Complexity Theory in Axiomatic Design

by

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Submitted to the Department of Mechanical Engineering
in partial fulfillment of the requirements for the degree of

Doctor of Philosophy in Mechanical Engineering

at the

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

May 2003

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Abstract

During the last couple of decades, the term complexity has been commonly found in use in many fields of science, sometimes as a measurable quantity with a rigorous but narrow definition and other times as merely an ad hoc label. With an emphasis on pragmatic engineering applications, this thesis investigates the complexity concept defined in axiomatic design theory to avoid vague use of the term 'complexity' in engineering system design, to provide deeper insight into possible causes of complexity, and to develop a systematic approach to complexity reduction.

The complexity concept in axiomatic design theory is defined as a measure of uncertainty in achieving a desired set of functional requirements. In this thesis, it is revisited to refine its definition. Four different types of complexity are identified in axiomatic design complexity theory: time-independent real complexity, time-independent imaginary complexity, time-dependent combinatorial complexity and time-dependent periodic complexity. Time-independent real complexity is equivalent to the information content, which is a measure of a probability of achieving functional requirements. Time-independent imaginary complexity is defined as the uncertainty due to ignorance of the interactions between functional requirements and design parameters. Time-dependent complexity consists of combinatorial complexity and periodic complexity, depending on whether the uncertainty increases indefinitely or occasionally stops increasing at certain point and returns to the initial level of uncertainty. In this thesis, existing definitions for each of the types of complexity are further elaborated with a focus on time-dependent complexity. In particular, time-dependent complexity is clearly defined using the concepts of time-varying system ranges and time-dependent sets of functional requirements.

Clear definition of the complexity concept that properly addresses the causes of complexity leads to a systematic approach for complexity reduction. As techniques for reducing time-independent complexity are known within and beyond axiomatic design theory, this thesis focuses on dealing with time-dependent complexity. From the definition of time-dependent complexity, combinatorial complexity must be transformed into periodic complexity to prevent the uncertainty from growing unboundedly. Time-dependence of complexity is attributed to two factors. One is a time-varying system range and the other is a time-dependent set of functional requirements. This thesis shows that achieving periodicity in time-varying system ranges and maintaining functional periodicity of time-dependent sets of functional requirements prevent a system from developing time-dependent combinatorial complexity. Following this argument, a re-initialization concept as a means to achieve and maintain periodicity is presented. Three examples are drawn from different fields, tribology, manufacturing system, and the cell biology, to support the periodicity argument and illustrate the re-initialization concept.

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Professor Seth Lloyd
Dr. Hilario Larry Oh
Dr. Jeffrey Thomas

Acknowledgments

I have to confess that writing this section of acknowledgement was indeed most challenging and exciting at the same time. Looking back last six years of my life at MIT gave me an overwhelmingly long list of people that more than deserve a place for their name in this humble thesis.

This thesis would not have been possible without the help of Professor Nam Suh. Ever since the very first day I met him in the fall of 1997, he has always believed in what I am doing and trusted what I am able to do, oftentimes more than I do to myself. That has been the thrust with which I was able to go through so many ups and downs. Most importantly, knowing his trust and confidence in me made me comfortable criticizing myself when necessary. He is more than a thesis advisor to me: a true teacher only those most fortunate can have.

Dr. Hilario Larry Oh is another figure in my life at MIT whose influence goes well beyond this thesis work. I got to know Larry as my mentor for projects I carried out under MIT-SVG partnership program, and soon he became the one I discuss almost everything with. Our conversations, may it be a technical or non-technical, never ended within thirty minutes, and we really enjoyed bouncing ideas back and forth. In addition to all the technical lessons he taught me, I will always remember his passion, creativity, and vision.

I would like to say very special thanks to my other committee members, professor Jung-Hoon Chun, professor Seth Lloyd, and Dr. Jeffrey Thomas. They helped me through by guiding my work to be on the right track, providing fresh perspectives, and raising critical questions. I truly appreciate their support and patience.

Former and current members of the axiomatic design group gave me a great memory of fun as well as contributions to this thesis. Dr. Derrick Tate was a Ph.D. student when I joined the group, and offered hands so many times when I needed. Jinpyung Chung has been my office-mate making my life in 31-061 more than enjoyable, and most happily made it to Ph.D. degree together at the same time. I am glad that I have become a close friend with Jason Melvin who consistently showed his ingenuity and many other things that made me want to learn from him. Hrishkesh Deo always asks critical questions that other people take for granted. I would also like to thank Dr. Rajesh Jugulum and Dr. Il-Yong Kim for their support and encouragement.

Times spent with my friends – Yongsuk, Sangjun, Sokwoo, Daekeun, Soohaeng, and all the KGSAME members – here are a big part of my memory at MIT. They are better persons than I am in many ways, and nurtured me to become a better person.

My parents have been the greatest to me. I know that they are the ones mostly delighted by my becoming a Ph.D. than anyone else in the world. They are my teachers, friends, and supporters throughout my entire life. As a parent myself, they are role models that I would like to see myself being any close to.

The most graceful and exciting things that have happened to me during my years at MIT are my becoming a husband of Alice Haeyun Oh and a father of the most precious girl Herin. This thesis would not have any meaning without them in my life. Alice is the one that made this thesis possible. I cannot say in words how much I love Alice and Herin. Ever since they came into my life, they are the reasons of my life.

Contents

List of Figures

List of Tables

Chapter 1

Introduction

The term, complexity is commonly found in use throughout virtually all fields of science including physics, biology, sociology, to name a few. In the discipline of engineering, the term “complex” or “complexity” are also getting popular as the objects of its study tend to demand unprecedented way of treatments in many ways. Despite the abundance of currently available diverse definitions and descriptions, this thesis introduces still another concept that shares the name, complexity. The scope of complexity concept discussed in this thesis remains mainly in the pragmatic domain, particularly in the area of engineering system design. The peculiarity of the complexity concept discussed in this thesis lies in its emphasis on the relative aspect with respect to the functional requirements, i.e. what we want to achieve/know. It has its theoretical basis on axiomatic design theory. In this introductory chapter, common notions of complexity concept are discussed with some examples, followed by a brief survey of some of the formal complexity study. Next, the complexity is discussed in relation to engineering design. In particular, the concept of complexity defined in axiomatic design theory is briefly introduced, and the difference in its perspective from the others is emphasized.

1.1 General Concept of Complexity

Arguably the most fundamental question around the subject matter is "what is complexity?" What makes us to judge that an entity – may it be a system, phenomenon, problem, etc. – is complex? Perhaps in its most naïve sense, it is the difficulty in dealing with[1] the entity under consideration. Indeed, many of formal and informal complexity discussions are centered around this basic notion of difficulty. Efforts are focused on characterizing and quantifying the difficulty. It is not surprising to see that there is no consensus on the definition of complexity so far. For example, some people define complexity as a degree of disorder. Others use the minimum length of description to define complexity. The amount of resource such as time or memory to solve a certain problem is another example of complexity definition. Combination of these definitions yields still other definitions. Before discussing some of the existing concept of complexity, let us begin with a few examples to highlight some of the issues in defining the concept[2].

- A cell vs. the potato it was taken from

If the complexity is some absolute quantity inherent to a system, it would make most sense to say that a subsystem cannot be more complex than the system that it is a part of. For example, regardless of the complexity of a cell, an organism is more, or at least no less, complex than a cell since a cell cannot take away the complexity of other cells. However, that is not obvious any more when viewed from different frameworks. It seems natural to judge a cell under a microscope to be much more complex than the potato it was taken from. That is because human beings intrinsically choose different framework with a different level of granularity to filter out unnecessary information. This example suggests that defining problem itself play an important role in understanding complexity and that absolute complexity concept could be inappropriate way of looking at the problem.

[1]This vague expression is intentionally used because depending on the context, it could be 'understanding', 'solving', 'modeling', etc.

[2]These examples are taken from [11]

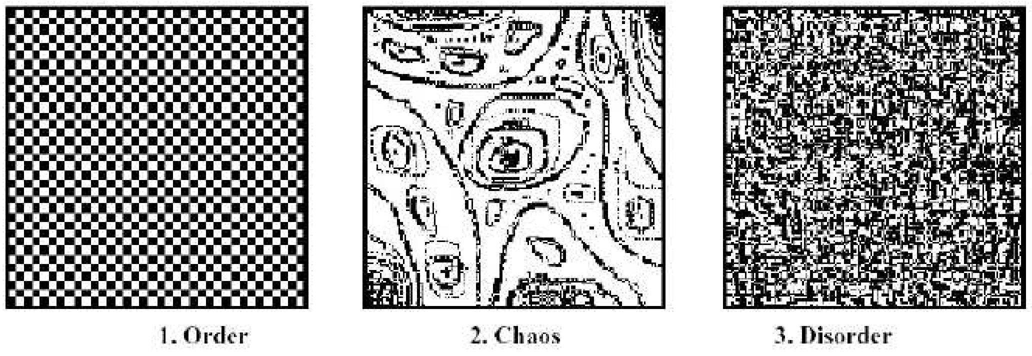


Figure 1-1: Order, midpoint between order and disorder, and disorder. Figure is taken from [1]

- Motion of the planets

One may want to ascribe its complexity to the system of planets itself and quantify its absolute value. That can possibly be done by relying on some framework for which privilege is claimed, e.g. entropy. As a consequence of doing so, one would have to consider all feasible models as equally complex since a model is simply a description of what is being modeled. Thus, no matter what language is used to model the system, the complexity of a system itself should remain same. However, clearly different degree of complexity, in some sense, is perceived depending on the required specificity and the models chosen to describe a system. For example, motion of the planets will be considered extremely complex to describe if the required error level is prohibitively tight. On the other hand, it can be as simple as a drawing on the back of the envelop if all we need is a very general description. Perceived level of complexity also depends on the model chosen to describe the system. For example, the exact motions of the planets seem so complicated when we describe them in terms of epi-cycles but much simpler in terms of ellipses. From this example, we can see that the absolute complexity concept can potentially overlook important part of the problem such as specificity and the role of model and tool.

- Ordered/disordered pattern

When asked which of the above three patterns in figure 1-1 is most complex, we can possibly consider how difficult it is to represent each pattern or to generate them. Instead of a single right answer, there could be several different opinions depending on the assumptions or a particular perspective held by an observer. For example, one might say the second (middle) is the most complex pattern since both the first and the third have simple rules behind – simple repetition and randomness (no-rule), respectively –, and thus easy to generate. On the other hand, in terms of representation, the third figure is most complex because there is no regularity at all. Therefore, it is almost incompressible, which means the amount of required information is that of raw data. This example shows that depending on the particular aspect of the problem, the same system – pattern in this case – can present different level of complexity.

As shown in the above examples, there could be a large ambiguity in discussing the concept of complexity unless it is carefully delineated: for example, the language (model) chosen to describe the object, scale or level of detail, specificity, and particular aspect being considered must be part of the complexity discussion. Even though the different concepts share the same name, complexity, and probably address same fundamental question of difficulty, they rely on different assumptions and therefore lead to different implications. To avoid such ambiguity, many of the formal complexity research have attributed the complexity to some absolute quantity by considering only one aspect of the problem, considering the minimal size, or limiting its scope to a certain type of problems/systems. This will be evident by looking at the well-established complexity measures. Formal ways of discussing the subject of complexity fall largely into three classes of definition: probabilistic, algorithmic, and computational approaches. Here, a few examples in each of three classes are presented.[3]

Examples of the first class include Boltzmann-Gibbs entropy and Shannon Information. Equation 1.1 is the general form of the definition of entropy in statistical mechanics:

$$S = -k \sum p_i \ln p_i \tag{1.1}$$

[3]For more comprehensive survey of complexity measures, readers are recommended to refer to [11].

where k is an arbitrary constant that depends on the unit of entropy, and p_i is the probability that particle 'i' will be in a given microstate for a certain macrostate.

Since the advent of statistical mechanics in the late 1800's and early 1900's, entropy, which has its root in the second law of thermodynamics, has been considered an effective measure of disorder. The concept gained a statistical interpretation, i.e. a measure of uncertainty about the actual microscopic state of a system. This aspect of entropy interpretation was enriched when Shannon introduced the information theory in the middle of twentieth century [12],[13],[14]. It is by no means a coincidence that Shannon's entropy shares the same mathematical formula as Shannon proved in [15]. Not only is the mathematical formula common to both concepts, but also is the core interpretation: they measure uncertainty. It is this statistical interpretation that enabled the concept of entropy to reach the realm outside the thermodynamics and to have become one of the norms in the complexity discussion later on. The central idea of this approach is that the more disordered a system is, the more information is needed to describe it and thus the system is more complex.

Algorithmic complexity[4] and logical depth are the examples of the second kind. Algorithmic complexity, sometimes called AIC (Algorithmic Information Content) or Kolmogorov complexity, is defined for a string of symbols, x, as the length of the shortest program that instructs a Turing machine to produce output x and then halt [17],[16]. Formally,

$$K_U(x) = \min_{p:U(p)=x} l(p)$$

where U is a universal computer (or Turing machine), p is any program that print x and halt, and $l(p)$ is the length of a program. A Turing machine is an abstract representation of a computing device. It uses an infinite tape as its unlimited memory. It has a read/write head that can read and write symbols and move around on the tape. Initially the tape contains only the input string and is blank everywhere

[4]The ideas of this algorithmic complexity, i.e. minimal length of description, were put forth independently and almost simultaneously by Kolmogorov, Solomonoff and Chaitin [16]. Thus, it is commonly referred to as Kolmogorov complexity or Kolmogorov-Solomonoff-Chaitin complexity.

else. The machine scans the tape, writes information on the tape if necessary, and changes its state. It continues computing until it decides to produce an output. From the definition, we see that the algorithmic complexity does not require probability distribution when defining complexity. Another example of algorithmic definition of complexity is logical depth, which is a variation of algorithmic complexity. Bennet[18] defines logical depth as the running-time to generate an object in question with near-incompressible program. Strictly, the depth of a string x at level s is

$$D_s(x) = \min\left\{T(p) \middle| |p| - |p^*| < s \land U(p) = x\right\}$$

where $T(p)$ is the time taken by program p, and p^* is the shortest program that computes x. It is thus combination of Kolmogorov complexity and computational complexity which is discussed below.

The last class is computational complexity. The computational complexity is the amount of time, memory, or other resources required for solving a computational problem with respect to the size of a problem [17]. This is a very useful concept when analyzing computational algorithms. Computational algorithms are evaluated based on the running time to reach a solution for a problem. In turn, once the best algorithm is discovered for a certain type of problems, such problems are categorized to different classes of computational complexity, e.g. class P, NP, etc. Here, the term complexity is equated to the difficulty of solving a problem strictly in terms of computational resources, given the best algorithm known so far. Therefore it has well-bound scope of application: the study of computation and computational algorithm.

As mentioned earlier, the desire that has led to the above formulation is to attribute complexity purely objectively to a physical process or a system and thereby eliminate potential ambiguity in the concept. Such absolute-complexity standpoint excludes the relative aspect of the problem, which is indeed a crucial part of the problem from a pragmatic standpoint. From a pragmatic perspective, complexity is a quantity or quality perceived by an observer (or designer, solver, etc.). Therefore, various factors involved in the act of observation must be taken into account in assessing the complexity. From this reasoning, some claim that the complexity is more critically dependent on the choice of a particular model rather than what is being modeled itself. In other words, complexity can be defined only in relation to scientific modeling. In that sense, the complexity is defined as “the difficulty associated with a model’s form when given almost complete information about the data it formulates” [11], or “the property of a real world system that is manifest in the inability of any one formalism being adequate to capture all its properties” [19]. The previous examples of “planet motion” and “cell vs. potato” may illustrate the point.

After all, from axiomatic design standpoint whose emphasis is pragmatically on design, one of the most valuable outcomes of the study of complexity would be the deep insight into the causes of complexity, but it can hardly be found in the existing complexity discussions. Substantial part of complexity study is to quantify (or sometimes just describe) the difficulty associated with the object under question. It is as much or even more of interest in engineering domain to know what causes the difficulty and, from there, to figure out how to actually reduce the level of difficulty.

1.2 Object of Complexity Measure: Complexity of What?

An attempt to answering the first question – what is the complexity? – leads to another question: “of what entity are we discussing the complexity?” To list a few, it could be the complexity of a natural system, phenomenon, artifact, or a computational problem. In discussing complexity, especially complexity measure, the object of being complex – the entity that is judged to be complex – is oftentimes implicit. A metric that works very well for a certain subject may not be suitable at all for the complexity of other subjects. Indeed, that accounts for much of the confusion, and explains why a survey of wide range of complexity definitions seems unproductive. The question, ‘what is complexity’ is ambiguous until the target of the question is specified. Once we specify the object to which the concept of complexity is applied, then the first question can be rephrased such that it becomes a more tangible question. For example, why is 'this pattern' complex, and how complex? What is the complexity of 'this computational problem'? Depending on the context of the complexity question, it focuses on a certain aspect of the problem and accordingly requires different formulation. Consequently, it carries different meaning with, possibly, a unique metric. Therefore, one of the very first steps in discussing the subject matter of complexity is to clarify the object of the question, i.e. 'complexity of what?'

Table1.1 summarizes such relationship. In the left column listed are various definitions or measures[5] of complexity, and the right column shows what is considered by the relevant complexity concept. As you can see, some of the complexity concept, for example cognitive complexity, has a very specific target application that is behavioral personality. On the other hand, a size as a complexity measure is very general concept that can be used in many different context: for example, size of rules is used to represent complexity of a pattern, and size (number) of variables to indicate complexity of modeling.

Even without knowing details about the individual complexity measures listed in the table, one can immediately tell that many of the complexity measures have limited scope, which, if not properly recognized, would create unnecessary confusion. For example, albeit they share part of their name, cognitive complexity is not even close to Kolmogrov complexity in any dimension. The objects of the study for two complexity concepts are so remote that each measure aims completely different aspect of the problem. In many cases, an attempt to use one complexity definition for different object results in an inappropriate force-fitting. Therefore, any complexity definition or measure must be understood in the context of its intended use, and care should be given when discussing different kinds of complexity definitions and measures.

Secondly, many of the measures address the complexity indirectly. Some of them are potentials that can lead to large complexity but not necessarily. For example, 'size' of the problem, i.e. number of variables or dimensions, is the potential at-

[5]These complexity definitions and/or measures are taken from Edmonds' survey [11].

Complexity definition/measure	**Object**
Kolmogorov complexity (AIC), Shannon's information/entropy	An object with information, e.g. string bit, pattern
Size (size in many different context)	General
Variety, Irreducibility	(Biological) System
Dimension, Ability to surprise, Irreducibility	System (as an object of modeling)
Connectivity, Cyclomatic number, Ease of decomposition	System with network characteristic (components are interconnected)
Stochastic complexity	Physical processes or data
Size of rules (or grammars), Midpoint between order and disorder, Logical depth, Sophistication	Pattern (if viewed as a result of production rules in a language)
Boltzmann-Gibson entropy, AIC, Improbability, Thermodynamic depth, Total information	(Thermodynamic) System or state
Sober's minimum extra information, Expressivity, Logical complexity, Kemeny's measures, Goodman's complexity	Statement, Language, (Theory)
Size of minimal characteristic matrix	Logic
Cognitive complexity	Personality, Cognitive/behavioral
Time (processing/execution/preparation)	A task
Resources (time/memory/others), Ignorance, Information in loose sense	Solving a problem

Table 1.1: Various complexity definitions/measures and their intended application

tribute for a complex system, but having a large size does not necessarily mean that the system is complex. One can augment this simple measure by including additional features such as connectivity, cyclomatic number, but still it cannot be the sufficient condition for being complex. Some of them are more of an indicator or symptom of being complex. Such measures include computational time, length of description (information), irreducibility, logical depth, and ease of decomposition. They are useful in comparing two or more objects that present same symptoms. For example, computational time measure enables us to say that problem A is more complex than problem B because it takes longer time to solve. However, these concepts generally have a strict scope due to the fact that they tend to focus on a specific symptom.

Lastly, based on the observation on table 1.1, it seems that for most of the complexity concepts, the name, complexity has been given to those concepts and measures *ad hoc*. In other words, in the course of the study of particular subject, the associated difficulty – in any sense – is termed as complexity of the entity and a suitable metric is provided. It explains very well the lack of universal complexity definition and measure. That, in turn, justifies our attempt to introduce another concept of complexity.

1.3 Complexity in System Design

Having emphasized the significance of clarifying the object of complexity measure, this thesis limits the scope within engineered system, a system being a collection of physical/non-physical entities (design parameters) that cooperatively deliver overall functional requirements. This would include typical engineering systems, biological systems, and economic systems, while excluding patterns, logic, and computational problems. It will be discussed in section 1.5 that the complexity concept presented in the thesis is based on the axiomatic design framework. In this section, current notion of complexity in the context of engineering system (design) is briefly discussed.

In engineering system design, the term 'complex' is often considered as a synonym of 'complicated' just as they are in English dictionaries.[6] However, if we want to be precise in using the terminologies, there is a subtle difference in their connotations. A distinction can be made between the two by emphasizing the aspect of 'interdependence' of complex system. Complex systems have components whose behavior is dependent on the interactions with other components in the system. On the other hand, the interactions of the components of complicated systems are simply additive, and thus their behavior is generally independent of those interactions. In other words, the properties of an individual component in a complex system cannot be determined without considering the whole system, whereas they can be determined from local consideration for a complicated system. Yet, the use of the terminologies is relatively casual and so is the meaning of them. Thus they are considered as synonyms by and large. Little work has been done to define 'complexity' in the context of engineering design, and we simply depend on its ordinary dictionary definition. Despite the lack of formal definition, it is well accepted that modern engineering systems are becoming more and more 'complex'. Typical examples of using the term 'complexity' or 'complex' would be 'Boeing-737 is a complex system'; 'An automobile is less complex than an aircraft'; 'This manufacturing system is complex'; 'A large system has large complexity'; 'A system with modular design has low complexity.'

We can immediately generate a list of intuitive reasons that lead to the above example sentences.

- has a large physical structure (e.g. huge part count)
- performs many different functions as a result of a collective behavior
- has multi-level interactions or chained interactions among its constituents
- does not have clear cause-effect relations
- is difficult to understand/make

The first thing you may notice from the list is that the last one is the essential attribute of so-called complex system and the other four are potential causes of it.

[6]**complex**: Consisting of parts or elements not simply coordinated, but some of them involved in various degrees of subordination; complicated, involved, intricate; not easily analyzed or disentangled. **complicated**: Consisting of an intimate combination of parts or elements not easy to unravel or separate; involved, intricate, confused. From Oxford English Dictionary 2nd edition, 1989.

Without difficulty in understanding (or making, operating, etc.), a system is not said to be complex. For example, the intricate interactions among its component is not sufficient reason for a system to be complex. Secondly, the notion of complexity, in the context of engineering system design, almost always implies the system concept. Indeed, the first three are commonly considered as the attributes of a 'system.' Thus, the complexity is a property of a system. Then, the last two will be what distinguishes between a complex system and non-complex system.

Having said so, complexity is the property of a system that makes it difficult to *understand* as a whole through the collection of knowledge about its constituents. Or in other words, one of the essential characteristics of complex system is its emergent (or collective) behavior that is not readily understandable/predictable from individual components' properties. "Understanding" means being able to explain its causality and thus being able to predict its behavior (output) given initial conditions (input). When it is properly understood, one should be able to explain 'how' the overall behavior is produced from the collective behavior of its components. It is particularly challenging to understand a complex system because "pushing on a system 'here' often has effects 'over there' due to interdependence of parts."[7] So, the complexity is a property of a system that makes it difficult to understand the causality between its overall function or behavior and the components' properties given external condition or input. Figure1-2 summarizes such a general concept of complexity.

Although the above paragraph defined complexity based on the common intuitive reasons why a system is complex, it is merely an elaboration of what is defined in an ordinary dictionary. Indeed, this loose definition represents general notion of the complexity in engineering system design as suggested by a couple of examples.

This informality may be an indication of its inclination to pragmatism in engineering domain. With this casual treatment of the subject, the complexity is only a metaphor to signify the difficulty associated with a system under question. It does not matter whether the system is truly complex or not, or exactly how complex a system is. What matters is the particular aspect that the so-judged complex system

[7]Cited from http://necsi.org/guide/whatis.html

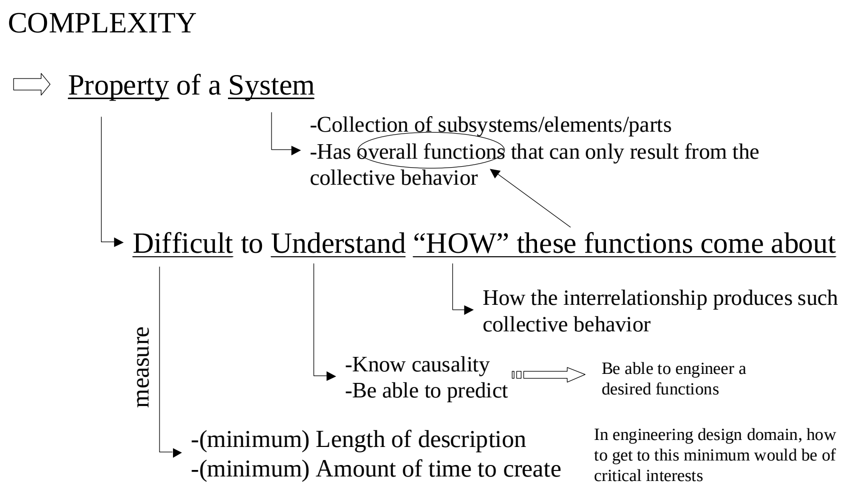

COMPLEXITY
Property of a System
-Collection of subsystems/elements/parts
-Has overall functions that can only result from the collective behavior
Difficult to Understand "HOW" these functions come about
How the interrelationship produces such collective behavior
measure
-Know causality
-Be able to predict
Be able to engineer a desired functions
-(minimum) Length of description
-(minimum) Amount of time to create
In engineering design domain, how to get to this minimum would be of critical interests

Figure 1-2: General complexity concept in the context of engineering system

has, e.g. large and interconnected structure. The message to convey when saying 'this is a complex system' is most likely that this system is not subject to straightforward engineering practice. In that sense, the complexity *per se* is a conceptual (qualitative) property that may be distinguished by several measurable or observable quantities, and apparently unavoidable property that must be handled sophisticatedly. Each of those measurable quantities is the subject of a specific technique. Then the collection of the activities and the results is abstracted as "coping with complexity." Therefore, the study of complexity almost immediately becomes the study of complex systems, i.e. how to cope with complexity. However, the lack of fundamental definition of the complexity itself results in failure to address one important question: how to reduce complexity. Without understanding the complexity itself, we cannot deal with this critical question. That is why we introduce the complexity in axiomatic design theory.

1.4 Objectives: Why do we introduce the concept of complexity?

We see that there exist wide range of complexity definitions as briefly discussed in section 1.1 and 1.2. Despite the abundance of diverse definitions and descriptions currently available, this thesis still introduces another different concept that shares the common name, complexity. That is justified by the fact that most of the existing complexity concepts are defined *ad hoc* per each of the research objectives, and thus the result of such research is not readily transferable to our concern, i.e. engineering system design. Not only is it justified, but also the effort is motivated by the lack of established complexity concept in engineering design domain. The peculiarity of the complexity concept introduced here lies in its emphasis on the relative aspect with respect to functional requirements, i.e. what you want to achieve/know. This complexity concept has its theoretical basis on axiomatic design theory.

The introduction of axiomatic design's complexity concept is intended to achieve the following objectives:

- To avoid the vague usage of the term in the engineering design discipline.

As mentioned in section 1.3, the term is commonly used in a vague way within engineering domain. Such a vague use of the term may be justified by the intrinsic pragmatism of the discipline. However, it probably has been so simply due to the lack of proper definition of it. A clear definition of complexity cannot be counter-productive since it will open up a channel for communication by eliminating the ambiguity and the confusion caused by it. To this end, the complexity concept defined in axiomatic design theory can definitely contribute.

- To acquire deeper insight into possible causes of complexity in engineering design.

The next step of the research is to investigate the potential causes of complexity. What makes this system complex? Is there a sufficient condition for being complex? Or a necessary condition? The understanding of the cause of complexity will bear significant practical implications. Once this insight is gained, it will be reflected to the design process for complexity reduction.

- To develop a systematic approach to complexity reduction.

The ultimate goal of the complexity study here is to develop a systematic approach to reduce complexity. Note that this objective assumes that the complexity is a reducible quantity. That is very different from many other complexity concepts which attribute complexity purely objectively to the physical entity itself. Also, it implies that the complexity is the consequence of certain causes that are subject to engineering activities.

This thesis explores the axiomatic design's complexity concept in detail to make some progress toward these goals.

1.5 Complexity in Axiomatic Design

The very first appearance of the concept of complexity in axiomatic design can be found in [20], and two years later came a more detailed discussion as a book chapter in [2]. Axiomatic design theory[8] is centered around the concept of functional requirement (FR), design parameters (DP) and their quantitative/qualitative interrelations represented by design matrix ([DM]). A set of functional requirements, {FR} are the functional deliverables of the designed artifact, and design parameters {DP} are the means to achieve the {FR}. Then, [DM] signifies how they are related; it could have a form of sensitivity, model, transfer function, or qualitative description. Then, a design is defined as an interplay between functional domain – FR domain – and physical domain – DP domain –, and it is represented by a hierarchy of {FR}, {DP} and [DM] as shown in figure 1-3.

There are two design axioms, namely Independence axiom and Information axiom:

> **Independence Axiom**: Maintain the independence of functional requirements
>
> **Information Axiom**: Minimize the information content

A design that satisfies the two design axioms is considered a good design that delivers intended functions with minimum information content.

[8]Those who are not familiar with the subject are recommended to refer to [2]

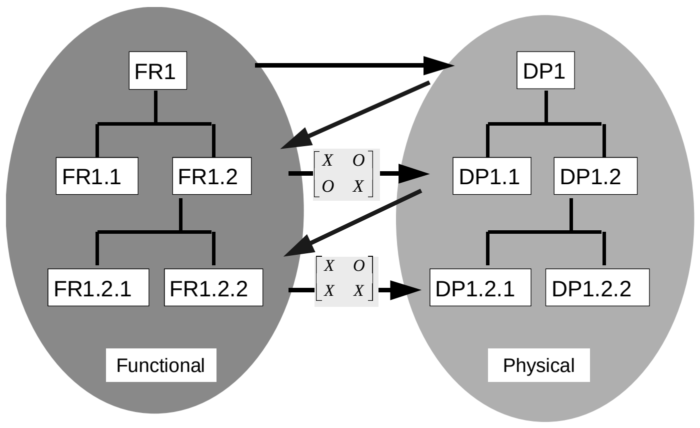


Figure 1-3: Representation of a design in axiomatic design theory

Suh defined complexity in axiomatic design while trying to answer to the question "why some sets of {FR} are more difficult to achieve than others?" Having said the ultimate goal of design is to satisfy the desired functional requirements, the answer he came up with is 'complexity'. It is defined as *a measure of uncertainty in achieving a set of desired functional requirements* [2]. In other words, complexity is the property of a design – represented by {FR}, {DP} and [DM] – that makes the desired functional requirements difficult to achieve (or improbable to satisfy). Note that uncertainty of achieving desired functional requirement is interpreted as the unlikelihood of achieving them. Usually, 'uncertainty' implies being not sure about the outcome, and thus uniform probability about the outcome, as in a fair coin toss, is most uncertain with the least predictability. However, axiomatic design theory is extremely interested in 'getting head' in an arbitrary coin toss, i.e. how 'likely' it is to obtain head in a coin toss. Therefore, when we say 'it is very uncertain to get head in a coin toss,' it means 'it is very unlikely to get head' rather than 'it is not certain whether we are going to get head or tail.' In other words, if the probability of head is 0.001, it is quite certain that the outcome will be the tail side. Saying that it is very 'uncertain' to get head out of the coin toss may sound strange since we are quite 'certain' that we will not get head as the outcome. It is clearer to say it is very unlikely to get head out of the coin toss. To summarize, "a measure of uncertainty in achieving desired functional requirement" must be understood as "a measure of un-likelihood of achieving desired functional requirement.

At a first glance, this particular definition of complexity – large complexity means large uncertainty – may seem counter-intuitive. One might say, for example, biological systems have been evolved into more and more *complex* system to cope with surrounding environment more effectively, i.e. to reduce the uncertainty in achieving the desired functional requirement – survival. So, uncertainty in achieving a desired set of functional requirements is reduced as a system becomes more complex. In other words, as a result of decreasing the uncertainty, systems typically become more complex. Some extends this argument further to state that complexity is the result of robustness: 'the robustness drives the complexity' [21]. The above argument is partially right in that as a result of an effort to reduce uncertainty, some physical quantities tend to increase. Such quantity could be number of components, interconnectivity of its network representation, number of functions, etc. But, axiomatic design's complexity concept does not necessarily agree with the reasoning that those quantities represent the complexity. In axiomatic design's complexity perspective, a single-cell amoeba can be more complex than a human being with more than tens of trillions of cells; a bicycle can be more complex than Boeing 737. These seemingly counter-intuitive statements can be understood when considering its relative aspect of function-oriented definition. It emphasizes functional view over physical view. Being physically complicated is not necessarily equated to being complex. It is also a strictly relative concept while most of us are used to an absolute or objective complexity concept such as size. Axiomatic design complexity can be discussed only after the system's functional requirements are properly defined. In other words, the complexity discussion must be put into some context. On what basis are we discussing the complexity of an amoeba vs. a human being? Recall the cell-potato example presented early in this chapter. On what basis is the comparison justified? Is it meaningful to compare a cell with potato within a single framework? Axiomatic design complexity is always relativised with respect to system's functional requirements. Complexity of a system strongly depends on the question we pose. Based on axiomatic design's complexity definition, the antonym of 'complex' is not 'simple' but 'certain' in terms of achieving a desired set of functional requirements.

Axiomatic design complexity can also be justified in intuitive sense, i.e. complicated $\approx$ complex if the realization process, {DP}-{PV} is taken into account. Achieving FR is essentially a two-step process: come up with means to achieve a desired set of FRs, and physically realize the mechanism of {DP}$\rightarrow${FR}. If the embodiment process, {DP}-{PV}, is complicated, then the second step can present large uncertainty. Take, for instance, a high-precision machine such as a lithography machine used in semiconductor manufacturing process. If there is a well-established mechanism that guarantees the conformance of its output to the desired design range, then it is not considered complex anymore within {FR}-{DP} domain. But, physically realizing the mechanism in {DP}-{PV} domain can still be very challenging with large uncertainty due to tight manufacturing tolerance, which eventually adds to the uncertainty of achieving FRs. In that sense, defining complexity as uncertainty in achieving functional requirements is largely consistent with the common notion of being complex. In this thesis we will limit the scope within the functional and physical domain.

As discussed above, axiomatic design's complexity concept implies difficulty: particularly the difficulty in achieving {FR}. The difficulty is assumed to have roots in the uncertainty of the way {FR} is achieved, i.e. {FR}-{DP} relation. To summarize, the following are the aspects of the axiomatic design complexity definition:

Difficulty: It all began with the question about difficulty in achieving {FR}. Degree of difficulty associated with the task of achieving functional requirements of a system is a function of design – {FR}, design range, {DP}, and [DM]. Complexity is measured for the task of achieving functional requirements, and thus is a function of design.

Uncertainty: By the definition of axiomatic design complexity, it is a measure of uncertainty in achieving the {FR}. As noted earlier in this section, this uncertainty is interpreted as un-likelihood of FR's being achieved. The ultimate goal of design is to achieve desired {FR}, and thus any uncertainty in accomplishing the goal is considered to incur complexity. When the uncertainty of achieving the desired {FR} is low, we tend to perceive the design as a non-complex design since we expect it to happen without a need to do extra work.

Relativity: It should be emphasized that axiomatic design's complexity is a relative concept rather than absolute one. It should always be discussed in relation to the functional requirements in question, design range of each functional requirement, and the specific {DP} chosen for the design. For example, measuring 60 seconds within +/- 5 second is quite a simple design task if we are simply given any kind of timepiece with a second hand, while measuring 60 seconds within +/- 0.1 second with the same equipment is rather formidable. Before specifying all the constituents of a design – {FR}, design range, {DP}, [DM] –, complexity cannot be assessed. Axiomatic design complexity concept opposes to the idea that the complexity is inherent quantity (or quality) that can be attributed to an object.

Information (Information content): Information is an effective measure of uncertainty since it is what is required to resolve any uncertainty. In that sense, complexity should be proportional to the information [22]. Axiomatic design theory also has the quantity called information content which is quite similar to that of Shannon's. Since axiomatic design complexity is explicitly defined in terms of uncertainty, it is natural to relate complexity to information. Indeed, as will be shown later, one of the four kinds of axiomatic design complexity is directly measured by the information content.

Ignorance: Uncertainty can be increased by the ignorance or lack of knowledge about a design. Here, the term 'ignorance' is used in a semantic sense, not as a measure of lack of syntactic information. For example, the semantic ignorance about a coin toss is 'not knowing the probability of head or tail,' whereas a syntactic ignorance would mean 'not knowing the result of a coin toss given the probability of head or tail.

Having defined the complexity as a measure of uncertainty in achieving the desired functional requirements, Suh came up with different kinds of uncertainties, and thereby identified four types of complexity.

- Time-independent real complexity

- Time-independent imaginary complexity
- Time-dependent combinatorial complexity
- Time-dependent periodic complexity

First, based on the dependence on time, he defined time-independent complexity and time-dependent complexity. Time-independent complexity, as its name suggests, captures the complexity of a system where describing its functional requirement set or determining overall uncertainty in achieving those functional requirements does not require time dimension. Time-independent complexity is embedded in its design – how a given set of functional requirements is achieved by design parameters –, and remains constant unless the design changes. On the other hand, time-dependent complexity involves time as one of its determinants. Note that it does not necessarily mean that the complexity is an explicit function of time. Rather, it means that in determining the complexity of a given system (design), one has to pay attention to the change of functional requirements or their behavior with time.

Time-independent complexity is further divided into real complexity and imaginary complexity, depending on its root cause. Time-dependent complexity is also divided into two different kinds: combinatorial complexity and periodic complexity. Each of these four types of complexity is discussed in the following chapters. In Chapter 2, time-independent complexity is revisited to discuss its meaning with more detail, followed by detailed discussion on time-dependent complexity in Chapter 3 and 4.

