A Note on Competitive Investment under Uncertainty

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Uncertainty over future output prices or input costs can affect investment by a risk-neutral firm in two opposing ways. First, it can increase the value of the marginal unit of capital, which leads to more investment. This only requires that the stream of future profits generated by the marginal unit be a convex function of the stochastic variable; by Jensen's inequality, the expected present value of that stream is increased. This result was demonstrated by Richard Hartman (1972) and later extended by Andrew Abel (1983) and others. In their models, constant returns to scale and the substitutability of capital with other factors ensure that the marginal profitability of capital is convex in output price and input costs. Even with fixed proportions, however, this convexity can result from the ability of the firm to vary output, so that the marginal unit of capital need not be utilized at times when the output price is low or input costs are high.[1]

If investment is irreversible and can be postponed, a second effect of uncertainty is to create an opportunity cost of investing now, rather than waiting for new information to arrive before committing resources. This increases the full cost of investing in a marginal unit of capital, which reduces investment.[2] Hence the net effect of uncertainty on irreversible investment depends on the size of this opportunity cost relative to the increase in the value of the marginal unit of capital. The sign of the net effect is significant because of its possible policy implications. Pindyck (1988) and Giuseppe Bertola (1989) developed models in which a firm faces a downward-sloping demand curve and found the net effect to be negative.

Ricardo Caballero (1991) has recently studied investment by an individual firm facing a stochastically shifting demand curve. He shows that, as returns to scale become constant and the elasticity of demand faced by the firm rises, this opportunity cost of investing now rather than waiting approaches zero. Thus, in the limit of constant returns and an infinitely elastic demand curve, an increase in uncertainty tends to raise rather than decrease current investment, even if that investment is irreversible.

It is important to stress that Caballero's limiting result treats the firm in isolation and describes the effect of a *mean-preserving* increase in price uncertainty, and not an increase in *industry-wide* demand uncertainty. This note shows that, in a competitive market, the key interactions between irreversibility and uncertainty occur at the industry level and can only be understood by making price and industry output endogenous.[3] Doing so restores the positive opportunity cost associated with irreversible investment.

In addition, Caballero's result is based on a model with convex adjustment costs. An innovative aspect of the model is that these costs can be asymmetric, thereby allowing for partial or complete irreversibility; complete irreversibility corresponds to a cost of

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[1]Then the marginal profit of capital at each time t in the future is $\max[0,(p_t - c_t)]$, where c_t is variable cost. Thus, a unit of capital represents a set of all options on future production, which are worth more the greater the variance of p_t or c_t.

[2]For a detailed discussion of this point and a survey of the recent literature on irreversibility and its implications for investment and market evolution, see Pindyck (1991).

[3]Studies that make price endogenous include Steven Lippman and Richard Rumelt (1985), Avinash Dixit (1989, 1991), and John Leahy (1990).

downward adjustment that is infinite. The model is particularly useful in that it allows one to study the sensitivity of investment to the extent of asymmetry in adjustment costs. However, as with other models of this kind, the size of the firm would be unbounded were it not for adjustment costs: it is only adjustment costs that determine firm size. This role of adjustment costs is crucial to the results of Caballero and earlier authors regarding the effect of uncertainty on investment. This note also shows that, while helpful for studying the behavior of a firm in isolation, this adjustment-cost framework (used widely in empirical studies of investment) is inconsistent with a competitive market equilibrium and, hence, with the behavior of a competitive firm.

I. Adjustment Costs, Competition, and Irreversible Investment

A firm that has constant returns to scale everywhere and faces an infinitely elastic demand curve will have a profit function that is linear in the capital stock. Hence, convex costs of some kind are needed to bound the size of the firm; otherwise, the firm would expand indefinitely if the present value of the stream of incremental profits from a unit of capital exceeded the cost of the unit. In most adjustment-cost models of competitive investment, the convexity of adjustment costs limits the size of the firm by making the marginal cost of investment an increasing function of the level of investment. However, because adjustment costs are a function of only the level of investment (and not the stock of capital), investment in each period is independent of investment or the stock of capital in any other period. Hence, there are no linkages between decisions in one period and the next.

