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Jointly Produced Exhaustible Resources[1]

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Natural resources are often produced jointly from composite ores, which in turn are extracted from fixed reserve endowments. In this paper market behavior is examined for such resources, and it is shown how the price of each resource will depend on its demand, and the demands and storage costs for the other resources present in the ore. The measurement of resource scarcity is discussed and the effects of uncertainty over future resource demands are examined. It is shown that the competitive market will still extract, produce, and store at socially optimal rates if firms are risk neutral and the average cost of storage is constant. Policy implications are noted, particularly with reference to government stockpiling programs.

1. INTRODUCTION

A number of exhaustible resources are discovered and/or produced jointly. For example, oil and natural gas are discovered jointly and sometimes (in the case of "associated" natural gas) produced jointly, natural gas and helium are produced jointly, and a wide variety of minerals are found together and must be separated from the same ore deposits.[2] Perhaps of future importance, the deep seabed "manganese nodules" contain manganese, copper, nickel, and cobalt, and the economic viability of exploiting this resource will depend on the prices of these four metals and their joint production characteristics. In this paper the implications of joint production for resource market behavior and for government policy are examined.

A number of issues arise when exhaustible resources are jointly produce. First, even if extraction costs are constant, the net prices of the individual resources need not rise at the rate of interest. Instead the resource price trajectories—and hence any levels of private storage—are interdependent. An objective here is to characterize the behavior of prices and storage for jointly produced resources in unregulated markets.

Second, resource "scarcity" and the depletion of the reserve endowment have been a concern of policy. But conventional measures of resource "reserves" have limited economic meaning when resources are jointly produced, and the question arises as to how resource scarcity should be defined and measured.

Third, uncertainties may exist over the future demands for some of the individual resources. How will this affect market behavior, and will the competitive market extract, sell, and store resources at socially optimal rates when such uncertainties are present? If not, what is the proper role for government policy intervention?

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[2]See, for example, Harris and Skinner [7].

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This last issue is especially significant, since an important form of government intervention in resource markets has been the accumulation and management of "strategic" stockpiles. Currently the government owns and/or controls stockpiles of a wide variety of resources, many of which were originally implemented for defense preparedness. However, these stockpiles have also been used to "stabilize" prices, and now that most of them far exceed foreseeable military needs, arguments for their expansion are increasingly made on economic grounds.[3] In particular, it is argued that economic growth and changing technologies make the future demands for some of these resources highly uncertain. The question, however, is whether uncertainties are a cause of market failure, implying a need for government intervention.

Helium and its joint production with natural gas is just one example for which all of these issues are relevant, and for which the public policy problem is timely. Almost all helium is found in natural gas deposits in the United States, so that helium reserves will be exhausted once our natural gas reserves are exhausted. Producers of gas can separate the helium and sell it or store it, or the helium can be dispersed into the atmosphere when the gas is purified or burned. Currently helium demand is small, but some geologists and scientists argue that changing technologies may make future demand much larger. It is this uncertainty over future helium demand that led to the government's Helium Conservation Program, under which the Bureau of Mines purchases and stores helium. Government storage in 1978 was about 40 billion cubic feet, or some 60 times that year's consumption. Under debate is whether the stockpile is sufficient, or should be greatly increased in size.[4] This paper tries to provide an analytical background for that debate, and others like it.

In the next section we develop a simple deterministic model for jointly produced resources, and show how competitive prices, production levels, and sales depend on relative demands and storage costs. In Section 3 we argue that resource "rent" is the appropriate measure of *in situ* scarcity, and we show how rents can be defined and calculated for the composite ore and the constituent resources. Section 4 is concerned with uncertainties over future resource demands; we show that competitive resource exploitation is still socially optimal if firms are risk neutral, and although deviations from social optimality may exist if firms are risk averse, their direction is ambiguous. In the last section the results of the paper are summarized and their policy implications are discussed.

2. A SIMPLE MODEL OF JOINT RESOURCE PRODUCTION

We begin with a model describing a market that is competitive at all levels: ownership and extraction of reserves of composite "ore," joint production (or "separation") of the individual resources from extracted ore, and storage and/or

[3] For an overview of government policy in this area, and a discussion of some of the economic issues, see the report by the Office of Technology Assessment [17].