1.6 Summary

During the last couple of decades, the term, complexity, has been commonly found in use, sometimes as a measurable quantity with rigorous definition and also as merely an ad hoc label. Among the various definitions of the concept, well-known formalisms are found in the probabilistic definition – e.g. entropy and information –, algorithmic definition – AIC –, and computational definition – computational complexity. Most of these definitions have attributed the complexity to some absolute quantity by considering only one aspect of the problem, considering the minimal size, or limiting its scope to a certain type of problems/systems. Consequently, it is uncommon that a complexity concept defined in a particular research context can be applied to other circumstances. It is more evident when cross-examining various complexity concepts.

In engineering, partially due to its natural inclination to practicality and also due to the lack of fundamental concept, complexity per se does not convey much significance and the emphasis is on the study of coping with complex systems. It is understandable to some degree, but prevents the development of systematic understanding of the subject. Axiomatic design's complexity concept aims to achieve the following goals:

- To avoid the vague usage of the term, complexity, in the engineering design discipline.

- To acquire deeper insight into possible cause of complexity in engineering design.

- To develop a systematic approach to complexity reduction.

AD complexity is defined as a measure of uncertainty in achieving the desired functional requirements. It encompasses the aspect of difficulty, uncertainty, relativity, ignorance, and information, which are also part of general notion of complexity. The peculiarity of AD complexity concept can be well understood once these aspects are recognized.

There are four different sub-categories of AD complexity: time-independent real complexity, time-independent imaginary complexity, time-dependent periodic complexity, and time-dependent combinatorial complexity. The following chapters discuss each of these complexity concepts in detail.

Chapter 2

Time-independent Complexity

In axiomatic design theory, complexity is defined as a measure of uncertainty in achieving a desired set of functional requirements. Directly from this definition, time-independent complexity requires the uncertainty be time-independent. Time-independent complexity, as its name suggests, is the complexity where uncertainty of achieving the desired functional requirements does not change over time. In other words, time-independent complexity captures the complexity of a system in which determining overall uncertainty in achieving the functional requirements does not require time dimension. Uncertainty in achieving functional requirements is represented probabilistically by a resultant system range. Therefore, for a system with time-independent complexity, its system range does not change with time. Non-probabilistic factors also contributes to a system's uncertainty. Since the uncertainty is considered always in relation to a desired set of functional requirements, time-independent complexity implies that the functional requirements are also time-independent. In other words, describing its functional requirement set does not involve time factor. Time-independent complexity is embedded in its design – how a given set of functional requirements are achieved by design parameters –, and remains constant unless the design changes.

Time-independent complexity is further divided into real complexity and imaginary complexity, depending on its root cause. Time-independent real complexity is related to the uncertainty that arises from the random nature of a system, and thus essentially equivalent to the information content. Time-independent imaginary complexity, on the other hand, is due to the ignorance of the design. Ignorance, in semantic sense, is difficult to effectively quantify, but axiomatic design's imaginary complexity addresses only one particular type of ignorance, i.e. lack of knowledge about the structure of design matrix.

2.1 Real Complexity

Time-independent real complexity is related to the uncertainty that arises from the random nature of a system. Randomness of a system may come from the variation of input(design parameters), design matrix and noise factors. All of these contribute to the variation of functional requirements, which represents the uncertainty of a system. Since this type of uncertainty is mostly inevitable and it actually exists in a system, it is called real complexity. Suh defined real complexity as *a measure of uncertainty when the probability of achieving the functional requirements is less than 1 because the common range is not identical to the system range* [2]. This definition can be restated as 'the complexity caused by system range's being outside of the design range.' By this definition of real complexity, it is essentially equivalent to the information content. First, let us review the information content in axiomatic design.

2.1.1 Information Content in Axiomatic Design and its Computation

Information content is defined, for a functional requirement, as the negative logarithm of p_s, probability of achieving the functional requirement. That is,

$$I(FR) = -\log_2 p_s \tag{2.1}$$

$$p_s = \begin{cases} \int\limits_{\substack{design\\range}} f(FR)\mathrm{d}FR & \text{for continuous } FR \\ \sum\limits_{\substack{\{i|FR_i \in \\ design\ range\}}} p(FR_i) & \text{for discrete } FR \end{cases} \tag{2.2}$$

where, f is a probability density function for continuous FR, and p is a probability mass function for a discrete FR. p_s is called probability of success, and it is simply the probability that the functional requirement value is within the specified design range as shown in figure 2-1.

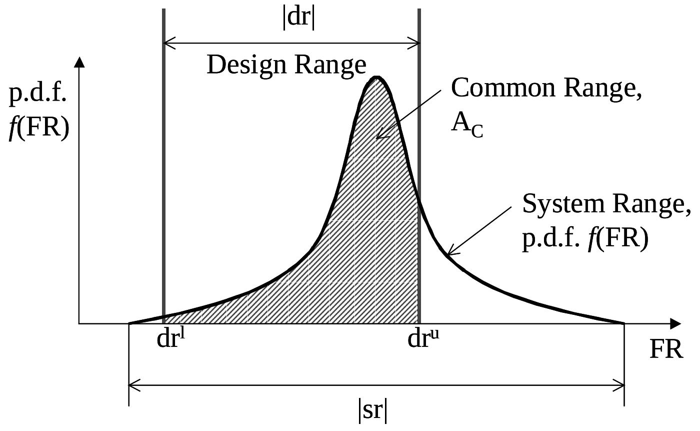


Figure 2-1: p_s is the probability that a functional requirement (FR) is within the specified design range.

Design range bounds acceptable values of the functional requirement, and it is specified by designer(s). For example, suppose a required functional requirement is 'rotate a spindle constantly at 300rpm.' Given the nominal value of the functional requirement, designer(s) specifies limits, typically upper and lower, within which the functional requirement value is considered acceptable: e.g. +/- 10rpm. System range represents the actual (or estimated) distribution of output functional requirement value. Depending on variation of the chosen design parameter and its mechanism to deliver the functional requirement, the functional requirement value follows certain distribution. The resulting distribution of the functional requirement is called a system range. The portion of the FR distribution, $f(FR)$ that rests within the design range is called a common range, and the common range represents the probability that the output FR has an acceptable value. For most cases, evaluating system range or $f(FR)$ requires simulation or experiment using a prototype. In some cases where the relationship between FR and DP can be analytically represented by a function u as $FR = u(DP)$, p can be estimated from the design equation and the distribution of design parameter. Let g be the probability density function of DP and $FR = u(DP)$ has inverse[1] $DP = v(FR)$, then it is not difficult to show that the probability density function of FR, $f(FR)$ is

$$f(FR) = |v'(FR)| \times g[v(FR)] \tag{2.3}$$

For example, in a simple linear case $FR = a \times DP$ with $g(DP)$, probability density function of DP, the derived density function for FR is

$$f(FR) = \frac{1}{|a|} g\left(\frac{FR}{a}\right) \tag{2.4}$$

Therefore, if design equation and probability density of DP are known, then system range of the FR can be easily estimated. p_s, then, can be computed by integrating the system range over the design range.

In the above definition of information content, a functional requirement that has a range of continuous or discrete values is interpreted into a binary variable, i.e. success or failure, using design range as a boundary. In some cases, however, a functional requirement itself has a binary output. For example, a functional requirement, 'pass current through when X happens' has a binary output if the amount of current and the time it takes are not the concern. Thus, its output simply takes value 1(success) or 0(failure). The probability of success, p_s is,

$$p_s = \lim_{N \to \infty} \frac{N_{success}}{N} \tag{2.5}$$

Information content is defined in the same way as in equation (2.1).

The above definition of the information content for a single functional requirement

[1]For simplicity, we assume that there exists a single-valued inverse.

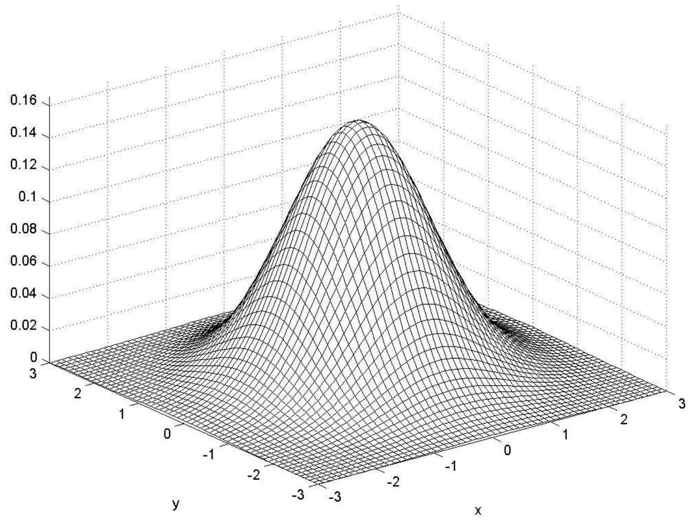


Figure 2-2: 2-FR joint probability density function, $f(FR_1, FR_2)$

can be expanded to multi-functional requirements using joint probability.

$$I(FR_1, FR_2, \cdots, FR_n) = -\log_2 p_{1,2,\ldots,n} \tag{2.6}$$

The joint probability that all the functional requirements are satisfied, $p_{1,2,\cdots,n}$ is

$$p_{1,2,\ldots,n} = \int_{design\ hyperspace} f(FR_1, FR_2, \cdots, FR_n)\mathrm{d}FR_1\mathrm{d}FR_2 \cdots \mathrm{d}FR_n \tag{2.7}$$

$f(FR_1,\ FR_2,\ \ldots,\ FR_n)$ is a joint probability density function for n FRs, and $p_{1,2,\ldots,n}$ is the probability that all the functional requirements are within their design ranges. Note that even a single FR's being outside of design range (hyperspace) is considered failure. An example of joint probability density for a simple 2-FR case is shown in figure 2-2.

In case of uncoupled design, the functional requirements are *statistically independent*[2] to each other, and thus the joint probability density function (or mass function for discrete FR) is simply the product of individual probability density functions. Therefore, the probability of success and information content for an uncoupled sys-

[2]Statistical independence is different from functional independence in Axiom 1 (Independence axiom).

tem with multiple FR is,

$$\begin{aligned}
p_{1,2,\ldots,n} &= \int\limits_{dr_1}\int\limits_{dr_2}\cdots\int\limits_{dr_n} f(FR_1, FR_2, \cdots, FR_n)\mathrm{d}FR_1\mathrm{d}FR_2\cdots\mathrm{d}FR_n \\
&= \int\limits_{dr_1} f_1(FR_1)\mathrm{d}FR_1 \times \int\limits_{dr_2} f_2(FR_2)\mathrm{d}FR_2 \cdots \int\limits_{dr_n} f_n(FR_n)\mathrm{d}FR_n \qquad (2.8)\\
&= p_1p_2\cdots p_n
\end{aligned}$$

$$I(FR_1, FR_2, \cdots, FR_n) = I(FR_1) + I(FR_2) + \cdots + I(FR_n) \qquad (2.9)$$

Each of the probability distribution functions, f_i, can be derived from corresponding DP's distribution as in equation (2.3). When a design is decoupled or coupled, however, total information content is not readily computable because the FRs are not statistically independent. In decoupled case, one can alternatively evaluate conditional probability according to the sequence in the design matrix. Suppose a hypothetical design with n FR and n DP, and the design matrix is lower triangular matrix. Then, probability of success is

$$\begin{aligned}
p_{1,2,\ldots,n} &= \int\limits_{dr_1}\int\limits_{dr_2}\cdots\int\limits_{dr_n} f(FR_1, FR_2, \cdots, FR_n)\mathrm{d}FR_1\mathrm{d}FR_2\cdots\mathrm{d}FR_n \\
&= \int\limits_{dr_1}\cdots\int\limits_{dr_n} f(FR_n|FR_1, \cdots, FR_{n-1})f(FR_{n-1}|FR_1, \cdots, FR_{n-2}) \qquad (2.10)\\
&\quad \cdots f(FR_2|FR_1)f(FR_1)\mathrm{d}FR_1\mathrm{d}FR_2\cdots\mathrm{d}FR_n
\end{aligned}$$

Integration is carried out sequentially starting from $f(FR_n|FR_1, \cdots, FR_{n-1})$. The conditional probability density function can be derived by equation (2.3) by simply taking all other FRs as constants. By the time of the last integration of $f(FR1)$, it only involves one variable FR_1. For example, consider the following 2-by-2 linear decoupled design:

$$\begin{Bmatrix} FR_1 \\ FR_2 \end{Bmatrix} = \begin{bmatrix} a & 0 \\ b & c \end{bmatrix} \begin{Bmatrix} DP_1 \\ DP_2 \end{Bmatrix} \qquad (2.11)$$

Assume that DP_1 and DP_2 are independent and let g_1 and g_2 be the probability density function of DP_1 and DP_2 respectively. According to equation (2.10), $p_{1,2}$, probability of success for both FRs is

$$\begin{aligned} p_{1,2} &= \int\limits_{dr_1}\int\limits_{dr_2} f(FR_1, FR_2)\mathrm{d}FR_1\mathrm{d}FR_2 \\ &= \int\limits_{dr_1}\int\limits_{dr_2} f_{2|1}(FR_2|FR_1)f_1(FR_1)\mathrm{d}FR_1\mathrm{d}FR_2 \qquad (2.12)\\ &= \int\limits_{dr_1}\left\{\int\limits_{dr_2} f_{2|1}(FR_2|FR_1)\mathrm{d}FR_2\right\} f_1(FR_1)\mathrm{d}FR_1 \end{aligned}$$

From the design equation (2.11), we can express DP_2 as a function of FR_1 and FR_2.

$$\begin{aligned} DP_2 &= \frac{1}{c}FR_2 - \frac{b}{c}\cdot DP_1 \qquad (2.13)\\ &= \frac{1}{c}\left(FR_2 - \frac{b}{a}FR_1\right) \end{aligned}$$

Thus, if DPs are statistically independent, the conditional probability density function $f_{2|1}(FR_2|FR_1)$ is, by equation (2.3),

$$f_{2|1}(FR_2|FR_1) = \frac{1}{|c|}g_2\left(\frac{FR_2 - {}^{b}\!/_{a} \times FR_1}{c}\right) \qquad (2.14)$$

Finally, $f_{2|1}$ and f_1 are plugged into equation (2.12) to compute $p_{1,2}$.

$$p_{1,2} = \int\limits_{dr1}\left[\int\limits_{dr2} \frac{1}{|c|}g_2\left(\frac{FR_2 - {}^{b}\!/_{a} \times FR_1}{c}\right)dFR_2\right]\frac{1}{|a|}g_1(\frac{FR_1}{a})dFR_1 \qquad (2.15)$$

In many cases, it seems reasonable to assume that DPs are statistically independent to each other because in axiomatic design, DPs are selected by designers typically as independent design variables. Yet, there may be situations where DPs are not independent, and need more general approach to estimate $p_{1,2}$. Then, equation (2.3) can be extended to multiple FR case. It does not require independence of DPs. Given DP_1 and DP_2 with joint probability density function $g(DP_1, DP_2)$, and $FR_1 = u_1(DP_1, DP_2)$ and $FR_2 = u_2(DP_1, DP_2)$ each of which has the single-valued inverse $DP_1 = v_1(FR_1, FR_2)$ and $DP_2 = v_2(FR1, FR2)$, the joint probability density function $f_{1,2}$ of FR_1 and FR_2 is

$$f_{1,2}(FR1, FR2) = |J| \;\; g\left[v_1(FR1, FR2), v_2(FR1, FR2)\right]$$

$$J = \begin{vmatrix} \frac{\partial DP1}{\partial FR1} & \frac{\partial DP1}{\partial FR2} \\ \frac{\partial DP2}{\partial FR1} & \frac{\partial DP2}{\partial FR2} \end{vmatrix} \tag{2.16}$$

Then, $f_{1,2}$ is integrated over the design range (space) to estimate the probability of success.

Frey [23] developed an intuitive way to compute information content. The idea is twofold: 1) map the design range of FRs into DP space via design matrix, and 2) integrate the joint probability density function of n DPs over the mapped space. The joint probability density function of DPs is simply the product of all of the individual density function of DP if they are statistically independent to each other. This is particularly easy when the design is decoupled.

2.1.2 Information Content and Information Theory

Since information content shares its name and mathematical formula with well-known definition of information in traditional information theory – hereinafter referred to as Shannon information –, it seems appropriate to clarify the difference between the two.

The way the information content is defined in axiomatic design theory is that we look at the event of a functional requirement's 'success,' success being a functional requirement value falls within the specified design range. Define a binary random variable u_i for FR_i,

$$u_i = \begin{cases} 1 & \text{with } p_i \\ 0 & \text{with } 1 - p_i \end{cases} \tag{2.17}$$

where p_i is the probability of success for FR_i. Whether FR_i is a discrete or continuous variable, u_i is always a binary variable of success(1) or failure(0). The information content is, then, written as

$$I(FR_i) = -log_2(p_i) = I(u_i = 1) \tag{2.18}$$

As you can see, the information is measured not directly for the variable FR_i, but for the interpreted variable u_i which is the result of imposing a design range over the distribution of FR_i. It is the information of the event that a functional requirement is achieved. Using the coin-toss analogy, it is the information of getting head side in a coin toss. Having defined a binary random variable, both information content in axiomatic design and Shannon information are exactly same quantity in regard to their definition. However, interpretation of the information is subtly different. Information content in axiomatic design is considered as the amount of 'information' required to resolve the uncertainty in achieving the functional requirement. On the other hand, Shannon information is typically perceived as the amount of 'surprise' by the happening of particular event. Besides the subtle difference in interpreting the quantity, the information theory rarely discusses the quantity as it is. The theory is based on another quantity, entropy, rather than the information of individual event. Shannon entropy of a random variable X, $H(X)$ is defined as

$$H(X) = -\sum p_i \cdot log_2 p_i = E[I] \tag{2.19}$$

It can be regarded as average information of a random variable X. Since u_i is also a random variable, the entropy can be written for u_i:

$$H(u_i) = -\{p(u_i = 1) \cdot log_2 p(u_i = 1) + p(u_i = 0) \cdot log_2 p(u_i = 0)\} \tag{2.20}$$

But, axiomatic design theory does not employ the concept of Shannon entropy in discussing information content. The reason why axiomatic design theory does not define/use similar quantity as H is because of the way the information is intended to be used. The following figures illustrate the point.

Figure 2-3(a) shows that information content, $I(u_i = 1)$ is zero when p_i is 1, and

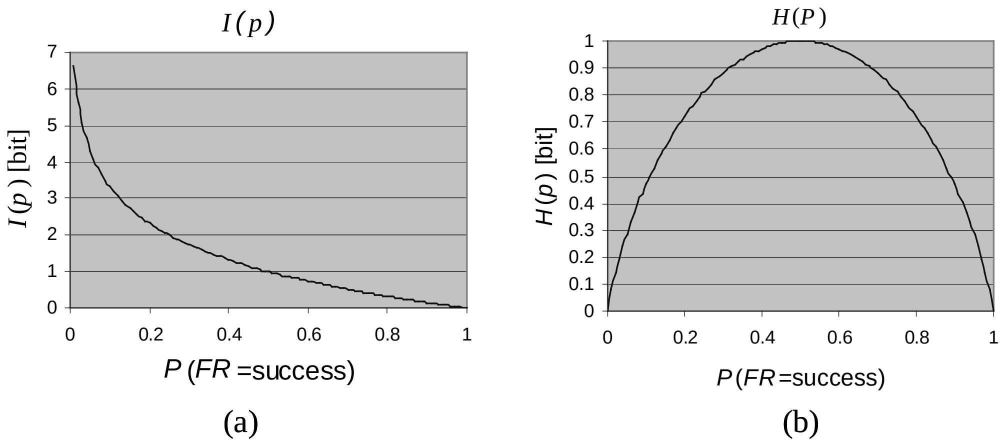


Figure 2-3: (a)Information content vs. (b)Entropy

increases logarithmically as p_i decreases. On the other hand, $H(u_i)$ reaches maximum when p_i is 0.5 and is zero when it is either 0 or 1. Information content measures the uncertainty of the occurrence of "success," whereas the entropy measures the uncertainty of the event itself. It can be rephrased as following: the information content measures the uncertainty in achieving a functional requirement, whereas the entropy measure the uncertainty in *knowing* whether a functional requirement is achieved or not. Because of the way axiomatic design interprets the information content – something necessary to resolve uncertainty in achieving functional requirement –, the entropy is not quite appropriate quantity to be used in its context. Instead, it simply uses the information as the metric for uncertainty in achieving a functional requirement.

Since FR itself can be considered a random variable, it is possible to define entropy for FR. For a continuous FR_i, the entropy is defined as

$$h(FR_i) = \int f(FR_i) \cdot log_2 f(FR_i) \mathrm{d}FR_i \tag{2.21}$$

This quantity does not fit in to the context of information content discussion, either. First of all, the above definition does not involve the design range which is of critical interest from axiomatic design standpoint. One might suggest that we do integration only over a design range. That is quite arbitrary, and does not resolve the problem:

when all FR values are within design range, this value is the same as equation (2.21), while we want the quantity to be, say, zero. Secondly, some of the properties of h are not acceptable in the context: for example, $h(FR_i + \text{constant}) = h(FR_i)$, but the shift in the functional requirement value has significant effect on the success or failure of the design.

As discussed above, the emphasis on success, i.e. achieving functional requirement, makes the information content a judgemental concept instead of neutral one. Consequently, entropy concept becomes irrelevant. The fact that axiomatic design does not use the concept of entropy prevents the result of information theory from being transferred and utilized since the information theory is based on the entropy. The information content in axiomatic design simply changes the scale of the probability of success, and thus its implication does not quite go beyond the straight probability discussion. In fact, that partially explains why the concept of complexity is introduced in axiomatic design.

2.1.3 Dealing with Real Complexity

Typically, certain amount of real complexity is unavoidable because a system is subject to the randomness such as input variation and it has non-zero sensitivity with respect to the variation while design range is finite. Large amount of real complexity implies that the task of achieving functional requirements of a system is difficult and thus it is perceived as a complex task. Thus, there is a definite need for some methodologies to reduce this type of complexity. Indeed, addressing the issue of reducing the real complexity has been popular topics of engineering design research. To illustrate these methods, it is useful to write down a 'loose[3]' mathematical expression.

$$\overrightarrow{FR} - \overrightarrow{FR}^* = \left.\frac{\partial \overrightarrow{FR}}{\partial \overrightarrow{DP}}\right|_{\vec{DP}=\vec{DP}^*} (\overrightarrow{DP} - \overrightarrow{DP}^*) + \left.\frac{\partial \overrightarrow{FR}}{\partial \vec{n}}\right|_{\vec{n}=0} \Delta\vec{n} + \left.\frac{\partial \overrightarrow{FR}}{\partial \vec{C}}\right|_{\vec{C}=\vec{C}^*} (\vec{C} - \vec{C}^*) \tag{2.22}$$

The term on the left-hand side represents a deviation of the functional require-

[3]It is loose in a sense that the expression is not mathematically rigorous.

ments from its target value, $\overrightarrow{FR}^*$. The objective of engineering effort is to minimize, if not eliminate, the magnitude of this term. Right-hand side of the equation is the Taylor expansion of the functional requirements, and the terms are clustered to three categories: input (design parameter) variation, noise, and compensation. One evident source of output variation is the input variation, i.e. the deviation of the input variable from its nominal value. It is represented by the first term on the right-hand-side. Some of the output variation can be attributed to noise. Noise is the random variation of parameters other than the input variables, and its impact on the output variation is determined by its magnitude and the sensitivity of the output with respect to the noise factors. The last term represents the compensation factor, which is used to counteract to the variation induced by the first two terms. In the broadest sense, there are three approaches to deal with the output variation: eliminate the source of variation, desensitize the system, and compensate for the variation.

- Eliminate the source of variation

Eliminating the source of variation is to reduce or eliminate $\Delta\vec{n}$ or $\Delta\overrightarrow{DP}$. This approach is usually effective in the first-order reduction of the variation. The strategy is to identify the most troublesome noise factor and/or input variation, which has a large sensitivity associated with it, and to either directly reduce the magnitude of it or focus on the root cause of such variation. Examples of this approach include statistical process control (SPC) and Poka-yoke. SPC is dedicated to detect abnormal – statistically out of control – behavior of a system response. Once the system is determined to experience abnormal behavior, every effort is made to find assignable cause of the problem and eliminate it. Poka-yoke is a Japanese word that is interpreted as "mistake-proofing device." It is aimed to achieve 100 percent inspection, and its central idea is to systematically prevent any mistake from occurring in manufacturing process. Unfortunately, this approach of eliminating the source of variations is generally limited since one may not have any control over those variations. Also, as the level of variation becomes smaller, it can be unreasonably costly to reduce the magnitude of the variation by these methods.

- Desensitize the system

Desensitizing the system is commonly known as robust engineering. As opposed to the case of eliminating the source of variation, it focuses on the sensitivity portions of the first two terms of the right-hand-side of the equation (2.22). As mentioned above, it is mostly inevitable to have certain level of noise factor or input variation even after all the effort to eliminate them. Having assumed so, what can be done to deal with those 'leftover' variations is to make the system insensitive to them so that the effect of the variation – noise and input variation – is minimized in the output variation. Oftentimes, it is very economical solutions compared to the first approach, especially when the desired level of output variation is very low. Robust design, also known as Taguchi method, is a famous example of this approach.

- Measure and compensate

Compensation is another strategy to deal with the variation. In some cases, previous two approaches do not deliver the desired level of output variation. Then, the remaining option is to measure the deviation and compensate for it. Sometimes, for example in customized product manufacturing, the compensation approach is the most economical option. This is represented by the last term on the right-hand-side of the equation (2.22). First, one has to find some parameters that can be used as "compensators", and determine appropriate values for them to cancel the effect from noise factors and input variations. That is,

$$\left.\frac{\partial\overrightarrow{FR}}{\partial\vec{C}}\right|_{\vec{C}=\vec{C}^*}(\vec{C}-\vec{C}^*) = -\left(\left.\frac{\partial\overrightarrow{FR}}{\partial\vec{n}}\right|_{\vec{n}=0}\Delta\vec{n} + \left.\frac{\partial\overrightarrow{FR}}{\partial\overrightarrow{DP}}\right|_{\vec{DP}=\vec{DP}^*}(\overrightarrow{DP}-\overrightarrow{DP}^*)\right) \quad (2.23)$$

This approach is sometimes called as active cancellation, and a well-known example of this approach is noise cancellation system that can be found in a luxury vehicle or audio system. Another example of this approach is very mundane manufacturing technique: shimming.

2.2 Imaginary Complexity

In section 2.1, the uncertainty caused by the random nature of a system was discussed. It is represented by the probability of success of a set of desired functional requirements, and this type of uncertainty is defined as a real complexity. The second type of time-independent complexity in axiomatic design theory, imaginary complexity, is defined as *uncertainty that arises because of the designer's lack of knowledge and understanding of a specific design itself.*

2.2.1 Ignorance in Design Process

Since the definition of imaginary complexity is centered on 'ignorance', which is quite an ambiguous concept, it is necessary to explain what it really is. Recall that in axiomatic design theory, a design is represented by a hierarchy of {FR}, {DP} and [DM] as shown in figure 1-3. Thereby, ignorance, or lack of understanding can come from these three sources.

Ignorance of {FR} is related to the failure to properly understand and define a design task. For example, missing some of the essential functional requirements or misinterpreting them, yields incomplete design as it addresses simply wrong problem. Resulting design does not deliver truly desired functional requirements. While capturing all of the essential functional requirements is critical, it should be noted that having unnecessary functional requirements is not desirable as well since a larger set of functional requirements tend to increase the uncertainty. To capture the functional requirements no more and no less than necessary can be very challenging task. It is particularly so when sociological/psychological factors are considered: for example, in many consumer electronics, technologically unnecessary - or rarely appreciated - functions can be added for marketing purpose. Missing some of those technically-inessential functional requirements may lead to disastrous business consequence. The importance of selecting the best set of functional requirements cannot be overemphasized. Specifying excessively tight design range is another type of ignorance of {FR}. Obviously, that will make the design problem more challenging than it should be.

One can make any design problem arbitrarily difficult by requiring that its output be infinitely precise. Therefore, it is crucial to know appropriate level of design range for functional requirements. Unfortunately, the problem of assessing the right level of design range throughout the system hierarchy has not been clearly understood yet, which is part of the reason why tighter specification is common in practice. Overall, ignorance of {FR} increases uncertainty of a system above what it really is, and thus contributes to imaginary complexity.

Lack of knowledge required to synthesize or identify proper {DP} will increase the uncertainty. This type of ignorance can be due to the sheer absence of technological expertise in a particular subject matter or inability to predict emergent effect when a particular solution is implemented. Oftentimes, advances in technology resolve the problem. On the other hand, as seen from the planet motion example in section 1.1, it could be due to wrong choice of model to solve the problem. The uncertainty caused by the above is not inevitable, and thus imaginary.

Yet, these types of ignorance are very difficult to effectively measure because the context of such ignorance is in the semantic domain. Therefore, even though we can qualitatively argue that the uncertainty is the consequence of the lack of knowledge on the subject matter, any degree of quantification seems formidable as of now. However, the uncertainty caused by these types of ignorance is still part of imaginary complexity in a qualitative sense. For the sake of quantification, complexity discussion in axiomatic design theory limits the scope of the term to the ignorance of the [DM] structure [2]. It focuses on the iteration in the design process, and comes up with a particular measure for it.

2.2.2 Iteration in Design Process and Imaginary Complexity

Almost inevitably, design process involves some iterations. Iteration in design process may have different meanings depending on the context, and here, it strictly refers to the iteration to reach a set of target values of responses through successively changing values of a set of inputs. Iteration process - number of iterations, sequence, convergence, etc. - is dictated by the interrelationship among the inputs and target responses. In axiomatic design framework, design parameters and functional requirements correspond to inputs and target responses respectively, and a design matrix is used to represent the interrelationship among the FRs and DPs. For uncoupled design, which has a diagonal design matrix, DPs and FRs have one-to-one relationship, and thus it does not require any iteration in getting each of the functional requirements on target value. For decoupled design, some of DPs affect more than one FRs with one-to-many relationships. As a consequence, there exists a certain sequence only through which target values for FRs can be reached without iteration. In case of a coupled design, the desired FR values are obtained through a number of iterations.

In the attempt to satisfy all the desired functional requirements, one has to follow a certain sequence dictated by the structure of the design matrix, if there exists such sequence. Failure to do so will cause unnecessary iterations or even non-convergence as shown in Figure 2-4, which eventually increases the uncertainty in achieving the desired functional requirements. Therefore, it is important to understand the structure of design matrix to avoid undesirable consequence. Based on the above reasoning, the probability of selecting a correct sequence is associated with the uncertainty in achieving functional requirements. The imaginary complexity is defined as following:

$$C_I = \log_2 \frac{1}{p(\text{selecting a correct sequence})} \tag{2.24}$$

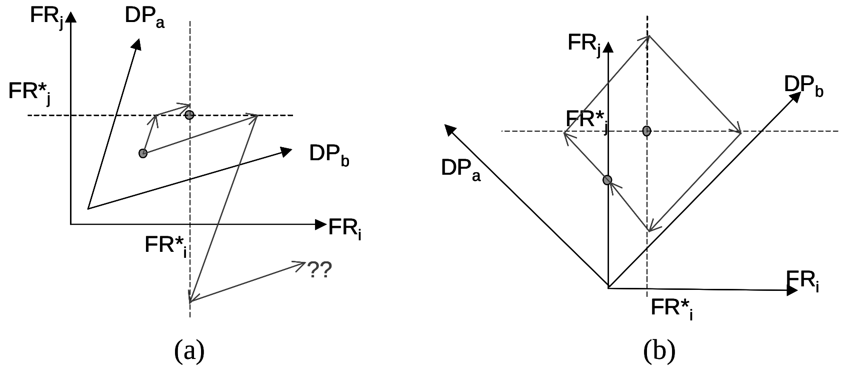


Figure 2-4: Design iteration can be non-converging by (a) selecting inappropriate DP for FR or (b) singularity

For uncoupled design, p(selecting a correct sequence) is 1 since any arbitrary sequence is valid. Iteration is not needed, which means no uncertainty associated with additional iteration. Thus, C_I is 0. For a coupled design, the probability is not defined because there exists no right sequence in coupled design. In a decoupled design, the probability is $(z/n!)$ where z is the number of valid sequences and n is the total number of functional requirements.

It is interesting to note that the probability is not a function of the size of design matrix. It is rather a function of the precedence relationships among FRs and DPs, which is roughly the number of off-diagonal elements in the matrix. For example, a 2×2 decoupled design has the same probability as a 7×7 matrix with one off-diagonal element. For 2×2 decoupled matrix, z is 1 and n is two. Thus, the probability of selecting the correct sequence is

$$p = 1/2! = 0.5 \tag{2.25}$$

For 7×7 matrix with one off-diagonal element, there is only one precedence relationship as in 2×2 case. Therefore, the probability should be also 0.5. z is $5!\times({}_6C_2 + 1)$, and n is 7. The probability is

$$\begin{aligned} p &= \frac{5! \times \left\{ \begin{pmatrix} 6 \\ 2 \end{pmatrix} + 6 \right\}}{7!} \\ &= 0.5 \end{aligned} \tag{2.26}$$

which agrees with our intuition.

The definition of C_I has some problems. First of all, strictly equating the probability of finding a correct sequence to the probability of success is a bit of overstatement. Selecting wrong initial sequence does not exclude the possibility of reaching a solution. It is very possible that the solution can be obtained through some iteration even when the initial sequence tried is not the correct one. Also, the probability of $(z/n!)$ assumes completely random trial when changing DP values to satisfy functional requirements, which is rarely the case in practice. Therefore, it is rather a lower bound – worst case – in the presence of decoupled elements. The fact that it is only applicable to decoupled design also weakens the utility of the concept. More fundamental problem is, though, that the quantity C_I carries little practical meaning because once we are able to measure C_I, it is not there anymore: measuring C_I requires *knowledge* about the structure of design matrix, which will, in turn, eliminate C_I.

In spite of some controversial aspects in its metric, C_I, the concept of imaginary complexity is still valid in a pedagogic sense. It is very acceptable that failure to recognize the structure of design matrix, i.e. interactions among FRs and DPs, will cause some uncertainty even when the design solution is indeed good. That is clearly undesirable and must be avoided. Therefore, the main message that needs to be emphasized regarding the imaginary complexity is "in order to minimize the uncertainty in achieving functional requirements, one has to know what (s)he is doing before doing it."

2.3 Information Content vs. Complexity

Having followed the discussion on real complexity (and imaginary complexity), one might suspect that the concept of complexity is not different from the information content: complexity is defined as a measure of uncertainty, and the information content is defined in terms of probability of success for a functional requirement(s) that is, in fact, uncertainty.

Let us revisit the definition of complexity in axiomatic design. Complexity is a certain quality or quantity, perceived in regard to a designed system, that is a *consequence* of the uncertainty in achieving a desired set of functional requirements. Thus, strictly speaking, complexity is not the same as uncertainty. Since it is such an abstract concept, there is no *direct* measure for it. In search of a metric, it is forced to consider a reasonable surrogate. By hypothesizing that complexity is proportional to the uncertainty in achieving functional requirements, the uncertainty itself is chosen as a measure of complexity. Therefore, uncertainty is a measure of complexity, and at the same time, complexity is a measure of uncertainty.

Generally, uncertainty is directly related to probability, and it is true that probability is a good metric for uncertainty. However, uncertainty in axiomatic design can be understood in a probabilistic context and a non-probabilistic context. In probabilistic sense, the uncertainty is related to the probability of success p_s. Zero probability gives infinite uncertainty, and probability of one yields zero uncertainty. By the probabilistic nature of the concept, this type of uncertainty has its cause in the randomness around a design, i.e. DP variation, noise, etc. Naturally, this type of uncertainty is measured directly by information content, $I = log_2 1/p_s$. In non-probabilistic context, the uncertainty cannot be easily quantified by probability. The uncertainty in this context is improbability or un-likelihood that can be attributed to non-probabilistic factors such as ignorance. In spite of the lack of quantitative measure, the existence of this type of uncertainty is out of doubt. Information content, while a reasonable measure for uncertainty in probabilistic context, cannot extend further to reach non-probabilistic regime of uncertainty. That is the main reason to introduce the concept of complexity.

Imaginary complexity would not have been realized if we had focused only on information content, and neither would time-dependent complexity concept. The contribution of complexity concept is that it enables to investigate the subject of uncertainty in various dimensions in addition to probabilistic perspective.

2.4 Summary

Time-independent complexity is the complexity where uncertainty in achieving functional requirement is time-independent. It implies that its system range does not change over time and that functional requirements are also time-independent. Two types of time-independent complexity is defined in axiomatic design theory: real complexity and imaginary complexity.

Real complexity accounts for random nature of a system, and is defined as *a measure of uncertainty when the probability of achieving the functional requirements* *is less than 1 because the common range is not identical to the system range.* By its definition, it is equivalent to information content since information content measures the uncertainty in the same probability context. Information content is defined as a function of probability of success. Estimating probability of success typically requires simulations or experiments, but if design equations are known and simple, it can be easily derived. Information content is almost same as Shannon information, but two concepts are subtly different because of the way information is interpreted.

Reducing real complexity is essentially reducing variation in functional requirements. Common engineering research and practice include 1)eliminating source of variation, 2)desensitizing a system, and 3)compensating for the variation. Based on technical and economic consideration, these three approaches must be combined for optimal result.

Imaginary complexity, on the other hand, accounts for the uncertainty due to ignorance. Lack of proper understanding of functional requirements, lack of knowledge or wrong choice of design parameters, and ignorance of design matrix structure increase the uncertainty. Even though the ignorance is difficult to effectively measure, the existence and its effect on uncertainty is out of doubt. Quantitative definition of imaginary complexity in axiomatic design is limited and imperfect, but the message is still valid.

Chapter 3

Time-dependent Complexity

As opposed to time-independent complexity, time-dependent complexity requires time dimension as one of its determinants. Although it may not be an explicit function of time, the behavior of functional requirements necessitates time factor for proper description of it. According to Suh's definition [2],[24], for time-dependent complexity, the uncertainty changes as a function of time. Why would the uncertainty change over time? The time-dependency can come from either 1) time-varying system range or 2) unpredictability of functional requirements in future.

