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Optimal sizing of energy storage for efficient integration of renewable energy

Pavithra Harsha Munther Dahleh

***Abstract*— In this paper, we study the *optimal storage investment problem* faced by an owner of renewable generator the purpose of which is to support a portion of a local demand. The goal is to minimize the long-term average cost of electric bills in the presence of dynamic pricing as well as investment in storage, if any. Examples of this setting include homeowners, industries, hospitals or utilities that own wind turbines or solar panels and have their own demand that they prefer to support with renewable generation. We formulate the optimal storage investment problem and propose a simple *balancing control* for operating storage. We show that this policy is optimal for constant prices and some special cases of price structures that restrict to at most two levels. Under this policy, we provide structural results that help in evaluating the optimal storage investment uniquely and efficiently. We then characterize how the cost and efficiency of storage, dynamic pricing and parameters that characterize the uncertainty in generation and demand impact the size of optimal storage and its gain. One surprising result we prove is that for storage to be profitable under the balancing policy the ratio amortized cost of storage to the peak price of energy should be less than $\frac{1}{4}$.**

I. INTRODUCTION

The increasing demand for electricity coupled with environmental concerns have motivated the need for smart grids that aim towards integrating vast amounts of renewable energy with the grid. According to the US Department of Energy, by 2030, 20% of the US electricity portfolio should consist of wind energy [1], while the current levels contribute to just around 2% [2]. Similar aggressive targets have been set across the world for different forms of renewable energy.

Renewable energy sources are non-dispatchable sources that are both uncertain and intermittent. For example, it is not uncommon to see an 80-90% drop in generation in short durations of time. Large scale integration of such sources can lead to several issues including increased need for reserves, large ramps and uncertainty leading to stability concerns and costly upgrades in the transmission network [3]. Energy storage technologies can address all these concerns because they decouple the time of generation and consumption. But the high capital costs continue to be the main barrier in this direction. However, there are significant on-going efforts to create *economical* energy storage technologies such as batteries, flywheels, compressed air energy storage, capacitors, fuel cells, and biomass, in that, storage is bound to become an integral part of the future grid.

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Pavithra Harsha is a research staff member at the IBM T. J. Watson Research Center at Yorktown Heights. This work was done when she was a post-doctoral associate at the Laboratory of Information and Decision System (LIDS) at MIT. Email: pharsha@us.ibm.com

Munther Dahleh is a professor of Electrical Engineering and Computer Science at MIT. Email: dahleh@mit.edu

With millions of dollars in government subsides being spent on renewable energy installation and integration and several additional millions being spent on the development of storage devices, it is crucial to understand the interaction between storage and renewables and in particular, the tradeoff between the value that storage creates and its capital cost. It is also important to understand the other factors that affect the gain from storage and their impact.

Our work focuses on this interplay between storage and renewables. We study the setting where a renewable generator aims to support a portion of a local (elastic) demand by storing any excess generation in a storage device. Our goal is to minimize the long term average expected cost of demand that is not satisfied by renewable generation but satisfied by energy from the electric grid and any cost associated with investment in storage. We refer to this problem as the *optimal storage investment problem*. Examples of this setting include homeowners, industries, hospitals, game parks or utilities who own renewable generation facilities, have their own demand and prefer to use renewable generation to minimize their cost using storage devices. These settings will get more prevalent with city governments investing in microgrids, homeowners getting subsidies for investing in solar panels, utilities for renewable generation sites and so forth.

In this paper, we formulate the optimal storage investment problem as an average cost infinite horizon stochastic dynamic programming problem. In the light of smart meters that can price electricity at different times of day differently, we assume that prices are exogenous and stochastic revealed prior to consumption. We propose a simple storage management policy which we refer to as the *balancing control*. It sequentially performs *balancing* which is satisfying the demand with renewable generation and *storing* which is storing the excess generation, in that order, at peak prices and the reverse order at lower prices. We show that this simple control is optimal under (a) constant prices and (b) a special case of two level pricing scheme i.e., when the lower price is zero. We then prove structural results under this control policy that help in computing the *unique* optimal storage size efficiently.