[4] Of course it is not even clear that the government should be storing any helium, since private firms can (and do) store helium themselves in the same Cliffside field used for storage by the government, so there are no diseconomies of small scale. For an overview of the helium conservation problem, see the Helium Study Committee Report [10]; and for an economic analysis (based on a deterministic model), see Epple and Lave [5].

sales of the resources.[5] We assume that the composite ore is extracted at an average and marginal cost $c(R)$ that is a decreasing function of the (known) reserve level R. We also assume that after extraction the ore is "separated" to yield the individual resources, which may be stored. Finally, we assume that the cost of producing resource i at a rate x_i depends on the rate of ore extraction (and consumption) q, but not on the production rates x_j, $j \neq i$. In particular, there are n cost functions $C_i(x_i, q)$ with smooth and continuous derivatives $\partial C_i/\partial x_i > 0$, $\partial^2 C_i/\partial x_i^2 > 0$, $\partial C_i/\partial q < 0$, $\partial^2 C_i/\partial q^2 > 0$, $\partial^2 C_i/\partial q\, \partial x_i < 0$, $(\partial/\partial q)(C_i/q) < 0$, and $C_i(0, q) = 0$, $C_i(x_i, 0) = \infty$.[6]

In this model one can think of resources as being consecutively separated from the ore, and separation can be complete or incomplete, with one or more individual resources partially or totally discarded. (This is indeed the case for most minerals, and for natural gas and helium.) These assumptions permit considerable generality in the specification and estimation of cost functions for particular resource markets. A useful functional form that satisfies the assumptions, for example, is

$$C_i(x_i, q) = c_i x_i + x_i(x_i/\beta_i q)^m.$$

Note that for large values of m this approximates a pure fixed proportions technology, with β_i the amount of resource i that can be recovered from 1 unit of ore. For smaller values of m, this cost function says that varying amounts of resource i can be recovered from 1 unit of ore, but the average cost rises as the amount recovered rises.

Letting p_i, q_i, S_i, and k_i be price, sales, storage, and storage cost, respectively, for resource i, the competitive equilibrium is given by

$$\underset{(q, x_1, \dots, x_n, q_1, \dots, q_n)}{\text{Max}} \int_0^\infty \left[\sum_i p_i q_i - c(R)q - \sum_i C_i(x_i, q) + \sum_i k_i S_i \right] e^{-rt}\, dt \quad (1)$$

subject to

$$\dot{R} = -q, \; R(0) = R_0 \quad (2)$$

$$\dot{S}_i = x_i - q_i, \; S_i(0) = 0, \; i = 1, \dots, n \quad (3)$$

and R, q, x_i, q_i, $S_i \geqslant 0$. Since the industry is assumed competitive, the maximization is done with the p_i's taken as given, but in equilibrium the p_i's and q_i's satisfy the market demand conditions $q_i = q_i(p_i)$, $i = 1, \dots, n$.

[5] We use the word "ore" loosely here. It can refer to a mineral deposit containing a number of compounds in various concentrations for which metals or other resources can be refined, or it can refer to a composite reserve of, say, natural gas and helium. Also, the model ignores common access problems. The reader can assume that reserves of ore are unitized, or that they are socially managed, or that individual producers own identical reserves and face identical marginal extraction costs $c_i(R_i)$, so that $c(R)$ applies in the aggregate. (Note that this aggregation issue disappears if marginal extraction cost is constant, i.e., $c'(R) = 0$.)

[6] Note that with $c(R) > 0$ there are economies of scope, as defined in Willig [21]. This occurs not because of a sharing of facilities or management in producing $x_1, \dots, x_n$ but because of a "sharing" of composite ore.

The Lagrangian for this problem is

$$L = \left[\sum_i p_i q_i - c(R)q - \sum_i C_i(x_i, q) - \sum_i k_i S_i\right] e^{-rt}$$
$$- \lambda q + \sum_i \mu_i (x_i - q_i) + \sum_i \theta_i S_i \tag{4}$$

with $\theta_i = 0$ if $S_i > 0$, $\theta_i \geqslant 0$ if $S_i = 0$. Maximizing L with respect to q gives

$$\lambda = \left[-\sum_i (\partial C_i/\partial q) - c(R)\right] e^{-rt}. \tag{5}$$

Note that λ is the discounted shadow price of a unit of *in situ* ore. Now define

$$\tilde{p} = -\sum_i (\partial C_i/\partial q). \tag{6}$$

so that $\lambda e^{rt} = \tilde{p} - c(R)$. Since $\dot{\lambda} = -\partial L/\partial R = c'(R)qe^{-rt}$, $\tilde{p}$ satisfies the usual rule:

$$\dot{\tilde{p}} = r[\tilde{p} - c(R)]. \tag{7}$$

Observe that $\tilde{p}$ is the total value of a marginal unit of extracted ore, and λe^{rt} is the rent component of that value. In other words, $\tilde{p}$ is the market price at which competitive mining firms would sell to competitive processing firms that separate out the individual resources; equivalently, it is the *transfer price* that vertically integrated firms should use to value the ore. Put another way, given the competitive production levels $x_1, \ldots, x_n$, $\tilde{p}$ is the marginal benefit to producers from a unit of ore as measured by the total production costs savings resulting from that unit.[7]