Time-dependent complexity is divided into two different kinds: combinatorial complexity and periodic complexity. In a combinatorial complexity case, the uncertainty grows indefinitely whereas it stops increasing and returns to the initial state occasionally in case of periodic complexity. In some sense, a system with a combinatorial complexity behaves in a similar way as a chaos system does. Chaos, or sometimes simply called a dynamic instability, is defined as the extreme sensitivity to the initial condition. Two nearly indistinguishable initial conditions would result in two final predictions which differ vastly from each other.

One of the most interesting issues in the study of time-dependent complexity system is whether or not the presence of combinatorial complexity leads to unmanageable design task and, if it does, whether or not it can be reduced to certain degree.

In this chapter, the origin of time-dependency is discussed: time-varying system range and unpredictability of functional requirements in future. Then, functional periodicity is introduced to define the concept of periodicity, semi-periodicity, and aperiodicity. This discussion will serve as the theoretical basis for the work presented in the remaining of this thesis.

3.1 Time-dependent Complexity

By Suh's original definition, complexity is a measure of uncertainty in achieving a desired set of functional requirements. This definition implies that the complexity is a consequence of being uncertain about delivering required FRs or that the uncertainty is a symptom of a system's being complex. In section 2.3, it is argued that complexity is a measure of uncertainty and at the same time, the uncertainty is a measure of complexity. It is particularly explicit for time-independent real complexity: complexity is measured by the information content, which measures the (probabilistic) uncertainty. Either way, it is clear that time-dependency of complexity is related to time-dependency of uncertainty. In other words, for time-dependent complexity, the uncertainty changes as a function of time. So, if it changes with time, the direction of change is going to be of our concern. If it decreases, then it is less of a problem. For such case, major attention will be given to the initial uncertainty and the rate at which it decreases. One example of such systems is a speech recognition system with learning capability. The uncertainty in performing a given task, recognizing natural spoken language, becomes smaller and smaller with time[1]. Unfortunately, there are much more instances where the uncertainty increases with time. Systems with dynamic instability, which many real world systems are considered as, present increasing uncertainty. In a mechanical system, wear of its components typically increases uncertainty in its functionality. This ever-increasing complexity can possibly pose serious problems such as sustainability and stability. It is this increasing uncertainty that axiomatic design's time-dependent complexity discussion has its focus on.

[1]Performance of such system improves as it processes more and more reference data. It is assumed that the amount of data it processes is proportional to the time of its operation.

Recall that there are two types of uncertainty: quantitative (quantifiable) and qualitative (non-quantifiable). The first type of uncertainty is subject to probabilistic treatment, and thus is related to time-varying system range. The second type is explained in terms of the unpredictability of functional requirements in future.

3.1.1 Time-varying System Range

Time varying system range is illustrated in figure 3-1 for a simple single-FR case. Given a design range, suppose that a design solution initially yields a system range indicated by a solid line. For a variety of reasons, after a certain time period, the system range can change and deviate from its initial distribution as indicated by dotted lines, thereby changing the probability of success. The new probability of success at time t is,

$$p_s^t = \int_{\substack{design \\ range}} f^t(FR)\mathrm{d}FR \tag{3.1}$$

Since FR is a function of design matrix and design parameter, this change can be due to the shift in the mean of design parameter, increased variation of design parameter, and/or change in design matrix element. Uncoupled n-FR design is equivalent to n single-FR problems and thus treated similarly as a single-FR case. The overall probability of success is simply the product of the individual probability of success for n FRs.

For decoupled and coupled n-FR design, change in any one of the FRs always implies resulting change in the joint probability density of n FRs as well as its own marginal probability. That is also the case with uncoupled design, but there is one obvious but important difference. Unlike uncoupled design, the change in one FR in a decoupled/coupled design may be associated with other FR's distribution depending on the structure of design matrix. For example in a hypothetical design of equation (3.2), change in DP_1 affects both FR_1 and FR_2 distributions whereas change in DP_2 results only in FR_2 distribution. This should be obvious once the design matrix structure is understood.

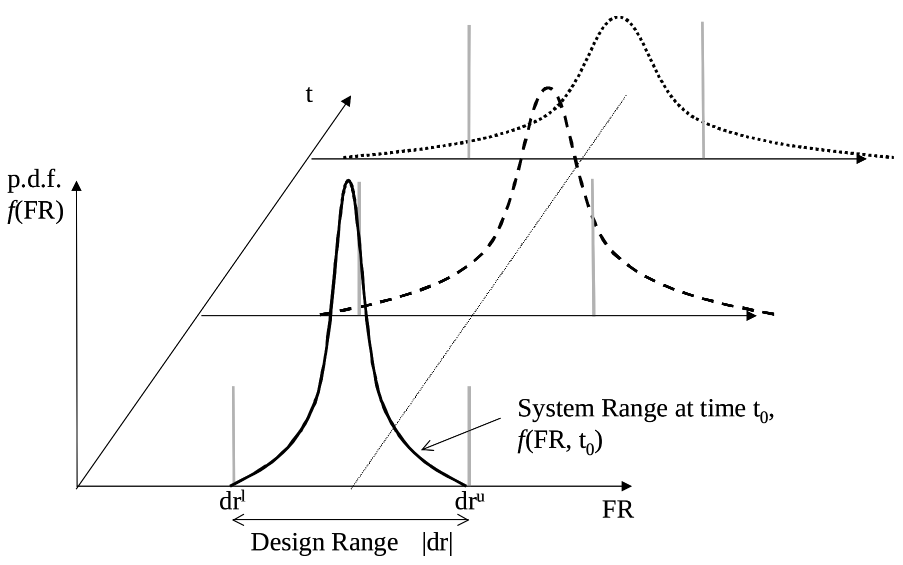


Figure 3-1: Time-varying system range for a single FR

Time-varying system range for multiple FR

Less obvious is the fact that monitoring individual marginal probability could be quite misleading in making decisions regarding the change of system range. Even though the FRs being monitored individually seem well under control, the overall probability of success could be decreasing. In chapter 2, it was shown that correct computation of probability of success for multi-FR, unless uncoupled, requires joint probability density function be evaluated rather than marginal probability density function. It has to be emphasized again that all of the relevant functional requirements have to be considered simultaneously, i.e. one must pay attention to the joint probability rather than marginal probability. Consider, for example, the following 2-FR decoupled design.

$$\begin{Bmatrix} FR_1 \\ FR_2 \end{Bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{Bmatrix} DP_1 \\ DP_2 \end{Bmatrix} \tag{3.2}$$

For the sake of simplicity, assume that DP_1 and DP_2 initially follow uniform distributions U[-1,1] and U[0,1.5] respectively. The joint probability density function for (FR_1, FR_2) is a uniform probability of $\frac{1}{3}$ over the range shown in figure 3-2(a). Marginal probability density functions for FR_1 and FR_2 are shown in figure 3-2(b)

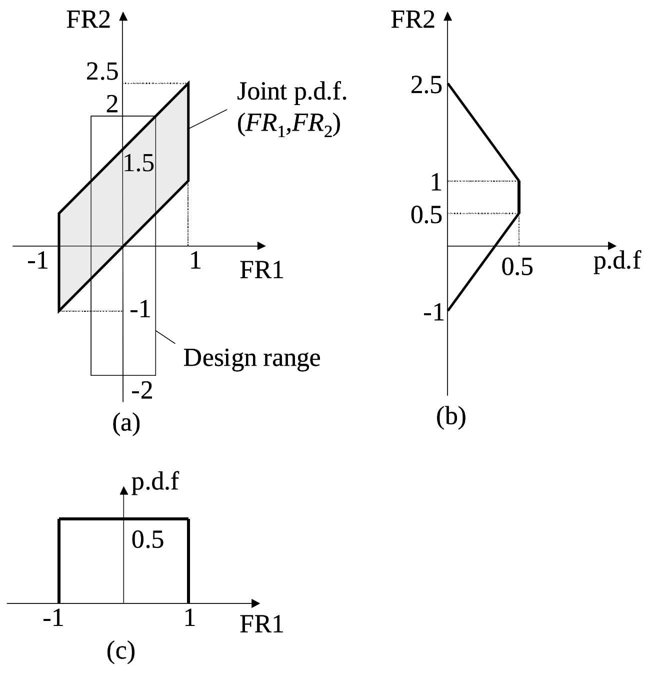


Figure 3-2: (a) Joint p.d.f. of (FR_1, FR_2) is defined over the parallelograms, (b) marginal p.d.f. of FR_2, and (c) marginal p.d.f. of FR_1

and (c) respectively. Suppose that p.d.f. of DP_2, after some time t, has been changed to U[-1,1.6]. Then, the joint probability density of (FR_1, FR_2) changes accordingly as shown in figure 3-3(a). The value of new p.d.f. is $p = \frac{1}{5.2}$, and the parallelogram is also changed. Note that the marginal probability density function for FR_1 still remains same for both cases: U[-1,1]. Therefore, until the change in FR_2 or the joint distribution of (FR_1,FR_2) is monitored, the overall system's deterioration is hidden. Monitoring the marginal distribution of FR_2 will resolve the problem only partially: it could lead to a wrong conclusion about the system's performance. Suppose the design range for FR_1 and FR_2 is [-0.5,0.5] and [-2,2] respectively. This design range corresponds to a rectangle on FR_1-FR_2 plane as shown in figure 3-2 and 3-3. Table 3.1 summarizes the probability calculation for the system before and after the change. As you can see, the marginal probability for FR_1 remains same at 0.5 after the change, and it even slightly increases by 0.007 for FR_2. Given such information, it is likely to conclude that the system is producing better output. On the contrary, however, the overall probability of success, i.e. the probability determined by joint distribution, shows slight decrease from 0.5 to 0.499. This example shows the effect of statistical

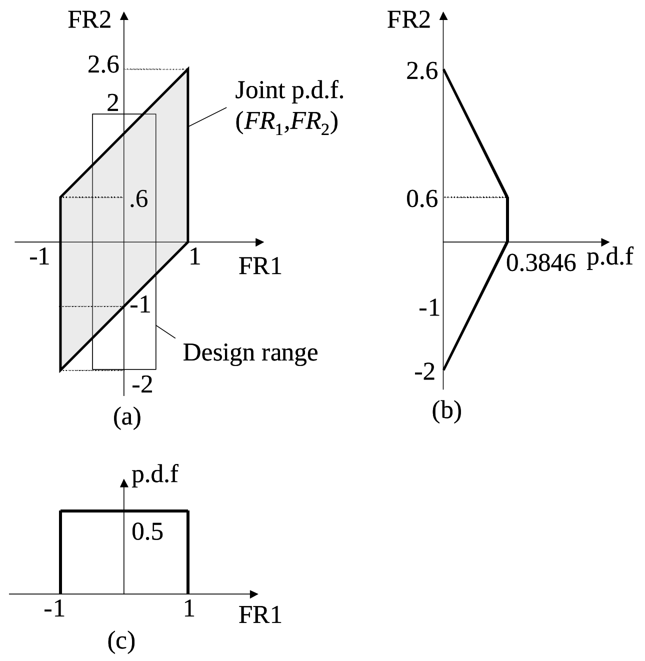


Figure 3-3: (a) Modified joint p.d.f. of (FR_1, FR_2) is defined over the parallelograms, (b) marginal p.d.f. of FR_2, and (c) marginal p.d.f. of FR_1

	p_{FR1}	p_{FR2}	$p_{FR1} \times p_{FR2}$	$p_{FR1,FR2}$
Before	0.5	0.9583	0.4792	0.5
After	0.5	0.9654	0.4827	0.4990

Table 3.1: Marginal probability of success for FR_2 increases while the overall probability of success decreases.

dependence of two functional requirements and clearly points out why we should monitor joint probability rather than individual marginal probability.

Dealing with time-varying system range

An apparent issue in dealing with time-varying system range is how to bring the shifted or widened distribution back to initial distribution. First thing that needs to be done is to detect this change in system range which may require continuous monitoring effort such as Schwartz control chart. Control chart is intended to detect a deviation from a process's normal output distribution. It reveals mean shift and increase of variation in output, and differentiates systematic change of the process output from normal, acceptable disturbance. Note that the control chart is irrelevant

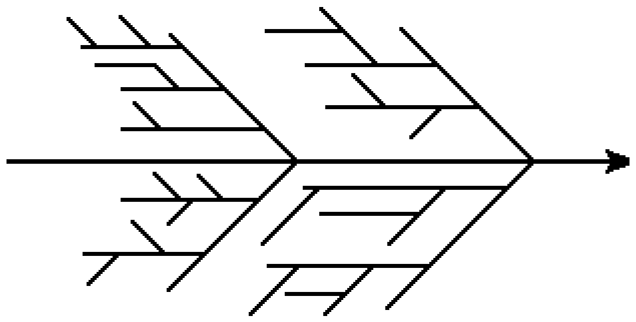


Figure 3-4: Cause-and-effect diagram

of design range. To be more useful, the information from control chart should be used along with design range because it puts the information in the proper context. As discussed above, for more than one FR case, unless the system is uncoupled, functional requirements have to be monitored jointly, i.e. its joint distribution must be monitored rather than individual marginal distribution.

Once deterioration from the original distribution is discovered, assignable causes must be identified. A cause-and-effect diagram[2](figure 3-4) is an example of common practices to find root causes of the problem. Methods like Schwartz control chart and cause-and-effect diagram are commonly termed as statistical process/quality control (SPC/SQC). A complete decomposition of functional requirements and design parameters along with associated design matrix should be very helpful when identifying the root cause of the problem. One of the reasons why it is typically hard to find out the root cause is that they attempt to link directly between a symptom (high-level FR) and low-level parameters. If the architecture of the system is well understood and represented by {FR}-[DM]-{DP} hierarchy, it quickly narrows down the scope of the problem of locating the root cause [25]. Indeed, the flow-diagram representation of system architecture in axiomatic design can be used as the platform of a cause-and-effect diagram.

We know some of the common causes of varying system range always exist in reality. Examples are tool wear, machine component wear, fatigue, etc. To avoid the cost of system range deterioration, which could sometimes be significant, preventive

[2]It is interchangeably called 'fishbone diagram' or 'Ishikawa diagram.'

maintenance can be carried out instead of waiting until the change has happened due to the common causes. Obviously, frequency of preventive maintenance and the cost associated with it must be traded off against the cost of losing process capability.

Although sometimes it may be the only option in practice, SPC approach to recover original system range is not a fundamental solution to the problem. Fundamental solution that must be sought for whenever and wherever feasible is to make a design more robust and reliable as much as possible. Following example illustrates how a design change effectively delays the system range deterioration.

Example 3-1: Decoupling a Knob Design

Let us consider the "Knob design" example presented in [2]. It has to satisfy two functional requirements:

FR1 = Grasp the end of the shaft tightly

FR2 = Turn the shaft

DPs are chosen such that the two FRs are satisfied. Refer to figure 3-5 for illustration:

DP1 = Interference fit (between the shaft and the inside diameter of the knob)

DP2 = Flat surface

Design (a), one of the implementation of the above DPs, is a decoupled design[3] : when the torque is applied by the knob, the grip force on the shaft decreases with the increase of the slot opening as a result of normal load acting on the flat surface. As a result of coupling in design (a), the knob will slide off the shaft after certain period of use. In other words, FR1, grasping force, is getting out of design range as time elapses. One way of dealing with this problem is to simply increase the wall thickness

[3]In [2], design matrix element (2,1) is said to have a small value, and accordingly the design was considered 'weakly-coupled'. For the sake of our discussion, it can be ignored because the problem occurs due to the dependence of FR1 from FR2 - design matrix element (1,2).

of the knob. It will increase the stiffness of the knob such that it has bigger resistance to slot opening, and thus will sustain longer period of use. However, it induces another problem: to make the interference fit work without excessive force requirement in manufacturing process, a tight control over the part dimensions must be maintained. This will bring up the process control issue, and likely incur additional cost in manufacturing. More fundamental approach is to get rid of the root cause, which is, in this case, the dependency of FR1 on the FR2. Such a design is presented in (b). In this design, the slot terminates where the flat part of the knob begins. Because the flat surface is completely away from the slot, the turning action does not force the slot to open and, therefore, the axial grip is not affected. Achieving an uncoupled design by the slight design change delays the system range deterioration.

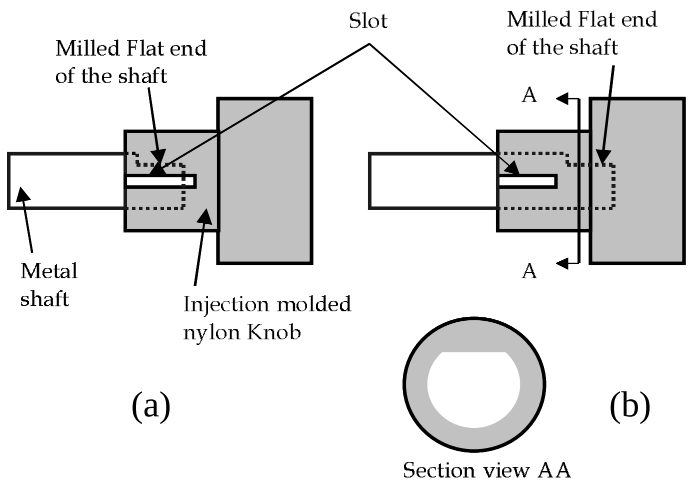

Slot
Milled Flat end of the shaft
Milled Flat end of the shaft
A
A
Metal shaft
Injection molded nylon Knob
(a)
(b)
Section view AA

Figure 3-5: Knob design example (adapted from [2])

3.1.2 Unpredictability of Functional Requirements in Future

Previous section discusses the time-dependent complexity caused by time-varying system range. It implicitly assumes that functional requirements and their design ranges are static. The second type of the source of time-dependency is the unpredictability of FRs in future. This unpredictability is related to time-varying attribute of functional

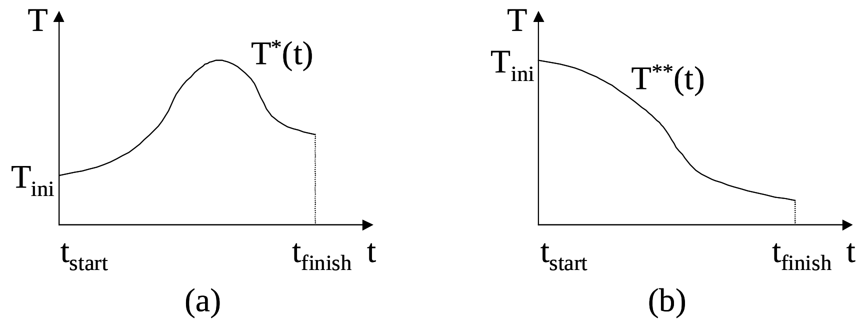


Figure 3-6: Desired FR values are changing as a function of time, and thus dynamic FR. From design perspective, however, T*(t) is not different from T**(t).

requirements. The complexity in axiomatic design can only be defined in relation with functional requirements, and thus the functional requirements' dependence on time makes associated complexity also dependent on time. It is possible that, even without time-varying system range, there is time-dependence due to functional requirements. There can be different types of time-varying behavior of functional requirements.

Functional requirements are considered time-dependent when 1) target FR values are time-varying and 2) the elements of a set, {FR}, changes. The former was called dynamic FR [26], and the latter was discussed by Suh[27] when defining *large* system. Following example illustrates the concept of dynamic FR.

- FR: Control temperature profile of a heating chamber

This FR can be re-stated as "$T = T^*(t)$ while $t_{\text{start}} \leq t \leq t_{\text{finish}}$ ", and shown in figure 3-6. As you can see, the functional requirement is surrogated by a specific variable, i.e. T in this example, and the target value of it is varying as a function of time. A sophisticated version of this FR can be found in photoresist processing system in semiconductor manufacturing, and a mundane example would be a toaster oven in your kitchen. Once the function is initiated, its target value is moving along a specified trajectory as a function of time. It is obviously time-dependent in view of its operation – FR value is changing over time –, but if we consider designing process, time factor is not particular concern for this type of FR. For example, designing for a trajectory T^* is not different from designing for T^{**} in figure 3-6. Also, time t is not critical factor in a sense that achieving the FR - $T(t) = T^*(t)$ - at any time t_1 is not necessarily more uncertain than achieving the FR at other time t_2. If it does,

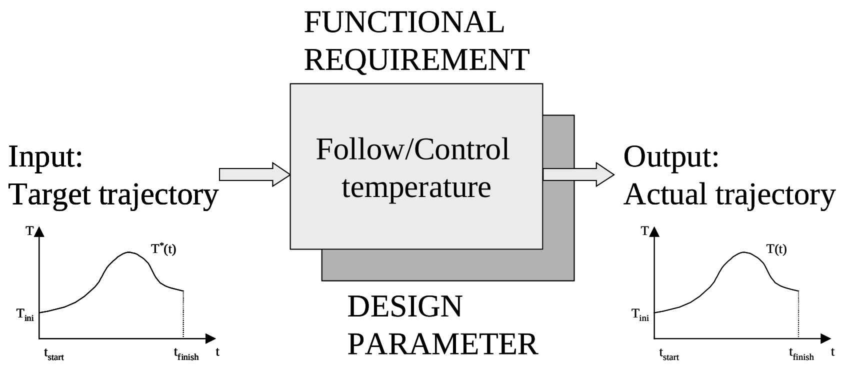


Figure 3-7: A representation that separates design process and operational signal flow. Time-dependence is now part of the input instead of FR.

it is more likely due to the value T than t. When the uncertainty does increase as t increases, then this problem becomes the same type as time-varying system range problem. The main concern of design is to come up with design parameter(s) to cover the entire operating range, $[T^l, T^u]$, with required error level, $\varepsilon = T(t) - T^*(t)$. It is most likely that this functional requirement is accompanied by static (constant) design range, which makes this type of problem less time-dependent. Therefore, even though the functional requirement value is time-varying in operation, its design is more like a static functional requirement from design perspective in a sense that we deal with the same functional requirement – that FR happens to be two dimensional – all the time. It can be shown more clearly if we take slightly different representation of FR-DP with operational input and output concept. In representation shown in figure 3-7, functional requirement is “Follow/Control temperature within the range $[T^l, T^u]$ with error level $\varepsilon \leq \varepsilon^*$.” While the functional requirement is semantically stated, the diagram representation treats the FR box as a kind of transfer function. It takes the input, which is temperature trajectory, and the functional requirement is achieved to produce actual output trajectory. Design parameter that can achieve this function is shown in the box beneath the FR box. So, FR-DP mapping is represented vertically, while the signal[4] flows horizontally. Thereby, design for this functional requirement - vertical interaction - is separated from time-dependent aspect of operational input/output, and thus time-independent.

[4]It can also be material or energy.

The other type of time-dependency of FR arises when the elements of a set, $\boldsymbol{FR}$, changes as a function of time. It is meaningful only when multiple number of functional requirements exist. Functional requirements are time-dependent in a sense that they are relevant at certain time (or duration), and become irrelevant at other time. For example, suppose a set $\boldsymbol{FR}$ consists of seven member FRs, i.e. $\boldsymbol{FR} = \{FR_a, FR_b, FR_c, FR_d, FR_e, FR_f, FR_g\}$. At different times, different subset of FR needs to be satisfied. That is,

$$\begin{array}{lll} t = t_1 & \boldsymbol{FR} = & \{FR_b, FR_c, FR_d, FR_g\} \\ t = t_2 & \boldsymbol{FR} = & \{FR_a, FR_e\} \\ \vdots & \vdots & \end{array}$$

At a particular time, $\boldsymbol{FR}$ consists of different combinations of FRs, signifying time – or sequence – as an important factor. In this case, one of the immediate issue is to avoid coupling at every t_i. First of all, number of available DPs must be no fewer than number of FRs at any time. Theorem 1 in axiomatic design states that "when the number of DPs is less than the number of FRs, either a coupled design results or the FRs cannot be satisfied." The theorem is modified to include this time-dependent characteristics:

> Theorem 1′: When the number of DPs is less than the number of *active* FRs *at any given time*, either a coupled design results or the FRs cannot be satisfied *for the duration.*

While we want enough number of DPs as a necessary condition to satisfy functional requirements, it is not desirable either to have too many DPs. Having more than necessary DPs results in redundant design and potentially inefficient design. By the fact that not all FRs are active at the same time, it may be possible to use some of DPs to satisfy multiple FRs without causing coupling. Using the same DP for more than one functional requirement is sometimes called 'function-sharing.' By the modified theorem 1, each FR's belonging to different subset $\boldsymbol{FR}$ is the necessary condition for successful function-sharing. In order to maintain optimal number of DPs – neither fewer nor more than number of FR at any given time –, prediction of $\boldsymbol{FR}(t)$ is necessary. If a system's $\boldsymbol{FR}$ is unpredictable, it is difficult to achieve the goal. Therefore, the predictability of $\boldsymbol{FR}(t)$ becomes important in a system with time-dependent $\boldsymbol{FR}$.

Time-dependency of $\boldsymbol{FR}$ implies that the functional requirements are recurring. The recurrence of functional requirements brings up a couple of issues which have not been considered for time-independent complexity. As discussed above, predictability of the pattern of $\boldsymbol{FR}(t)$is an important factor. Recurrence of FRs naturally relates to periodicity of $\boldsymbol{FR}$. In some cases, periodicity, which is defined in section 3.2, may be necessary condition for a system to sustain its functionality without becoming chaotic. Sometimes, sequence of functional requirements' occurrence is of a concern. Identifying correct precedence relationships between FRs and mechanism to ensure such precedence relationships are among the important design tasks. Triggering and stopping each of FRs at right moment is required. Completeness of $\boldsymbol{FR}$ is also an essential problem. All FRs must be achieved when they become present. In some cases, they have to be achieved once and only once (or twice and only twice, etc.) within a certain period. The completeness of recurrence of FRs naturally relates to periodicity of $\boldsymbol{FR}$, and it is indeed the basis for defining periodicity.

Time-dependency of FR

In this section, u_i, the binary random variable that was defined in section 2.1.2, is revisited to represent time-dependency of functional requirements and periodicity. Recall u_i for time-independent functional requirements. u_i for FR_i, was defined as following:

$$u_i = \begin{cases} 1 & \text{with } p_i \\ 0 & \text{with } 1 - p_i \end{cases} \tag{3.3}$$

The information content is, then, written as

$$I(u_i = 1) = -log_2(p_i) \tag{3.4}$$

If we consider u_i as a state variable, then for multiple number of functional requirements, a state vector $\mathbf{u}$, can be defined:

$$\mathbf{u} = \{u_1, u_2, \ldots, u_N\} \tag{3.5}$$

For time-independent case, the concerns are what is the probability of $\mathbf{u} = \{1, 1, \ldots, 1\}$ and how to increase the probability.

For time-dependent case, functional requirements are time-dependent in a sense that they are present or absent at particular time period. The definition of u_i is modified to include time factor:

$$u_i(t) = \begin{cases} u_i & \text{for } t \geq \tau_i \\ 0 & \text{for } t < \tau_i \end{cases} \tag{3.6}$$

where, τ_i is the time when FR_i emerges (or is decided whether achieved or not achieved). This definition basically accounts for the fact that an FR does not exist prior to certain time, t, which is peculiar to time-dependent FRs.

Then, the FR state vector, $\mathbf{u}$ becomes

$$\mathbf{u}(t) = \{u_1(t), u_2(t), \ldots, u_N(t)\} \tag{3.7}$$

By definition of $u_i(t)$, $\mathbf{u}(0) = \{0, 0, \ldots, 0\}$ and after certain time T, when all the functional requirements are achieved, $\mathbf{u}(T) = \{1, 1, \ldots, 1\}$. Note that time-dependency implies that functional requirements are recurring, i.e. they appear repeatedly and need to be achieved every time. Otherwise, introducing time factor may have not been necessary. Recurrence of functional requirements requires the issues such as periodicity be resolved.

A set of functional requirements is said to be deterministic if all of τ_i are known *a priori*. In such a deterministic set of functional requirements, $\mathbf{u}(t)$ is evidently a function of time. For example, assume, $0 < \tau_1 < \tau_2 < \tau_3 < \ldots < \tau_N$.

$$\mathbf{u}(0 < t < \tau_1) \;=\; \{0, 0, \ldots, 0\}$$

$$\begin{aligned}\mathbf{u}(\tau_1 < t < \tau_2) &= \{u_1, 0, \ldots, 0\} \\ \mathbf{u}(\tau_2 < t < \tau_3) &= \{u_1, u_2, 0, \ldots, 0\} \\ \mathbf{u}(\tau_N < t) &= \{u_1, u_2, \ldots, u_N\}\end{aligned}$$

Given that time-dependent FR implies recurrence, it is obvious that if τ_i is constant for all i, then the system is periodic.

Now that $\mathbf{u}(t)$ is defined, we can define probability of success for time-dependent situation. Since $u_i(t < \tau_i)$ is zero by definition, if we simply define the probability of success as in time-independent case, it is always zero until $t > \tau_N$. Therefore, we need to introduce a modified version of the probability of success. In doing so, it seems reasonable to define "success" at any given moment as achievement of the ideal state at that moment, ideal state being a state where all the *present* functional requirements are achieved. Then, probability of success becomes the probability of $\mathbf{u}(t) = \mathbf{u}^*(t)$, where * indicate the ideal state. That is,

$$p_s(t) = p[\mathbf{u}(t) = \mathbf{u}^*(t)] \tag{3.8}$$

For example, in the deterministic case,

$$\begin{aligned}\mathbf{u}(\tau_2 < t < \tau_3)^* &= \{1, 1, 0, \ldots, 0\} \\ \mathbf{u}(\tau_2 < t < \tau_3) &= \{u_1, u_2, 0, \ldots, 0\}\end{aligned} \tag{3.9}$$

The probability of success at time $\tau_2 < t < \tau_3$, $p_s(\tau_2 < t < \tau_3)$ is $p(u_1 = 1, u_2 = 1, 0 = 0, \ldots)$. Since {0=0} is redundant, $p_s(\tau_2 < t < \tau_3)$ is $p(u_1 = 1, u_2 = 1)$. That is the joint probability of both FR_1 and FR_2 being achieved. $p_s(t < \tau_1)$ is 1 since $\mathbf{u}(t) = 0$ and $\mathbf{u}(t)^* = 0$. $p_s(t > \tau_N)$ is $p(u_1 = 1, u_2 = 1, \ldots, u_N = 1)$, which is the probability of all FRs' being achieved. The probability of success, as defined above, is time-dependent, and that is one of the characteristics of time-dependent complexity.

3.2 Functional Periodicity

Having identified two different kinds of time-dependency – time-varying system range and time-dependent $\{\boldsymbol{FR}\}$ –, this section introduces the concept of periodicity. In axiomatic design, time-dependent complexity is divided into two different kinds: periodic complexity and combinatorial complexity. Criteria for differentiating the two time-dependent complexity is the existence of periodicity.

First, the apparent periodicity can be defined by system range's behavior. As shown in figure 3-1, time-varying system range changes and deviates from its initial desired distribution, decreasing probability of success. A system is said to have a periodicity, and thus periodic complexity, if the system range regains its initial state periodically (see figure 3-8). It should be noted that the periodicity is not defined in terms of time as opposed to conventional perception of periodicity. The periodicity in axiomatic design complexity discussion does not require regular interval – period – as part of its definition. Definition of periodicity is based purely on the ability to regain the initial state. When the system range retrieves its initial state, it is said to be *reinitialized*. For this to happen, either internal of external action has to be taken, and such action is called reinitialization.

Secondly, periodicity can be defined based on the time-dependency of functional requirements. In the previous section, time-dependency of FR was represented by time-varying set of $\{\boldsymbol{FR}\}$, $\mathbf{u}(t)$. Here, $\mathbf{u}(t)$ is used again to define periodicity, particularly functional periodicity. Depending on its pattern of appearance, three different types of functional periodicity can be defined: semi-periodic, periodic, and aperiodic.

Periodic There exist T_i such that $\mathbf{u}(0) = \mathbf{u}(T_1) = \mathbf{u}(T_2) = \cdots$ with regular transition pattern

Semi-periodic There exist T_i such that $\mathbf{u}(0) = \mathbf{u}(T_1) = \mathbf{u}(T_2) = \cdots$ without regular transition pattern

Aperiodic None of the above

If a system's functional requirement set has the initial state where no FR is present

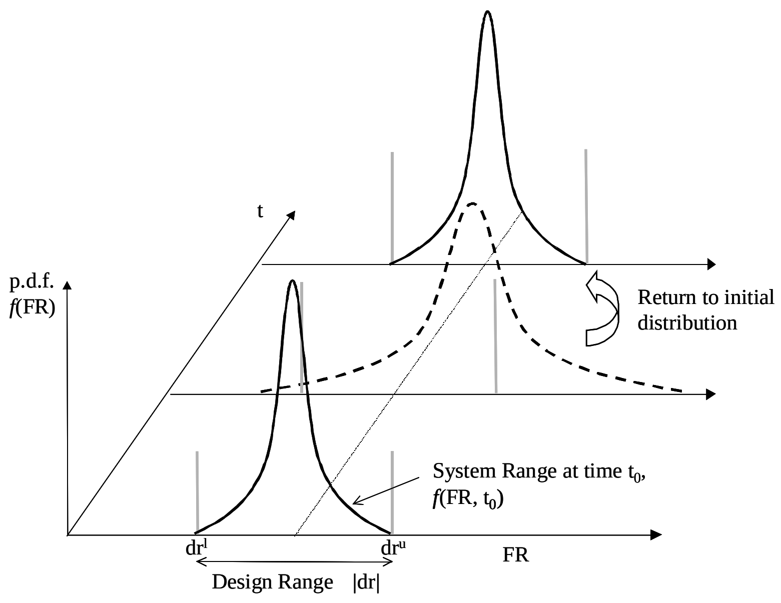
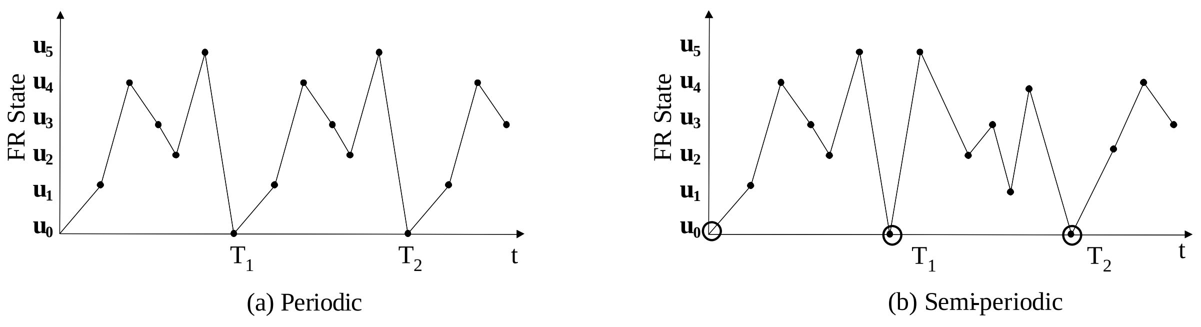

t
p.d.f.
f(FR)
Return to initial distribution
System Range at time t0, f(FR, t0)
dr^l
dr^u
FR
Design Range |dr|

Figure 3-8: System range regains its initial distribution after it has been degraded. A system with this characteristics is said to have periodicity.

and final state where all the FRs are accomplished once (or twice, thrice, etc.), and they occur repeatedly, it is said to have semi-periodicity. If the pattern between the initial state and final state is consistent, then it is periodic. If none of these conditions are met, such system is said to be *aperiodic*. See figure 3-9 for illustration. Figure 3-9(a) represents periodic $\{\boldsymbol{FR}\}$. A system begins with null state, $\mathbf{u}(0)$, and reaches fully-active state, $\mathbf{u}(T-\delta)$. Each of five FRs becomes active (relevant) in the same sequence – FR1-FR4-FR3-FR2-FR5 – for every period. In semi-periodic case, the system begins with null state and reaches fully-active state as with periodic case, but the transitional pattern is different from one period to another: FR1-FR4-FR3-FR2-FR5 for the first period, and FR5-FR2-FR3-FR1-FR4 for the next period. Still all five FRs become active exactly once in a period. In that sense, the state at $t = T_1$ is not different from that of $t = T_2$. Figure 3-9(c) shows no regularity at all, and thus called aperiodic.

Aperiodic behavior is simply the behavior that occurs when no variable describing the state of the system's functional requirements undergoes a regular repetition of values. Aperiodic behavior never repeats and it continues to manifest the effects of any small perturbation; hence, any prediction of a future state in a given system that



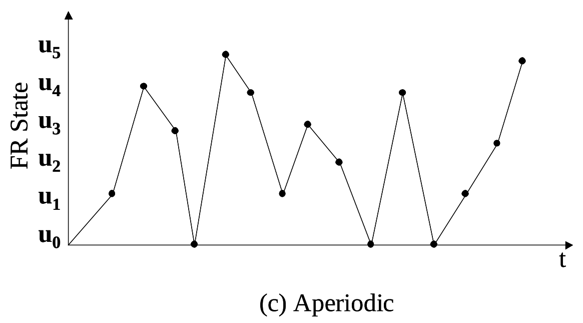


Figure 3-9: Illustration of Periodic/Semi-periodic/Aperiodic $\mathbf{u}(t)$

is aperiodic is impossible unless it is completely deterministic.

It should be noted that the value T_k does not have to be a constant. The length of the interval between null state to the next null state does not have to be a constant value. This is equivalent to saying that we are interested only in the sequence and/or completeness of $\{\boldsymbol{FR}\}$, not in the actual time interval. Since we have not found alternative terminology for proper name for it, we have no choice but to use the term, periodicity. But, unlike conventional periodicity concept, the periodicity here does not necessarily require regular interval as part of its definition. In other words, the periodicity here means the functional periodicity rather than the temporal periodicity. The length of period is indeed determined by the functional period.