In the remaining part of the paper, we explore the tradeoffs and impact of the different factors that govern the size of optimal storage and its gain. First, we prove a rather interesting result that shows that under the balancing policy it is imperative for the ratio of the per-period amortized cost of storage to the peak price of energy be less than $\frac{1}{4}$ for storage to be a profitable investment. To the best of our knowledge, this is the first theoretical tight upper bound on the cost of storage independent of any assumptions on the distribution of uncertainties. Next, for a specific distribution of the net gap between renewable generation and demand (independent and identically distributed (i.i.d) uniform), we study the impact of the following factors on optimal storage: (a) the statistics of the uncertainties that characterize the renewable generation and demand; (b) the cost and efficiency of storage; and (c) dynamic pricing and any uncertainties associated with the prices themselves.

The remainder of the paper is organized as follows. In section 2 we provide a brief literature review of related work. In section 3, describe our model. In section 4 we propose the balancing policy, discuss its performance guarantees and then prove structural results that help towards the computation of the optimal storage investment. In section 5, we focus on tradeoffs and impact of different factors on optimal storage and conclude in section 6.

II. Related Work

Interaction between renewable energy and storage has been the subject of many papers. Prior work has mostly focused on the setting where renewable generators participate in conventional electricity markets by bidding and entering into forward contracts that have associated penalties if contracts are breached [4], [5], [6], [7], [8]. Researchers show that using storage can increase the economic value of these operations. With the exception of Kim and Powell [7] and Bitar et al. [8] who use dynamic programming techniques, the methodology used is to solve a deterministic version of the problem over a sample path and then take an average of the revenue over different sample paths. However, as pointed by [7], this approach does not produce an admissible policy as the decision depends on the sample path.

Our setting differs from the above in that we consider renewable generators that directly face demand and use storage to reduce the amount of conventional generation used. Authors Brown, Lopes and Matos in [9] consider a similar setting which also aims to find the optimal storage size but our work is different from theirs partly in the setting and very much in the methodology: first, their setting is that of an isolated power system but ours is not, in particular, because we allow demand to draw electricity from the grid at prices that maybe stochastic in nature; and, second, they use the sample path approach discussed above and we use that of dynamic programming.

In all above papers including ours, authors assume that market prices are independent of the renewable generation but Sioshansi's work shows that high wind energy penetration can suppress the prices of energy and in particular, storage can help mitigating this effect [10].

We would like to point the readers that the models that we consider and those by [7], [8] have a very similar structure to the problems in classical inventory theory including the well-known newsboy problem [11]. The main differences though are that storage is limited in size, has associated ramp constraints and conversion losses. In addition, in the energy setting there are a lot more uncertain quantities such as renewable generation, demand and price. These result in different dynamics and newer challenges. A key question that has not be tackled in inventory theory literature is what factors affect the size of optimal storage investment and how. This is the focus of our study.

III. Model

Consider a renewable generator the purpose of which is to satisfy a local demand. Any excess generation is assumed to be lost unless stored in a storage device and that any excess demand that is not satisfied from renewable generation is supported by other generators connected to the electric grid at prices which are revealed before consumption. Our goal is to identify the optimal size of storage to invest in so as to minimize the long term average cost of electric bills along with any cost related to storage investment. We refer to this problem as the *optimal storage investment problem*. Below we state our assumptions, describe the way we model storage and then formulate the investment problem.

We assume the renewable generation and price are known exogenous stochastic processes that are independent of each other denoted by W_t and p_t. We assume that demand is also an exogenous stochastic process that may be elastic in which case it depends on the price. We denote it by $D_t(p_t)$. We assume that the purpose of storage is to store renewable generation only and not store energy from the electric grid. This is because we already account for elastic demand.

Any storage can be characterized using the following parameters: energy rating, power rating, efficiency and total ownership cost of storage. *Energy rating* is the net capacity or size of storage represented by S. *Power rating* specifies the rate at which storage can be charged or discharged. This can be the same or different for charging and discharging cycles. We denote it by R_i and R_o for each respectively. *Efficiency* primarily consists of the conversion losses and are typically much higher than any dissipation losses. We denote it by ρ. This is also commonly referred to as the roundtrip efficiency because it is the product of two conversion losses: converting renewable energy to its stored form and the reverse. And finally, the *total ownership costs* is the amortized per unit cost of capital and may include any operational and maintenance charges denoted by c. This is a general model of storage and our goal is to find the optimal S given the other parameters.