The remaining first-order conditions are

$$-(\partial C_i/\partial x_i)e^{-rt} + \mu_i = 0, \tag{8}$$

and

$$p_i e^{-rt} - \mu_i = 0, \tag{9}$$

so that

$$p_i = \partial C_i/\partial x_i, \qquad i = 1, \ldots, n \tag{10}$$

[7]Ignoring storage for simplicity so that $x_i = q_i$, the added profit to producers of the n resources from an extra unit of ore is

$$d\Pi/dq = \sum_i [p_i - \partial C_i/\partial q_i](dq_i/dq) - \sum_i (\partial C_i/\partial q).$$

Competitive producers will set q_i so that p_i equals marginal cost $\partial C_i/\partial q_i$, and therefore would pay the price $\tilde{p} = -\Sigma_i(\partial C_i/\partial q)$ for the ore. Also, note that if extraction cost is constant, (7) is just the Hotelling [12] r-percent rule.

that is, the price of each resource is equal to its marginal production cost, given the optimal input of ore.

Differentiating (9) with respect to time and combining with $\dot{\mu}_i = -\partial L/\partial S_i = k_i e^{-rt} - \theta_i$ gives the standard condition for the price of a storable good in a competitive market:

$$\dot{p}_i \leqslant rp_i + k_i, \qquad i = 1,\ldots, n. \tag{11}$$

Note that (11) holds with equality if $S_i > 0$, that is, with storage available at constant average cost, the competitive rate of capital gain on any stored unit will just equal the total cost of holding the unit, namely the interest cost plus the direct storage cost.

A set of terminal conditions completes the description of competitive market behavior. Let T be the time that extraction of ore ceases, and $T_i \geqslant T$ be the times at which sales q_i cease. Then $\tilde{p}(T) - c[R(T)] = q(T) = x_i(T) = 0$, and $q_i(T_i) = S_i(T_i) = 0$, $i = 1,\ldots, n$ with $T_i > T$ only if $S_i(T) > 0$.[8]

In summary, market behavior is described by a total of $2n + 2$ differential equations and $2n + 1$ static identities and behavioral equations that can be solved for the following $4n + 3$ variables: $\tilde{p}$, q, R, x_i, q_i, p_i, S_i. The differential equations are (2), (3), (7), and (11) and the remaining equations are (6), (10), and the demand functions $q_i = q_i(p_i)$. Finally, if storage is zero for $m \leqslant n$ resources, (3) and (11) are dropped for $i = 1,\ldots, m$ and replaced with $x_i = q_i$.

Note that the extent to which the demand for one resource will affect the price behavior of the other resources depends strongly on the availability and cost of storage. Consider a composite containing one resource in high demand (e.g., natural gas), and a second resource whose demand today is small but very inelastic, or is expected to be much larger in the future (e.g., helium). Suppose storage of the second resource is costly. Then relative to what would prevail if they could be produced independently, jointness of production will reduce the current price of the second resource and speed up its exhaustion, and at the same time will raise the current price of the first resource and delay its exhaustion. The cheaper the cost of storage, the more this effect is reduced.

The interrelationships of prices and production levels are shown graphically in Figs. 1a and 1b for the example of natural gas (q_1) and helium (q_2). In the figures we assume no storage, so that $x_i = q_i$. Observe that as depletion of the natural gas–helium composite ensues, the marginal cost curves $\partial C_i/\partial q_i$ move to the left. The price of natural gas keeps rising, but the price of helium remains constant for the first several periods since the marginal cost curve $\partial C_2/\partial q_2$ is flat at its intersection with the demand curve. (Because of natural gas demand, a large quantity of the composite is being extracted relative to what is needed for helium production.) Only in the later periods (as exhaustion of the composite nears) does the price of helium rise, and then it rises rapidly.

Of course this provides an incentive to store helium. If helium could be stored cheaply, then its production would initially be much larger (with most going into storage), and sales would be somewhat smaller, with price starting out higher and continually rising at a percentage rate $r + k_2/p_2$. Also, since production of the

[8]These conditions hold if the market demand functions $q_i(p_i)$ intersect the price axes at finite points $\bar{p}_i = \partial C_i(x_i, 0)/\partial x_i < c(0)$. Otherwise R, q, and the q_i's can all approach zero asymptotically.

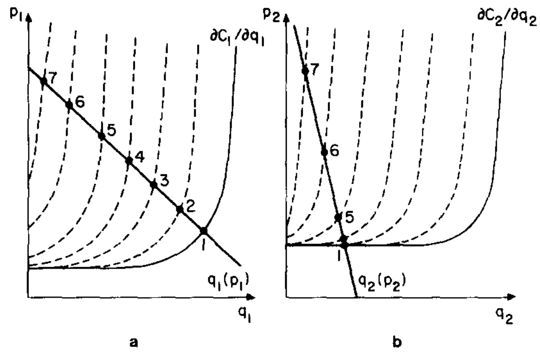

FIG. 1. (a) Natural gas and (b) helium.

composite would be initially greater, the price of natural gas would rise more rapidly from a lower initial level.