Recall the probability of success for time-dependent case:

$p_s = p(\mathbf{u}(t) = \mathbf{u}(t)*)$

If it is either periodic or semi-periodic, we can say $\mathbf{u}(0) = \mathbf{0}$ without loss of generality. For periodic case, it is obvious. For semi-periodic case, since $\mathbf{u}(T_1) = \mathbf{u}(T_1) = \cdots = \mathbf{c}$, where $\mathbf{c}$ is a constant vector, we can introduce a new variable, $\mathbf{w} = \mathbf{u}(T_i) - \mathbf{c}$ and arbitrarily set the constant such that $\mathbf{w}$ is zero. Therefore, by the definition, $p_s(0) = 1$, which implies that the uncertainty for a periodic or semi-periodic $\mathbf{u}(t)$, the uncertainty grows within a limited interval and comes back to 0.

3.3 Summary

By definition of complexity – measure of uncertainty in achieving functional requirements –, time-dependent complexity can be attributed to two factors. One is varying system range, which is related to probabilistic aspect of uncertainty, and the other is time-dependency of functional requirement.

The uncertainty of achieving functional requirements is time-dependent if p_s, probability of success is time-dependent. p_s changes with time if the system range of FR is time-varying. For a system with time-varying system range, the change of system range must be detected. Proper corrective actions are required to bring the degraded system range back to the initial, desirable state. Statistical process control is one example toward such goal. Although it may not be always possible, fundamental design solution must be sought to prevent system range deterioration. The uncertainty is also time-dependent when functional requirements set, $\boldsymbol{FR}$ is time-dependent. A functional requirement becomes relevant at certain time and irrelevant at other time. Consequently, a system presents different subset of $\boldsymbol{FR}$, each of which consists of different combination of FRs. Time-dependency of $\boldsymbol{FR}$ can be clearly represented by using the modified function state variable $u_i(t)$.

Apparent periodicity can be observed if a time-varying system range regains its initial distribution periodically. Periodicity is the consequence of corrective actions or events, and they are termed reinitialization. Depending on the transition pattern of $\mathbf{u}(t)$, three different types of functional periodicity are defined: periodic, aperiodic, and semi-periodic. In case of periodic and semi-periodic $\boldsymbol{FR}$, the initial and final state can be defined and the system returns to initial state repeatedly. If the transition pattern from initial to final state is regular, it is said to be periodic. Aperiodic $\boldsymbol{FR}$ does not have clearly defined initial and final state. It should be noted that the periodicity concept in axiomatic design does not require regular time interval as part of its definition. It is defined based on functional requirements, and thus called a functional periodicity. The length of period is indeed determined by functional periodicity.

Next chapter further discusses functional periodicity and its role in system's stability/sustainment, and the work presented in this chapter serves as theoretical basis for the discussion.

Chapter 4

Periodicity, Predictability and Complexity

In chapter 3, origins of time-dependent complexity was discussed. They are time-varying system range and time-dependent behavior of ***FR***. As discussed in previous chapters, complexity is a measure of uncertainty and real uncertainty is represented by the relationship between system range and design range. Thus, time-varying system range necessarily change the complexity as a function of time. On the other hand, by the fact that complexity can only be defined in relation to ***FR***, time-dependent behavior of ***FR*** renders associated complexity time-dependent.

Two different types of time-dependent complexity are defined in axiomatic design's complexity theory: combinatorial complexity and periodic complexity. If the uncertainty associated with a system's functional requirement increases without bound as time elapses, it is referred to as time-dependent combinatorial complexity. On the other hand, if the uncertainty returns to the initial level from time to time and thereby does not increase indefinitely, it is time-dependent periodic complexity. As part of time-dependent complexity definition in chapter 3, concept of functional periodicity was introduced. Whether a system presents periodicity in terms of time-varying system range and time-dependent behavior of ***FR*** determines the type of complexity.

This chapter discusses the role of functional periodicity in system's stability or sustainability, and the re-initializing a system as a means of introducing/maintaining periodicity is discussed.

4.1 Combinatorial vs. Periodic Complexity

Complexity is a measure of uncertainty in achieving functional requirements. From the definition of complexity, time-dependent complexity is related to the uncertainty that changes as a function of time. Depending on the type of such change, two different types of time-dependent complexity are identified: combinatorial complexity and periodic complexity. Combinatorial complexity increases continuously while periodic complexity ceases to increase at certain point and returns to initial level of uncertainty.

Combinatorial complexity increases indefinitely as time elapses due to two reasons: 1) system range's continuous drift-away from design range, and 2) unpredictability of functional requirements set $\boldsymbol{FR}(\mathrm{t})$.

In some cases, physical phenomena render the system range continue to drift away from its design range both in mean and variation. There exist wide range of such phenomena: for example, wear and fatigue in mechanical component, saturation in desired chemical reaction, and some unknown factor that forces us to reboot our PCs from time to time. Most of these phenomena move the original system range consistently away from the design range, and thus increase uncertainty indefinitely.

Unpredictability of a set of functional requirements $\boldsymbol{FR}(\mathrm{t})$ in future also causes combinatorial complexity. The goal of design is to achieve a set of $\boldsymbol{FR}(\mathrm{t})$ with as much certainty as possible. Prerequisite of achieving a set of $\boldsymbol{FR}(\mathrm{t})$ with certainty is knowing about $\boldsymbol{FR}(\mathrm{t})$ with certainty. If we do not know exactly when and which functional requirements to satisfy, it will be difficult to achieve $\boldsymbol{FR}(\mathrm{t})$ with high certainty. Thus, functional requirements set $\boldsymbol{FR}(\mathrm{t})$ that is unpredictable inevitably increases uncertainty in achieving $\boldsymbol{FR}(\mathrm{t})$. Sometimes, the initial uncertainty amplifies the uncertainty that follows in time since decisions made to deal with initial uncertainty affect later functional events. In this sense, the uncertainty increases as its decision steps (& time) increases combinatorially.

Periodic complexity, on the other hand, refers to the uncertainty that stops increasing at some point and returns to initial (or near initial) acceptable level of uncertainty. Managing airline flight schedule is a good example [2]. During a day, many unexpected events happen to number of flights: for example, mechanical problem with an aircraft, delays due to air traffic at heavy-loaded airport, and bad weather. When those events happen, a series of unplanned actions – flights are cancelled, crews are relocated, passengers are routed to other flights, aircrafts are redirected, etc. – are taken, which results in the loss of regular aircraft/crew/passenger schedule. However, since most part of the airline schedule is periodic each day, the uncertainty during one day and its consequences are not allowed to extend to the next day. Thus, the schedule typically is able to recover from turmoil at the end of the day to near-normality next morning.

From the above description of the combinatorial and periodic complexity, it is clear that avoiding combinatorial complexity and achieving periodic complexity prevents uncertainty from ever-increasing and thus helps maintaining complexity under manageable limit. A system with combinatorial complexity needs to be somehow transformed to a system with periodic complexity.

4.2 Transformation of Complexity

Combinatorial complexity problem must be transformed to periodic complexity problem to prevent uncertainty from increasing indefinitely. In chapter 3, time-varying system range and time-dependence of functional requirements set $\boldsymbol{FR}(\mathrm{t})$ are identified as origins of time-dependency of complexity. Both of them contribute to development of combinatorial complexity. The transformation requires different methods depending on the origin of time-dependence. Still, the overall goal remains the same: introduce periodicity in its uncertainty.

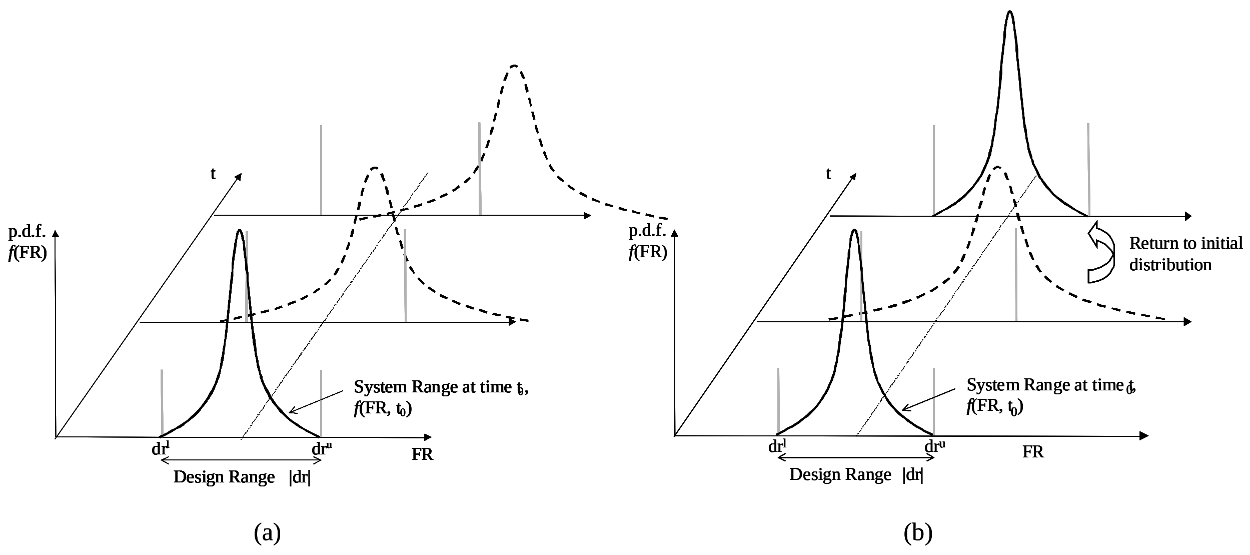


Figure 4-1: (a) System range continues to degrade: Combinatorial complexity. (b) System range regains its initial distribution after it has been degraded: Periodic complexity.

4.2.1 Time-varying System Range

Relationship between time-varying system range and combinatorial complexity is straightforward. If the system range deviates from the desired design range in terms of mean and variation (variance) and the deviation continues to grow as a function of time, the system has combinatorial complexity (see figure 4-1(a)). When a system experiences this type of complexity, its functional requirements are becoming more difficult to achieve as time elapses. Periodically recovering the initial system range becomes a central issue. Once that is achieved as in figure 4-1(b), it is equivalent to transforming the combinatorial complexity to periodic complexity. The act of achieving this transformation is referred to as re-initialization, in a sense that the system is re-initialized to its initial state in terms of the associated uncertainty.

Re-initialization to Recover Initial System Range

As mentioned earlier, the deterioration of the system range is oftentimes due to some physical phenomenon such as wear. In such cases, a bruteforce approach to re-initialization is preventive maintenance, one of the common engineering practices. Preventive maintenance generally apply to design parameters or process variables, for example component change and reconditioning of elements. Since it involves external agent – people or other system –, it is external re-initialization. By periodically performing preventive maintenance, a system is re-initialized and its system range returns to initial or near-initial state. Sometimes preventive maintenance includes inspection to determine whether the system range is deteriorated. If the system range is determined to be acceptable, then the actual re-initialization is postponed until next inspection. In many cases, however, it simply means unconditional re-initialization. Adopting conditional or unconditional re-initialization in preventive maintenance depends on the cost of inspection, re-initialization and failure. Since preventive maintenance typically requires downtime of the system, the cost of downtime is another important factor in the decision. The interval between preventive maintenance is closely related to the system's reliability, i.e. how fast the system range gets deteriorated. One interesting example was presented in section 3.1.1. In the knob example, functional coupling is identified as a key to increase the reliability of the design over long period of use. When the coupling is eliminated, system range deterioration is significantly delayed.

For a high-volume manufacturing system, statistical process control is another common practice that aims to maintain a desired system range. Statistical process control intends to detect deterioration of system range as immediately as possible. In order to systematically detect the abnormal behavior of the system range, one of the most important tools in statistical process control is Schwartz control chart. The control chart is designed to detect a deviation from a process's normal output distribution. It reveals mean shift and increase of variation in output, and differentiates systematic change of the process output from normal, acceptable disturbance. Once abnormal change in system range is determined to be a systematic deterioration, corrective actions are taken to remove cause of the change. Cause-and-effect diagram is one way to find the cause.

Methods like preventive maintenance and statistical process control are external re-initialization in that it requires some type of external agents to re-initialize the system. The external re-initialization is perfectly acceptable way to bring the system range back to the initial state. However, it is much better if the system has some kind of autonomous mechanism that re-initializes itself before the system range becomes too bad. Sometimes, this type of internal re-initialization can be achieved by actively seeking design feature for periodicity. Following example demonstrates a way of internal re-initialization by introducing geometrical periodicity.

Example 4-1: Design of Low Friction Surface

Friction at the sliding interface of metals is caused by the following mechanisms [28]:(a) plowing of the surface by wear debris and other particles, (b) removal of asperities by asperity interactions at the interface, and (c) adhesion of the sliding interface. While all three mechanisms collectively contribute to the friction at the interface, plowing by particles is a dominant factor for most engineering applications [29]. Particles are generated when asperities are removed by asperity interactions. Plowing by the particles generated earlier or by asperities also generates additional particles.

Wear particles entrapped at the interface penetrate into the surface under a normal load. When the interface slide against each other, these particles plow the surface so that work by external agent is required. The work done per unit distance slid is what is known as the frictional force. The friction force, thus, depends on the penetration depth. Indeed, friction coefficient increases nonlinearly as a function of the depth of penetration of the wear particle. As the interface slide continuously, wear particles may agglomerate as shown in figure 4-2. As the particles agglomerate, the applied normal load is carried by a smaller number of larger particles rather than by a large number of small particles. The agglomerated particles penetrate deeper into the surface than the smaller particles do, resulting in increased friction force.

Having understood the mechanism of surface friction, relevant design

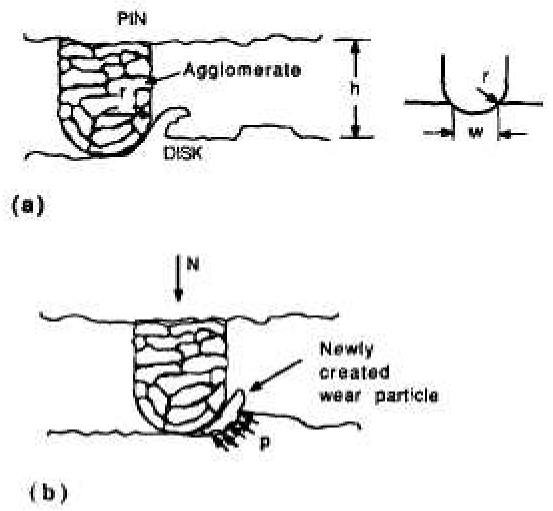


Figure 4-2: (a) An agglomerate wear debris is shown as a cylindrical shape, and (b)wear particles may agglomerate to form larger particles at the sliding interface when there is sufficient pressure to deform the particles and cause bonding. Figure taken from [3].

range is the size of agglomerated wear particles. Then, the system range, actual size of wear particle, drifts out of the design range as the interface slides continuously. Thus, it has to be re-initialized to bring the system range back to initial system range. That is to maintain the agglomerate particle size below the desired level. Re-initialization is achieved by creating undulation on the surfaces. Central idea is that the agglomerated particles fall into the pockets of undulated surface before they become too large. Figure 4-3 compares the particle agglomeration on the flat surface and prevention of it on the undulated surface. As illustrated by the figure, undulated surface plays a role of internal re-initialization and thereby achieves low friction coefficient. □

4.2.2 Time-dependent Functional Requirement

Time-dependent functional requirement set $\boldsymbol{FR}$ incurs time-dependent complexity. This is evident if we represent $\boldsymbol{FR}$ using $\mathbf{u}(t)$. Probability of success is clearly time-dependent as the ideal function state $\mathbf{u}^*(t)$ is time-dependent. In this case, whether a system has combinatorial or periodic complexity depends mainly on the type of

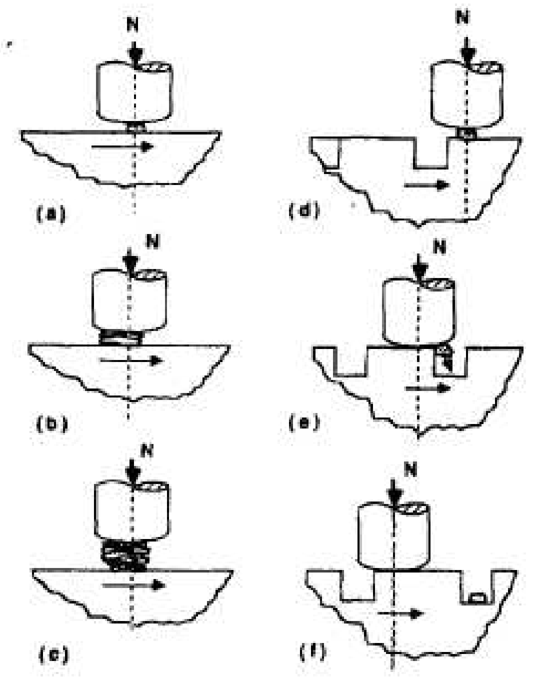


Figure 4-3: (a)-(c): Schematics of wear particle agglomerations on a normal surface, (d)-(f): Particle agglomeration is prevented by undulated surface. Figure taken from [3].

periodicity present in time-dependent functional requirements set $\boldsymbol{FR}$. In chapter 3, three types of periodicity were defined for $\boldsymbol{FR}$: periodic, aperiodic, and semi-periodic.

It can be shown that periodic and semi-periodic $\boldsymbol{FR}$ result in periodic complexity whereas aperiodic $\boldsymbol{FR}$ leads to combinatorial complexity. Recall the probability of success for time-dependent case:

$$p_s(t) = p(\mathbf{u}(t) = \mathbf{u}^*(t)) \tag{4.1}$$

For periodic and semi-periodic $\boldsymbol{FR}$, $\mathbf{u}(0) = \mathbf{u}(T_1) = \mathbf{u}(T_2) = \cdots$ by definition of functional periodicity. Also, $\mathbf{u}(0) = \mathbf{0}$ and $\mathbf{u}^*(0) = \mathbf{0}$ without loss of generality. Thus, probability of success for periodic $\boldsymbol{FR}$ at $t = T_i$, $P_s(T_i)$ is 1. Since we have the information about the initial state – $\mathbf{u}(0), \mathbf{u}(T_1), \cdots$ – we can arbitrarily set $\mathbf{u}(0) = 0$, which gives the above property. P_s begins with the value 1 at $t = T_i$, and decreases until $t = T_{i+1}$, but becomes 1 at $t = T_{i+1}$. In other words, the uncertainty of the system returns to zero every time the system gets back to its initial state. Thus for both periodic and semi-periodic $\boldsymbol{FR}$, uncertainty grows only within a limited interval. Since the uncertainty in periodic and semi-periodic $\boldsymbol{FR}$ is periodically reset to zero, it is clearly periodic complexity.

On the other hand, probability of success for aperiodic $\boldsymbol{FR}$ continues to decrease since $\mathbf{u}^*(t)$ increases in size continuously. In periodic or semi-periodic case, there always is a final state where all the functional requirements are achieved, and the final state triggers the initial state of next period. That way, the uncertainty is limited to one period. Aperiodic $\boldsymbol{FR}$, however, does not have such clear initial and final state. Instead of considering probability of success in a single period with N number of functional requirements, it has to take into account a continuous series of functional requirements to measure probability of success. Therefore, the probability of success is continuously decreasing as we look at certain point in further downstream.

Periodic, semi-periodic, and aperiodic $\boldsymbol{FR}$ has another important implication in affecting the type of complexity. It is the unpredictability of $\boldsymbol{FR}$. Unpredictable $\boldsymbol{FR}$ inevitably increases uncertainty in achieving $\boldsymbol{FR}$. Achieving $\boldsymbol{FR}$ with high certainty requires complete knowledge about $\boldsymbol{FR}$. Thus, if $\boldsymbol{FR}$ is not predictable, it necessarily incurs uncertainty in achieving them. Therefore, by changing unpredictability to predictability, we can eliminate some uncertainty that is due to not knowing $\boldsymbol{FR}$. We can prevent the system from going into certain states that can lead to fatal situation.

Periodic $\mathbf{u}(t)$ implies predictability. If a system has periodic $\boldsymbol{FR}$, then knowledge about a single period brings the predictability of the whole $\mathbf{u}(t)$. Even when little is known about $\boldsymbol{FR}$ at the beginning, an observation of a single period gives complete information about $\boldsymbol{FR}$ in future. On the other hand, unpredictable $\boldsymbol{FR}$ implies aperiodicity in $\mathbf{u}(t)$. These can be expressed in propositional form:

$$\begin{array}{lcll} \text{(Periodicity)} & \longrightarrow & \text{(Predictability)} & \cdots\text{(i)} \\ \text{(Unpredictability)} & \longrightarrow & \text{(Aperiodicity)} & \cdots\text{(ii)} \end{array}$$

where ‘A $\longrightarrow$ B’ means ’A implies B’.

Having said predictability of $\boldsymbol{FR}$ is prerequisite to reduce uncertainty in achieving $\boldsymbol{FR}$, the first proposition states periodicity is one way to reduced uncertainty. Proposition (ii) is not quite interesting as it is, but its contraposition is. The contraposition of a proposition $p \longrightarrow q$ is $\sim q \longrightarrow \sim p$ where $\sim$ indicates negation. A proposition and its contraposition are equivalent. Thus, the contraposition of the proposition (ii) is:

$$\sim\text{(Aperiodicity)} \quad \longrightarrow \quad \sim\text{(Unpredictability)} \quad \cdots\text{(iii)}$$

Negation of unpredictability is predictability. Thus, what this proposition says is 'if $\mathbf{u}(t)$ is not aperiodic, then it is predictable.' Therefore, it states that periodic or semi-periodic $\mathbf{u}(t)$ is sufficient condition for predictability, which in turn leads to reduced uncertainty. Note that the inverse of the proposition (ii) – (Aperiodicity) $\longrightarrow$ (Unpredictability) – is not necessarily true: for example, some ***FR***, if they are deterministic and non-chaotic, are predictable even when they are aperiodic. So, unpredictability ***FR*** means non-deterministic(or chaotic) as well as aperiodic ***FR***.

Based on the above argument, if $\mathbf{u}(t)$ is periodic, ***FR*** is completely predictable. If it is semi-periodic, it is still predictable in a sense that it is known that $\mathbf{u}(t)$ returns to initial and final state periodically. When $\mathbf{u}(t)$ is aperiodic, it may or may not be predictable depending on other factors.

Both probability of success argument and predictability argument favor periodic and semi-periodic $\mathbf{u}(t)$ over aperiodic $\mathbf{u}(t)$, and thus justifies the effort to introduce functional periodicity in ***FR***. If complete periodicity cannot be achieved, semi-periodicity should be pursued to obtain at least partial predictability. Avoiding aperiodic $\mathbf{u}(t)$ plays an important role in preventing the development of combinatorial complexity.

Re-initialization to maintain periodicity in ***FR***

Functional periodicity in ***FR*** ensures that the ideal function state $\mathbf{u}^*(t)$ is known at recurring initial states and that ***FR*** is (at least partially) predictable. Thereby, it prevents a system from developing combinatorial complexity. Establishing the initial state of ***FR*** in a system is called re-initialization. As with system range re-initialization, functional periodicity can be obtained by external or internal re-initialization. External re-initialization requires some intervention from outside of a

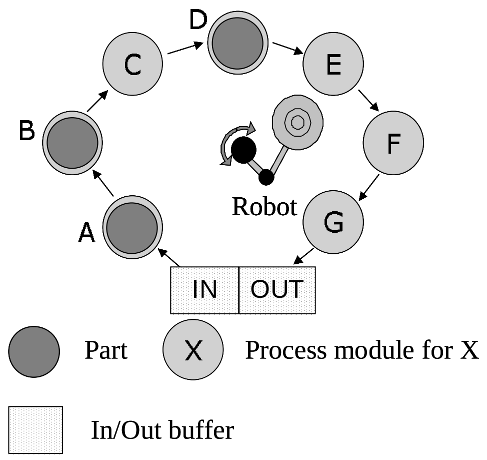


Figure 4-4: A cluster tool consists of a series of process modules and a transporter surrounded by them. Scheduling of part transport in a cluster tool has time-dependent functional requirements.

system to gain periodicity. Example of this type of re-initialization is discussed in chapter 5. On the other hand, sometimes functional periodicity is inherent to a system and maintaining such periodicity is sufficient, which calls for internal re-initialization. Following example illustrates maintaining functional periodicity in a system.

Example 4-2: Periodicity in Part-transport for a Cluster tool

A cluster tool is characterized by a transporter (robot) surrounded by multiple process modules as shown in figure 4-4. Parts are continuously fed into the tool to go through a series of processes. A robot is responsible for moving parts from one module to the next module when process is finished on the current module. Scheduling of part transport in a cluster tool is a typical example of time-dependent functional requirement in that each of the transport task represents individual sub-level functional requirements and they occur at different times. Figure 4-5 shows that different transport tasks, numbered squares, occur at different times as parts are processed within the system.

Sometimes more than two parts are finished nearly at the same time and compete for the robot. Such situation is referred to as transport conflict.

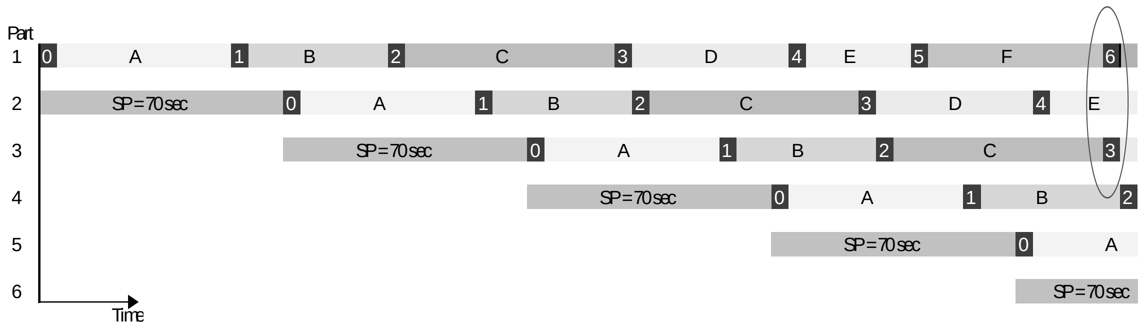


Figure 4-5: As parts are fed to the system continuously, transport conflicts develop in downstream.

The goal of scheduling is to effectively resolve the conflicts to achieve maximum productivity. Suppose that the process time for each of the processes are as follows (in seconds): P_A=50, P_B=40, P_C=60, P_D=45, P_E=30, P_F=50, P_G=50. Transport time between modules is the time it takes for a robot to move to a module to pick up a part therein and move to the subsequent process module to place the part. Transport time for all transport tasks is assumed to be 5 seconds. Given the process and transport time, maximum steady state throughput time is [30]:

$$P_{bottleneck} + 2 \cdot \text{transport time} = 70 seconds$$

In steady state, the system cannot take new parts at faster pace than its maximum throughput rate. Thus, the shortest sending period – time between successive parts enter the system – is 70 seconds. As parts are continuously fed into the system with 70 seconds sending period, transport conflict starts to develop in the downstream of part flow as indicated by a circle in figure 4-5.

The conflicts can be resolved by various means: rules such as priority can be applied whenever the robot encounters a conflict [31], or intentional delay times can be inserted to some of the process modules [32],[33]. Since all the process times are constant and so is sending period, the system is

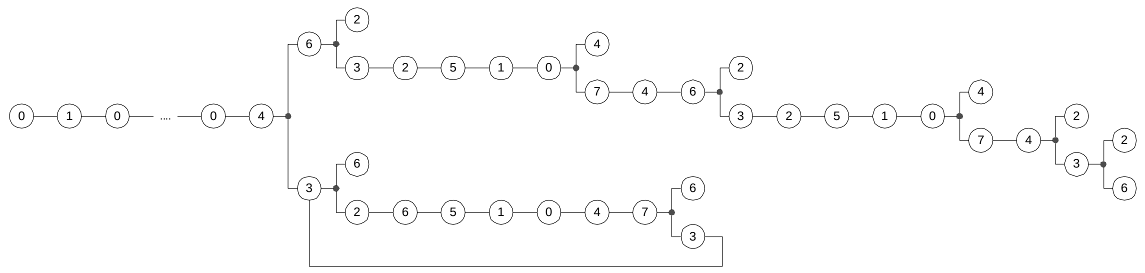


Figure 4-6: Depending on the decisions made upstream, subsequent conflicts patterns may develop into periodic or aperiodic behavior.

expected to have periodic transport pattern. However, if the conflicts are handled erroneously, e.g. applying wrong or inconsistent rules, the transport functional requirements can develop aperiodic behavior. Figure 4-6 depicts a decision tree. Each of the numbered circles represents a particular transport functional requirements. A conflict is shown as a bifurcation node where a decision has to be made to resolve the conflict. It should be noted that the decision made at a node in upper stream affects the subsequent conflict patterns. While the lower branch from the first conflict node ends up with the periodic pattern of transport functional requirements as expected (3-2-6-5-1-0-4-7), aperiodic pattern may develop if wrong decision is made at one of the conflict nodes. If the system develops such aperiodic behavior of transport functional requirements, predictability is lost and probability of success is decreasing as time elapses as discussed earlier, which in turn lead to combinatorial complexity of the system.

In case of rule-based scheduling scheme, a good set of rules must be applied in a consistent manner in order to avoid aperiodic functional requirements. A good set of rules need to be comprehensive, i.e. every possible conflict has to be addressed, and also need to be verified to guarantee the maximum throughput rate. Except for very simple cases, it is time-consuming to develop such a good and verified set of rules. A better alternative is to prevent transport conflicts from occurring at the first place. That can

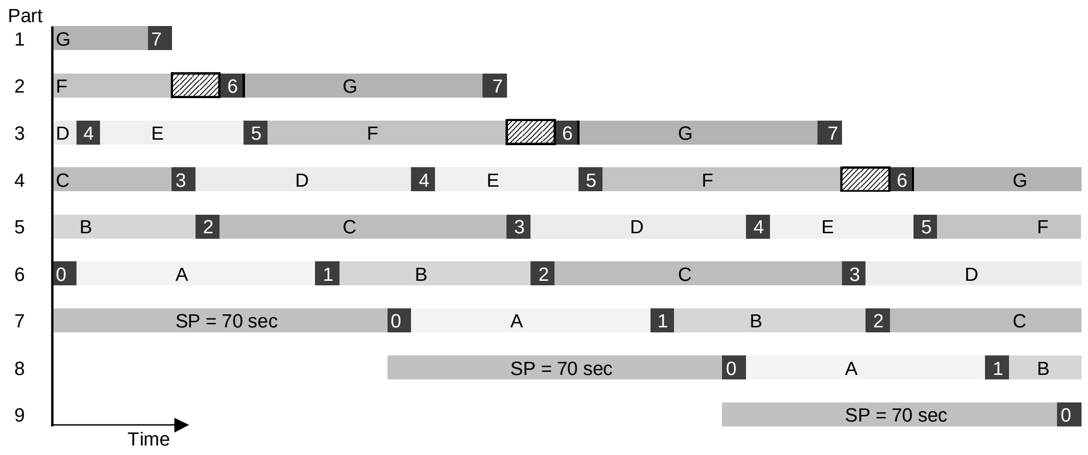


Figure 4-7: 10 seconds of delay time at the end of process step F eliminates transport conflict.

be achieved by designing in some intentional delay times to some of the process steps [32],[33].

Figure 4-7 shows that intentional delay times of 10 seconds to process step F eliminates potential transport conflicts. Resulting transport pattern yields complete periodicity of (3-2-6-5-1-0-4-7), and it is repeated over and over. Since there is no conflict situation, it will not need rules to resolve conflicts. In addition, all parts experience exactly same delay times at the same process steps, eliminating uncertainty in product quality. As a consequence of maintaining periodicity in the system, it simplifies the scheduling activity and potentially improves product quality. □

Functional periodicity is inherent to many of the biological systems. Reproduction of a certain organelle in a cell consists of a series of events that form a periodic cycle. Human behavior typically follows a circadian cycle in everyday life. Many plants and some animals have an annual cycle for their survival. Nature develops various mechanisms to end one functional cycle and start a new one. Most of the biological systems involve some form of re-initialization such as sleeping and hibernation. Maintaining functional periodicity by re-initialization is crucial in sustaining life, and failure to do so results in the death of individual cells and sometimes even the death of organisms. Following example describes how a cell maintains functional periodicity by means of

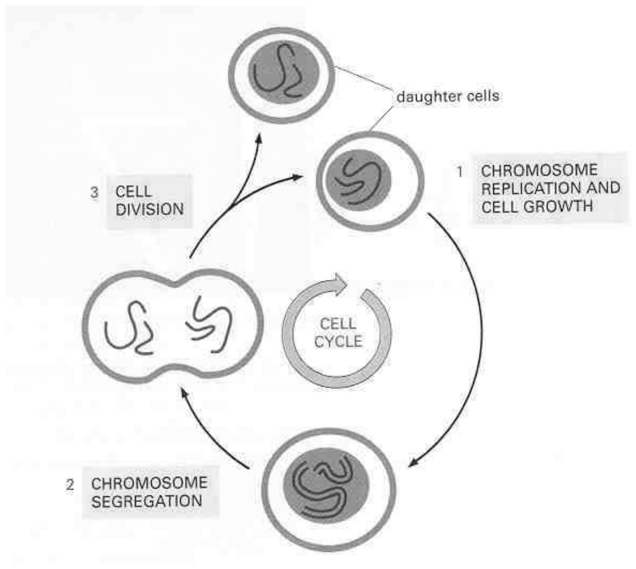


Figure 4-8: During a cell cycle, chromosomes are replicated and the duplicated chromosomes are separated to be inherited to each of the daughter cells (taken from [4]).

re-initialization.

Example 4-3: Re-initialization of the Cell Cycle

One of the central mechanisms of living cell is its reproduction through cell duplication and division. A cell, when it is in proliferation, duplicates and divides itself to form daughter cells. Each of the daughter cells has to inherit exactly one copy of DNA from their parent cell to survive and properly perform its functions. Critical events that have to happen during cell reproduction are, thus, precise replication of its chromosomes, which contain DNA, and exact partition of the duplicated chromosomes into two identical offspring (see figure 4-8). During the process, other intracellular substances such as mitochondria are also distributed to the daughter cells. A series of events occur during the process in an orderly sequence, forming what is known as the *cell cycle.*

Depending on characteristic events, the cell cycle is typically divided into four phases. Chromosomes are duplicated during S phase, and nuclear division followed by cell division occurs during M phase. Between M and

S phase are called G1 and G2 phase respectively to allow some time for a cell to grow and get ready for the crucial events in M and S phase. Thus, cells in proliferation undergoes a cycle of G1-S-G2-M phases repeatedly. A new cell begins its own cycle at G1 phase. When the cell grows enough and the extracellular environment is favorable, it commits itself to the subsequent phases by entering S phase. Transition from one phase to the next is controlled by various factors. Two most fundamental components in the cell cycle control mechanism are cyclin and Cdk (Cyclin dependent kinase). Different cyclins interact with different Cdk partners to form specific cyclin-Cdk complex to play dominant role in each phase. Concentration of specific cyclin rises and falls as the cell cycle progresses: for example, in vertebrate cells, cyclin A becomes abundant during S phase and binds Cdk2 to form S-Cdk. While timely activation of cyclin-Cdk complex is critical in the cell cycle progress, it is just as important to bring the activity of all of the cyclin-Cdk complexes at the end of the cycle. Otherwise, the cell cycle might progress abnormally, resulting in incomplete daughter cells. In other words, state of a cell in terms of Cdk activity has to be re-initialized at the end of one cycle.

One of the re-initializing mechanisms comes from a protein complex, called Cdc20-APC. Cdc20-APC is a protein complex responsible for triggering separation of duplicated chromosomes (sister chromatids). While it initiates sister chromatids separation, it also inactivates M-Cdk by ubiquitylating[1] M-cyclin(cyclin that is present in high concentration during M phase). Thus, as Cdc20-APC become more active toward the end of M phase, M-Cdk level decreases. Interestingly, activation of Cdc20-APC requires M-Cdk, and thus decreased level of M-Cdk leads to decreased Cdc20-APC activity. By this feedback mechanism, the level of M-Cdk and APC activity decrease simultaneously (see figure 4-9). As Cdc20-

[1]Ubiquitylation refers to a process where a protein complex is marked as targets for destruction by a special protein, called ubiquitin.

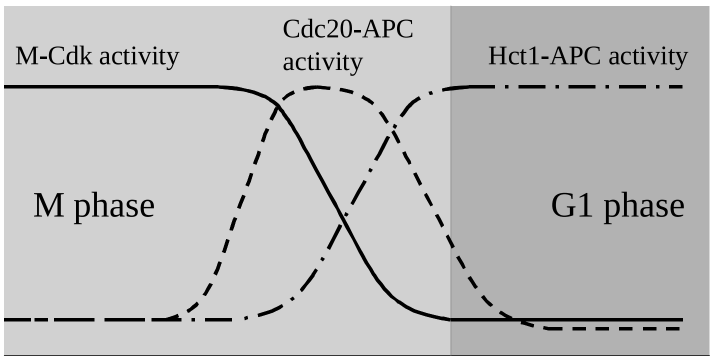


Figure 4-9: M-Cdk activity is decreases toward the end of M phase by Cdc20-APC, followed by Hct1-APC. This Cdk-suppressing mechanism re-initializes the level of Cdk activity as a new daughter cell starts its own cycle. Figure is taken from [5].

APC activity is inactivated, a cell needs different mechanisms to suppress Cdk activity during G1 phase. They are Hct1-APC complex and CKI: Hct1-APC complex that initiates the ubiquitylation of M-cyclin and CKI (Cdk inhibitory protein) accumulation. The two and M-Cdk are mutually inhibitory: Hct1-APC and CKI suppress M-Cdk, and M-Cdk suppreses Hct1-APC and CKI. By this mutual inhibition, these mechanisms become active with decrease in M-Cdk level (see figure 4-9). Thus as M-Cdk level decreases In animal cells, these mechanisms are in effect and restraining Cdk activity until late G1 phase. As a result, a new daughter cell starts its own cell cycle with same initial state and thereby each cycle is independent from the cell cycle of the previous generation.

