(\cos \psi y + \frac{\theta}{2\psi}\sin \psi y\right), \tag{A.10}$$

where

$$\psi \equiv \sqrt{\frac{\theta\lambda}{\beta} - \frac{\theta^2}{4}}.$$

The coefficients $B(\psi)$ are identified by the initial condition, yielding:

$$B(\psi) = \frac{2}{\pi}\frac{\psi^2\sigma_b^4}{\psi^2\sigma_b^4 + \beta^2}. \tag{A.11}$$

Combining (A.9), (A.10) and (A.11) we obtain the solution for $g(y,t)$:

(A.12)

$$g(y,t) = \theta e^{\theta y} + \frac{2}{\pi}e^{-(\beta\theta/4)t + (\theta/2)y}\int_0^\infty \frac{\psi e^{-(\beta\psi^2/\theta)t}}{(\psi^2 + \theta^2/4)}\left(\psi \cos \psi y + \frac{\theta}{2}\sin \psi y\right) d\psi.$$

The expression for the conditional expectation is now obtained by solving the integral in the expression:

$$\mathrm{E}[B(t)|B(0)=U]=U\int_{-\infty}^{0}e^{y}g(y,t)\,dy.$$

Hence,

$$\text{(A.13)}\quad \mathrm{E}[B(t)|B(0)=U]=U\left[\frac{2\beta}{\sigma_b^2+2\beta}+e^{-\beta^2 t/2\sigma_b^2}\int_0^{\infty}\lambda(z)\,e^{-\sigma_b^2 zt/2}\,dz\right]$$

where

$$\text{(A.14)}\qquad \lambda(z)=\frac{z^{1/2}}{\left(z+\beta^2/\sigma_b^4\right)\left(z+\beta^2/\sigma_b^4+2\beta/\sigma_b^2+1\right)}.$$

We now substitute eqn. (A.13) back into (10) evaluated at $x=u$, which yields:

$$W(u)=\frac{U\Lambda(\beta,\sigma_b,\delta+\gamma)}{(\delta+\gamma)},$$

where

(A.15)

$$\Lambda(\beta,\sigma_b,\delta+\gamma)\equiv\frac{2\beta}{\sigma_b^2+2\beta}+\frac{2(\delta+\gamma)}{\pi}\int_0^{\infty}\frac{\lambda(z)}{\sigma_b^2 z+\beta^2/\sigma_b^2+2(\delta+\gamma)}dz.$$

The first term in the expression for $\Lambda(\cdot,\cdot,\cdot)$ summarizes the impact of the various parameters on the ergodic mean, while the second term encompasses the transition from the value of $B(t)$ at entry and its unconditional (ergodic) mean. Clearly, the latter will be more important when firms give more weight to the short run, i.e., when $(\delta+\gamma)$ is large.

Given $\Lambda(\cdot,\cdot,\cdot)$, the value of U can be found, as before, from the free entry condition:

$$\text{(A.16)}\qquad U=F\frac{\delta+\gamma}{\Lambda(\beta,\sigma_b,\delta+\gamma)}.$$

We have again arrived at an expression for U (and thus the optimal investment rule), but this time by deriving the path for the expected marginal revenue product and utilizing the free entry condition.

B. *The Data and Calculation of* $b(t)$. Our raw database was originally developed by Brian K. Sliker at M.I.T., and is used with his permission. We calculate $b(t)$ based on (24) using two- and four-digit SIC data for the real value of output (*ROUTPUT*), real inputs of capital (*RK*), materials (*RMAT*), and labor hours

(*TOTHRS*), and the corresponding price deflators. *TOTHRS* is the sum of hours for production workers (*PWHRS*) and nonproduction workers (*NPWHRS*), where the latter is estimated as the product of nonproduction worker employment (*NPWEMP*) and average hours per employee for production workers (the mean of *PWHRS/PWEMP*).