What does this model say about the desirability of government storage programs? Under our assumption that the average cost of storage is constant, the government can store resources no more cheaply than private firms. In this case it is easy to show that competitive market behavior as described above is socially optimal, in the sense of maximizing the present value of the flow of consumer plus producer surplus. There is then no role for a government storage program, or any other form of intervention.

In the case where a producer is a monopolist supplier of some or all of the n resources, the transfer price $\tilde{p}$ again follows Eq. (7). However, assuming the demand functions are independent, for the monopolist-supplied resources, Eqs. (10) and (11) are replaced by

$$\mathrm{MR}_i = \partial C_i / \partial x_i \tag{10'}$$

and

$$\dot{\mathrm{MR}}_i \leqslant r\,\mathrm{MR}_i + k_i. \tag{11'}$$

Although $\tilde{p}$ will still follow the r-percent rule, the composite ore will be extracted more slowly when there is monopoly power in markets for the produced resources, and $\tilde{p}$ begins rising from a *lower* initial level. This occurs because monopoly power reduces the marginal value (to the monopolist) of a unit of ore as an input to resource production. Since the monopolist will produce the resources more slowly, he views the *in situ* ore as less scarce.[9]

3. MEASURING RESOURCE SCARCITY

Economists, geologists, and resource consumers have all been concerned with measuring the scarcity of various natural resources. Unlike the estimates of "proved," "probable," or "potential" reserves used by geologists, resource rent (price—or

[9]Clark and Munro [3] demonstrate an analogous result for a renewable resource market. They show how *monopsony* power in the processing sector reduces the rate of harvesting from a fishery, and they consider whether that reduced rate will be close or equal to the socially optimal rate.

marginal revenue in a monopolistic market—net of marginal extraction cost) is a logical measure of *in situ* scarcity because it represents the opportunity cost of extracting a unit of resource.[10] But how should rent be calculated for resources that are found and produced jointly?

First note that for the composite ore, rent is clearly given by $\lambda e^{rt} = \tilde{p} - c(R)$, as in (5) and (6). Denote the full cost of producing all n resources by $C = c(R)q + \Sigma_i C_i(x_i, q)$. Then the full marginal cost of producing a unit of resource j is

$$\mathrm{MC}_j = \frac{\partial C}{\partial q}\frac{dq}{dx_j} + \frac{\partial C}{\partial x_j} = c(R)\frac{dq}{dx_j} + \frac{dq}{dx_j}\sum_i \frac{\partial C_i}{\partial q} + p_j. \tag{12}$$

Thus the rent associated with resource j is just

$$h_j = p_j - \mathrm{MC}_j = (\tilde{p} - c)\frac{dq}{dx_j}, \tag{13}$$

that is, the rent on the ore times the amount of ore needed to (efficiently) produce one extra unit of the resource.[11]

Calculating the rent h_j requires three numbers: the "yield" of resource j per unit of ore, the marginal extraction cost for the ore, and the market or transfer price of the ore $\tilde{p}$. If data on $\tilde{p}$ is not available, the cost functions $C_i(x_i, q)$ can be estimated and then, using data for $x_i, \ldots, x_n$, and q, $\tilde{p}$ can be calculated from (6). Alternatively, an optimizing process model describing resource production can be used to estimate $\tilde{p}$, again from (6). A process model would also yield estimates of the *efficient* yields dx_i/dq.

Equation (13) also serves to illustrate why estimates of potential reserves, crustal abundance, and related "scarcity" measures have little economic meaning. In particular, the equation shows how *in situ* scarcity for any one resource depends in part on the demands and storage costs for the other resources in the composite ore. To see this, consider again the example of natural gas (x_1) and helium (x_2).

Suppose the current demand for natural gas increases. This increases the price of natural gas, but it will also increase the price $\tilde{p}$ and rent for the composite, and reduce the efficient yield dx_2/dq. As can be seen from (13), the result is an *increased scarcity of helium*. Although the demand for helium is the same, the increased demand for natural gas means the composite will be extracted more rapidly, so that *in situ* helium is more scarce.

Similarly, *in situ* scarcity is also dependent on storage opportunities and costs for the produced resources. For example, suppose a new technology is developed that allows firms to store helium cheaply. This will reduce the price $\tilde{p}$ and rent for the

[10] A measure of resource scarcity should reflect the present value of current and future sacrifices required to obtain in a unit of resource. As explained in Brown and Field [1] and Fisher [6], rent provides such a measure in an *in situ* context. As shown in Pindyck [18], when there is exploration, rent is equal to marginal discovery cost plus the opportunity cost of an additional unit of cumulative discoveries.