Chapter 6 discusses periodicity in the cell cycle in detail. □

4.3 Summary

Combinatorial complexity increases continuously while periodic complexity ceases to increase at certain point and returns to initial level of uncertainty. A system has combinatorial complexity when (1) its system range continues to drift away from design range and (2) the functional requirements set $\boldsymbol{FR}(\mathrm{t})$ is unpredictable. In order to prevent uncertainty from ever-increasing and to maintain complexity under

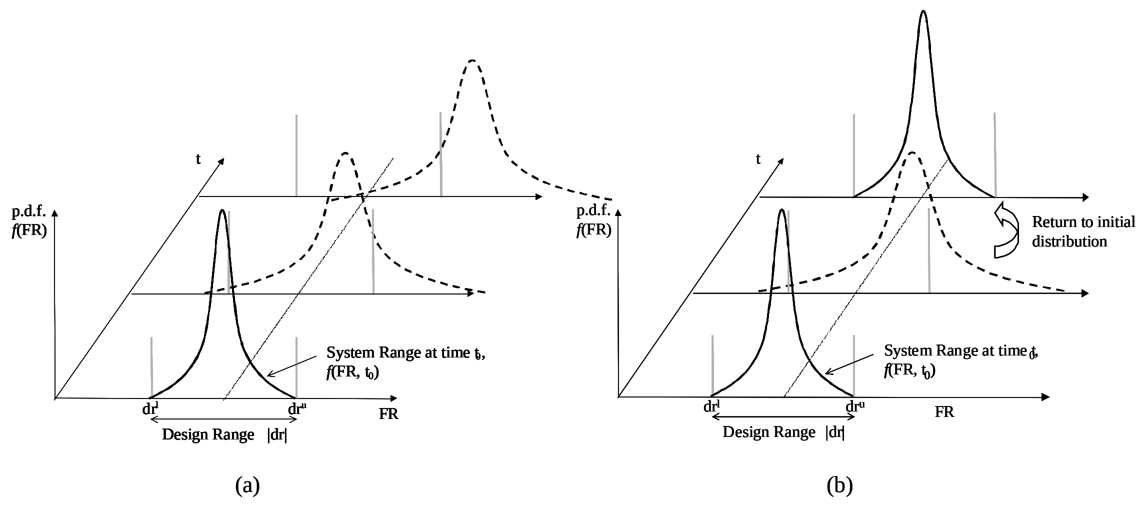

p.d.f.
f(FR)
t
System Range at time t0, f(FR, t0)
dr^l
dr^u
FR
Design Range |dr|
(a)
Return to initial distribution
(b)

Figure 4-10: (a) Combinatorial complexity. (b) Periodic complexity: re-initialization

manageable limit, combinatorial complexity should be avoided.

When a system experiences combinatorial complexity due to time-varying system range issue, functional requirements are becoming more difficult to achieve as time elapses. Thus, periodically recovering the initial system range is important. Once that is achieved as in figure 4-10(b), it is equivalent to transforming the combinatorial complexity to periodic complexity. The act of achieving this transformation is referred to as re-initialization, in a sense that the system is re-initialized to its initial state in terms of the associated uncertainty. Re-initialization of time-varying system range can be done externally or internally. Preventive maintenance and statistical process control are examples of external re-initialization. Clever design as in knob example in section 3.1.1 can help minimizing the cost of external re-initialization.

Whether a system has combinatorial or periodic complexity can also depend on the type of periodicity present in time-dependent functional requirements set $\boldsymbol{FR}$. Periodic and semi-periodic $\boldsymbol{FR}$ result in periodic complexity whereas aperiodic $\boldsymbol{FR}$ leads to combinatorial complexity. For periodic and semi-periodic $\boldsymbol{FR}$, probability of success can be defined to be 1 repeatedly at T_i when initial state is established since we have information about the initial states. Aperiodic $\boldsymbol{FR}$, on the other hand, does not present clear initial and final state, and thus it has to take into account a continuous series of functional requirements to measure probability of success. Therefore, the probability of success is continuously decreasing as we look at certain point in further downstream. Relationship between ***FR*** periodicity and complexity can be argued by unpredictability of ***FR***. Unpredictable ***FR*** inevitably increases uncertainty in achieving ***FR***. Thus, by changing unpredictability to predictability, we can eliminate some uncertainty that is due to not knowing ***FR***. Periodic or semi-periodic ***FR*** implies at least partial predictability in ***FR***. When ***FR*** is aperiodic, it may or may not be predictable depending on other factors. Therefore, from both probability or predictability arguments, functional periodicity is preferred.

This chapter discussed the relationship between the type of complexity – combinatorial or periodic – and the origins of complexity – time-varying system range and functional periodicity of ***FR***. Following chapters present examples from different discipline to demonstrate the practical implication of the functional periodicity.

Chapter 5

Periodicity in a Simple Manufacturing System

This chapter presents an example of scheduling of part-transport in an integrated cluster tools to illustrate the concept of time-dependent complexity transformation. The example demonstrates the re-initialization of functional period as a means of transforming a combinatorial complexity into a periodic complexity. Also, it shows the consequence of the loss of functional periodicity in an integrated cluster tool with variation.

When two different machines or subsystems with two different throughput rates are joined together, they must be integrated to maximize the productivity of the entire system. Consider a simple discrete-product manufacturing system, Z, consisting of subsystem X and subsystem Y, which are physically connected to each other to make products continuously. When the last operation in subsystem X is completed, the part is transported to subsystem Y for the first operation in subsystem Y. Both subsystems can have many physical modules in them to process a part by subjecting it to a variety of different processes that require different process times. The process time in each machine is different and specific to the part to be made. Throughput rates of each subsystem may vary from the nominal values because of the various factors such as process time variation.

Suppose that the throughput rate of subsystem X is α and that of subsystem Y is

β. If $\alpha < \beta$ ($\alpha > \beta$), then the maximum throughput rate of the system Z should be equal or very close to α (β). When $\alpha = \beta$, the maximum rate of the system should be nearly equal to α or β. In other words,

$$\text{Throughput(Z)} \approx \min[\alpha, \beta]$$

However, the throughput rate of system Z may be substantially less than this theoretical maximum throughput rates due to many factors such as the random variation in the process times, conflicts in scheduling the pick-up time of the robots, and constraints imposed on the system operations. Now the question is what causes positive Δ, where $\Delta = \min[\alpha, \beta]$ - Throughput(Z), and how to minimize it.

Section 5.1 presents general background of scheduling part-transport in a cluster tool. Then, section 5.2 discusses a particular example of part-transport scheduling in an integrated cluster tool with process time variation, followed by a summary of the chapter.

5.1 Background: Scheduling of a Cluster Tool

5.1.1 Throughput Rate of a Cluster Tool

A cluster tool refers to a group of process modules organized around a transporter to sequentially perform a series of process steps on a part. This type of system is commonly found in use for semiconductor manufacturing process. Figure 5-1 shows a cluster tool with five process modules organized around one transporter. Parts enter through [In] buffer and exit through [Out] buffer. Once the transporter takes a part from the [In] buffer, the part is transported sequentially through a series of modules to sequentially go through different processes. The throughput rate of cluster tool is well studied in [34],[30],[35],[36]. In this chapter, we follow Perkinson's notation and result. Perkinson [30] shows that the maximum steady state throughput rate is

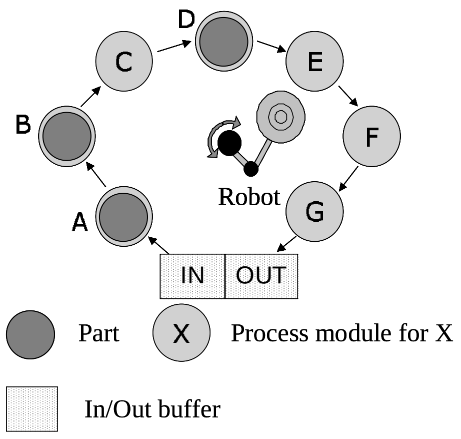


Figure 5-1: Cluster tool with seven process steps: A cluster tool is characterized by a a group of modules organized around a transporter.

determined mainly by the minimum fundamental period, FP^*, as

$$\begin{aligned} &\text{Maximum throughput rate} = \tfrac{1}{FP^*} \\ &FP^* = Max\left[\tfrac{(P_i+4T)}{n_i}\right] \qquad\qquad i = 1, \cdots, N \end{aligned} \tag{5.1}$$

where P_i is process time of process step i, T is a transport time, n_i is the number of modules that perform process step i, and N is the number of different process steps. The process time is the time from when a part enters a module for processing to when the part is ready to exit. The transport time is the time it takes for the transporter to travel from current location to destination module with or without a part and place or pick up the part. The fundamental period, FP is defined as the elapsed time between the completion of two consecutive parts. It is determined by a bottleneck process, and sets the upper bound of the throughput rate of a cluster tool. In equation (5.1), all the transport tasks are assumed to take same time T for simplicity, but it is likely that different transport tasks require different times mainly due to the path length is different. To allow different transport times for different transport task, equation (5.1) can be modified as following:

$$FP^* = Max\left[\frac{P_i + MvPk_i + MvPl_{i+1} + MvPk_{i-1} + MvPl_i}{n_i}\right] i = 1, \cdots, N \tag{5.2}$$

where $MvPk_i$ is the time it takes for a transporter to move to process i module and pick up a part, $MvPl_j$ is the time it takes for the transporter to move to process j module and place a part. (+1) and (-1) in the subscripts indicate the next and previous process step, respectively. For example, when $i = 1$, $(i-1)$ is [In] and $(i+1)$ is 2. One of the assumptions made in deriving equation (5.1) and (5.2) is that when it is empty-handed, the robot waits at its current position, and only begins moving toward a destination module after the process therein is complete. It is sometimes called as simple scheduling. Another assumption is that the system is in 'process-limited mode' of operation, meaning that one of the process modules is a bottleneck of the system as opposed to the transporter being a bottleneck. For further details on determining fundamental period, readers are recommended to refer to [30],[32].

5.1.2 Scheduling for Deterministic System

Scheduling in a cluster tool can be defined as assigning a sequence of transport tasks to transporter(s). It can be a fixed sequence repeated over and over, or can be a long sequence without a repeatable pattern. The most fundamental functional requirement for scheduling task is to 'transport a part from one station to the next station when the current station finishes its processing on the part.' In rare cases, a transporter may be able to simply respond to all the transport requests without any trouble. If that is the case, a schedule is simply the result of a transporter's responding to transport requests. One extreme example of such case is a system with only one process step with one transporter. All it needs to do is to pick a finished part up from the process module and place it to [Out] buffer, and pick up a new part from [In] buffer to place it into the process module. With no complication, the simple sequence repeats itself. However, except for a few instances, it is very likely that there is a situation where two or more parts need to be transported nearly at the same time. Such situation is called a conflict: a conflict between multiple transport tasks that share the same transporter. Those conflicts can be resolved by various means, and depending on the way they are resolved, different schedules will be obtained. Thus, scheduling becomes an active design task rather than passive outcome of transporter's reactions to transport requests. Depending on which part is picked first by the transporter, the subsequent pattern of the part flow and the subsequent schedule of the transporter motion will be different. Each one of the decision points is a bifurcation point and therefore, the number of the possible combinations for the flow paths for the parts and for the transporter motion will increase with the number of such decisions made. Such a system is defined as a time-dependent combinatorial complexity system – a system in which follow-on decisions are always affected by previous decisions, increasing the number of possible combinations.

Reacting to the conflict based on predetermined priority rules is one way to resolve the conflicts. Planning a synchronized transport/process pattern is another way of handling the problem [33],[32],[37]. Typically, the goal of scheduling is to achieve the throughput time[1] as close to FP^* as possible. In most cases, however, that is not the only objective. Oftentimes, other objectives, such as minimizing makespan[2] is pursued, which makes the scheduling an optimization problem. Also, it is common that scheduling is subject to process constraints, e.g. 'a part has to leave the process module immediately when the module finishes its process on the part.'

A cluster system is said to be deterministic if all the process times are constant, transport times are constant and it does not have random events such as machine breakdown. Although it is far from realistic assumption, a deterministic model characterizes a system in terms of upper/lower bound of the system performance. For deterministic system, the maximum performance – upper bound of the throughput rate – is known to be FP^* which can be readily computed by equation 5.1 and 5.2. Unless the system is handled erroneously, a deterministic system reaches a steady state with its own fundamental period. Then, the goal of scheduling is to minimize Δ = (Actual throughput time) $- FP^*$ while satisfying other objectives and constraints.

As mentioned previously, top level functional requirement for scheduling is to 'transport a part from one station to the next station when the current station finishes

[1]Throughput time is (throughput rate)$^{-1}$. Sometimes, it is referred to as cycle time.
[2]Makespan: Time it takes for a system to process a batch, i.e. from when the first part enters the system to when the last part of the batch leaves the system

its processing on the part.' Given the process recipe and system configuration, a fixed set of sub-level functional requirements can be explicitly defined:

Transport a part from [In] to M_1	$=$	FR_0
Transport a part from M_1 to M_2	$=$	FR_1
Transport a part from M_2 to M_3	$=$	FR_2
	$\vdots$	
Transport a part from M_N to [Out]	$=$	FR_N

Since the functional requirements appear at different times, $u_i(t)$ can be used to appropriately represent these FRs. Recall the function state variable, $u_i(t)$ defined by equation 3.6 and 3.7 in chapter 3.

$$u_i(t) = \begin{cases} u_i & \text{for } t \geq \tau_i \\ 0 & \text{for } t < \tau_i \end{cases} \tag{5.3}$$

$$\mathbf{u}(t) = \{u_0(t), u_1(t), \ldots, u_N(t)\} \tag{5.4}$$

Then, within one period, $\mathbf{u}$(t) changes from the initial state, $\mathbf{u}(0) = \{0, 0, \cdots, 0\}$ to the final (completion) state, $\mathbf{u}(\mathrm{T}) = \{1, 1, \cdots, 1\}$. The number of total (feasible & infeasible) transition patterns is $(N+1)!$. Particular transition pattern corresponds to a particular schedule. Some of these patterns represent feasible patterns, and others do not. Obviously, exhaustive search for feasible transition patterns and selecting the best one by evaluating all of them is not an economic way to obtain the solution. The priority-based 'if-then' type of reactive scheduling can be used as an alternative, but its developing process could be time-consuming because every single conflict situation has to be addressed by rules. Indeed incomplete set of rules can develop time-dependent combinatorial complexity in a system, and eventually fail the system. Furthermore, it does not ensure the throughput rate until the priority rules are validated collectively. Better alternative is to deploy a deterministic scheduling scheme. Since the deterministic system is completely predictable, we can effectively maintain

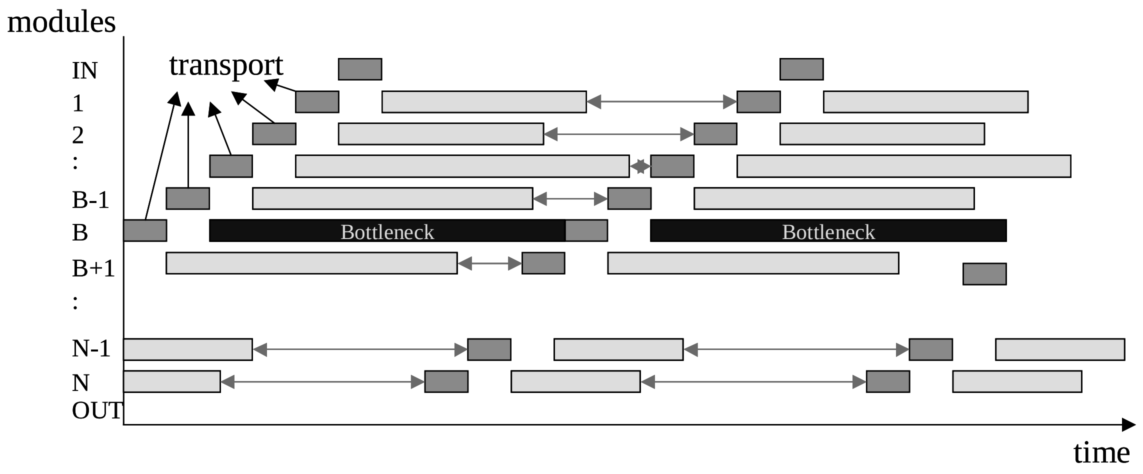


Figure 5-2: If there is no process constraint and thus delays are allowed at all the process steps, there exists a trivial schedule that achieves FP^*. In the figure, horizontal arrows indicate delay times, and all the process times are extended to have the same length as bottleneck process.

functional periodicity of transport tasks and obtain planned and thus predictable result by means of scheduling.

In a deterministic scheduling, process constraint can become a determining factor for the maximum throughput it can achieve. Process constraint regarding scheduling is represented in terms of maximum delay time of the service of its transport request after the process is finished. Sometimes these constraints leave the scheduling with only one option of slowing down the system in resolving transport conflict.

If there is no process constraint, i.e. delays are allowed at all the process steps, FP^* is always achievable. The most straightforward scheduling solution is to run all the processes with $P_i = n_i FP^* - 4T$. In essence, what it does is extending non-bottleneck process time a little bit such that all the process steps have same time as bottleneck process (see figure 5-2 for illustration).

Then, the resulting schedule will be of the following transition pattern:

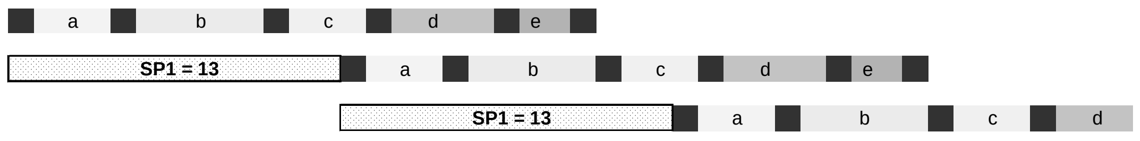
$$\begin{aligned}\mathbf{u}(0) &= \{0, 0, \ldots, 0\} \\ \mathbf{u}(\tau_1) &= \{0, 0, \ldots, 0, 1\} \\ \mathbf{u}(\tau_2) &= \{0, 0, \ldots, 0, 1, 1\}\end{aligned}$$

$$\begin{aligned}\mathbf{u}(\tau_3) &= \{0, 0, \ldots, 0, 1, 1, 1\} \\ &\vdots \\ \mathbf{u}(\tau_{n+1}) &= \{1, 1, \ldots, 1\}\end{aligned}$$

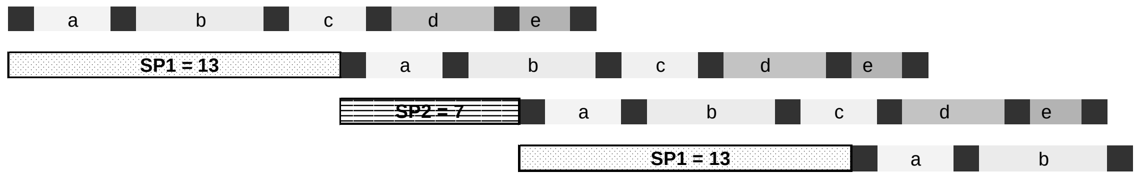
Although it may not be the best schedule in a sense that the delay times may be excessive, it certainly is one of the scheduling solution that deliver FP^*.

At the other extreme where all the delays are completely prohibited, FP^* is unlikely to be achievable. If there exists any conflict when the system runs at throughput rate of $(FP^*)^{-1}$, then the only degree of freedom to resolve conflict is its sending period – time between part-feeding into the system. Since the sending period cannot be shorter than FP^*, using longer sending period is the only way to resolve conflicts. In such case, r-part cyclic schedule is known to be equal or better than a single-part cyclic schedule [38],[39][40]. In r-part cyclic schedule, r parts enter a system and r parts leave the system within one period whereas a single-part cycle is defined by one-part leaving and entering a system (see figure 5-3). In terms of sending period, r-part cycle has r different sending periods that repeat as a whole set. 1-part cyclic schedule is a special case of r-part cyclic schedule. In some cases, r-part cycle is better than 1-part cycle, delivering higher throughput rate. Since 1-part cycle is a special case of r-part cycle, r-part cycle can be said to be equal or better than 1-part cyclic schedule. Consider, for example, a system with five process steps each with a single process module [38]. Processing times P_i are as following: P_1=3, P_2=5, P_3=3, P_4=4, P_5=2. As shown in figure 5-3, while 1-part cycle delivers throughput time of 13 time units, the 2-part cycle that consists of sending periods of 7 and 13 units is able to deliver throughput time of average of 10 units.

In r-part cyclic schedule, a set of r sending periods repeat themselves as the system processes more than r parts. On the contrary, there can be a schedule that is perfectly feasible for r parts but not repeatable for subsequent parts. This type of schedule is called acyclic schedule. If we know that there will be exactly k parts to be processed, we can come up with k-part acyclic schedule. In terms of maximum throughput rate, this k-part cyclic schedule is equal or better than r-part cyclic schedule ($r \leq k$) [40].



(a) Best 1-part cycle at SP = 13 time unit



(b) Best 2-part cycle at SP1 = 13 time unit and SP2 = 7 time unit

Figure 5-3: (a) 1-part cycle gives a throughput rate of $\frac{1}{13}$ while 2-part cycle shown in (b) yields higher throughput rate of $\frac{1}{10}$

That is because the repeatability is, in a sense, a constraint in finding a scheduling solution. 10-part cyclic schedule for 10 parts to process may be different from acyclic schedule, and acyclic schedule may be better than cyclic schedule. However, it should be noted that obtaining r-part acyclic schedule becomes computationally hard as r gets large [40],[41].

Less extreme is the scheduling with some process constraints. Some of the process steps require zero delay time for the part transport, but others are not so critical that they can be used as temporary buffers. In such case, whether FP^* is achievable or not depends on the conflict pattern with respect to the process constraints. For example, suppose the process step j requires zero delay and conflict occurs between transport $[j \rightarrow (j+1)]$ and transport $[(j-1) \rightarrow j]$. If it were not for the constraint, delaying the transport $[j \rightarrow (j+1)]$ will resolve the conflict without necessarily slowing down the system. However, due to the constraint, the only way to resolve the conflict is to increase the sending period until the conflict is gone. In this example, the process constraint on j is the limiting factor for the system's throughput rate. If none of process constraints is limiting factor, then the system will be able to achieve FP^* by properly assigning delay times to non-critical process steps [32],[37].

If process constraint is limiting factor, it is easy to see that multi-part cycle may deliver better throughput by the same reasoning in the case with fully-constrained scheduling. r-part cycle implies a set of r sending period and possibly a set of r different delay times at non-critical process steps. However, in some cases, non-uniform delay times for different parts could result in inconsistent quality since different parts will experience different delay times.

To summarize, if delays are allowed at all the process stations, maximum throughput rate can always be achieved at $(FP^*)^{-1}$. If a system is fully constrained, FP^* is unlikely to be achievable since even a single conflict cannot be resolved without slowing down the system. For such system, multi-part cycle solution is equal or better in terms of throughput rate than one-part cycle solution. However, finding the global optimum solution, i.e. finding r^* for r-part cyclic schedule that yields the best throughput rate, is known to be computationally hard problem. If a system is partially constrained at some of its process stations, the existence of schedule solution to achieve FR^* depends on constraints: process constraints may not be a limiting factor and thus do not rule out a schedule for FP^*. Or they render the scheduling problem to be similar to fully-constrained problem and thereby force to slow down the system. r-part cycle may give a better result. But then consistency in product quality may be a problem because parts in different groups will experience different delay times. In any case, preserving periodicity can take the benefit of the predictability in a deterministic system, and result in predictable system performance.

Non-deterministic case is different form deterministic case, and the solution obtained from the deterministic case would not work. Next section discusses the scheduling problem for non-deterministic problem, and an interesting example is presented to illustrate the consequence of the loss of periodicity. Also, a means of maintaining functional periodicity, re-initialization, is discussed.

5.2 Maintaining a Periodicity in a Manufacturing System

When two different subsystems with two different throughput rates are joined together, they must be integrated to maximize the productivity of the entire system. Consider a simple system consisting of subsystem X and subsystem Y, which are physically connected to each other to make a product. Both subsystems may consist of many physical modules to process a part by subjecting it to a variety of different processes that require different process times. The process time in each module is different and specific to the part to be made. In addition, although the nominal processing times are set for each subsystem, the throughput rate of each subsystem may vary from the nominal value if there are variations in the processing times.

Obviously the modules in subsystems will complete their processes at different times because parts arrive in each module at different times and also because their process times are different. Furthermore, there will be inevitable random variations in the process times of individual modules in the subsystems. Some of these parts will be finished nearly at the same time, waiting to be picked up by a transporter – hereinafter referred to as a robot – for the next operation. The role of scheduling is to resolve such conflicts. Conflicts should be resolved such that the system does not have a time-dependent combinatorial complexity since a system with combinatorial complexity may not be able to achieve the maximum theoretical throughput rate. For such a system to have the maximum productivity, it must be transformed into a system with time-dependent periodic complexity. The reduction of the system complexity through the transformation will maximize the productivity. In deterministic case, the conflict pattern is predictable a priori, and thus appropriate schedule can be constructed off-line to avoid combinatorial complexity. To create a system with time-dependent periodic complexity in the presence of random variations, we introduce the concept of "re-initialization." Upon re-initialization, the initial states may vary due to unexpected variations in process times as well as other reasons. Re-initialization requires the following: (1) existence of a functional period in which all the functions

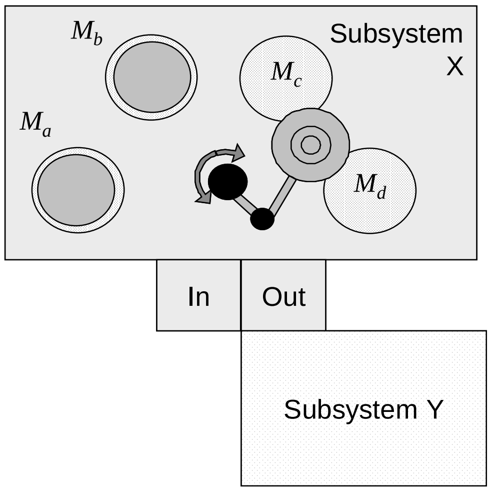


Figure 5-4: Subsystem X and Y are joined together to subject wafers to a series of processes.

are repeated, (2) at the beginning of each period, establishment of new initial conditions, and (3) determination of the best schedule for the entire process from the newly established initial conditions. The period begins when a key function – either triggered internally or externally – re-initializes the system. We will demonstrate the process using an example.

5.2.1 Example: Wafer Processing System

Consider a hypothetical wafer processing system[3]. The system is to be created by integrating two subsystems X and Y. Figure 5-4 depicts a physical configuration of the system under consideration. The configuration is characterized by a transporter surrounded by multiple process machines – cluster tool. Subsystem X performs four different functions in a given sequence, a, b, c, and d, to produce patterns on a wafer continuously. Subsystem Y takes the semi-finished part from subsystem X and subjects them to subsequent processes. Processes a through d are performed in subsystem X by machine M_a through M_d, respectively. Each machine M_i processes only one part at a time. A robot located in subsystem X transports parts between machines in subsystem X, and passes it on to subsystem Y. Process sequence along with specified

[3]This example is based on [42]

process times is often called a process recipe. The number of machines for each of process steps is determined based on the process times and required throughput rate. If a particular process step takes too long to meet required throughput rate, then additional machine is added to the process step. For example, if the throughput time requirement is 60 seconds and process a takes 80 seconds, then two machines, $M_{a,1}$ and $M_{a,2}$ are required to meet the target throughput time. This set of machines to perform the same process step, $\{M_{a,1}, M_{a,2}\}$, is referred to as station a, S_a. Given the system configuration along with process sequence, functional requirements for the transporter is explicitly stated as following:

Transport a wafer from [In] to S_a	$=$	FR_0
Transport a wafer from S_a to S_b	$=$	FR_1
Transport a wafer from S_b to S_c	$=$	FR_2
Transport a wafer from S_c to S_d	$=$	FR_3
Transport a wafer from S_d to [Out]	$=$	FR_4

The process time P_i for each of the process steps is the time between receipt of a part and completion of the processing at machine M_i (when the part is ready to leave M_i). The cycle time CT_Y of subsystem Y is defined as the time between receipt of a part at subsystem Y and removal of the part from subsystem Y (when subsystem Y is ready to take a new part).

In this example, we assume that the cycle time of subsystem Y varies significantly compared to the variation of process times of subsystem X. Also, process c is assumed to be critical such that a part in machine M_c must be removed as soon as process c is completed. It is further assumed that considerations of economic efficiency render the maximum utilization rate of subsystem Y highly desirable.

Equation (5.2) can be modified to better model the throughput of the integrated system. In a particular setup of this example, FP of the whole system is determined by the slower of the two subsystems. If subsystem X determines the pace of the integrated system, fundamental period of the system is given by

$$\begin{aligned} FP &= FP_X \\ &= Max\left[\frac{P_i + MvPk_i + MvPl_{i+1} + MvPk_{i-1} + MvPl_i}{n_i}\right] \\ & \quad i = a,\ b,\ c,\ d \end{aligned} \tag{5.5}$$

If subsystem Y is slower than subsystem X, the fundamental period of the integrated system is given by

$$FP = FP_Y = CT_Y + MvPk_{Y-1} + MvPl_Y \tag{5.6}$$

where $MvPk_{Y-1}$ is the time for the robot to move to the last process stage S_d in subsystem X and pick up a part, and $MvPl_Y$ is the time for a robot to move the part to subsystem Y (Out). The fundamental period of the overall system, FP, is given by the larger of FP_X and FP_Y. That is,

$$FP = Max\left[FP_X, FP_Y\right] \tag{5.7}$$

The following three cases illustrate the re-initialization of systems to achieve time-dependent periodic complexity for different subsystem throughput relationships.

Case 1 Subsystem X is slower than subsystem Y: $FP_X > \max[FP_Y]$

Case 2 Subsystem X is faster than subsystem Y: $FP_X < \min[FP_Y]$

Case 3 Both subsystems are at approximately same speed: $FP_X \approx FP_Y$, i.e. $\min\{FP_Y\} < FP_X < \max\{FP_Y\}$

In each case, the maximum productivity (i.e. throughput rate) is attained when the operations of the subsystems are subject to a repeated re-initialization implemented after the completion of subsystem cycle. Re-initialization introduces periodicity and thus changes the scheduling problem from that of a time-dependent combinatorial complexity to a time-dependent periodic complexity problem. Each of the cases will be considered separately.

Station		P_i or CT_Y	Number of machines	$MvPk_i$	$MvPl_i$
In		-	1	5	-
	a	30	1	5	5
X	b	40	1	5	5
	c	60	1	5	5
	d	80	2	5	5
Y		60±5	1	-	5

Table 5.1: Parameters for Case 1

Case1: $FP_X > \mathbf{max}[FP_Y]$

Case 1 addresses a system in which FP_X is larger than FP_Y. As a consequence of subsystem Y's being faster than subsystem X, subsystem Y has to wait until a next part finishes its processes in subsystem X. In other words, subsystem Y is operated in a 'starved' mode. Therefore, it does not matter when subsystem Y finishes its process and requests a new part as long as this relationship between FP_X and FP_Y holds. In short, the variation of CT_Y does not affect subsystem X's operation. The maximum productivity is achieved simply when the throughput of the integrated system reaches that of subsystem X. Table 5.1 shows the process times for processes a, b, c, d, the cycle time for subsystem Y, the number of machines for each process, and the associated transport times.

According to equations (5.5), FP_X is 80 seconds. As stated earlier, CT_Y is assumed to vary within ±5 seconds, i.e. $CT_Y = 55 \sim 65$ seconds. Thus, by equation 5.6, FP_Y is $65 \sim 75$ seconds. Given FP_X and FP_Y, the fundamental period FP of the integrated system is 80 seconds.

As mentioned above, subsystem Y operates in a starved mode since FP_X is larger than the maximum FP_Y. Therefore, variation in subsystem Y's cycle time, 55∼65 seconds, will not affect the scheduling. Even if subsystem Y requests a wafer as it is ready to process the next one, no wafer is available from subsystem X, and thus subsystem Y has to wait for a while. The variation in CT_Y needs not be considered in subsystem X's transport scheduling. Since there is no variation issue in scheduling, the steady state scheduling can be directly used.

To solve this problem, it is very useful to visualize the situation using a timing

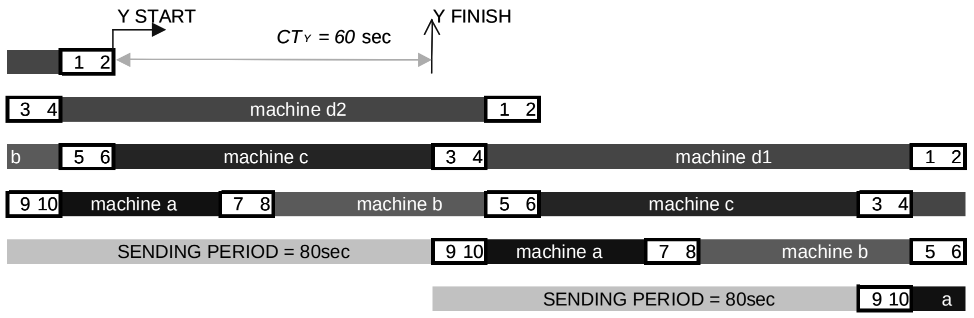


Figure 5-5: Part-flow timing diagram. Each row represents the individual panels being processed by different machines. Transport task (3 4)-(9 10) and (1 2)-(5 6) are in conflicts.

diagram. Figure 5-5 is a part-flow timing diagram. The horizontal axis represents time and the vertical axis (row) represents different parts processed by the system. In particular, the first row represents the flow of a first part processed by the system, the second row represents the flow of the second part processed by the system, etc. Since FP is constant at 80 seconds, the incoming parts are assumed to be 80 seconds apart from each other in steady state. In other words, the sending period is set to equal FP of the system. Figure 5-5 shows that there are two transport conflicts between transport task [1 2]-[5 6] and [3 4]-[9 10], and the conflicts occur repeatedly. It is clear that, depending on which part is picked up first by the robot at the moment of the conflict, the subsequent pick-up schedule will be affected by the decision. Thus, the number of possible routes for the robot increases as additional decisions are made at the time of the conflicts. The system is therefore subject to time-dependent combinatorial complexity. To reduce the number of possible combinations and thereby facilitate the robot scheduling, this time-dependent combinatorial complexity problem must be converted into a time-dependent periodic complexity problem. If a sending period and delay times at some of the process steps are appropriately selected, the complexity of the subsystem X can be converted into a time-dependent periodic complexity.

From Figure 5-5, one can easily see, by inspection, that 10 seconds of post-process delay time in machine M_b and another 10 seconds post-process waiting time in M_d will simply resolve these conflicts without the need for real-time decision-making. In other words, those waiting times prevent conflicts from occurring at the first place.

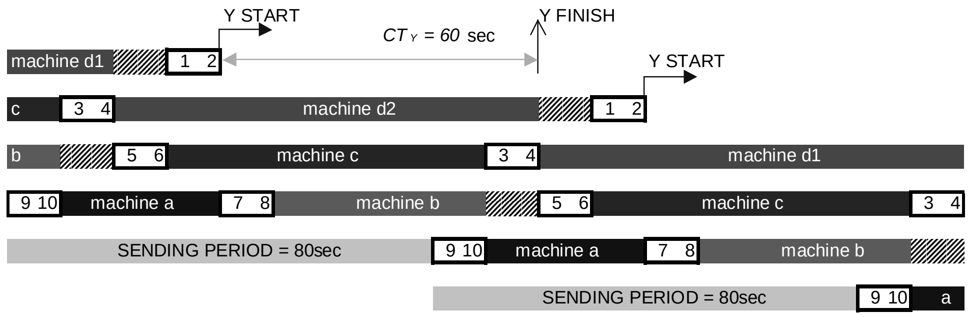


Figure 5-6: 10 seconds post-process delay times at process b and d resolve the conflicts.

Figure 5-6 shows such scheduling solution with the waiting time indicated by hatched area. Note that the time subsystem Y starts processing is determined by subsystem X, which is not affected by the variation in CT_Y. This solution gives the following periodic $u(t)$:

$$\begin{aligned}
\mathbf{u}(0) &= \{0,0,0,0,0\} \\
\mathbf{u}(\tau_0) &= \{1,0,0,0,0\} \\
\mathbf{u}(\tau_3) &= \{1,0,0,1,0\} \\
\mathbf{u}(\tau_2) &= \{1,0,1,1,0\} \\
\mathbf{u}(\tau_4) &= \{1,0,1,1,1\} \\
\mathbf{u}(\tau_1) &= \{1,1,1,1,1\} \\
\mathbf{u}(FP) &= \{0,0,0,0,0\}
\end{aligned}$$

Case 1 is equivalent to a scheduling problem with no variation since subsystem X, that has no process time variation, determines the pace of the whole system. Therefore, steady-state scheduling method such as [32] can be used directly to provide scheduling solution as in Figure 5-6.