We calculate the labor and materials shares by setting α_L and α_M equal to the mean values of *TLC/NOUTPUT* and *NMAT/NOUTPUT* respectively, where *TLC* is total labor costs, *NMAT* is the nominal value of materials inputs, and *NOUTPUT* is the nominal value of output. Letting $\phi = \alpha_L/(\alpha_L + \alpha_M)$, we then compute the Solow residual, s_t, as:

$$s_t = y_t - (1 - \alpha_K)h_t - \alpha_K k_t,$$

where $y_t = \log(ROUTPUT)$, $h_t = \phi l_t + (1 - \phi)m_t$, $l_t = \log(TOTHRS)$, $m_t = \log(RMAT)$, and $k_t = \log(RK)$. Finally, $b(t)$ is given by:

$$b(t) = \log\left[(1 - \alpha_K)^{(1-\alpha_K)/\alpha_K}\alpha_K\right] + (1/\alpha_K)s_t$$

$$- \frac{1 - \alpha_K}{\alpha_K}\left[\phi(\log TLC - l_t) + (1 - \phi)\log PMAT - \log POUTPUT\right].$$

REFERENCES

ABEL, A. B., "Optimal Investment Under Uncertainty," *American Economic Review* 73 (1983), 228–233.

BERNANKE, B. S., "Irreversibility, Uncertainty, and Cyclical Investment," *Quarterly Journal of Economics* 98 (1983), 85–106.

BERTOLA, G. AND R. CABALLERO, "Irreversibility and Aggregate Investment," *Review of Economic Studies* 207 (1994), 223–246.

CABALLERO, R. J., "Competition and the Non-Robustness of the Investment-Uncertainty Relationship," *American Economic Review* 91 (1991), 279–288.

DIXIT, A., "Hysteresis, Import Penetration, and Exchange-Rate Pass-Through," *Quarterly Journal of Economics* 104 (1989a), 205–228.

———, "Entry and Exit Decisions under Uncertainty," *Journal of Political Economy* 97 (1989b), 620–638.

———, "Irreversible Investment with Price Ceilings," *Journal of Political Economy* 99 (1991), 541–557.

———, "Investment and Hysteresis," *Journal of Economic Perspectives* 6 (1992), 107–132.

——— AND R. S. PINDYCK, *Investment Under Uncertainty* (Princeton: Princeton University Press, 1994).

HARTMAN, R., "The Effects of Price and Cost Uncertainty on Investment," *Journal of Economic Theory* 5 (1972), 258–266.

HOPENHAYN, H. A., "Entry, Exit, and Firm Dynamics in Long Run Equilibrium," *Econometrica* 60 (1992a), 1127–1150.

———, "Exit, Selection, and the Value of Firms," *Journal of Economic Dynamics and Control* 16 (1992b), 621–653.

KARLIN, S. AND H. M. TAYLOR, *A Second Course in Stochastic Processes* (New York: Academic Press, 1981).

LAMBSON, V. E., "Industry Evolution with Sunk Costs and Uncertain Market Conditions," *International Journal of Industrial Organization* 9 (1991), 171–196.

———, "Competitive Profits in the Long Run," *Review of Economic Studies* 59 (1992), 125–142.

LEAHY, J., "Investment in Competitive Equilibrium: The Optimality of Myopic Behavior," *Quarterly Journal of Economics* 108 (1993), 1105–1133.

——— AND T. WHITED, "Some New Evidence on the Relationship Between Investment and Uncertainty," unpublished, Harvard University, 1993.

LIPPMAN, S. A. AND R. P. RUMELT, "Industry-Specific Capital and Uncertainty," unpublished, UCLA, Sept. 1985.

MAJD, S. AND R. S. PINDYCK, "Time to Build, Option Value, and Investment Decisions," *Journal of Financial Economics* 18 (1987), 7–27.

MCDONALD, R. AND D. R. SIEGEL, "The Value of Waiting to Invest," *Quarterly Journal of Economics* 101 (1986), 707–728.

PINDYCK, R. S., "Irreversible Investment, Capacity Choice, and the Value of the Firm," *American Economic Review* 78 (1988), 969–985.

———, "Irreversibility, Uncertainty, and Investment," *Journal of Economic Literature* 29 (1991), 1110–1152.

———, "A Note on Competitive Investment Under Uncertainty," *American Economic Review* 83 (1993), 273–277.