[11] Rent is an appropriate scarcity measure only when referring to the resource *in situ*. If we refer to the resource as a consumption good, price is the appropriate scarcity measure. In fact, as shown by Heal [8], Pindyck [18], and Fisher [6], if extraction costs rise rapidly enough as depletion ensues, rent can fall over time. This just means that the opportunity cost of resource extraction is falling because resource use is decreasing as extraction costs—and price—rise, so that the resource is indeed becoming less scarce *in situ*.

composite, and also increases the efficient yield dx_2/dq, thereby *reducing the scarcity of helium*—even though that scarcity is measured on an *in situ* basis.

Finally, *in situ* scarcity will be affected by government policies. For example, government *subsidies* for helium stockpiling will increase the price $\tilde{p}$ and increase the efficient yield dx_2/dq, thereby increasing the scarcity of natural gas, but having an ambiguous effect on the scarcity of helium.

4. DEMAND UNCERTAINTY

For many resources future demands are subject to considerable uncertainty. In an earlier paper [19], I examined the implications of demand (and reserve) uncertainty for resource exploration and production, but in the context of an individually produced resource. In the case of jointly produced resources there are two questions of interest. First, how do uncertainties over future demands for one or more of the resources affect the expected evolution of prices and production for all of the resources in the composite ore? Second, might such uncertainties imply that competitive market exploitation of the resources is not socially optimal, so that some kind of government intervention is called for? Note that this second question is at the heart of the public policy debate over helium conservation and resource stockpiling in general—does the considerable uncertainty over *future* demand justify government subsidies for helium separation and storage?

Our analysis of uncertainty follows the same approach as in Pindyck [19]. In particular we write the market demand functions as

$$p_i = p_i[q_i, y_i(t)], \qquad i = 1, \ldots, n \tag{14}$$

with $\partial p_i/\partial q_i < 0$, and $\partial p_i/\partial y_i > 0$. Here the $y_i(t)$ are continuous-time stochastic processes of the form

$$dy_i = \alpha_i y_i dt + \sigma_i y_i dz_i, \qquad i = 1, \ldots, n \tag{15}$$

and $z_i(t)$ is a Wiener process, that is, $dz_i = \varepsilon_i(t)\sqrt{dt}$, where $\varepsilon_i(t)$ is a serially uncorrelated normal random variable with zero mean and unit variance, so that $E[(dz_i)^2] = dt$. We also assume that the dz_i are uncorrelated, that is, $E[dz_i dz_j] = 0$ for $i \neq j$. Equation (15) therefore implies that current demands are known, that random fluctuations in demand occur continuously over time,[12] and that uncertainty about future demand grows with the time horizon.[13]

If firms are risk neutral, they maximize the expected value of the integral in (1), again subject to (2) and (3), but also subject to the stochastic differential equations

[12] The assumption that random fluctuations in demand occur continuously over time is justified here given that new technologies (that might alter resource demands) may appear suddenly, but are usually commercialized slowly.

[13] In my earlier paper [19], I wrote demand as $p = y(t)f(q)$, so (14) generalizes this by allowing the stochastic growth variables to enter demand additively, multiplicatively, or nonlinearly. Equation (15) is the limiting form as $h \to 0$ of the difference equation $[y_i(t+h) - y_i(t)]/y_i(t) = \alpha_i h + \sigma_i \varepsilon_i(t)\sqrt{h}$, with $E[\Delta y_i/y_i] = \alpha h$ and $\mathrm{Var}[\Delta y_i/y_i] = \sigma_i^2 h$. Note that $y_i(t)$ is log-normally distributed, with $E_0[\log(y_i(t)/y_i(0))] = (\alpha_i - \frac{1}{2}\sigma_i^2)t$, and $\mathrm{Var}[\log(y_i(t)/y_i(0))] = \sigma_i^2 t$. Generally we would expect $\alpha_i > 0$ so that demand has a positive deterministic drift as a result, say, of economic growth. For an introduction to stochastic processes of the form of (15), see Chapter 5 of Cox and Miller [4].

(15), and the conditions $R, q, x_i, q_i, S_i \geqslant 0$. In the Appendix we derive the interior solution to this stochastic optimization problem, based on the assumption that the inequality $q > 0$ holds up until an (unknown) terminal extraction time T.[14] There we show that the transfer price $\tilde{p}$ follows the r-percent rule in expected value terms, and competitive extraction, production, and storage are at socially optimal rates.