Case2: $FP_X < FP_Y$

Table 5.2 shows the process times for processes a, b, c, d, the cycle times for subsystem Y, the number of machines for each process, and the associated transport times. Based on the equations (5.5)-(5.7), the fundamental period of the integrated system

Station		P_i or CT_Y	Number of machines	$MvPk_i$	$MvPl_i$
In		-	1	5	-
	a	30	1	5	5
X	b	40	1	5	5
	c	50	1	5	5
	d	80	2	5	5
Y		80±5	1	-	5

Table 5.2: Parameters for Case 2

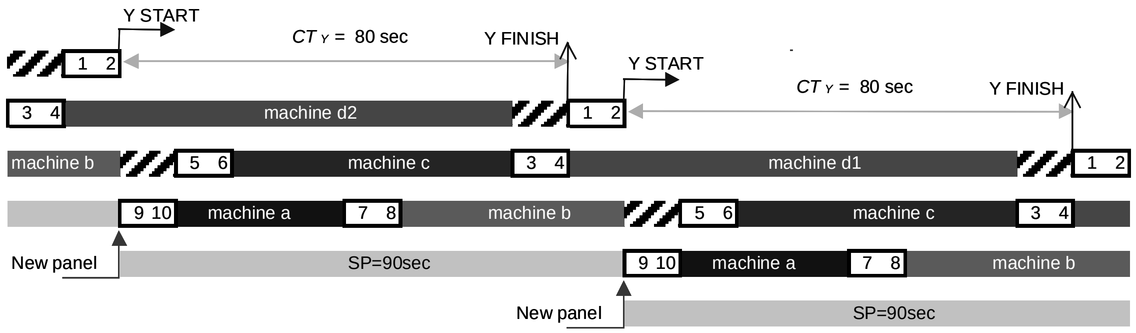


Figure 5-7: Steady state scheduling solution with CT_Y at constant 90 seconds

is determined by subsystem Y to be 85$\sim$95 seconds. If CT_Y were constant at 80 seconds, which would yield 90 seconds of FP_Y, assigning appropriate delay times with the system's sending period set to 90 seconds would simply solve the scheduling problem as in case 1. The pattern shown in Figure 5-7 would be one of legitimate scheduling solutions that repeat itself. However, as opposed to Figure 5-7, the pattern of timing diagram under the variation will not be the same for each period. Subsystem Y takes a part from subsystem X once every 85$\sim$95 seconds non-deterministically, and thus the temporal location of the transport task from subsystem X to Y, [1 2], is not fixed with respect to other transport tasks. Therefore, the constant waiting time solution as in Figure 5-7 is not valid, and the schedule for robot motion must be recomputed each time subsystem Y picks up a semi-finished part from subsystem X. Recall that there are two constraints on subsystem X. First, the part just processed at machine M_c must be immediately picked up for transport. Second, a part and the robot must be available for subsystem Y when it is ready to take the part. Depending on the variation in the cycle time CT_Y and the inherent conflict patterns, the task of scheduling can be significantly unfavorable. In particular, the difficulty in scheduling results from the randomness in the transport conflict pattern due to CT_Y variation. The variation in CT_Y along with decisions made at 'current' conflict can lead to combinatorial complexity. As previously described, time-dependent combinatorial complexity problem must be converted into a periodic complexity problem. The conversion requires that a functional period be imposed on the system. In such a period, the same set of tasks are performed cyclically and, therefore, a limited number of scheduling possibilities exist. The period is initiated by an internal or external key event, or re-initializing event, that needs to be appropriately defined. In this example, a part-request from subsystem Y, FR_4 is chosen to be the re-initializing event for three reasons. First, subsystem Y limits the pace of the integrated system. Also, the pace of the integrated system has to be adjusted to accommodate the variations in the cycle time of subsystem Y. Third reason is that a basic constraint on the system requires that delivery of a part to subsystem Y be completed immediately upon the request. Because a part-request issued by subsystem Y is treated as the re-initializing event, the length of each period depends on CT_Y. Even though the length of each period is generally different, the same set of functions, i.e. transport tasks, are completed by the robot in the subsystem X within each period.

The central idea is to force the system to have a periodicity whose initiation is defined by the re-initializing event that is FR_4. At the time of re-initialization, the current state of subsystem X – which machines are available and the remaining process times of occupied machines – is determined. Appropriate delay times are then calculated for each of the occupied machines, which determines a schedule for the current period. First, to ensure that the robot is always available during the time slot of the next re-initializing event, a no-transport-time interval is defined:

$$No\_transport\_time =$$
$$\{t | t \in [(MvPk_{Y-1} + MvPl_Y) + min[CT_Y], (MvPk_{Y-1} + MvPl_Y) \cdot 2 + max[CT_Y]]\}$$

where t=0 at the moment of the current re-initializing event. Transport of parts must be scheduled such that none of the transport tasks is assigned during this time interval

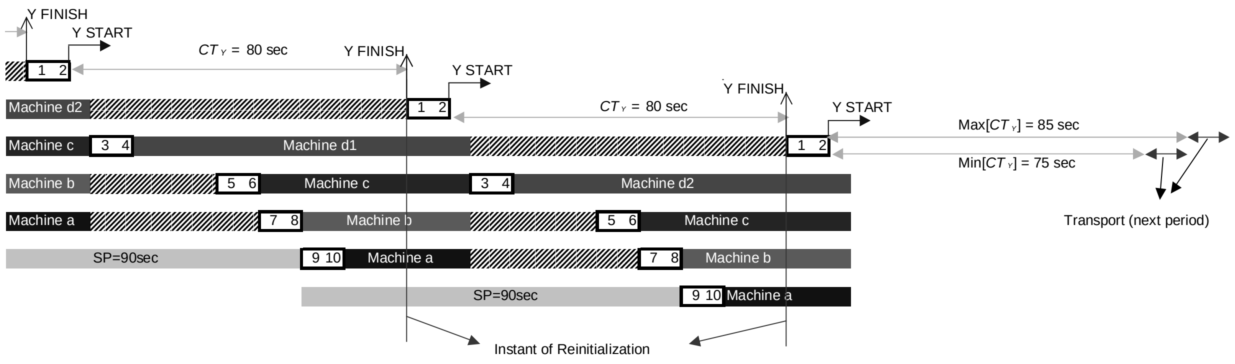


Figure 5-8: Variation in subsystem Y's cycle time

to ensure immediate delivery of parts to subsystem Y. Second, assign all transport tasks which are determined at the instant of re-initialization (i.e., pre-fixed transport tasks). Pre-fixed tasks include transport [1 2] by the definition of the re-initializing event, and transport [3 4] due to the process constraint at machine M_c. Remaining transport tasks are then allocated accordingly. Figure 5-8 depicts a part-flow timing diagram for the steady state operation, showing a potential variation in cycle time CT_Y at the right end. The vertical lines indicate the time when subsystem Y requests a part from subsystem X and therefore represent the moment of re-initialization. As shown in the last period, due to the variation in CT_Y, subsystem Y requests a semi-finished part from subsystem X at some time at least 85 seconds but no more than 95 seconds after the re-initialization (75$\sim$85 second cycle time plus 10 second transport time for [1 2]). Therefore, the transport task is scheduled so that the robot is available for the period from 85 seconds to 105 seconds after the re-initialization.

After a part-request is issued by subsystem Y (vertical line), a renewal signal is generated to re-initialize the database of processes. First, the state of each machine M_i is identified as busy or idle, and empty or occupied. For example, at the onset of the second re-initialization (second vertical line), the following information is identified:

Empty	{}
Busy	{ M_a, M_b, M_c, M_{d2} }
Occupied	{ M_{d1} }

Once busy machines are identified, their remaining process times are monitored.

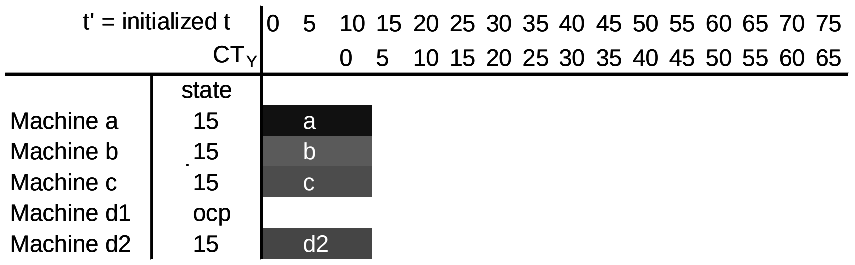


Figure 5-9: Information at the instant of re-initialization

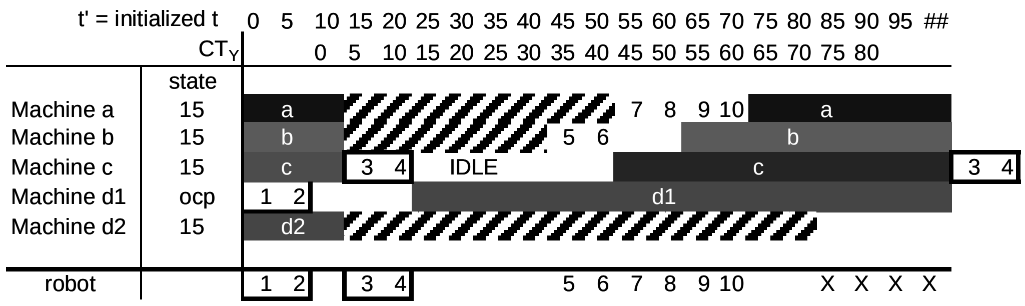


Figure 5-10: [1 2], [3 4] are pre-fixed. No-transport-time is indicated by X's.

Figure 5-9 depicts the remaining process times for the busy machines. Based on this information, the transport schedule is constructed. In this example, no-transport-time is $\{t'|85 < t' < 105\}$. Transport task [1 2] and [3 4] are pre-fixed tasks. Transport task [1 2] occurs 0 to 10 seconds after the moment of re-initialization. Another task, [3 4] must occur from 15 to 25 seconds after re-initialization because the part in machine M_c must be removed as soon as process c is complete. The allowable transport timeslots are computed and the remaining transport tasks are assigned in the timeslots. One possible schedule is shown in Figure 5-10 in which the X's signify the no-transport-time interval. Transport task [5 6] is delayed for a while due to the no-transport-time condition for the following task [3 4] in the next period. This delay indeed ensures every transport task appears exactly once in a period as well as maximizes utilization of subsystem Y. Transport tasks [7 8] and [9 10] simply follow task [5 6] at the earliest possible time according to fundamental conditions for part transport – the current machine is finished, the next machine is empty, and the robot is available.

By applying the above-mentioned approach repeatedly, schedule for the next pe-

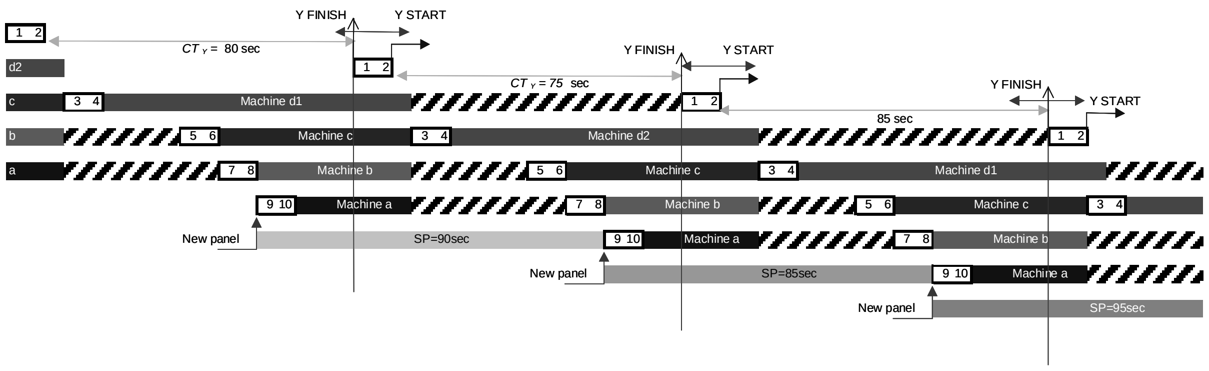


Figure 5-11: Resulting schedule for case 2

riod can be determined relatively easily regardless of when subsystem Y picks up the part exiting subsystem X. Figure 5-11 depicts multiple intervals with different cycle times CT_Y for subsystem Y, and it is clear that the variation is well accommodated. Indeed, every interval is independent from the previous intervals except for the immediately preceding interval. In other words, the initial state of $\mathbf{u}(0)$ is attained repeatedly so that the effect from the variation of the CT_Y does not propagate to the later periods.

Case3: $FP_X \approx FP_Y$

Case 3 is a mixture of case 1 and case 2 because FP_Y is sometimes less than the FP_X and at other times is larger than FP_X. It is tempting to think that the effect of the period where subsystem Y is slower is canceled out by that of faster periods over the long run, and thus it is possible to operate subsystem X at subsystem Y's pace. Unfortunately, the faster period, when CT_Y has the value of $\min[CT_Y]$, cannot be used to directly offset the slower period, when CT_Y is $\max[CT_Y]$, because the duration of the actual cycle time CT_Y for the next period is not known a priori. Since it is not known when subsystem Y will request its next part, subsystem X has to be ready to deliver a part at the earliest possible request time by subsystem Y if it is to satisfy the constraint of maximum utilization of subsystem Y. Therefore, it is equivalent to operating subsystem X at the faster pace than its maximum capability, which cannot be achieved.

Station		P_i or CT_Y	Number of machines	$MvPk_i$	$MvPl_i$
In		-	1	5	-
	a	30	1	5	5
X	b	40	1	5	5
	c	50	1	5	5
	d	80	2	5	5
Y		60±5	1	-	5

Table 5.3: Parameters for Case 3

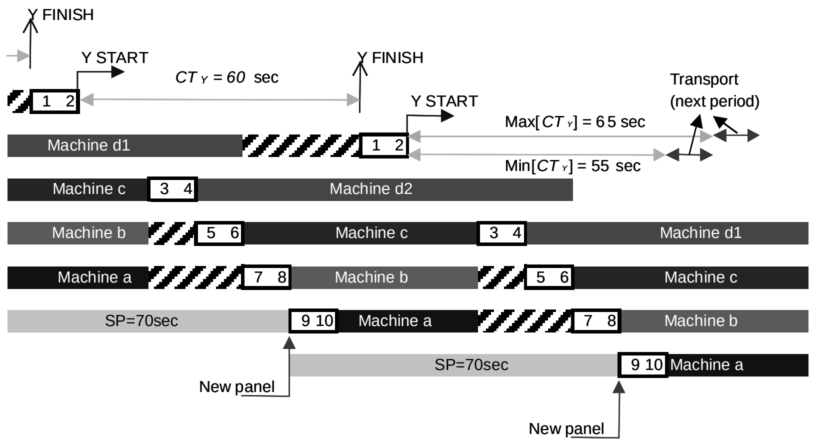


Figure 5-12: Steady state operation with sending period of 70 seconds

The process times, number of machines and associated transport times are shown in Table 5.3. According to equation (5.5) and (5.6), FP_X is 70 seconds, and FP_Y is 65∼75 seconds. That is, the pace of the integrated system is sometimes determined by subsystem X, and at other times by subsystem Y. Figure 5-12 depicts one mode of steady state operation of subsystem X with a sending period equal to nominal FP of 70 seconds. Limited to the illustrated instance, subsystem X appears capable of providing a part to subsystem Y even if subsystem Y has a cycle time CT_Y of 55 seconds. In particular, the fundamental conditions for part-transport are satisfied because a part is ready at machine M_{d2}, the robot is available, and subsystem Y is ready to accept a part. Would it be acceptable as it appears? Would subsequent intervals with CT_Y of 65 seconds compensate for this short period?

Referring to Figure 5-13 - 5-19, however, it can be shown that subsystem X cannot sustain a high system throughput over many intervals. In Figure 5-13 - 5-19, we begin with depicting a situation where CT_Y is first assumed to be 55 seconds which is

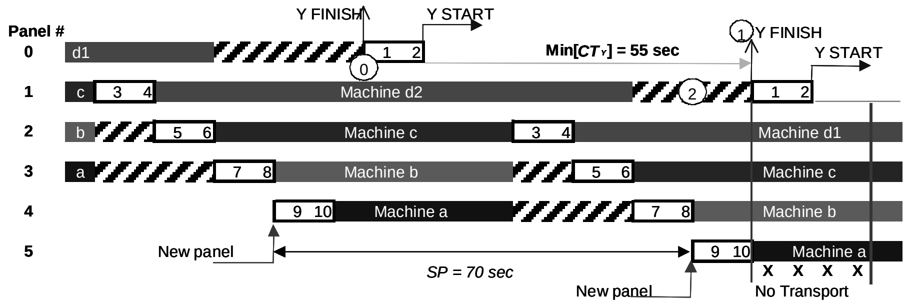
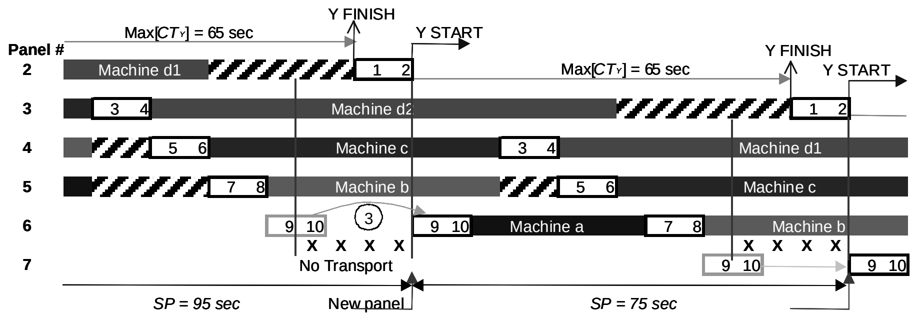


Figure 5-13:



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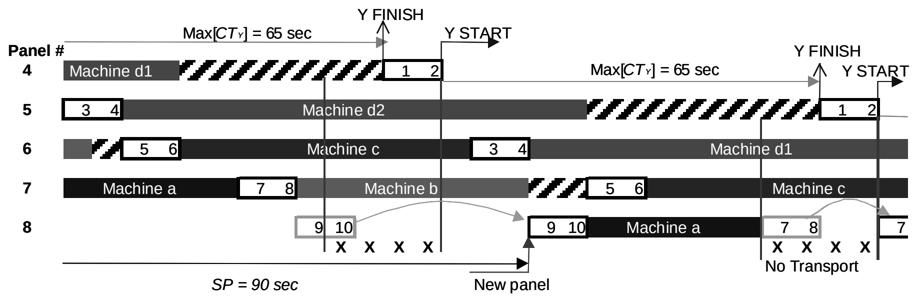

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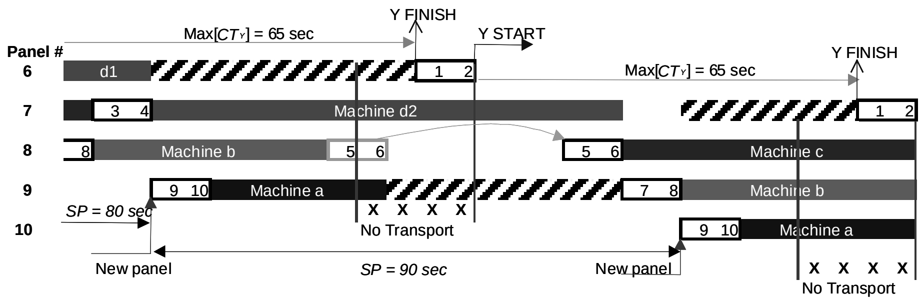

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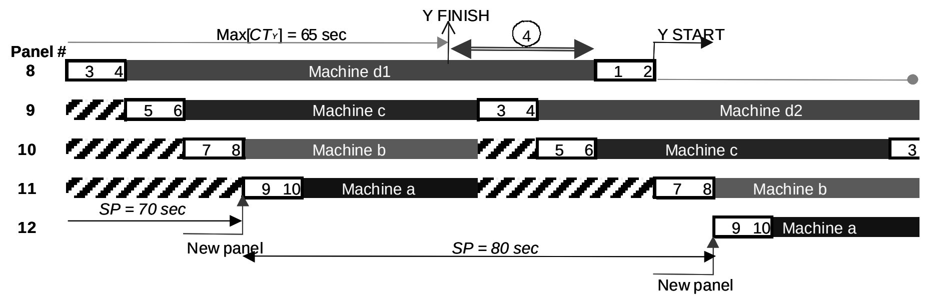

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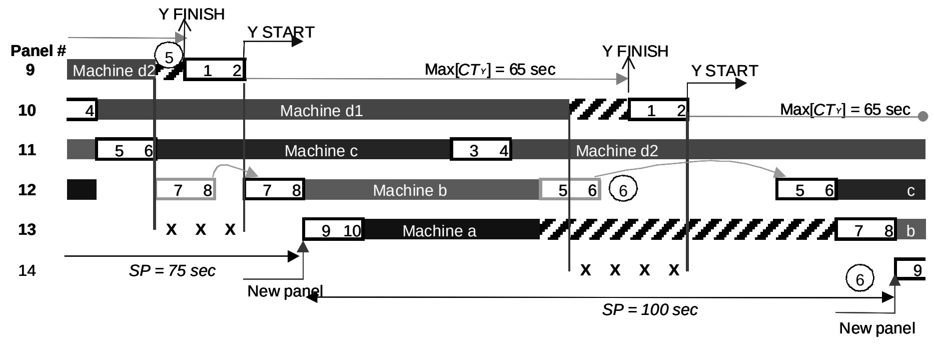

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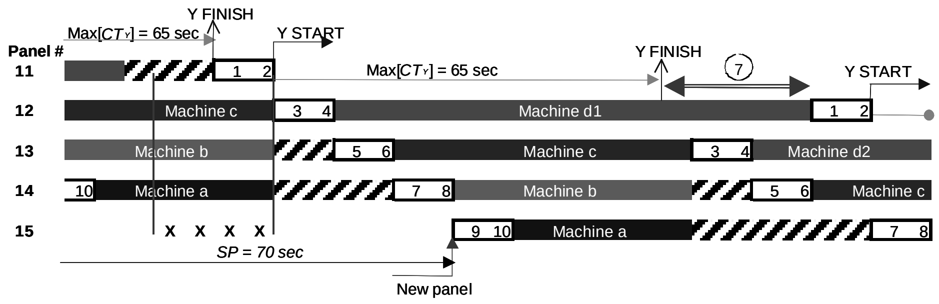


Figure 5-19:

min[CT_Y], and kept to 65 seconds, max[CT_Y] afterwards. In other words, subsystem Y is once faster than subsystem X, and it becomes slower afterward. In constructing the timing diagram, we used simple fundamental conditions of part-transport along with no-transport-time that was defined in case 2. Figure 5-13 - 5-19 is a series of part-flow timing diagrams arranged in chronological order. Figure 5-14 immediately follows Figure 5-13 in time; Figure 5-15 immediately follows Figure 5-14, and so on. Each row in the figures is numbered according to a specific part number and the horizontal axis represents increasing time. A throughput time is defined as the period of time between a "Y finish" and the immediately following "Y finish", for example time from ⓪ to ① in Figure 5-13.

In Figure 5-13, subsystem Y requests a part after it finishes its process with 55 second CT_Y (see ①). Subsystem X is able to deliver a part for this early request because machine M_{d2} has completed its process and waits for part number 1 to be picked up (see ②). Thus, when subsystem Y completes its cycle, part number 1 is immediately provided. As a result, the throughput time for this particular period (time from ⓪ to ①) is 65 seconds. In Figure 5-14, CT_Y is shown to be 65 seconds. Note that there are only four transport tasks, i.e. [1 2], [3 4], [5 6], and [7 8] in the first interval of Figure 5-14. It is required that a no-transport-time interval of 20 seconds (indicated by two vertical lines and the X's) be available to handle variations in the cycle time CT_Y of subsystem Y. Consequently, transport task [9 10] cannot be performed during the first interval of Figure 5-14 and is instead delayed to the next period (see ③).

The effect of the incomplete interval is first manifested in the elongation of subsequent sending periods as well as the immediate increase of it as shown in Figure 5-14. In order to feed parts in time to subsystem Y at its 65 seconds of cycle time, the system must have 75 seconds or shorter sending period. But, as seen from the following figures, some of the subsequent sending periods are longer than the 75 second. Unless there is a sufficient number of short sending periods to compensate for those long sending periods, the integrated system will not be able to produce parts per FP of 75 seconds due to the shortage of parts introduced into the system. For the intervals up to and including Figure 5-16, subsystem X manages to follow the rate of request from subsystem Y. In Figure 5-17, however, there is no part in subsystem X ready to satisfy a part-request from subsystem Y (see ④). As a result, subsystem Y has to wait for its next part to be available. In the first interval shown in Figure 5-18, subsystem X regains its ability to immediately provide a part to subsystem Y (see ⑤). But, the no-transport-time condition in the next interval produces a long sending period of 100 seconds (see ⑥), which returns the system back to the shortage state. Figure 5-19 shows another instance of delay in part delivery from subsystem X to subsystem Y (see ⑦). The pattern depicted in Figure 5-19 is the same as that of Figure 5-17, thus it is obvious that the system achieves a steady state and that the further extension of the part-flow timing diagram from Figure 5-19 is simply a repetition of Figure 5-17 - Figure 5-19.

In constructing the diagram, it has been assumed that cycle time of subsystem Y, CT_Y is 65 seconds except for the first interval. Subsystem X is faster than subsystem Y if CT_Y is 65 seconds: FP_Y = 75 seconds > FP_X = 70 seconds. Thus it might be expected that subsystem X would evolve to a steady state operation in which the fundamental period of the total system is 75 seconds, and that subsystem X would always be able to feed a part to subsystem Y in time. Obviously, that will be the case if the cycle time CT_Y is 65 seconds without exception. Temporal location of the no-transport-time is fixed relative to the other robot moves, thus the same part-flow pattern is established as the system reaches steady state. Such a case is trivial since it is merely a transition from one steady state ($FP = 70$sec) to another ($FP = 75$sec). Figure 5-13 - 5-19 demonstrates, however, that a single occurrence of a 55 second CT_Y combined with an attempt to run subsystem X above its maximum speed – applying no-transport-time interval – results in a degradation of system performance. The system cannot even maintain a fundamental period of 75 seconds which it should be able to achieve. Every fourth interval, subsystem Y must wait for an additional 25 seconds (see ④ and ⑦). Thus its average throughput time in steady state is 81.25 seconds (i.e., (75+75+75+100)/4 seconds).

In this example, the longer interval of subsystem Y does not compensate for the functional incompleteness in the period with shorter CT_Y. Even in the period with longer CT_Y, in order to cover the full range of unknown variation in cycle time CT_Y, subsystem X should complete its cycle (processes and transports) within the minimum cycle time CT_Y so that there is enough time to accommodate the no-transport-time interval. If subsystem X cannot complete a cycle within one interval of CT_Y, it inevitably violates one of the requirements for re-initialization: existence of a functional period in which all the functions are repeated. The scheduling problem becomes a time-dependent combinatorial complexity problem lacking proper periodicity. As a result, chances for non-optimal scheduling decisions increase and the overall system performance can degrade. The root cause of inability to achieve the desired productivity is the attempt to run subsystem X beyond its maximum rate with a presumption that slower periods and faster periods would nullify the effect of each other. The attempt to overrun subsystem X caused an incomplete cycle. Consequently, it yields a low throughput rate, which is the opposite of what was intended. This implies that subsystem X can absorb subsystem Y's cycle time variation only when subsystem X is sufficiently faster than subsystem Y. Under such condition, it can be re-initialized with appropriate period and, thus, assure the periodicity of robot's transport functional requirements. In the present example, subsystem X must be re-initialized conditionally: if subsystem Y requests a part at a pace faster than subsystem X's maximum speed, then initialization must wait until it reaches FP_X, i.e. subsystem X is ready. It, in turn, means that subsystem X must always be given enough time to complete its functional cycle before going into the next period. That is,

$$t_{ini} = \begin{cases} t_{request} & \text{if } t_{request} \geq FP_X \\ FP_X & \text{if } t_{request} < FP_X \end{cases} \tag{5.8}$$

Under this limitation, the scheduling procedures used in case 2 can be applied to the present example. Figure 5-20 shows the remaining process times and state of each machine at the moment of initialization (see ⓪ in Figure 5-13). The no-transport-time interval is determined to be the duration between 70 seconds and 85 seconds after the moment of re-initialization. The time from 65 seconds to 70 seconds

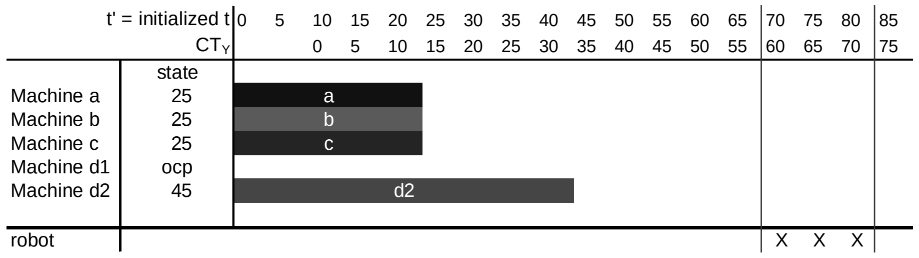


Figure 5-20: Information at the moment of re-initialization

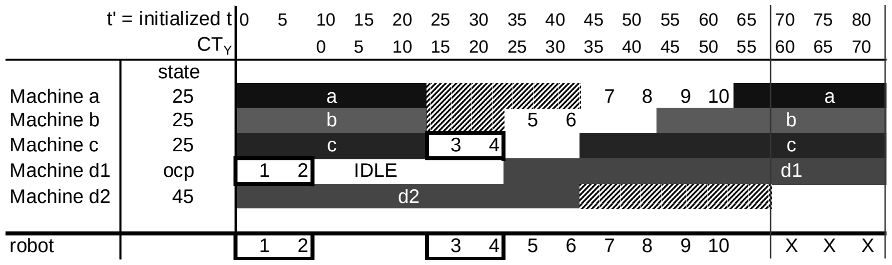


Figure 5-21: Resulting schedule for a single period

is excluded from the no-transport-time interval because, as previously described, the re-initialization cannot occur before the FP_X of 70 seconds has expired. Pre-fixed transport tasks [1 2] and [3 4] are allocated outside of the no-transport-time interval. The other transport tasks are assigned based on the fundamental conditions of part transport. A finished part in machine M_{d2} does not leave until another part request is issued by subsystem Y. One possible schedule for a period is shown in Figure 5-21, and the same approach can be repeatedly applied to successive periods.

5.3 Summary

A system is often constructed by integrating various subsystems. One aspect of system integration requires a coordination of various machines and subsystems with varying process or cycle times. Scheduling of part-transport is one of the critical problems. Top level functional requirement for part-transport is to 'transport a part from one station to the next station when the current station finishes its processing on the part.' Given the process recipe and system configuration, a fixed set of sub-level functional requirements can be explicitly defined. Since the functional requirements appear at different times, $u_i(t)$ is appropriate representation for these FRs, and thus the scheduling problem is subject to time-dependent complexity. In order to ensure the productivity and reliability of such system, periodicity must be introduced by re-initializing the subsystems on a periodic functional interval and by eliminating scheduling conflict through the introduction of intentional delay time as decouplers. This is equivalent to preventing a system from developing time-dependent combinatorial complexity and thereby increases predictability of system performance and productivity.

Three different cases of the scheduling of the integrated system with a varying cycle time are considered as an illustrative example. Case 1 is equivalent to a scheduling with no variation, and thereby well-studied deterministic scheduling techniques apply to obtain periodic schedule with predictable throughput rate. In case 2, the pace of the integrated system is determined by a subsystem that has cycle time variation. Deterministic scheduling solution does not apply to this case due to the cycle time variation. In order to satisfy one of the system requirements – maximize the utilization of subsystem Y –, no-transport-time interval is defined as part of the scheduling. Since the temporal location of no-transport-time interval is not fixed relative to other transport functional requirements, the operation of subsystems are controlled by re-initializing the system periodically by identifying the completion of a given functional cycle. Re-initialization introduces periodicity and thus changes the scheduling problem from potentially time-dependent combinatorial complexity problem to a time-dependent periodic complexity problem. The re-initialization simplifies the scheduling problem and ensures the throughput performance.

Case 3 is particularly interesting because it clearly shows the consequence of breakdown of inherent functional periodicity of transport functions. An occurrence of incomplete functional period results in reduced throughput performance. Functional periodicity is lost in the example because the slower subsystem, subsystem X, is not given enough time to complete all of its transport functions, which violates one of the requirements for re-initialization: existence of a functional period in which all the functions are repeated. By modifying the triggering condition for re-initialization, the system is able to meet the requirements for re-initialization and functional periodicity is maintained.

Chapter 6

Periodicity in a Biological System: The Cell Cycle

Periodicity is abundant in biological systems. Reproduction of a certain organelle in a cell consists of a series of events that form a cycle. Human behavior typically follows a circadian cycle in everyday life. Many plants and some animals have an annual cycle for their survival. It is an interesting arena to investigate from axiomatic design's complexity standpoint to see whether this periodicity is indeed a fundamental requirement for biological systems or merely a consequence of other primary principles. A series of cyclic events that occur during cell reproduction, known as the cell cycle, is particularly interesting because it clearly demonstrates a crucial role of periodicity and the chaotic consequence of breakdown of the periodicity.

One of the central mechanisms of living cell is its reproduction through cell duplication and division. A cell, when it is in proliferation, duplicates and divides itself to form daughter cells. Each of the daughter cells has to inherit exactly one copy of DNA from their parent cell to survive and properly perform its functions. Critical events that have to happen during cell reproduction are, thus, precise replication of its chromosomes, which contain DNA, and exact partition of the duplicated chromosomes into two identical offspring. During the process, other intracellular substances are also distributed to the daughter cells. A series of events occur during the process in an orderly sequence, forming what is known as the *cell cycle*.

This chapter begins with a general background of cells, cell cycle, and its regulatory mechanism. An attempt is made to identify functional requirements of the cell cycle in order to better understand it from systems perspective – how the higher-level functions are related to molecular level interactions. Lastly, coordination of two independent cycles in the cell cycle, chromosome cycle and centrosome cycle, and the consequence of its failure is discussed to support the axiomatic design's view on to periodicity.

6.1 Background: Cells

A cell is a basic unit for a life form. Some living beings such as bacteria, yeast and amoeba consist of a single cell. Other creatures are made of a large number of cells. Groups of specialized cells are organized into tissues and organs in multicellular organisms. For example, C. *elegans*, one of nematode kinds, is known to have 959 cells. Human beings are estimated to have tens of trillions of cells. There are two distinct types of cells: prokaryotic cells, found only in bacteria, and eukaryotic cells, composing all other life-forms. Eukaryotic cell contains clearly defined nucleus enclosed by a nuclear membrane. Prokaryotic cell lacks such nucleus. Regardless of cell types and organisms, all of the living beings are products of repeated rounds of cell growth and cell division. Details of the process vary from organism to organism, but all of them are required to achieve a universal requirement: passing its genetic information on to the next generation. While many prokaryotic cells achieve the goal by relatively simple binary fission, eukaryotic cells have evolved a complicated network of regulatory proteins. This section presents a general background of a eukaryotic cell structure, the cell cycle, and the control of the cell cycle progression[1].

[1]Knowledge on the cell structure and the cell cycle is well established in the field of cell biology, and majority of the contents of this section is based on a reference, *Molecular Biology of The Cell*[6]

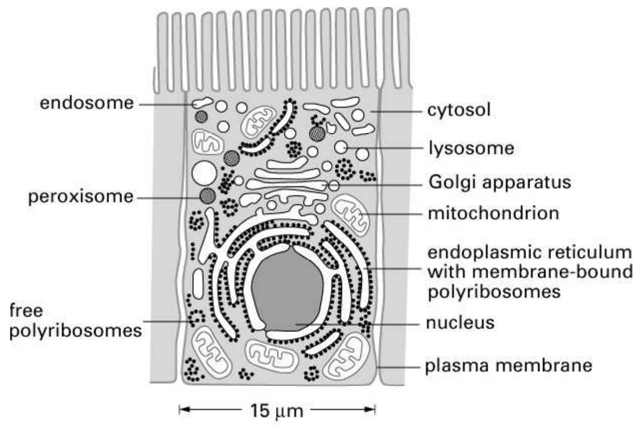


Figure 6-1: Schematics of the eukaryotic cell structure

6.1.1 Cell structure and the Cell Cycle

Major components of a eukaryotic cell include the nucleus, intracellular organelles, cytoskeleton, cytosol, and membrane. The nucleus contains chromosomes (mainly comprised of DNA) and is separated from other portion of a cell by a nuclear membrane. All other intracellular substances are referred to as a cytoplasm. The cytoplasm is comprised of organelles such as mitochondrion, that generates ATP fuel, and the cytosol, the fluid mass surrounding various organelles. It also contains cytoskeleton, a system of microscopic filaments or fibers. The cytoskeleton organizes other cell components, maintains cell shape, and is responsible for cell locomotion and for movement of the organelles within it. All of these components are enclosed by cell membrane to define a single cell. See figure 6-1 for illustration.

The outcome of cell reproduction by cell division is a pair of daughter cells, each with a complete set of intracellular components. It is well known that cell division is a cyclic event. A series of events occur during the process in an orderly sequence. Figure 6-2 illustrates the cell cycle. First, each of the daughter cells grow to be a mature cell for a next round of reproduction. When a cell is ready for cell division internally and externally, it starts to duplicate chromosomes. Once chromosomes are successfully duplicated, they are segregated to form two intact nuclei. Finally, the cell membrane is pinched in the middle to form two daughter cells of next generation. Note that most organelles and cytoskeleton are distributed to daughter cells as a

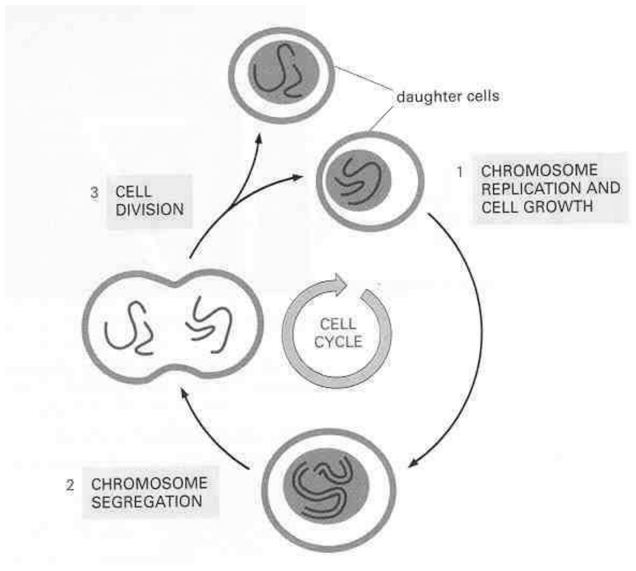


Figure 6-2: During a cell cycle, chromosomes are replicated and the duplicated chromosomes are separated to be inherited to each of the daughter cells. Figure is taken from [4].

consequence of cell division and continuously grow in the newly-formed daughter cell depending on the need from the cell, but chromosomes must be exactly duplicated during the cell cycle to form a complete DNA complement for each daughter cell by the time of separation. Thus, the most important events in the cell cycle are the duplication of chromosome and its proper segregation.

Biologists have explained the cell cycle in terms of two main phases and gap phases between the main phases. Two main phases are S phase – S for synthesis – where parent chromosomes are replicated and M phase – M for mitosis – where a cell with duplicated chromosomes is split into two individual daughter cells. Depending on the type of cell, it may have seemingly quiescent gap phases between S and M phases, called G1 phase before S phase and G2 after S phase. While it is proliferating, cell typically undergoes a cycle of G1-S-G2-M repeatedly. In some cases, however, cells partly exit from this normal cycle to a specialized, non-dividing state called G0. (See figure 6-3).