Below in our formulation, we model S as the maximum amount of *useful* energy that can be stored in storage and c as the amortized cost of *useful* storage of size S. The reason we do this is to explicitly avoid using two conversion losses. These parameters can always be modified by scaling without affecting the structure of the results in this paper.

We will now formulate the storage sizing problem as a discrete-time average-cost infinite horizon stochastic dynamic programming problem. We consider the time discretization to be in the order of hours. Although our model can be written at a finer granularity, making meaningful predictions at a finer grain may not be possible when one solves the storage investment problem several months in advance. Since the granularity of these discretizations are relatively small compared to the life-cycle of storage devices, we formulate the problem as an infinite horizon problem that amortizes the storage cost over several periods. Finally, we average the costs because between consecutive time discretizations the discount factors are close to 1.

Let X_t denote the level of useful energy in the storage at time t. We assume the following sequence of events in each period: at the beginning of period t, we are revealed exanté the value of the generation W_t, the price p_t and then the demand $D_t(p_t)$. Next, the decision, u_t, the amount to store is made. We allow this to be negative as we can extract energy from storage as well. We do not explicitly characterize the prediction models for W_t, p_t and $D_t(p_t)$. The components of such prediction models as well as their revealed exanté values at time t form the state of the system along with X_t. We assume that any uncertainty after the predictions are i.i.d. We now formulate the investment problem below:

$$\min_{S\geq 0, u_t} \lim_{T\to\infty} \frac{1}{T} E_{\mathbf{p},\mathbf{D},\mathbf{W}} \sum_{t=1}^{T} p_t \left[D_t(p_t) - W_t + u_t\right]^+ + cS$$

$$u_t \leq \min\{W_t, \frac{1}{\rho}(S - X_t), R_i\} \tag{1a}$$

$$u_t \geq -\min\{D_t(p_t), X_t, R_o\} \tag{1b}$$

$$X_{t+1} = X_t + \beta_t u_t, \tag{1c}$$

where $\beta_t = \rho$ if $u_t \geq 0$ and 1 otherwise. The amount to store, u_t, is by definition restricted by the size of storage, the wind, the demand and the ramp constraints as written in constraints (1a-1b). Constraint (1c) is the state update equation for the storage level which increments the current state by the amount of useful energy that can be stored which is $\beta_t u_t$. The first term in the objective is the penalty or the expenditure proportional to the unsatisfied demand in each period. The second term is the amortized cost which we assume has a linear form. Our goal is to identify the structure of the optimal policy given S and then optimize for S.

We assume that the renewable generation, demand and prices are stationary processes. Under a stationary control, the system evolves as a Markov Chain. Assuming that the Markov Chain has only one recurrent class and is aperiodic, the steady state distribution of the system exists and hence the limit. We refer the reader to Section 5.6 in book [12] for details on the continuous state Markov Chains.

IV. Main Results: Balancing control and storage size

In this section, we first propose the *balancing control*. Next, we discuss pricing schemes under which this control is optimal. And finally, we provide structural results under this control that allows one to compute the optimal storage size uniquely and efficiently.

A. Balancing Control

Consider the following stationary control policy that we refer to as the *balancing control*

- When the price is high, first satisfy the demand as much as possible using the renewable energy generated in the current period and then that available from storage. Next, store the excess generation, if any. In case of constant prices, we restrict to just this control.
- When the price is not high, first store all that is possible and then satisfy the demand with the excess generation.

Let p_H denote the highest price and let D_t^H denote the corresponding demand at the high price. Mathematically, the control, u_t^B, is as follows (the superscript B refers to the balancing control):

$$\text{Let } w_t^B = \begin{cases} (X_t + W_t - D_t^H)^+ - X_t & \text{if } p_t = p_H \\ W_t & \text{otherwise.} \end{cases}$$

In the absence of ramp constraints,

$$u_t^B = \min\left\{\frac{1}{\rho}(S - X_t), w_t^B\right\}, \tag{2}$$

and incorporating ramp constraints, we get

$$u_t^B = \min\left\{R_i, \frac{1}{\rho}(S - X_t), \max\{w_t^B, -R_o\}\right\}. \tag{3}$$

B. Optimality of the balancing control under special pricing schemes

Constant prices: For the case of constant prices, we choose the control that corresponds to the high price, p_H. The balancing control is optimal because there is no gain in storing any energy if you can satisfy demand now because in the future one can only sell it at the same price. In fact, the quantity of energy that can be sold decreases in the presence of conversion losses.