What if resource producers are risk averse, in that they maximize the expected integral of discounted utility $U[\Pi(t)]$, with $U' > 0$, $U'' < 0$? As shown in the Appendix, the competitive rates of extraction, production, and storage will now differ from the socially optimal rates, but in ways that depend on the utility function $U[\Pi]$ and the demand functions $p_i[q_i, y_i]$. If U is quadratic and the p_i's are linear in the y_i's *expected* demand growth (reflected in the deterministic drift parameters α_i) will cause competitive rates of extraction, production, and sales to be initially higher than the socially optimal rates. But *uncertainty* over demand growth (reflected in the variance parameters σ_i) will affect these rates only if marginal utility is nonlinear, or demands are nonlinear in the growth variables y_i. Further, if $U''' > 0$, demand uncertainty *reduces* initial rates of extraction and sales, leading to *overconservation*.[15] And $\partial^2 p_i/\partial y^2 < (>)0$ reinforces (tends to offset) this effect, so that the net effect of uncertainty can be ambiguous. Since there is little evidence that producers are indeed risk averse, and since risk aversion can have ambiguous effects (possibly leading to overconservation), it appears as a weak argument for underconservation by a competitive market.

5. CONCLUSIONS

We have seen that the competitive market price of extracted composite ore—or equivalently for integrated firms, the properly measured transfer price of the ore—will follow the usual r-percent rule. However, the prices of the individual resources need not follow an r-percent rule, whether or not storage is available. As shown by the example of Figures 1a and 1b, the relative demand functions for the resources might be such that the prices of one or more of them remain fixed over a period of time. Accounting for jointness of production might thus help explain some of the negative results that have been obtained in recent empirical tests of the Hotelling model.[16]

[14] This is a nontrivial assumption that greatly simplifies the analytical approach in the Appendix. We do not allow, for example, extraction to cease after drops in demand occur and then begin again if demand later rises. If this assumption is not made, that is, if resource owners can "sit" on reserves as an option on possible future production (should price rise), the rent associated with the composite ore will grow in expected value terms at a rate less than r. The competitive market, however, will still extract the ore at a socially optimal rate if producers are risk neutral. This is discussed in Pindyck [20].

[15] Uncertainty over future demands creates a "precautionary" reduction in current rates of extraction and sales if marginal utility is convex for much the same reason that a "precautionary" demand for savings in the presence of future income uncertainty requires a convex marginal utility. See Leland [14] for an analogous discussion of savings under uncertainty.

[16] See, for example, Heal and Barrow [9], and note that many of the resources they examined are metals jointly produced from mineral ores. Of course there may be other explanations for the long-term constant, declining, and U-shaped price profiles observed for many resources: the increased use of resource substitutes resulting from technological change; the durability of some resources, as discussed in Levhari and Pindyck [15]; and the process of reserve discovery and development, as discussed in Pindyck [18].

We have also seen that uncertainties over future resource demands do not imply a need for government intervention in a competitive market, and this has obvious policy implications, as exemplified by the debate over helium conservation. As Koopmans [13] observed, that debate has been noteworthy for being almost devoid of economic analysis or economic reasoning, and instead has focused largely on arguments over future demand projections. One recent study that does apply economic analysis to this problem is by Epple and Lave [5] who determined the optimal rate of helium production and storage (private or public) over time that would maximize consumer surplus net of cost. They took the rate of natural gas production to be exogenous and ignored uncertainty over future helium demand. In that context they found no justification for government involvement other than operating the Cliffside storage facility for private storage (so that average storage costs would indeed be constant for all firms). Of course uncertainty over future helium demand is indeed considerable, and this has been used as the major justification for a government program. But our results show that such justification is unwarranted, and thereby support and extend the policy conclusions of Epple and Lave.

As for stockpiles of other resources, some of them may indeed be justified for military needs, which create public good characteristics. But the policy question today is largely over whether these stockpiles should be increased in size *beyond* the military requirements. We find little economic justification for an affirmative answer to this question.[17]

Of course the model developed in this paper is a simple one, and ignores a number of the realities that characterize resource markets. For example, we have assumed that the ore is homogeneous, whereas in fact the constituent products of many ores vary widely across deposits. Also, we have considered jointness only in the separation and production of individual resources from a composite ore, whereas many resources are also (or instead) *discovered* jointly. A model that shows the interrelationship of resource exploration and production was developed in Pindyck [18] and it could be extended to include jointness in both the exploration and production processes. (In fact, it is just such a framework that is needed to model exploratory activity, reserve accumulation, and production for natural gas and oil.) Here we have characterized markets for jointly produced resources in only the most basic way.

APPENDIX

The Model with Demand Uncertainty

We analyze the model with demand uncertainty presented in Section 4. For risk-neutral firms the problem is to maximize the expected value of the integral in (1), subject to (2), (3), and (15), and with the integration up to an (unknown) terminal time T at which extraction ceases. We treat the problem as one in stochastic dynamic programming, following the approach in my earlier paper [19].