This, however, necessarily eliminates irreversibility from the problem. Irreversibility affects today's decisions when it causes those decisions to constrain behavior in the future under some states of nature but not under others. For example, consider the investment decision of a firm facing an uncertain future demand. If the firm invests a large amount this period, it would not want to disinvest next period if demand expands and so would not be constrained by irreversibility. However, the firm would be constrained if demand were to contract next period, because then it would want to disinvest and reduce its capital stock. Because next period's demand is uncertain, irreversibility leads the firm to invest somewhat less this period.

This effect can never arise when the size of the firm is constrained only by adjustment costs. Then, investment next period depends only on the realization of demand that period and on the adjustment-cost function; it is completely independent of investment this period. Hence the firm need only compare the marginal cost of investing to current and expected future marginal profits. If uncertainty increases expected future marginal profits, it will necessarily increase current investment.

Convex adjustment costs may indeed affect the rate at which firms invest (although simple "time to build" and the lumpiness of investment are likely to be more important constraints). It is unrealistic, however, to treat adjustment costs as the sole or main determinant of firm and industry size in equilibrium. In fact, a pure adjustment-cost model with constant returns to scale is inconsistent with a competitive market equilibrium. The reason is that, with free entry, a very large number of very small firms will come into the industry. (Very small firms would enter because they would have very small adjustment costs and hence lower total costs.) In the limit, the industry would be composed of an infinite number of infinitesimally small firms, and so each firm would have *no* adjustment costs.

Even if each firm is constrained to some minimum size, the possibility of entry by new firms or expansion of existing ones will ensure that investment decisions in the face of industry-wide uncertainty are intertemporally linked. As a result, industry-wide uncertainty will affect irreversible investment by a competitive firm with constant returns to scale much as it would a noncompetitive firm or a firm with decreasing returns. The reason is that, in each period, if demand increases existing firms will expand or new firms will enter until the market clears. From the point of view of an individual firm, this limits the amount that price can rise under good industry-demand outcomes. However, if investment is irreversible, there is no similar mechanism to prevent price from falling under bad demand outcomes. Each firm takes price as given, but it knows that the distribution of future prices is affected by the irreversibility of investment industry-wide. This reduces its own incentive to invest.

Thus, in a competitive equilibrium, uncertainty over market demand affects investment through the feedback of industry-wide capacity expansion and new entry on the distribution of prices.[4] The following example illustrates this with a simple two-period model that allows for this feedback.

II. An Example

As in Caballero's (1991) model, each firm is in place two periods, there is no depreciation or discounting, and the production function is Cobb-Douglas:

$$(1) \qquad q_i = AL_i^{\alpha}K_i^{1-\alpha} \qquad 0<\alpha<1$$

where q_i is the output of firm i. Since I concentrate on competitive firms facing industry-wide shocks, I begin with the *market* demand curve, rather than the demand curve facing an individual firm, and assume that it is isoelastic:

$$(2) \qquad P_t = Q_t^{-1/\varepsilon}Z_t$$

where ε is the elasticity of demand, and Z_t is a stochastic process, with $Z_1 = 1$. For simplicity, let Z_2 equal 0 or 2 with equal probability. I will compare this to the certainty case in which $Z_2 = 1$. Also, I will restrict the discussion to the case of complete irreversibility, with no cost of adjusting K_i upward.

Let there be a large number, N, of equal-size firms, so that each takes price as given, and $Q = Nq_i$. The profit function for each firm is then

$$(3) \qquad \Pi_i = hP^{\eta}K_i$$

where $h = (1-\alpha)A^{1/(1-\alpha)}(\alpha/w)^{\alpha/(1-\alpha)}$ and $\eta = 1/(1-\alpha) > 1$. Also, $q_i = BK_i$, where $B = h/(1-\alpha)$. With no loss of generality, I choose A so that $B = 1$. Note that the value of a marginal unit of capital is hP^{η}, whatever the firm or industry capital stock. This value is convex in P, so its expectation is increased by a mean-preserving spread in P; but as will be seen, this need not mean that uncertainty leads the firm to invest more.