During G1 phase, a cell typically grows in its size/mass as it prepares for cell division. At the same time, it constantly senses the extracellular environment for signals that inhibit or stimulate cell growth and division. If extracellular conditions

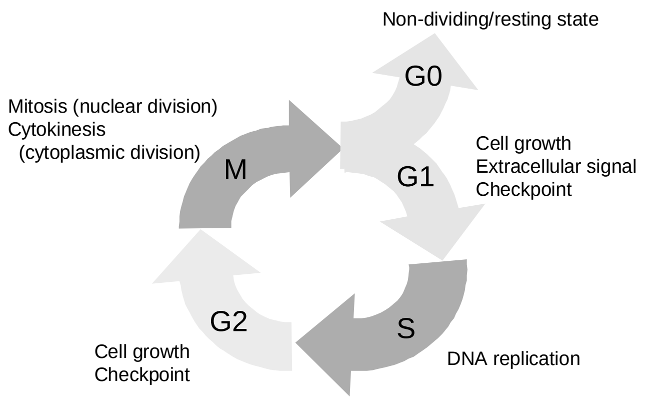


Figure 6-3: Cell cycle progresses through different phases: DNA is replicated during S phase, and the nucleus and cytoplasm divide in M phase. Partly to allow more time for growth, most cells have gap phase: G1 between M and S phase and G2 between S and M phase. Depending on the extracellular condition, some cells enter a specialized resting state, called G0.

are favorable and the cell has grown enough, then it commits itself to cell division process by entering S phase for chromosome duplication. As chromosomes are being duplicated inside a nucleus during S phase, a microtubule-organizing organelle, called the centrosome, is duplicated outside the nucleus. Later in M phase, centrosomes play a crucial role in segregating duplicated chromosomes. Completion of DNA replication in S phase leads to G2 phase where the cell is getting ready for actual separation. Then, the cell finally enters M phase. In the first half of M phase, the membrane of the original nucleus is broken down, and the replicated chromosomes are segregated and pulled apart to opposite sides of the cell. Then, new nuclear membrane is formed around each of the two chromosome sets to form two intact nuclei. This process of nuclear division is called mitosis. Following mitosis is the separation of the parent cell into two daughter cells each with one nucleus. This process of cytoplasmic division is called cytokinesis. After cytokinesis, two individual cells are formed and they are in their G1 phase and proceed to the next round of their own cell cycle.

6.1.2 Regulating the Cell Cycle: Cyclin and Cdk

The progress of cell cycle, e.g. from G1 to S phase, is precisely regulated by the cell cycle control mechanism. Considering that the correct completion of cell division is

Cyclin-Cdk	Vertebrates		Budding Yeast	
Complex	Cyclin	Cdk partner	Cyclin	Cdk partner
G1-Cdk	Cyclin D	Cdk4, Cdk6	Cln3	Cdk1
G1/S-Cdk	Cyclin E	Cdk2	Cln1,2	Cdk1
S-Cdk	Cyclin A	Cdk2	Clb5,6	Cdk1
M-Cdk	Cyclin B	Cdk1	Clb1,2,3,4	Cdk1

Table 6.1: Cyclins and their Cdk partners for vertebrate cell and budding yeast [10]

crucial for survival of both the individual cell and organism, it is not surprising to see that the cell cycle control system is meticulous in nature. Among a huge number of participants in cell cycle control, the most fundamental components are cyclin and Cdk (Cyclin dependent kinase). Cyclin gets its name from the fact that its concentration rises and falls in accordance with the cell cycle. Cdk, as the name suggests, is a protein kinase whose activation is (partially) dependent on cyclin. Different classes of cyclins are defined by the particular cell phase at which they interact with Cdk and perform their functions. In vertebrate cells, four different kinds of Cdks binds different cyclins while a single Cdk protein interacts with all classes of cyclins. For example, in a vertebrate cell, cyclin A binds Cdk2 during S phase to initiate DNA replication, and cyclin B interacts with Cdk1 during M phase. Cyclical changes in cyclin levels result in cyclical assembly and activation of the cyclin-Cdk complexes, which plays a crucial role in controlling progression from one stage of the cell cycle to the next. For simplicity, we refer to the different cyclin-Cdk complexes as G1-Cdk, G1/S-Cdk, S-Cdk, and M-Cdk, the names specifying the phases in the cell cycle at which they play a dominant role. The names of the individual Cdks and cyclins for vertebrates and budding yeast are listed in table 6.1. The activity of cyclin-Cdk is cyclic mainly due to the cyclic changes in cyclin concentration in cells. Then what controls the level of cyclin? The way cyclin level is regulated is twofold: 1) proteolysis (degradation), and 2) cyclin gene transcription. Proteolysis is a general process where a large protein complex in the cytosol, called proteasome, degrades a target protein by hydrolysis at one or more of its peptide bonds. At certain cell-cycle stages, a specific class of cyclins is marked - ubiquitylated - as targets of proteasome by a ubiquitin-dependent mechanism. Two enzymes - ubiquitin ligases - are important in cyclin destruction: SCF and APC. SCF is named after its three main protein subunits, and is responsible for the ubiquitylation and destruction of G1/S-cyclin. APC (Anaphase-promoting complex) is responsible for M-cyclin proteolysis in M phase. While SCF and APC are one part of cyclin regulation, control of cyclin synthesis is another important part. Cyclin levels in most cells are controlled not only by changes in cyclin degradation but also by changes in cyclin gene transcription and thereby cyclin synthesis. For example, transcription of genes encoding S cyclins is promoted by a gene regulatory protein, E2F. Promoted by E2F, increased level of S cyclin synthesis leads to the increase in S-Cdk activity. Although the details of this gene expression regulation remain unknown, it is certain that cyclin gene transcription provides an added level of regulation. In addition to the above primary determinant of cyclic activation of cyclin-Cdk, there are additional mechanisms involved in Cdk activity regulation. These mechanisms include CAK (Cdk-activating kinase), inhibitory kinase (e.g. Wee1 kinase), phosphatase (e.g. cdc25) and CKI (Cdk inhibitor protein). By interacting with cyclin-Cdk complex when called upon, they provide the ability of fine-tuning Cdk activity in the cell cycle. To summarize, the level of cyclin-Cdk activity is regulated by the following mechanisms:

- degradation of cyclin: SCF, APC
- synthesis of cyclin: gene transcription control (E2F)
- CAK (Cdk-activating kinase)
- CKI (Cdk inhibitor protein): p27, p21, p16
- Cdk phosphorylation/dephosphorylation: Wee1 inhibitory kinase, Cdc25 activating phosphatase
- and possibly other mechanisms that are yet to be described

6.1.3 Transition of Phases in the Cell Cycle

After the completion of a cell cycle, each of the daughter cells has to have inherited exactly one copy of chromosome as well as other organelles from their parent cell to properly function. Therefore, the chromosome replication is among the most critical events in a cell cycle. This critical event occurs during S phase (S for synthesis), which requires 10-12 hours and occupies about half of the cell-cycle time in a typical mammalian cell. The molecular details of the DNA duplication are beyond the scope of this chapter, and we concentrate on the regulatory mechanism from the cell cycle control perspective.

The S phase regulatory mechanism has two main functions. One is to initiate the chromosome replication process, and the other is to prevent it from happening more than once per cycle. The second task is as important as the first since hyper-replication of chromosomes results in an abnormal number of genes in one or both daughter cells and leads to serious damage to the cell itself and potentially the organism as a whole. Initiation of chromosome duplication requires the activity of S-Cdk. Chromosome replication begins at “origins of replication.” The origins of replication are scattered at various locations in the chromosome, and serve as the landing stations for chromosome replication ‘machine.’ A large, multi-protein complex known as the origin recognition complex (ORC) binds to the origin of replication. In early G1 phase, the level of Cdc6, one of the regulatory proteins, increases transiently. Cdc6 binds to ORC, and together they promote the formation of Mcm protein rings on the adjacent DNA, which later migrate along DNA strands. The resulting protein complex, bindings of ORC, Cdc6, and Mcm rings, is known as the pre-replicative complex (pre-RC). With pre-RC in place, the replication origin is ready to fire. The activation of S-Cdk in late G1 phase triggers the origin firing, assembling DNA polymerase and other replication proteins and activating the Mcm protein rings. S-Cdk is also responsible for preventing hyper-replication of chromosome. First, it disassembles pre-RC by causing the Cdc6 protein to dissociate from ORC after the origin has fired. At the same time, it phosphorylates Cdc6, which in turn triggers Cdc6 ubiquitylation by the SCF enzyme. As a result, the level of free Cdc6 protein decreases quickly after the initiation of chromosome replication. S-Cdk also phosphorylate some Mcm proteins. It causes the export of Mcm protein from the nuecleus, and thus enhances the preventive mechanism. G1/S-Cdk and M-Cdk also helps restraining the unwanted pre-RC assembly to make sure DNA replication does not occur after S phase. Having understood that S-Cdk is the key component in progression into S phase, what, then, activates S-Cdk? Activating S-Cdk requires another set of mechanisms which involve extracellular signal, and the detail is discussed later.

Having two accurate copies of the entire genome, the cell is (almost) ready to undergo M phase, where the duplicated chromosomes and other cell contents are distributed equally to the two daughter cells. M phase is further broken down to two stages: mitosis and cytokinesis. During mitosis, a series of events occur: the replicated chromosomes condense, the mitotic spindle assembles, nuclear envelope breaks down, microtubules attach to the chromosomes, the chromosomes are aligned at the equator of the cell, the sister chromatids abruptly separate and are pulled apart, and new nuclear envelopes are formed around each of the chromosome set completing nuclear division. Once the nucleus is divided, other substances are to be divided. During cytokinesis, the cytoplasm (extra-nuclear substances) is divided in two by a contractile ring of filaments, completing the cell division process for one generation.

M-Cdk is responsible for initiating a number of events during M phase: the replicated chromosomes have to be rearranged for separation, the microtubules have to be organized around centrosome to form mitotic spindle, the spindles are attached to the replicated chromosomes, and so on. Once the complex rearrangements of the chromosomes and mitotic spindle are completed, the sister chromatids are to be separated. It is not M-Cdk but the anaphase-promoting complex (APC) that triggers the separation, the most dramatic event in M phase. When duplicated, the two sister chromatids are bound tightly together by the action of cohesin complex. Active APC acts on a protein complex that dissociates cohesin from the chromosomes and thereby initiates the sister chromatids separation. The separation of sister chromatids is im-

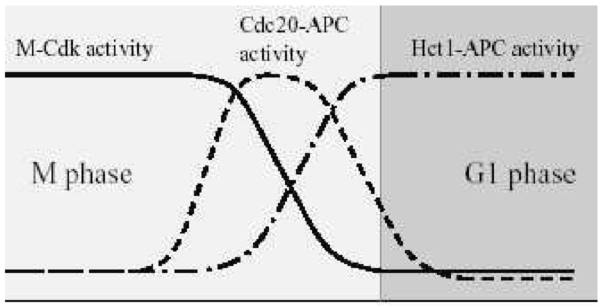


Figure 6-4: At the end of M phase, the activity of M-Cdk is suppressed by Cdc20-APC and Hct1-APC. Figure is taken from [6].

mediately followed by cytokinesis where cytoplasm is divided into two and thus two daughter cells are formed.

At the end of M phase when the chromosomes have been completely separated, the cell reverses the changes to prepare for cytokinesis. The spindle is disassembled, the chromosomes decondenses, and the nuclear envelope is re-formed. It is believed that M-Cdk inactivation is mainly responsible for this reversing process, and thus triggers the exit from mitosis. While Cdc20-APC complex initiates sister chromatids separation, it also inactivates M-Cdk by ubiquitylating M-cyclin. Thus, as Cdc20-APC become more active toward the end of M phase, M-Cdk level decreases. Interestingly, since the activation of Cdc20-APC requires M-Cdk, the activity of Cdc20-APC also decreases along with M-Cdk level. By this feedback mechanism, the destruction of M-cyclin leads to the inactivation of APC activity, which then allows the cell to quickly begin accumulating new M-cyclin for the next cell cycle (see figure 6.1.3). It is a useful setup for embryonic cells with no G1 phase, but not very useful for cells with G1 phase because they need to suppress Cdk activity for a while to allow for cell growth and to be regulated by extracellular signals. Therefore, cells with G1 phase need other mechanisms to ensure that Cdk reactivation is prevented after mitosis. There are three mechanisms that suppress Cdk activity at the end of M phase: Hct1-APC complex that initiates the ubiquitylation of M-cyclin, CKI accumulation (Sic1 in budding yeast cell and p27 in mammalian cell), and reduced transcription of M-cyclin genes. The first two are mutually inhibitory with respect to M-Cdk, i.e. Hct1-APC and CKI suppress M-Cdk, and vice versa. By this mutual inhibition, these mechanisms, unlike Cdc20-APC, become active with decrease in M-Cdk level (see figure 6.1.3). In animal cells, these three mechanisms are in effect and restraining Cdk activity until late G1 phase when G1-Cdk is activated.

As a fresh newborn, each of the two daughter cells is in G1 phase. Transition from G1 phase to S phase is particularly important since it is a commitment to the whole process of cell division. Indeed, the control of G1 progression and S phase initiation is often found disrupted in cancer cell.

6.2 Identifying Functional Requirements in the Cell Cycle

In this section, details of the events occurring in each of the cell phase are discussed. Especially, an attempt is made to identify functional requirements during cell cycle in relation to low level DPs such as cell regulatory proteins. Since a biological system is the result of over billions of years of evolution and what we observe now is the physical outcome of the process, we have relatively good understanding on high level functions and low level mechanisms. However, it is difficult to have systematically organized knowledge about its functions from top to bottom. It is similar to trying to understand a complicated electric circuit designed by someone else, that consists of many circuit elements such as resistors, capacitors, and inductors. By measuring resistance of a particular resistor, we know that across the resistor, there is certain amount of voltage drop. Or, by removing a specific resistor, we can speculate the resistor's function. At a high level, by observing what the system does as a whole, we are able to understand its high level functionality. However, there is a huge gap in between that makes it hard, for example, to troubleshoot.

Relating DPs to FRs in cell cycle is useful in two aspects. Firstly, it helps to understand a biological system in terms of functions rather than, for example, a signaling pathway. This leads to clear representation of functional periodicity in the cell cycle, as will be discussed. Secondly, it forces the abstraction of functional requirements to a higher level which at the same time the top level functional requirements are decomposed so that the available information is put into context. Also, some of the unrealized research questions can be raised by examining a system from functional perspective, which would be hard to do just through the interpretation of observed mechanisms.

6.2.1 Functional Decomposition for G1 phase

Top level FR-DP

Having understood the state of a cell at the end of M phase, it is not difficult to guess what needs to happen during G1 phase. It has to constantly monitor signals for cell cycle progression, and, once it comes, all three Cdk inhibitory mechanisms must be reversed so that S phase is quickly initiated. As a preparation for chromosome replication, it also has to form pre-RC during G1 phase. To maintain its size throughout the successive dividing processes, a typical cell must grow its size. Otherwise, the size of a cell becomes smaller and smaller as it divides itself. Note, however, in some cases cell does divide without growing (embryonic development), and grow without dividing (neuron). Another critical function is that a cell must prevent itself from entering S phase if there is something wrong with the cell. For example, a cell with DNA damage should not replicate itself to avoid creating malfunctioning daughter cells. In summary, the following is a list of functional requirements for G1 phase in particular relation to cell cycle regulation:

- FR1: Form DNA replication ‘machine’
- FR2: Grow cell to maturity
- FR3: Sense environment to determine the entry to next phase
- FR4: Cause cell cycle arrest/death if necessary
- FR5: Initiate exit-G1 & entry-S

Design parameters can be stated for each of the above functional requirements. Albeit abstract at this level of decomposition, explicitly stating design parameters helps organizing information into perspective.

- DP1: Pre-RC forming mechanism
- DP2: Extracellular signal (growth factors)
- DP3: Extracellular signal (mitogens)
- DP4: Cell cycle inhibitory mechanism / Apoptosis
- DP5: Rb-pathway

DP1, pre-RC forming mechanism, as discussed in 6.1.3, involves the activities of several proteins (e.g. Cdc6, Mcm) and is solely responsible for creating the pre-replicative complex at the origin of replication on DNA strand. DP2 is the extracellular growth factor (e.g. PDGF, EGF) , which stimulates cell growth by initiating intracellular signaling pathway. For single-celled organisms such as yeasts, cell growth depends only on nutrients. By contrast, animal cells require both nutrients and extracellular signals typically from other neighboring cells. DP3 is another type of extracellular substance, called mitogens. It initiates an intracellular signaling pathway, called Rb pathway, that leads to the break down of the blocking mechanisms that suppress the progress of cell cycle. Many extracellular signal proteins act as growth factor as well as mitogen. Also, growth factor stimulation also leads to increased production of the gene regulatory protein. DP4 is responsible for monitoring the anomaly in cell activity and taking appropriate action if necessary. This is very important FR-DP pair in regulating cell reproduction and suppressing cancer development, and the failing of this FR-DP is indeed commonly found in cancer cells. FR5 is the ultimate outcome of normal G1 phase, and Rb-pathway delivers the function.

The top level FR-DP pairs are decomposed to the next level to describe the system with further detail.

Decomposition of FR1-DP1: pre-RC forming mechanism

FR1-DP1 (pre-RC development) is decomposed to:

FR11:	Mark origin of duplication	DP11:	ORC - origin interaction
FR12:	Create Mcm helicase ring	DP12:	Cdc6-dependent mechanism

As discussed in background section, pre-RC creation requires ORC-origin interaction to mark the origin location. It is a stable interaction that marks a replication origin throughout the entire cell cycle. On the contrary, Cdc6-dependent mechanism is cell cycle dependent: Cdc6 is present at low levels during most of the cell cycle but increases transiently in early G1. Transcription of this protein was reported to be regulated in response to mitogenic signals through transcriptional control mechanism.

Decomposition of FR2-DP2: Response to extracellular growth factors

FR2-DP2 (growth factors) has the following sub FRs-DPs:

FR21:	Detect growth factor	DP21:	Growth factor receptor
FR22:	Stimulate protein synthesis	DP22:	Growth factor pathways
FR23:	Suppress cell cycle until it grows enough	DP23:	G1-cyclin inhibitory mechanism

To grow the cell, it has to increase the level of protein synthesis. This requires increased mRNA transcription and mRNA translation, which are stimulated by growth factors. For example, PI 3-kinase activates the S6 protein kinase and a translation initiation factor called eIF4E, which leads to increased mRNA translation. Most of these mRNA encode ribosomal components and as a result, protein synthesis increases. Since the exit from G1 phase begins by G1-cyclin accumulation, for a cell to have enough time to grow, it needs to suppress the available G1-cyclin level. DP23, G1-cyclin inhibitory mechanism is responsible for the function. It is quite interesting that G1-cyclin that is an important factor in determining dell division timing also holds an important role in cell growth regulation. For a cell that grows and divides, it is likely to hold the key to how cell growth and division is coordinated.

Decomposition of FR3-DP3: Response to mitogens

A cell also needs to react to the extracellular signal that promotes cell cycle progress.

FR31:	Detect mitogens	DP31:	Mitogen receptor
FR32:	Increase the level of G1-cyclin	DP32:	MAP kinase cascade

While growth factors primarily initiate kinases that stimulate protein synthesis, mitogen activates MAP kinase that leads to increased level of Myc gene. Myc increases the transcription of several genes, including the gene encoding cyclin D (G1-cyclin).

Decomposition of FR4-DP4: Cell-cycle inhibitory mechanism

Blocking cell cycle progress in undesirable events is crucial in maintaining survivability of cell itself and eventually organism. FR4-DP4 is decomposed to three sub-level FRs:

FR41:	Arrest cell cycle in the event of DNA damage	DP41:	p53-dependent mechanism
FR42:	Arrest cell cycle in the event of abnormal proliferation signal	DP42:	p19ARF mechanism
FR43:	Stop and terminally arrest when needed	DP43:	Intracellular stopping mechanism

p53, a gene regulatory protein, is a very important protein, which stimulates the transcription of several genes. One of these genes encodes CKI protein called p21. p21, as CKI protein, binds to G1/S-Cdk and S-Cdk and inhibits their activities which blocks entry into S phase. In normal conditions, a protein called Mdm2 binds to p53, ubiquitylating p53 for proteolysis. When DNA is damaged, it activates certain protein kinases that phosphorylate p53 and thereby reduce its binding to Mdm2. It leads to decrease in p53 proteolysis, and thus p53 concentration in the cell increases. The increase in p53 concentration results in rise of the level of p21, which inhibits Cdk activity. Just like DNA damage, abnormal proliferation signal - excessive mitogenic stimulation - causes p53 activation. The abnormal proliferation signal leads to the activation of cell-cycle inhibitor protein called p19ARF, which binds and inhibits Mdm2 resulting in p53 increase. DP43, stopping mechanism, is poorly understood. One possible explanation is a progressive increase in CKI p27. Another is 'replicative cell senescence (aging)' by a pair of sub-mechanisms.

Decomposition of FR5-DP5: Rb-pathway

Before decomposing FR5-DP5, examining the signaling pathway first will help understand the sublevel functional requirements. Recall that at the end of M phase, three Cdk-inhibitory mechanisms are in effect: low level of cyclin gene transcription, Hct1-APC, and CKI (p27 for mammalian cells), which collectively prevent Cdk activity. The activation of G1-Cdk is the very first step to reverse all three inhibitory mechanisms. The phosphorylation target of active G1-Cdk is retinoblastoma protein (Rb), an inhibitor of cell-cycle progression. During G1 phase, Rb binds to a gene regulatory protein called E2F and inhibits E2F activity. E2F promotes transcription of many genes that encode proteins required for S-phase entry such as G1/S-cyclins and S-cyclins. Thus, G1-Cdk with E2F as mediator stimulates the synthesis of G1/S-Cdk and S-Cdk. Then, what about those Cdk-inhibitory mechanisms? As with M-Cdk, G1/S-Cdk and S-Cdk also form positive feedback loops. The increase in these Cdk activities enhances the phosphorylation of Hct1 and p27, leading to their inactivation or destruction. Therefore, activation of G1/S-Cdk and S-Cdk result in more activation of themselves. Two more feedback loops enhance the process. Active E2F promotes the transcription of its own gene. G1/S-Cdk and S-Cdk enhance phosphorylation of Rb and thereby promote E2F release, which then increases gene transcription of G1/S-cyclin and S-cyclin. These enhancing feedback mechanisms ensure the rapid transition from G1 phase to S phase, once initiated by G1-Cdk.

The remaining part of the G1-S phase transition mechanism is G1-Cdk activation. In animal cells, activating G1-Cdk requires stimulation by the extracellular signals. As discussed in FR3-DP3 decomposition, mitogens promote the gene transcription for G1-cyclin. Not only does mitogen-dependent mechanism stimulate G1-cyclin syn-

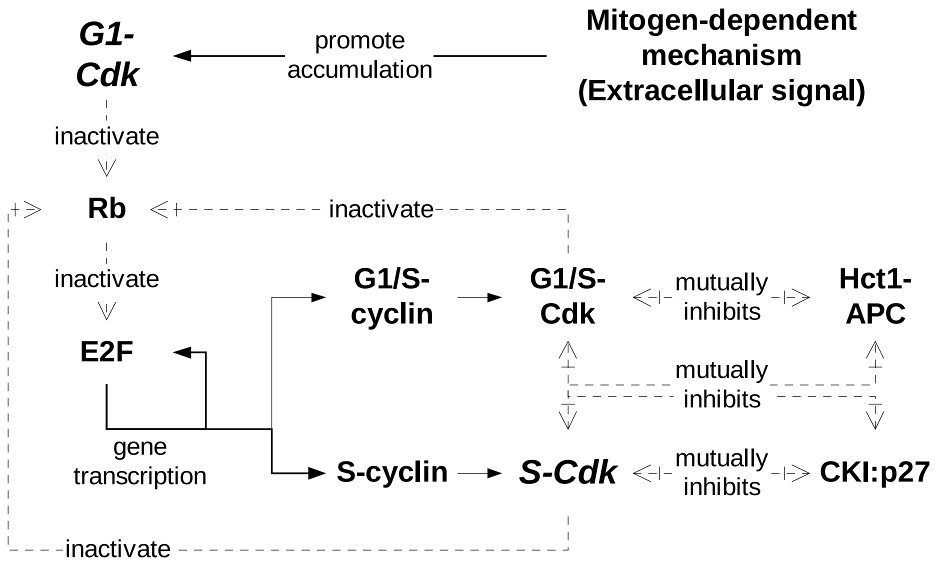


Figure 6-5: G1-Cdk triggers the phase transition from G1 to S by initiating a series of events that lead to increased level of S-Cdk activity.

thesis, but it also helps the transition by additional activities. The regulatory protein Myc increases the transcription of genes including a gene encoding a subunit of the SCF enzyme and E2F gene as well as G1-cyclin gene. SCF enzyme ubiquitylates CKI p27, leading to the increase in G1/S-Cdk activity. Increased level of E2F protein by transcription of E2F gene promotes its own gene transcription. The end result of the whole mechanism is the increased S-Cdk activity and consequently exit from G1 phase. Figure 6-5 shows the overall signaling pathway involved in G1-S phase transition. FR-DP representation of the above mechanism is shown below:

FR51:	Accumulate G1-Cdk	DP51:	Available G1-cyclin
FR52:	Inactivate Rb	DP52:	G1-Cdk
FR53:	Promote E2F synthesis	DP53:	Rb phosphorylation
FR54:	Promote S-cyclin synthesis	DP54:	E2F
FR55:	Activate S-Cdk	DP55:	S-cyclin
FR56:	Inactivate CKI (p27)	DP56:	G1/S-Cdk
FR57:	Inactivate Hct1	DP57:	S-Cdk
FR58:	Promote G1/S-cyclin synthesis	DP58:	E2F
FR59:	Activate G1/S-Cdk	DP59:	G1/S-cyclin

	ORC-origin interaction	cdc6 mechanism	Growth factor receptor	Extracellular growth factor pathway	Cln3 inhibitory mechanism	Mitogen receptor	Myc (gene regulatory protein)	p53-dependent mechanism	p19ARF->p53 mechanism	Intracellular stopping mechanism	Available G1-Cyclin	G1-Cdk	Inactivation of Rb	E2F	S-cyclin	G1/S-Cdk	S-Cdk	G1/S-cyclin
Mark origin of duplication	X																	
Form Mcm helicase		X				13	13									16	16	
Detect growth factor			X															
Stimulate cell growth			15	X		3												
Suppress cell cycle until grows enough				1	X		c											
Detect mitogen			14	2		X												
Increase the level of G1-Cyclin					b	4	X											
Arrest cell cycle in the event of DNA damage								X	O**									
Arrest cell cycle in the event of abnormal proliferation signals						d	5		X									
Stop and terminally arrest										X								
Accumulate G1-Cdk				O*	b		6				X							
Inactivate Rb											f	X						
Promote E2F protein							7					g	X	11				
Promote S-phase gene transcription													h	X				
Activate 'some' of S-Cdk								9	9					k	X			
Inactivate CKI (e.g. Sic1, p27)							8									X	12	
Deactivate Hct1 (APC-activating protein)																12	X	
Activate 'some' of G1/S-Cdk								10	10					m				X

Figure 6-6: Design matrix for FR-DP in G1 phase

Design matrix

Now that we have fair details on the functional requirements and design parameters, a design matrix is constructed to examine the interrelationship among FRs and DPs. The design matrix is shown in figure 6-6, and the explanations on each of non-zero design matrix elements are given as numbered annotation. Since the interactions indicated by diagonal elements (marked by 'X') are self explanatory, only the off-diagonal elements - cross interactions - are explained. Off-diagonal elements labeled with numbers indicate strong interactions, whereas alphabetic labels imply relatively weak interrelationships. Two of the zero elements are also explained for clarity. Note that the symbol 'A $\rightarrow$ B' means that A activates or increases the activity of B, whereas 'A $\dashv$ B' means inactivation or suppression of B by A.

1. G1-cyclin is synthesized in proportion to cell growth

2. Many extracellular growth factors activate the signaling protein Ras, which stimulates the MAP-kinase pathway to trigger cell-cycle progression

3. Many mitogens stimulate cell-growth by Mitogen → Ras → PI3-kinase

4. Mitogen → Ras → MAP-kinase → Myc → G1-cyclin gene transcription

5. Excessive Myc production induces cell-cycle arrest

6. Myc → G1-cyclin transcription → G1-cyclin → G1-Cdk

7. Myc → E2F transcription → E2F

8. Myc → SCF subunit transcription → SCF → p27 degradation

9. p53 → p21 transcription → p21 CKI ⊣ S-Cdk

10. p53 → p21 transcription → p21 CKI ⊣ G1/S-Cdk

11. E2F → E2F transcription

12. Symmetry: S-Cdk&G1/S-Cdk ⊣ Hct1-APC&CKI

13. Creating Mcm-halicase requires Cdc6 which in turn is regulated in response to mitogenic signal through transcriptional control mechanism involving E2F proteins.

14. Many growth factor receptors also recognize mitogens.

15. Receptor → Ras → PI 3-kinase

16. G1/S-Cdk and S-Cdk prevents Mcm ring formation after S phase initiation to prevent multiple DNA replication

a. Excessive Myc production induces cell-cycle arrest

b. G1-cyclin inhibitory mechanism poses a threshold level for G1-cyclin

c. Myc promotes G1-cyclin synthesis (Indeed, '1' is rather (3,3)-2-(5,5)-4-(6,6)-c)

d. Excessive Myc production induces cell-cycle arrest

f. G1-cyclin → G1-Cdk ⊣ Rb

g. G1-Cdk ⊣ Rb ⊣ E2F

O*. Extracellular GF eventually contributes to G1-Cdk accumulation by GF → Ras → MAP-kinase → Myc → G1-cyclin → G1-Cdk. However, that is too far-downstream, and that effect is represented by a chain DP21 $\rightarrow$ FR32 $\rightarrow$ DP32 $\rightarrow$ FR41

O**. p19ARF leads to p53 mechanism, but has nothing to do with DNA damage

Apparently, the above full design matrix looks like a coupled design matrix. Given that the cell cycle is successfully managed repeatedly in normal multicellular organisms, having a coupled design matrix may be explained by one of the following possible reasons. First, cells have evolved to have coupled relationships, but the cell cycle regulatory mechanism has been able to somehow manage the coupled relationships. If that is true, the cell cycle regulatory mechanism must involve intensive feedback mechanisms and they have to be tightly controlled. In other words, the existence of coupled design matrices is an evidence of complicated regulatory mechanism. Second, although it looks coupled at this level of decomposition, it is indeed a decoupled design if further decomposed. A theorem that addresses such a peculiarity was proposed by Melvin [26], and that may be the case with this design matrix. Thirdly, there is a possibility that the information captured by this design matrix is incorrect or incomplete because much of the details about the cell regulatory mechanisms are unknown yet. Testing each of the three hypotheses is an interesting problem from both cell biology and axiomatic design perspective.

6.3 Centrosome Cycle

The previous section discussed the cell cycle mainly from the perspective of the chromosome cycle. It focuses on the functional requirements to duplicate chromosomes and divide the replicated chromosomes into two daughter cells. As part of M phase FR-DP decomposition, the function of centrosome is briefly mentioned. While it is a part of chromosome cycle, centrosome has its own functional cycle for duplication and it is independent of the chromosome cycle in that the centrosome cycle can progress independent of chromosome cycle. This section discusses centrosome cycle and importance of maintaining synchronized periodicity between two cycles.

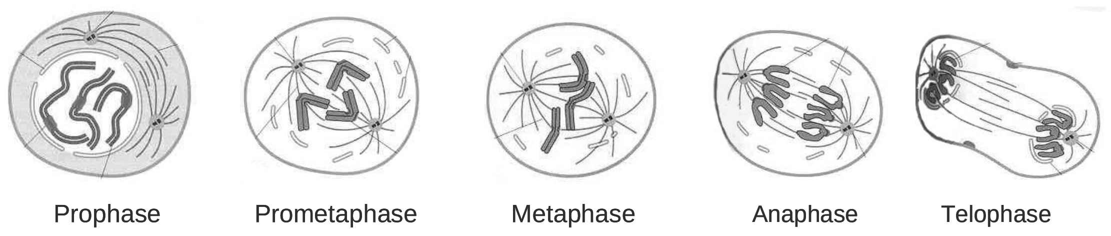


Figure 6-7: During mitosis, nuclear division, centrosomes shown as circles in this figure play an important role as microtubule-organizing center. Figure is taken from [7].

6.3.1 Centrosome: Microtubule Organizing Center

During mitosis, sister chromatids (duplicated chromosomes) are separated and pulled apart from each other to the opposite side of a cell. The mitotic spindle, an array of microtubules, performs the action of separating sister chromatids. Although in cells of higher plants and in many meiotic cells, bipolar spindles are assembled without centrosomes, in most animal cells, centrosome is known to play an important role in organizing microtubules to form the mitotic spindle [43],[44]. As a microtubule organizing center, the centrosome takes central stage during mitosis. Microtubules are nucleated within centrosome, and centrosome acts as a pole. As a nucleation site and mitotic spindle pole, it coordinates the microtubule activity during M phase.

Figure 6-7 shows a series of events during mitosis. In early M phase, each of a pair of centrosomes moves toward the opposite side of the cell, forming mitotic spindle poles at each side. As nuclear envelope breaks down, one end of mitotic spindle is attached to a sister chromatid's kinetochore while the other end is anchored at the spindle pole. Once the kinetochores on both sides of sister chromatids are attached to microtubules, the chromosome is said to have formed bipolar mitotic spindle attachment. By the action of attached mitotic spindle, the replicated chromosomes are aligned along a plane, called metaphase plate, that is nearly equidistant between the two spindle poles. With each of the centrosomes as a pole, kinetochore microtubules – microtubules that are attached to kinetochore – shrinks in length. Shortening of the length of kinetochore microtubules creates a pulling force that separates sister chromatids. Although the bipolar spindle may be formed without centrosomes in

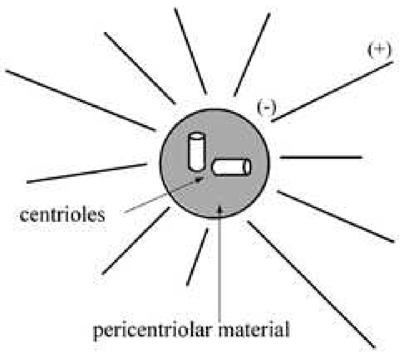


Figure 6-8: The centrosome consists of a pair of centrioles, surrounded by pericentriolar material. Lines in the figure are microtubues nucleated and anchored at pericentriolar material.

vertebrate cells whose centrosomes have been destroyed with a laser beam, the spindle is often mispositioned, resulting in abnormalities in cytokinesis [6].

The centrosome consists of a pair of centrioles, pericentriolar material (sometimes called pericentriolar matrix), and centrosomal domain (See figure 6-8). The centriole is a barrel-shaped, cylindrical organelle that is found in most animal cells. Although the functions of the centrioles within the centrosome remain unclear, centrioles are believed to be important in the assembly of pericentriolar material and the stabilization of the centrosomal structure [45],[46],[8]. Also, since centrosomes lacking centrioles do not reproduce in animal cells, centrioles seem to play an important role in centrosome reproduction [45]. The centriole pair is oriented perpendicular to each other, but the significance of this unusual orientation is still unknown [47]. Pericentriolar material surrounds the centrioles, and contains elements that nucleate and organize microtubules. Most importantly, it contains γ-tubulin complex that seems to serve as a template for microtubule polymerization. Instead of having a clearly defined boundary, the centrosomes extend out and integrate with cytoplasm, and these extensions are called outer centrosomal domain.

As a normal vertebrate cell enters M phase, a cell contains a pair of duplicated chromosomes and a pair of centrosomes. Thus, a cell has to duplicate exactly one copy of the existing centrosome prior to M phase. Duplication of centrosome begins with a separation of a centriole pair and forming a new daughter centriole from mother centrioles. Like chromosome replication, the centrosome is duplicated through a series of cyclic events, and the process is called centrosome cycle.

6.3.2 The Centrosome Cycle

Centrosome has its own cycle for its duplication. Normally, a single centrosome in a cell in G1 phase is duplicated exactly once to provide two centrosomes to form the poles of the mitotic spindle. Duplication of the single centrosome is initiated at the G1/S transition and completed before mitosis. At mitosis, the duplicated centrosomes are pulled apart to the opposite side of a cell, and plays a role of the mitotic spindle poles. When a cell is divided into two daughter cells at the end of M phase, each daughter cell receives one centrosome, and the centrosome begins its next cycle of duplication.

Figure 6-9 shows the centrosome cycle. Centrosome cycle begins with a loss of orthogonal connection between centrioles in a process called centriole disorientation. Once the centrioles are disoriented and exist as individual centrioles, a new daughter centriole is formed near one end of of parental centriole, and the nucleated daughter centriole is called a procentriole. Procentrioles then elongate until they reach full length. During the next round of duplication, both the the parental centriole and the daughter centriole from the current generation become parental centrioles for the next generation. Hence, one of the duplicated centrosomes is 'old', i.e. containing the oldest of the four centrioles, and the other is 'young.' While there exist two centriole doublet, the duplicated centrosomes remain associated until the cell enters M phase as indicated by the continuous pericentriolar material in figure 6-9 [47],[8]. In 'young' centrosome side, the parental centriole acquires a structure called distal appendages and cenexin. At the same time, pericentriolar material around young centrosome grows and acquires ϵ-tubulin that is initially present only in the old centrosome. As the cell enters M phase, the centrosomes are finally separated and move away to form mitotic spindle poles at the opposite side of the cell.