[17]Again, the optimality of the competitive market depends on the availability of storage at constant average cost. But where there are scale economies, the government can operate a storage facility without subsidizing the storage itself. This may be the preferred way to develop an oil stockpile. For a discussion of oil stockpiling and oil supply disruptions, see Hogan [11].

Letting $\Pi_d(t)$ be the integrand in (1), define the optimal value function

$$J = J(R, S_1, \dots, S_n, y_1, \dots, y_n, t) = \max_{(q, x_1, \dots, q_1, \dots)} E_t \int_t^T \Pi_d(\tau)\, d\tau. \tag{A.1}$$

The fundamental equation of optimality is[18]

$$0 = \max_{(q, x_1, \dots, q_1, \dots)} \Big\{ \Pi_d(t) + J_t - qJ_R + \sum_i (x_i - q_i) J_{S_i} + \sum_i \alpha_i y_i J_{y_i} + \tfrac{1}{2} \sum_i \sigma_i^2 y_i^2 J_{y_i y_i} + \sum_i \theta_i S_i \Big\} \tag{A.2}$$

with $\theta_i \geqslant 0$, $S_i \geqslant 0$. The first-order conditions are

$$\partial \Pi_d / \partial q = J_R \tag{A.3}$$

$$\partial \Pi_d / \partial x_i = -J_{S_i}, \qquad i = 1, \dots, n \tag{A.4}$$

$$\partial \Pi_d / \partial q_i = J_{S_i}, \qquad i = 1, \dots, n. \tag{A.5}$$

Next differentiate (A.2) with respect to R, and utilizing Ito's Lemma write the resulting equation as

$$\partial \Pi_d / \partial R + (1/dt) E_t d(J_R) = 0. \tag{A.6}$$

Now apply the differential generator $(1/dt)E_t d(\ )$ to both sides of (A.3) and combine with (A.6):

$$(1/dt) E_t d(\partial \Pi_d / \partial q) = -\partial \Pi_d / \partial R = c'(R) q e^{-rt}. \tag{A.7}$$

Similarly, differentiate (A.2) with respect to S_i, apply the differential generator $(1/dt)E_t d(\ )$ both both sides of (A.5), and combine to yield

$$(1/dt) E_t d(\partial \Pi_d / \partial q_i) + \partial \Pi_d / \partial S_i + \theta_i = 0. \tag{A.8}$$

Finally, note that (A.4) and (A.5) together imply

$$p_i = \partial C_i / \partial x_i, \qquad i = 1, \dots, n \tag{A.9}$$

as in the deterministic case.

To obtain the expected dynamics of price, substitute the partial derivatives of Π_d into (A.7) and (A.8):

$$(1/dt) E_t d\tilde{p} = r[\tilde{p} - c(R)] \tag{A.10}$$

and

$$(1/dt) E_t dp_i \leqslant rp_i + k_i, \qquad \text{with } S_i \geqslant 0. \tag{A.11}$$

Thus the deterministic results hold in expected value terms; the expected rate of

[18]Subscripts denote partial derivatives, for example, $J_R = \partial J / \partial R$. For an introduction to the techniques used in this Appendix, see Chow [2] and Merton [16].

change of the composite ore price $\tilde{p}$ follows the r-percent rule, and storage opportunities limit expected increases in resource prices to marginal storage and holding costs. Furthermore, by replacing the integrand in (1) with the discounted sum of consumer and producer surplus, it is easy to show that the competitive market extracts the ore and produces, stores, and sells the individual resources at socially optimal rates.

Now suppose that firms are risk averse. Then it is easy to show that (A.7)–(A.9) again hold, but with Π_d replaced by $U_d = U(\Pi)e^{-rt}$. However, the stochastic differentials $d(\partial U_d/\partial q)$ and $d(\partial U_d/\partial q_i)$ in (A.7) and (A.8) must be expanded using Ito's Lemma. Doing this gives the following equations analogous to (A.10) and (A.11):

$$\begin{aligned}\frac{1}{dt}E_t d\tilde{p} = {} & r(\tilde{p}-c) + (\tilde{p}-c)\frac{U''(\Pi)}{U'(\Pi)}\left[\sum_j k_j(x_j - q_j) - \sum_j \alpha_j y_j(\partial p_j/\partial y_j)q_j\right] \\ & - \tfrac{1}{2}(\tilde{p}-c)\frac{U''(\Pi)}{U'(\Pi)}\sum_j \sigma_j^2 y_j^2(\partial^2 p_j/\partial y_j^2)q_j \\ & - \tfrac{1}{2}(\tilde{p}-c)\frac{U'''(\Pi)}{U'(\Pi)}\sum_j \sigma_j^2 y_j^2(\partial p_j/\partial y_j)^2 q_j^2 \end{aligned} \tag{A.12}$$

$$\begin{aligned}\frac{1}{dt}E_t dp_i \leqslant {} & rp_i + k_i + p_i\frac{U''(\Pi)}{U'(\Pi)}\left[\sum_j k_j(x_j - q_j) - \sum_j \alpha_j y_j(\partial p_j/\partial y_j)q_j\right] \\ & - \tfrac{1}{2}p_i\frac{U''(\Pi)}{U'(\Pi)}\sum_j \sigma_j^2 y_j^2(\partial^2 p_j/\partial y_j^2)q_j \\ & - \tfrac{1}{2}p_i\frac{U'''(\Pi)}{U'(\Pi)}\sum_j \sigma_j^2 y_j^2(\partial p_j/\partial y_j)^2 q_j^2. \end{aligned} \tag{A.13}$$