First, consider the certainty case. Here, $P_2 = P_1$, and all investment occurs in period 1. Each firm will want to invest an infinite amount if $2hP_1^{\eta} > k$ and will invest nothing if $2hP_1^{\eta} < k$, where k is the cost of a unit of capital. Thus, in equilibrium, firms invest until price falls to the point that $2hP_1^{\eta} = k$. Hence $P_1 = P_2 = (k/2h)^{1/\eta}$. Industry investment in the first period is $I_1 = K_1 = Q_1 = (2h/k)^{\varepsilon/\eta}$. (Each firm's investment is just $1/N$ of this.)

Now suppose that Z_2 is unknown when firms invest in period 1; it can turn out to be 2 or 0, each with probability 0.5. Although Z_t is exogenous, P_t is determined as part of the market equilibrium. To find this equilibrium, one wants a distribution for P_t that results from Z_t and from firms' investment decisions, with those decisions based on this same distribution.[5]

I will surmise that equilibrium investment in period 1 is small enough so that, in period 2, firms invest some positive amount if $Z_2 = 2$ (whereas they invest nothing if $Z_2 = 0$). After solving for I_1, I will check that this is indeed the case. Then, if $Z_2 = 2$, firms will invest in period 2 to the point that the profit from a unit of capital equals its cost, that is, until $hP_2^{\eta} = k$, or $P_2 = (k/h)^{1/\eta}$. This implies that K_2 will equal

[4]This can also be the case under imperfect competition.

[5]This is easy to do for this simple example. Leahy (1990) solves a more general continuous-time problem.

$(P_2/2)^{-\varepsilon} = 2^{\varepsilon}(h/k)^{\varepsilon/\eta}$. Of course if $Z_2 = 0$, then $P_2 = 0$, $I_2 = 0$, and $K_2 = K_1$.

Given this distribution for price in period 2, risk-neutral firms will invest in period 1 to the point that the expected value of a unit of capital equals its cost:

$$hP_1^{\eta} + E_1(hP_2^{\eta}) = k \tag{4}$$

or

$$hK_1^{-\eta/\varepsilon} + 0.5k = k.$$

Hence $I_1 = K_1 = (2h/k)^{\varepsilon/\eta}$. Finally, I will check that I_2 is indeed positive if $Z_2 = 2$. If $Z_2 = 2$, then

$$I_2 = K_2 - K_1 = (2^{\varepsilon} - 2^{\varepsilon/\eta})(h/k)^{\varepsilon/\eta} > 0$$

since $\eta > 1$.

In contrast to Caballero's (1991) result, I have found that period-1 investment is the same when Z_2 is uncertain as it is when $Z_2 = 1$ with certainty. The reason is that while a mean-preserving spread in the distribution of P_2 increases the value of a unit of capital, a mean-preserving spread in Z_2 reduces the expected value of P_2. The equilibrium response of firms limits price increases under good outcomes of Z_2, but because of the irreversibility of investment, it does not limit price decreases under bad outcomes. In this particular example these two effects just offset each other, so investment is left unchanged.[6]

The Appendix extends this example to n periods and allows Z_t to follow a random walk; it begins at 1 and increases or decreases by 100 percent in each period. The Appendix shows that investment in period 1 is *lower* when Z_t is stochastic, as long as $n \geq 3$. Also, in any period, the difference between investment when future values of Z_t are known and investment when they are stochastic grows with the number of periods remaining. The reason is that the variance of future values of Z_t increases with the time horizon, but industry investment always limits price increases under good outcomes.[7]

III. Concluding Remarks

The simple example presented above shows how industry-wide uncertainty can have a negative effect on irreversible investment, even when firms are perfectly competitive and have constant returns to scale. That effect is mediated by the equilibrium behavior of all firms and the resulting impact on market price. In the two-period example above, that effect just offsets the increase in the value of a unit of capital that results from the convexity of the marginal profit function, so that period-1 investment is left unchanged by a mean-preserving spread in the demand shift variable. When the number of periods exceeds two, period-1 investment is lower when future demand is uncertain.