Two level pricing with lower level price equal to 0: At the high price, the balancing control is optimal for the same reason as the case of constant prices. (Note that this argument holds even if there are multiple price levels.) The control is optimal at the lower price which equal 0 because first there is no penalty of following any control at that price and second it is always in the best interest to have a higher level of storage to be prepared in the event of a high price.

Note that a low price of 0 may seem unrealistic at the but one can argue this as a special case of critical-peak-pricing (CPP) schemes when the peak price is much higher than the off-peak price in that the off-peak price can be ignored with regard to any investment decisions in storage.

Extending the balancing policy to account for multiple prices levels is a part of our future work that we pursuing. An interesting point to note in this case is that it suffices to study the case discrete set of price points because of the presence of conversion losses.

C. Optimal storage under the balancing policy

In this section, we restrict to the balancing policy and aim to find the optimal storage under this policy. We do not restrict the number of price levels but keep in mind that the balancing policy is optimal only under special cases.

Let $Z^B(S)$ be the first term in the objective function that corresponds to the cost of demand that is not satisfied by renewable generation under the balancing policy for a storage of size S. Below we show that $Z^B(S)$ is non-increasing and convex in S. In order to do so, we simplify the state

X_t and the objective $Z^B(S)$ under the balancing policy. For simplicity of exposition, we restrict to the case in the absence of ramp constraints but the proofs continue to hold even in their presence.

$$\begin{aligned} X_{t+1} &= X_t + \beta_t u_t^B \\ &= \begin{cases} \min\{S, \left[X_t + \beta_t(W_t - D_t^H)\right]^+\} & \text{if } p_t = p_H \\ \min\{S, X_t + \beta_t W_t\} & \text{otherwise.} \end{cases} \\ & \text{(substituting Eq. (2))} \end{aligned} \quad (4)$$

$$\begin{aligned} Z_t^B(S) &= p_t\left[D_t(p_t) - W_t + u_t\right]^+ \\ &= \begin{cases} p_H\left[D_t^H - W_t - X_t\right]^+ & \text{if } p_t = p_H \\ p_t\left[D_t(p_t) - \left(W_t - \frac{(S-X_t)}{\rho}\right)^+\right] & \text{otherwise.} \end{cases} \\ & \text{(substituting Eq. (2))} \end{aligned} \quad (5)$$

Theorem 4.1: $Z_t^B(S)$ is non-increasing in S $\forall t$.

Proof: In the interest of limited space, we just outline the proof. Consider two storage facilities of sizes S and S' respectively where $S \geq S'$. At time 0, we assume without loss of generality that the storage facilities are both empty. From Eq. (4) and a simple induction argument in the time dimension it is easy to observe that

$$0 \leq X_t^S - X_t^{S'} \leq S - S' \quad \forall t.$$

We use the LHS of the above inequality when $p_t = p_H$ and the RHS of the inequality when $p_t \neq p_H$ to obtain that $Z_t^B(S) \leq Z_t^B(S')$. ■

Since $Z_t^B(S)$ is non-increasing in S $\forall t$, $Z^B(S)$ is also non-increasing in S

Theorem 4.2: $Z^B(S)$ is convex in S.

Proof: Consider three storage facilities of sizes S_1, $S_2 = \lambda S_1 + (1-\lambda)S_3$ and S_3 respectively where $S_1 \geq S_3$ and $\lambda \in [0,1]$. Let ω be any instance of the sequence Y_t where $Y_t = (W_t, p_t, D_t(p_t))$ for $t = 1, 2, \ldots$. We assume without loss of generality that the storage facilities are all empty to begin with. We call this time, time 0. Let t_1 be the first instance of time when storage levels of the three facilities are different. Note that when the levels are different, it should be the case that $X_{t_1}^{S_3} = S_3, X_{t_1}^{S_2} = S_2$ and $X_{t_1}^{S_1} > S_2$. Let t_2 be the first time after t_1 when the storage levels are the same again (t_2 is the beginning of the next epoch after time 0). Note that when this happens $X_{t_2}^{S_1} = X_{t_2}^{S_2} = X_{t_2}^{S_3} = 0$. Also, since the storage level dropped it must be the case that $p_{t_2} = p_H$