Observe that if $\partial^2 p_j/\partial y_j^2 = 0$ for all j and $U''' = 0$, risk aversion implies a change in the expected rates of increase of $\tilde{p}$ and the p_j's, but because of the rates of deterministic demand growth α_j, and *not* because of uncertainty over demand growth. Expected increase in prices will imply, *relative to the risk-neutral case*, faster rates of ore extraction and resource production and consumption (and therefore lower initial prices), because of the incentive to shift profits to periods of higher marginal utility. *Uncertainty* over future demand affects extraction and production rates only if $U''' \neq 0$ or $\partial^2 p_j/\partial y_j^2 \neq 0$ for some j. Further, if the utility function is well behaved so that $U''' > 0$, this *reduces* the expected rates of increase of $\tilde{p}$ and the p_j's, which means *slower* rates of ore extraction and resource production and consumption. If $\partial^2 p_j/\partial y_j^2 < (>)0$, this effect is reinforced (at least partially offset). Thus the net effects of risk aversion can be ambiguous.

REFERENCES

1. G. M. Brown and B. C. Field, Implications of alternative measures of natural resource scarcity, *J. Polit. Econ.* **86**, 229–243 (1978).
2. G. C. Chow, Optimal control of stochastic differential equation systems, *J. Econ. Dyn. Control* **1**, 134–175 (1979).

3. C. W. Clark and G. R. Munro, Fisheries and the processing sector: Some implications for management policy, *Bell J. Econ.* **11**, 603–616 (1980).
4. D. R. Cox and H. D. Miller, "The Theory of Stochastic Processes," Chapman and Hall, London (1965).
5. D. Epple and L. Lave, Helium: Investments in the future, *Bell J. Econ.* **11**, 617–630 (1980).
6. A. C. Fisher, Measures of natural resource scarcity, *in* V. K. Smith, "Scarcity and Growth Reconsidered" (V. K. Smith, Ed.), Johns Hopkins Univ. Press, Baltimore (1979).
7. D. P. Harris and B. J. Skinner, "The Assessment of Long-Term Supplies of Minerals," unpublished, (1979).
8. G. Heal, The relationship between price and extraction cost for a resource with a backstop technology, *Bell J. Econ.* **7**, 371–378 (1976).
9. G. Heal, and M. Barrow, "Empirical Investigation of the Long-Term Movement of Resource Prices: A Preliminary Report," draft paper, Univ. of Sussex (June 1979).
10. Helium Study Committee, "Helium: A Public Policy Problem," National Research Council, Washington, D.C. (1978).
11. W. W. Hogan, Import management and oil emergencies, *in* "Energy and Security" (D. A. Deese and J. S. Nye, Eds.), Ballinger, Cambridge (1981).
12. H. Hotelling, The economics of exhaustible resources, *J. Polit. Econ.* **39**, 137–175 (1981).
13. T. C. Koopmans, Economics among the sciences, *Amer. Econ. Rev.* **69**, 1–13 (1979).
14. H. E. Leland, Saving and uncertainty: The precautionary demand for saving, *Quart. J. Econ.* **82**, 465–473 (1968).
15. D. Levhari, and R. S. Pindyck, The pricing of durable exhaustible resources, *Quart. J. Econ.* **96**, 365–377 (1981).
16. R. C. Merton, Optimum consumption and portfolio rules in a continuous-time model, *J. Econ. Theor.* **3**, 373–413 (1971).
17. Office of Technology Assessment, "An Assessment of Alternative Economic Stockpiling Policies," Report No. OTA-M-36 (August 1976).
18. R. S. Pindyck, The optimal exploration and production of nonrenewable resources, *J. Polit. Econ.* **86**, 841–861 (1978).
19. R. S. Pindyck, Uncertainty and exhaustible resource markets, *J. Polit. Econ.* **88**, 1203–1225 (1980).
20. R. S. Pindyck, The optimal production of an exhaustible resource when price is exogenous and stochastic, *Scand. J. Econ.* **83**, 277–288 (1981).
21. R. D. Willig, Multiproduct technology and market structure, *Amer. Econ. Rev.* **69**, 346–351 (1979).