In the example, I found the equilibrium distribution for price and the levels of investment in each period consistent with that distribution. Alternatively, I could have used the fact, demonstrated by Robert Lucas and Edward Prescott (1971), that the competitive equilibrium is the solution to the social-planning problem. The social planner will use the downward-sloping demand curve to calculate the optimal investment rule and hence will solve an optimal-investment problem that has the same structure as that of a monopolist with constant returns to scale.

The results in this note do not imply that industry-wide uncertainty will have identical effects on investment for a competitive firm and a monopolist. In Caballero and Pindyck (1992), a model with entry and exit is developed that helps clarify the differences between the competitive firm's response to

[6] If $Z_2 = 1$ with certainty, $P_2 = (k/2h)^{1/\eta}$, but if $Z_2 = 0$ or 2, $E_1(P_2) = \frac{1}{2}(k/h)^{1/\eta}$, which is smaller since $\eta > 1$. The expected marginal profit of capital is $k/2$ in both cases.

[7] Like the two-period example, the n-period example in the Appendix ignores depreciation and discounting. Hence if $Z_t = 1$ for all t, the value of a unit of capital grows linearly with n, but it grows less rapidly if Z_t is stochastic and expected future prices are lower. Including depreciation and discounting would reduce the depressive effect of uncertainty on current investment.

industry-wide uncertainty and that of a monopolist and also shows the effects of *firm-level* uncertainty (e.g., over the firm's productivity).

APPENDIX

This appendix extends the two-period example to n periods, where $Z_1 = 1$ and then, in each succeeding period, Z_t increases or decreases by 100 percent, with probability $\frac{1}{2}$ for each. Thus, $Z_2 = 0$ or 2. If $Z_2 = 0$, Z_t remains 0 for all future t, but if $Z_2 = 2$, then $Z_3 = 0$ or 4, and so on. As before, there is no depreciation or discounting. Hence in the certainty case ($Z_t = 1$ always), firms invest in period 1 to the point that

$$\text{(A1)} \qquad nhP_1^{\eta} = nh\left(K_1^{-1/\varepsilon}\right)^{\eta} = k$$

so $I_1 = K_1 = (nh/k)^{\varepsilon/\eta}$, $I_t = 0$ for $t > 1$, and $P_1 = P_2 = \cdots = (k/nh)^{1/\eta}$.

I will again find a solution for the stochastic case by surmising that investment is positive in a good state (i.e., when $Z_{t+1} > Z_t$) and then checking that this is indeed the case. First, in period n, the good state is that in which $Z_n = 2^{n-1}$. In this state, firms invest until $P_n = (k/h)^{1/\eta}$. Thus $K_n = (P_n/Z_n)^{-\varepsilon} = 2^{\varepsilon(n-1)}(h/k)^{\varepsilon/\eta}$, and $I_n = K_n - K_{n-1}$.

In period $n-1$, in the good state $Z_{n-1} = 2^{n-2}$, and firms invest to the point that $hP_{n-1}^{\eta} + E_{n-1}(hP_n^{\eta}) = k$, which implies that

$$\text{(A2)} \qquad h\left(2^{n-2}K_{n-1}^{-1/\varepsilon}\right)^{\eta} + 0.5k = k$$

or

$$K_{n-1} = \left[2^{\eta(n-2)+1}h/k\right]^{\varepsilon/\eta}.$$

Note that since $\eta > 1$, $K_n > K_{n-1}$ in a good state, as was surmised. In period $n-2$, in the good state, firms invest to the point that $hP_{n-2}^{\eta} + E_{n-2}[hP_{n-1}^{\eta} + hP_n^{\eta}] = k$, so that $K_{n-2} = [2^{\eta(n-3)+1}h/k]^{\varepsilon/\eta}$. In general, in a good state,

$$\text{(A3)} \qquad K_{n-m} = \left[2^{\eta(n-m-1)+1}h/k\right]^{\varepsilon/\eta}.$$

Finally, working back to period 1, $I_1 = K_1 = (2h/k)^{\varepsilon/\eta}$. Note that this is smaller than the certainty case when $n > 2$.

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