Despite its early discovery as old as late 19th century, research on centrosomes was not actively pursued except the work in early 20th century by Theodor Boveri

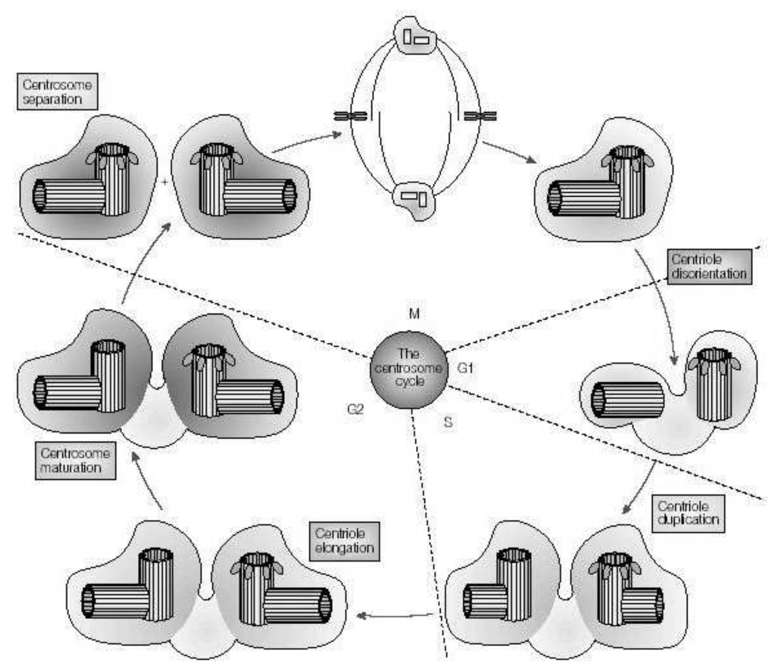


Figure 6-9: The centrosome is duplicated during S phase and separated later in M phase to organize activities of microtubules. Each of the two daughter cells receive one copy of the centrosomes. Figure is taken from [8].

[45]. It has been actively pursued only recently, sparked mainly by the demonstration that centrosomal abnormalities are frequent in many common cancers [8]. Centrosome research is now being extended to a molecular level, and yet much less is known about centrosome cycle relative to the fair amount of knowledge about chromosome duplication cycle. It was discovered several years ago that G1/S-Cdk and S-Cdk at the G1-S phase transition of the cell cycle allows centrosome duplication to proceed [48],[49],[50]. The earliest step that fails in the absence of Cdk2 activity is disorientation (splitting) of the centrioles [8],[48]. One of the downstream targets of Cdk2 activity is the protein nucleophosmin (NPM/B23). The work by Okuda et al[51] suggests that the protein nucleophosmin is associated with the centrosome during mitosis and needs to be removed to allow splitting of the centrioles. This explains the link between Cdk2 activity and the first step of centrosome cycle. Once the centrioles are split, new daughter centriole, procentriole, is nucleated. Although the regulation of new centriole formation is not well understood yet, a couple of mechanisms are known to play an important role in the process, including CaMK II (calmodulin-dependent

kinase II)[52] and zyg1 [53]. The procentrioles continue to grow to maximum length in parallel with the growth of pericentriolar material. With fully grown pair of centrosome, the last step of centrosome cycle is centrosome separation. It seems that there is a linkage between duplicated centrosome. A protein kinase, Nek2, along with its phosphorylation target C-Nap1, seem to regulate the linkage. Other factors that are likely to be part of the mechanism include a protein CP110, a ubiquitin-conjugating machine SCF complex, kinase Mps1p, and p53/p21 pathway. As mentioned earlier, understanding of centrosome cycle regulation is still at its early stage, and involves many unknowns. Yet, the most important and interesting fact about centrosome cycle is that its regulation, particularly synchronization with chromosome cycle, is crucial in maintaining cell's normality.

6.4 Synchronization of Centrosome Cycle and Chromosome Cycle

Because of the way centrosomes work during mitosis, it is crucial to have exactly one pair of centrosomes during M phase. Having two properly separated centrosomes at early M phase is required for a precise division of chromosomes. If there are more than two centrosomes (the extras called supernumerary centrosomes), missegregation of chromosomes is likely. Supernumerary centrosomes may produce multipolar spindles unless they fortuitously coalesce into a bipolar spindle. On the other hand, a single centrosome can induce formation of monopolar spindles. Multipolar or monopolar spindles do not properly direct cell division and generally provoke abnormal segregation of chromosomes [45]. The missegregation of chromosomes results in the aneuploidy (see figure 6-10), which, in most cases, leads to cell death due to the lack of essential chromosomes. However, some cells manage to function with defective chromosomes and rarely can acquire an advantage in terms of cell growth. In fact, aneuploidy is the most prevalent form of genetic instability found in cancer cells. Recent evidence argues that aneuploidy is a discrete event that contributes to malig-

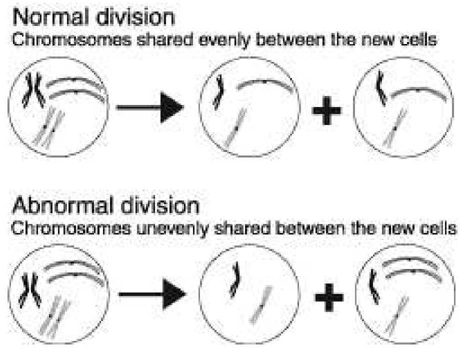

Normal division
Chromosomes shared evenly between the new cells
Abnormal division
Chromosomes unevenly shared between the new cells

Figure 6-10: Missegregation of chromosomes results in incorrect number of chromosomes in daughter cells.

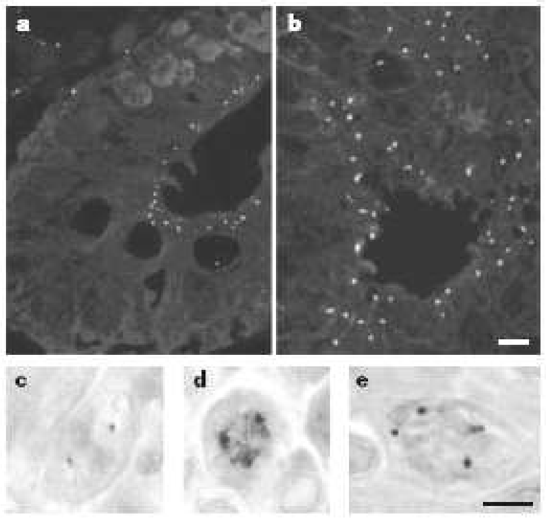
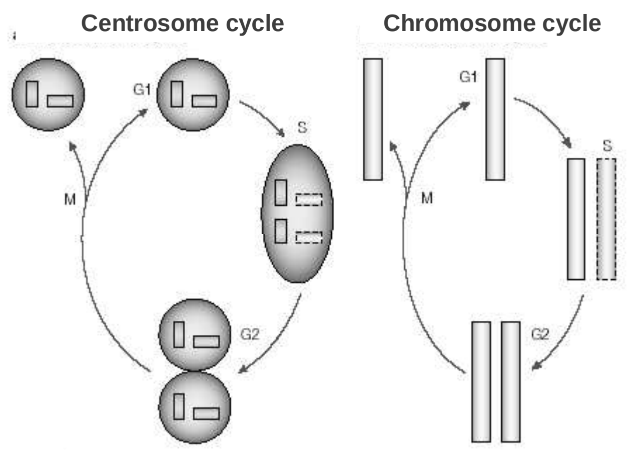


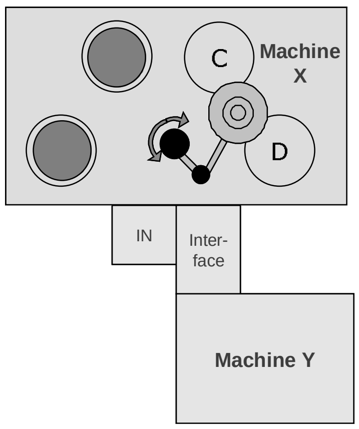
Figure 6-11: Centrosomal abnormalities are common in human tumor cells: Tumor colon tissues(b) contains amplified centrosomes compared to normal cells(a), indicated by bright spots; Human prostate tumor(d),(e) has multipolar spindles shown by dark spots while normal cell(c) has a bipolar spindle. Images are taken from [8].

nant transformation and tumor progression [45]. As mentioned above, it has long been recognized that errors in centrosome replication may be an important cause of aneuploidy and might thus contribute to cancer formation [45]. Centrosome abberation is common to most cancer types. Indeed, extra copies of centrosomes (supernumerary centrosomes) have been described for nearly all cancers that have been surveyed [8]. For example, figure 6-11 clearly shows centrosomal abnormalities in human tumors. Figure 6-11(a) and (b) are normal and tumor colon tissues from the same patient. Bright spots in the figure indicate centrosomes. The tumor cells contains amplified centrosomes that are larger and more numerous than those in the normal tissue. That is also the case with a human prostate tumor (figure 6-11 (d),(e)). While a normal cell has bipolar spindle indicated by two dark spots, there are multipolar spindles in dividing tumor cells. Although it is still being debated whether the centrosomal abnormality is the cause or consequence of cancer, there is no doubt that the two phenotypes enhances each other; centrosome aberrations will foster chromosome missegregation, regardless of whether they arose through deregulation of the centrosome cycle or as a consequence of another primary event [8].

To ensure that there are exactly two centrosomes at the beginning of M phase, synchronizing the chromosome cycle with centrosome cycle is critical. Figure 6-12(a) compares two cycles. In normal cells, exactly one copy of chromosomes is replicated during S phase, and at the same time, exactly one copy of centrosomes is duplicated. Thereby, by the time of G2-M transition, a cell contains a pair of duplicated chromosomes and centrosomes. That, in turn, ensures daughter cells receive one set of chromosomes along with one centrosome. If the coordination of the two cycles fails, it will lead to a change in ploidy. The coordination of these two subsystem cycles has an interesting similarities with manufacturing system example (see figure 6-12(b)): the integration of the two subsystems requires precise synchronization of the periodicity of the two sub-cycles for a proper functioning of the overall system. By recognizing the analogy, we can view the cell division cycle based on what is learned from the the manufacturing system scheduling example, i.e. periodicity. We can pose questions such as what controls the periodicity, what is the re-initialization agent? Do we have a control over the agent? Answers to these questions are at least partially available from current knowledge. The two cycles are independent in a sense that these two cycles can be dissociated at least experimentally during the rapid early nuclear divisions in the embryos of some species [54]. In human somatic cells, however, they were shown to be closely synchronized through the activity of Cdk2. As discussed in section 6.1.3 and 6.3.2, Cdk2 is known to act as a signal to trigger centrosome duplication as well as DNA replication at G1-S transition. This ensures one level of coordination between these two cycles by making the two cycles begin at the same



(a)



(b)

Figure 6-12: (a) A schematic comparison of centrosome cycle and chromosome cycle. This figure is taken from [8]. (b) Coordination of two cycles bears resemblance to the manufacturing system example presented in chapter 5.

time. Another part of synchronization is related to completing the duplication of both chromosomes and centrosomes before the cell cycle proceeds to mitosis. For the chromosome cycle, there is a dedicated mechanism responsible for monitoring the completion of chromosome replication, which is called a DNA replication checkpoint. It would be reasonable to expect that there is also a mechanism for centrosome cycle to ensure the completion of the centrosome duplication process. However, such a mechanism has not been identified yet. Instead, there is an indirect mechanism during mitosis of the cell cycle that monitors, in some sense, the completion of both cycles. It is called a spindle attachment checkpoint. At early mitosis, it prevents further cell cycle progress until all the chromosomes are properly attached to mitotic spindle. As you can imagine, though, it is unable to directly detect the number of centrosomes. Thus, the lack of dedicated monitoring mechanism for centrosome duplication seems to be one of the reasons for centrosomal abnormality. Finally, after mitosis and cytokinesis, both chromosome and centrosome cycles must return to their initial states so that newly-generated daughter cells begin their cell cycle with the same state. That portion of synchronization is ensured by the three Cdk-inhibitory mechanisms discussed in 6.1.3. The Cdk-inhibitory mechanism brings the activity

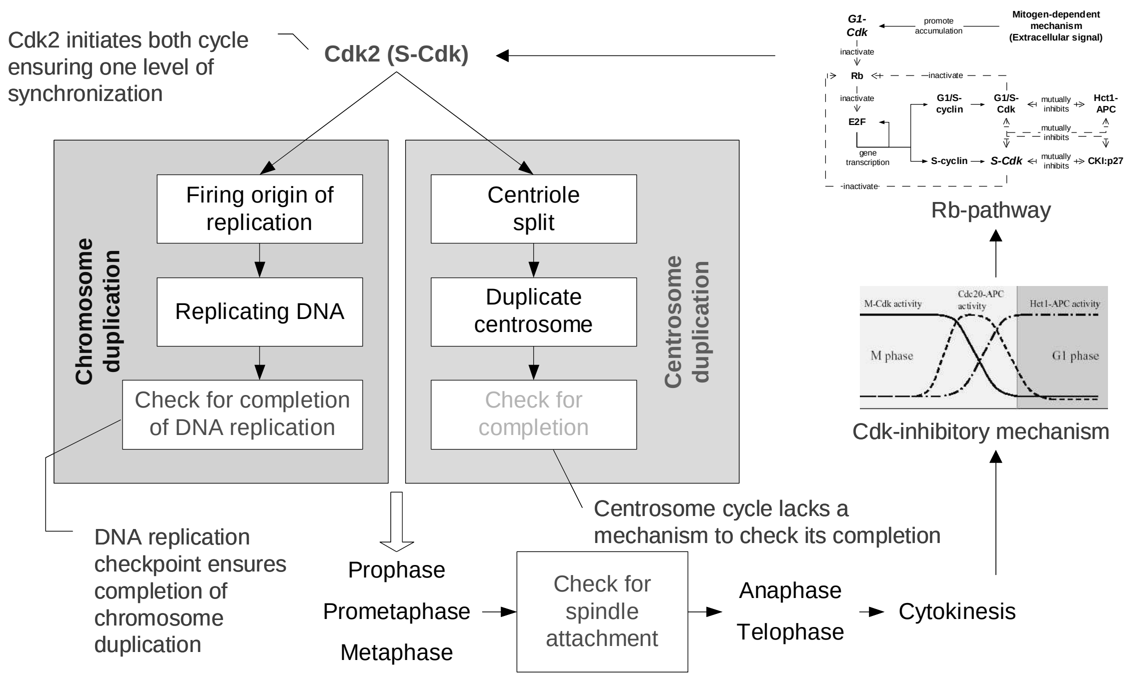


Figure 6-13: Synchronization of chromosome and centrosome cycles involve at least three mechanisms: S-Cdk acting as a signaling agent to initiate both cycles at the same time, checkpoints to ensure the completion of duplication process in both cycles, and Cdk-inhibitory mechanism to initialize the level of Cdk at the end of the cell cycle.

level of Cdk to zero, and thus re-initializes the cell cycle (and the subcycles) in terms of Cdk state. The level of Cdk activity turns into active state as the new cells prepares for the next round of division. Stimulated mainly by extracellular signals, the active Rb-pathway results in the increased level of S-Cdk. Figure 6-13 summarizes the coordination activity of a cell. Yet, details of these coordination mechanism is subject to further research, and it is likely to hold a key to identifying the causality between centrosomal abnormality and cancer development.

Centrosomal abnormality and resulting aneuploidy is a consequence of the breakdown of synchronized periodicity in chromosome-centrosome cycle. For example, overduplication of centrosomes within a single chromosome cycle has been proposed as one scenario. In most cases, this results in fatality in individual cells. Cells with genetic instability die or at least are not allowed to duplicate. But, rarely a few cells with genetic instability manage to survive. When they survive, some of them gain an unregulated periodicity. They proliferate and grow abnormally, which threatens the life of the organism. This shows a quite interesting similarity to the manufacturing system example discussed in chapter 5. In the scheduling example, a disturbance from one of the subsystems breaks down the inherent periodicity of the system. Because of the loss of periodicity, the system eventually yields suboptimal performance of the system. In case of cell, it leads to death of cell itself, even worse, death of organism due to cancer.

6.5 Summary

A cell under a microscope presents apparent morphological cycle, and the visually cyclic events has long been recognized for more than a century. Essential functions of the cell cycle are faithful replication of chromosomes and precise distribution of duplicated chromosomes to two daughter cells. Chromosomes are replicated during S phase, and they are divided into two daughter cells in M phase. G1 and G2 phases are common for cell growth and regulatory purpose. The progress of cell cycle is precisely regulated by intricate cell cycle control mechanism. The cell cycle control mechanism is in charge of ensuring the exactly one copy of chromosome is inherited to next generation by the end of each cell cycle. Thus, the essence of cell cycle regulatory mechanism is precise initiation and termination of functional requirements throughout the cycle.

FR-DP decomposition for G1 phase was presented to demonstrate the utility of axiomatic design approach in describing and understanding a biological system. First of all, by identifying and abstracting functions into a hierarchy, it can obscure molecular details for simplicity and clarity. Secondly, it reveals and highlights hidden FRs, DPs and their interactions. During the process of decomposition, it is required that all FRs be identified and corresponding DPs be specified along with interrelationships. Thus, an ambiguity related to any of those will stand out and requires attention. Lastly, by examining design matrix, it is possible to pose some hypotheses, and testing such hypotheses is an interesting problem from both cell biology and axiomatic design perspective.

Centrosome is known to play an important role as a microtubule organizing center in successful division of duplicated chromosomes. Centrosome is one of the two organelles – chromosome being the other – known to replicate exactly once during cell reproduction. Centrosome has its own duplication cycle which is independent from chromosome cycle. Coordination of these two cycles is crucial to produce normal daughter cells for next generation. Failure to synchronize the two cycles results in the breakdown of regular periodicity of the cell cycle, and leads to a genetic instability, aneuploidy. Aneuploidy, the abberation in chromosome number is commonly found in many cancer cells, and is believed to be one of the causes of cancer development. This example supports the periodic/combinatorial complexity argument in axiomatic design's complexity theory. To maintain periodicity is an important functional requirement in the cell division cycle rather than periodicity being a result or reflection of something else. By achieving periodicity, cells are able to manage the level of complexity for their survival. In addition, coordinating centrosome cycle and chromosome cycle is analogous to integrating multiple subsystems to form a manufacturing system that was discussed in chapter 5. By recognizing the analogy, it is possible to examine the problem with a new perspective. For example, what are the re-initializing signals, does either of the two cycles re-initialize the other, does the lack of monitoring mechanism for the completion of centrosome cycle explain the cause of aneuploidy? This type of new ways of looking at the problem can contribute to the study of biological systems.

Chapter 7

Geometric Periodicity in Designing Low Friction Surface

Previous two chapters present examples to discuss functional periodicity. Functional periodicity is considered in temporal domain in that it primarily deals with pattern of functional requirements' emergence. This chapter demonstrates periodicity concept in geometric domain using an example from the field of tribology. Design of low friction surface discussed by Suh [29] is presented as an example. As briefly mentioned in the example 4-1, in designing low friction surface, periodic complexity is achieved by establishing geometric periodicity at the sliding interface.

7.1 Background: Mechanism of Friction

When two surfaces slide against each other, the relative tangential motion is resisted by some force. The force resisting tangential motion is the frictional force. The frictional force depends on many factors such as normal load, surface roughness, and materials of the sliding surfaces. All of these factors except normal load is lumped into a characteristic parameter, friction coefficient. Although a simple description of friction force, $F_{friction} = \mu N$, has practical utility, the actual mechanism that cause frictional force is quite complicated. Friction at the sliding interface of metals is caused by the collective action of the three mechanisms [28]. First, wear debris and

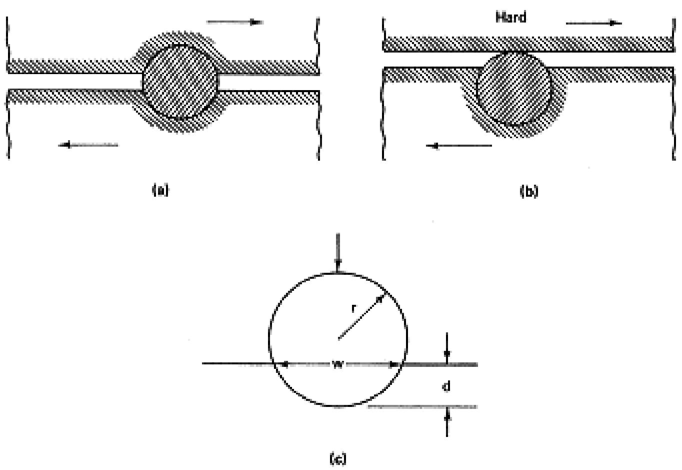


Figure 7-1: A spherical particle entrapped at the sliding interface is indenting into (a) two identical metals, (b) soft metal when one metal is much harder than the other. The dimensions of the particles are shown in (c). Figure is taken from [9].

other particles plow the surfaces as the interface slide against each other. Secondly, sliding interface cause asperity interaction by which the asperities are removed. Third, adhesion of the sliding interface contributes to the friction. If there is no plowing by particles, interface is perfectly smooth, and no adhesion between asperities, then the friction coefficient will be zero, i.e. no friction force. At the other extreme, with maximum plowing, maximum (steady state) roughness, and complete bonding, the friction coefficient will have the largest value. These three factors define friction space with upper and lower bound as mentioned above, and friction coefficient can take any value within the space. Among the three mechanisms, it is known that plowing by the entrapped particles is the most dominant factor in most engineering applications [29].

Wear particles at sliding interface are generated by the removal of asperities. Particles that have been generated earlier and exist at the interface also generate additional particles. Wear particles entrapped at the interface penetrate into the surface under a normal load as shown in figure 7-1. When the interface slide against each other, these particles plow the surface so that work by external agent is required to enable sliding. The work done per unit distance slid is what is known as the frictional force. The friction force, thus, depends on the penetration depth: the

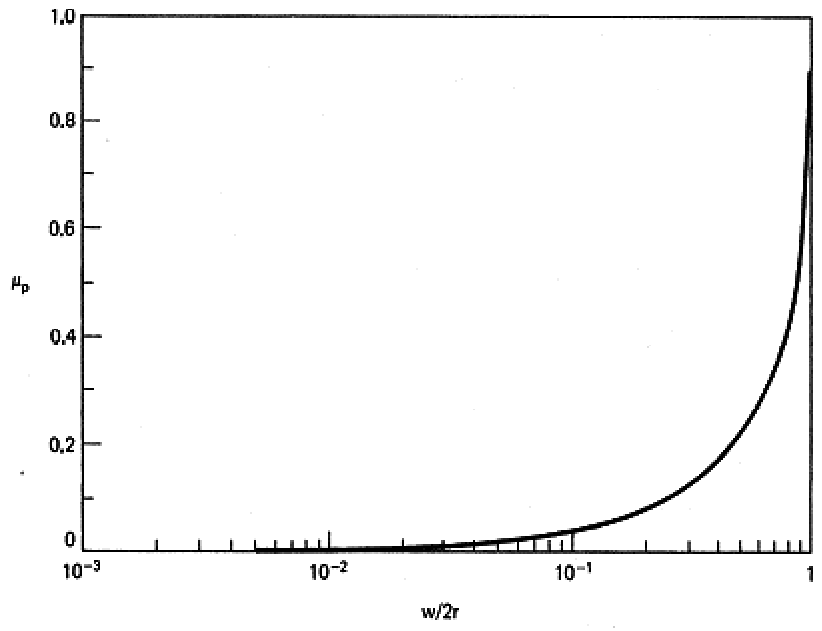


Figure 7-2: Friction coefficient due to plowing component increases nonlinearly as a function of the depth of penetration of the wear particle. Figure is taken from [9].

deeper particles penetrate into the surface, the more work is required to overcome resistance. Indeed, friction coefficient increases nonlinearly as a function of the depth of penetration of the wear particle (see figure 7-2).

7.2 Introducing Periodicity

As the interface slide continuously with wear particles entrapped between the sliding surfaces, wear particles may agglomerate as shown in figure 7-3. Under the normal load, a newly-generated particle undergoes plastic deformation and conforms to the work-hardened asperity of the existing agglomerate, a process similar to cold compaction of metal powders or coining process [3]. Sooner or later, the growing agglomerate reaches a point where it cannot sustain the stress due to increased moments. Then, the agglomerate breaks and new agglomerate begins to form. Thus, wear particle agglomeration has periodicity by itself. As many number of agglomerates follow the periodicity, it reaches steady state overall.

As the particles agglomerate, the applied normal load is carried by a smaller

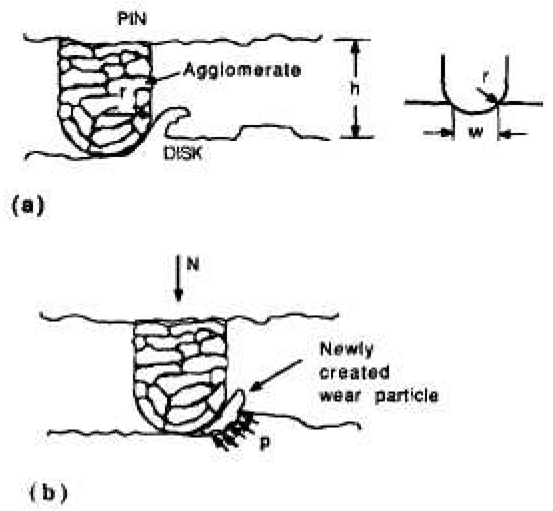


Figure 7-3: (a) An agglomerate wear debris is shown as a cylindrical shape, and (b)wear particles may agglomerate to form larger particles at the sliding interface when there is sufficient pressure to deform the particles and cause bonding. Figure is taken from [3].

number of larger particles rather than by a large number of small particles. Assuming the total wear volume is constant, the larger agglomerated particles penetrate deeper into the surface than the smaller particles do. Recall that the friction coefficient increases with the penetration depth of wear particle (figure 7-2). Therefore, friction coefficient is expected to increase as agglomeration occurs, and the experimental results reported in [3] show that is the case.

Natural periodicity of the particle agglomeration would result in the characteristic maximum size of the agglomerates at steady state, which, in turn would lead to steady state penetration depth. That will determine friction coefficient of the sliding interface. Then, if we want to lower the friction coefficient, introducing periodicity that is shorter – in terms of the size of agglomerates – than inherent periodicity.

7.2.1 Undulated Surface

Having understood the mechanism of surface friction, relevant design range is the size of agglomerated wear particles. Then, the system range, actual size of wear particle, drifts out of the design range as the interface slides continuously. Thus, it has to be re-initialized to bring the system range back to initial system range. That is to

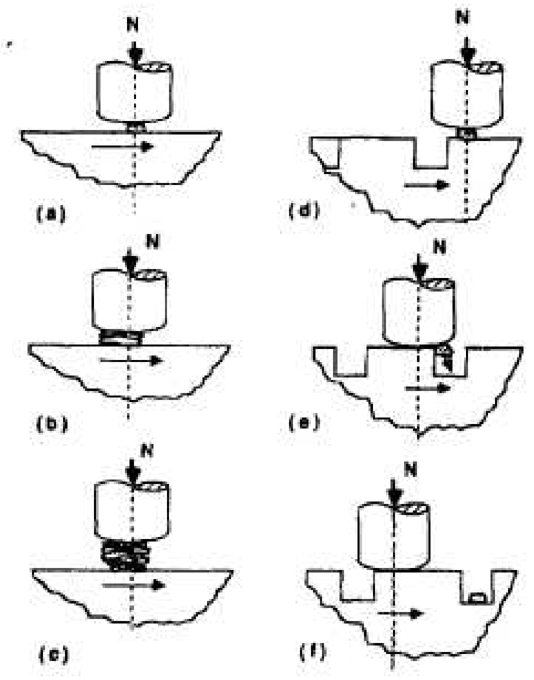
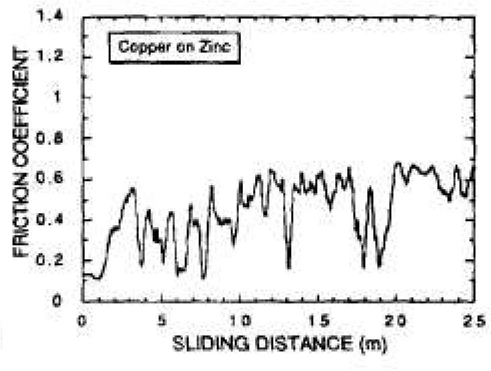


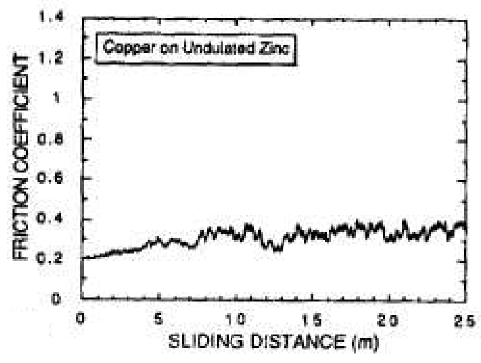
Figure 7-4: (a)-(c): Schematics of wear particle agglomerations on a flat surface, (d)-(f): Particle agglomeration is prevented by undulated surface. Figure is taken from [3].

maintain the agglomerate particle size below the desired level. One way to achieve re-initialization is to create undulation on the surfaces. Central idea is that the agglomerated particles fall into the pockets of undulated surface before they become too large.

Figure 7-4 compares the particle agglomeration on the flat surface and prevention of it on the undulated surface. As shown in figure 7-4 (a)-(c), initially small agglomerate entrapped at the interface of flat surface agglomerate and grow. It continues to grow until it reaches critical size and breaks. On the other hand, with undulated surface, small agglomerate is likely to fall into the pocket (or groove) before it becomes large. Thus, the particle size at the interface remains small. The size of the particles can be controlled by the geometry of the undulation. As a consequence of preventing wear particles from agglomerating to have large size, the friction coefficient is significantly reduced from that of flat surface interface. Experimental data in figure 7-5 clearly shows that the friction coefficient is much less for the sliding interface with undulation.



(a)



(b)

Figure 7-5: Friction coefficient versus sliding distance in copper on (a)flat Zinc surface and (b)undulated zinc, sliding at 2.5N normal load and 0.01m/s sliding speed [3].

7.3 Summary

Design of low friction surface was discussed to illustrate how combinatorial complexity can be transformed to periodic complexity. The transformation is achieved by introducing geometric periodicity into the sliding interface.

Knowing that plowing by the entrapped particles is the dominant contributor to the frictional force and that agglomeration of the wear particles increases the frictional force, the relevant design range is the size of wear particles. As the interface continue to slide against each other, the system range – actual size of agglomerate – moves out of the design range. Thus, it has to be re-initialized. Undulated surface plays a role of internal re-initialization and thereby maintains the particle size under the desired value. By introducing periodicity that is short enough to prevent excessive agglomeration, the interface achieves low friction coefficient.

Chapter 8

Conclusions

8.1 Complexity in Axiomatic Design

Although there exist many definitions for complexity concept, they do not properly address issues that are important from engineering design standpoint, for example, a relative aspect of design problem and causes of complexity. As a result, a concept of complexity has a limited utility, or more often it is used as a metaphor to indicate the difficulty associated in design/manufacturing/operation. The concept of complexity defined in axiomatic design theory provides a useful framework to effectively discuss causes of complexity and methods to reduce it.

Complexity in axiomatic design is defined as a measure of uncertainty in achieving the desired functional requirements. This definition of complexity incorporates attributes such as difficulty, uncertainty, relativity, ignorance, and information. The peculiarity of AD complexity concept can be well understood once those aspects of the definition are recognized. There are four different sub-categories of AD complexity: time-independent real complexity, time-independent imaginary complexity, time-dependent periodic complexity, and time-dependent combinatorial complexity (see figure 8-1). Time-dependent complexity consists of real and imaginary complexity. Real complexity accounts for random nature of a system, and is equivalent to information content since information content measures the uncertainty in the same probability context. Imaginary complexity, on the other hand, accounts for the

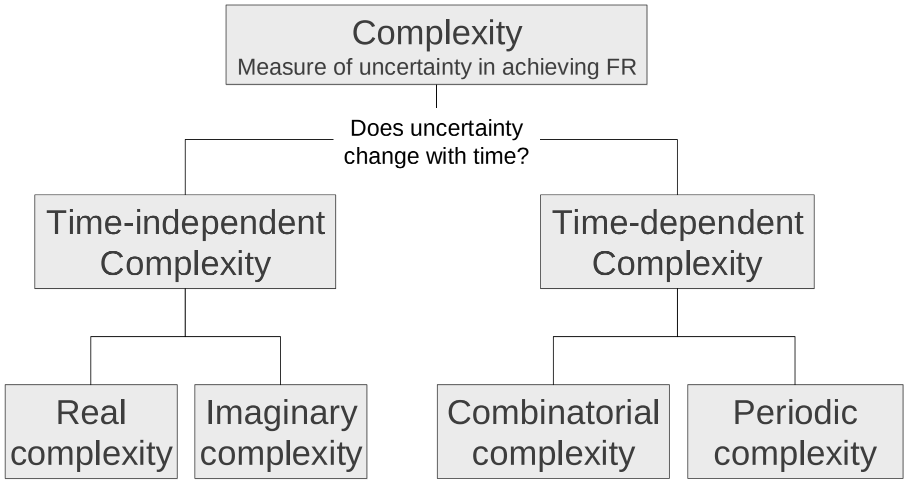

Complexity
Measure of uncertainty in achieving FR
Does uncertainty change with time?
Time-independent Complexity
Time-dependent Complexity
Real complexity
Imaginary complexity
Combinatorial complexity
Periodic complexity

Figure 8-1: Four types of complexity are identified in axiomatic design. Depending on the uncertainty's time-dependence, it is divided into time-independent and time-dependent complexity. Time-independent complexity consists of real and imaginary complexity, and time-dependent complexity has combinatorial and periodic complexity.

uncertainty due to ignorance, particularly the ignorance of the structure of design matrices. Time-dependent complexity is also divided into two different kinds: combinatorial complexity and periodic complexity. Uncertainty in a case of combinatorial complexity increases continuously while periodic complexity ceases to increase at certain point and returns to initial level of uncertainty. A system has combinatorial complexity when (1) its system range continues to drift away from design range and (2) the functional requirements set $\boldsymbol{FR}(\mathrm{t})$ is unpredictable.

It is clear that the causes of complexity are embedded in its definition. Real complexity is due to random variations associated in a design such as variations in DPs and noise factors. The cause of imaginary complexity is the ignorance of the structure of design matrices. For both combinatorial and periodic complexity, the causes are time-varying system range and time-dependent $\boldsymbol{FR}$. By identifying what causes complexity, it is possible to develop a systematic approach to complexity reduction.

8.2 Reduction of Complexity

Reducing real complexity is equivalent to reducing variation in functional requirements, which has been the topic of common engineering research and practices. They include 1)eliminating source of variation, 2)desensitizing a system, and 3)compensating for the variation. Based on technical and economic consideration, these three approaches must be combined to yield optimal result. Imaginary complexity must be eliminated by identifying the structure of design matrices and following one of the correct sequences dictated by design matrices.

By definition of time-dependent complexity, uncertainty of a system with combinatorial complexity increases indefinitely while in case of a system with periodic complexity, it ceases to increase at certain point and returns to initial level of uncertainty. In order to prevent uncertainty from ever-increasing and to maintain complexity under a manageable limit, combinatorial complexity should be avoided. Transforming combinatorial complexity into periodic complexity is referred to as re-initialization.

When a system experiences combinatorial complexity due to a time-varying system range issue, functional requirements are becoming more difficult to achieve as time elapses. Thus, periodically recovering the initial system range is important. Periodically bringing back the system range is equivalent to transforming the combinatorial complexity to periodic complexity. The act of achieving this transformation is one type of re-initialization, in a sense that the system is re-initialized to its initial state in terms of the associated uncertainty. Re-initialization of time-varying system range can be done externally or internally. While preventive maintenance and statistical process control are common examples of external re-initialization, the low friction surface design example illustrate how the system range can be self-re-initialized internally by design. A clever design as in knob example in section 3.1.1 can help minimizing the cost of external re-initialization by delaying the deterioration of system range.

Whether a system has combinatorial or periodic complexity can also depend on the type of periodicity present in time-dependent functional requirements set $\boldsymbol{FR}$.

It was shown by probability of success and predictability of $\boldsymbol{FR}$ that periodic and semi-periodic $\boldsymbol{FR}$ result in periodic complexity whereas aperiodic $\boldsymbol{FR}$ leads to combinatorial complexity. Functional periodicity is achieved by establishing appropriate initial states repeatedly in a system, and that is the second type of re-initialization.

Two examples are drawn from different fields to support the argument on the role of functional periodicity in achieving successful system functionality. First example is a scheduling of part-transport in an integrated system with process time variation. In the example, it was shown that break-down of functional periodicity in one of the subsystems results in sub-optimal throughput performance of the integrated system. In order to ensure the productivity and reliability of such system, functional periodicity must be maintained by re-initializing the subsystems on a periodic functional interval, meaning each and every functional requirement is completed in every period. This is equivalent to preventing a system from developing time-dependent combinatorial complexity and thereby increases predictability of system performance and productivity. Second example is drawn from biological world where periodicity is commonly observed at various scales. Among the abundant examples of biological systems with periodicity, the eukaryotic cell cycle is examined with particular emphasis on its functional periodicity. To maintain periodicity is an important functional requirement in the cell division cycle rather than periodicity being a result or reflection of something else. By achieving periodicity, cells are able to manage the level of complexity for their survival. In addition, coordinating centrosome cycle and chromosome cycle is analogous to integrating multiple subsystems to form a manufacturing system that was discussed in chapter 5. By recognizing the analogy, it is possible to examine the problem with a new perspective. This type of new ways of looking at the problem can contribute to the study of biological systems.

These examples illustrate a critical role of functional periodicity in achieving desired functional requirements. In many cases, loss of periodicity or lack thereof results in chaotic behavior of a system and even leads to failure of a system. Introducing functional periodicity by re-initialization ensures that a system does not develop a combinatorial complexity.

8.3 Suggestions for Future Research

This thesis has discussed axiomatic design's complexity concept to make progress toward the following goals: 1) to avoid the vague usage of the term in the engineering design discipline, 2) to acquire deeper insight into possible cause of complexity in engineering design, and 3) to develop a systematic approach to complexity reduction. While this thesis effectively addresses all three goals, further research will only improve our understanding and broaden the applicability of the concept. Development of systematic approach to complexity reduction must be on-going research topic.

In particular, developing quantifiable metric for time-dependent complexity is one of the challenging research topics along the line. Indeed, with a comprehensive complexity metric, the information axiom may be restated in terms of complexity. Another interesting topic is to investigate more on creating periodicity as opposed to maintaining inherent periodicity. For example, job shop scheduling requires defining or creating periodicity in the system which is not naturally present in it. Finally, it will broaden the scope of application of periodicity concept if periodicity in many different domains other than temporal domain is investigated.

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