By monotonicity of X_t with storage sizes (Theorem 4.1), t_2 is also the first time after t_1 when $X_t^{S_1} = 0$. This means during the periods, t such $t_1 \leq t < t_2$, $X_t^{S_1} > X_t^{S_2}$ i.e., they never meet. But this need not be the case between $X_t^{S_2}$ and $X_t^{S_3}$. They surely meet once before t_2 because of monotonicity of the storage levels and maybe more. With this we can now bound the gain of facility S_1 over S_2 and S_2 over S_3 in the periods between 0 to $t_2 - 1$ as follows:

$$\begin{aligned} Z_{[1,t_1-1]}^B(S_1) - Z_{[1,t_1-1]}^B(S_2) &= 0, \\ Z_{[t_1,t_2]}^B(S_1) - Z_{[t_1,t_2]}^B(S_2) &\geq p_H(S_2 - S_1), \\ Z_{[1,t_2]}^B(S_2) - Z_{[1,t_2]}^B(S_3) &\leq p_H(S_3 - S_2). \end{aligned}$$

Here, the subscript for Z^B represents the time interval over which the unsatisfied demand is computed. Multiplying the first two equations by λ and the last by $(1-\lambda)$ and adding all of them we get

$$\lambda Z_{[1,t_2]}^B(S_1) + (1-\lambda) Z_{[1,t_2]}^B(S_3) \geq Z_{[1,t_2]}^B(S_2).$$

Similarly, we can prove convexity for every epoch. This implies that the result holds in expectation and on average when T is chosen to be end of an epoch. But allowing T to ∞ allows the result to hold independent of the choice of T. Hence the theorem. ■

We have shown that $Z^B(S)$ is both non-increasing and convex. Along with a strictly convex amortized capital cost function for storage the objective of the optimal sizing problem is strictly convex. This means there exists a unique optimum S^* and this can be evaluated efficiently using gradient descent optimization. Some may argue that it appears to be futile to prove these results because the problem is a single variable optimization since we fixed the control. But the complexity comes from solving the steady state matrix equations for each value of S which is non-trivial task for state-spaces of large sizes.

V. Main Results: Tradeoffs of Storage Size with System Parameters

In this section, we focus on understanding the fundamental limits and tradeoffs of storage investment with the parameters of the problem under the balancing policy. We ignore the ramp constraints for simplicity of our analysis but our results continue to hold in their presence as well.

A. Storage size versus cost under the balancing control

Theorem 5.1: If storage is a profitable investment under the balancing policy then $\frac{c}{p_H} \leq \frac{1}{4}$.

Proof: Consider a storage of size S. Let $V_{\mathcal{D}}(S)$ refer to the total cost (i.e., objective) for some distribution $\mathcal{D}$ of the state space. Storage is a profitable investment only if

$$\begin{aligned} & V_{\mathcal{D}}(S) - V_{\mathcal{D}}(0) \leq 0 \\ \Longrightarrow\ & V_{\mathcal{D}}(S_{\mathcal{D}}^*) - V_{\mathcal{D}}(0) \leq 0 \text{ where } S_{\mathcal{D}}^* = argmin_S V_{\mathcal{D}}(S) \\ \Longrightarrow\ & \min_{\{\widehat{\mathcal{D}} | \mathcal{S}_{\widehat{\mathcal{D}}}^* = \mathcal{S}_{\mathcal{D}}^*\}} V_{\widehat{\mathcal{D}}}(S_{\widehat{\mathcal{D}}}^*) - V_{\widehat{\mathcal{D}}}(0) \leq 0. \end{aligned}$$

In the last equation, the maximum is over all distributions s.t. $S_{\widehat{\mathcal{D}}}^* = S_{\mathcal{D}}^*$. This means that it suffices to focus on set of all distributions for which the optimal storage size is fixed (hereon referred by just S). We want to show

$$\min_{\{\mathcal{D} | \mathcal{S}_{\mathcal{D}}^* = \mathcal{S}\}} V_{\mathcal{D}}(S) - V_{\mathcal{D}}(0) \leq 0 \implies \frac{c}{p_H} \leq \frac{1}{4}. \quad (8)$$

The proof will be constructive in the sense that we will first construct a distribution, $\mathcal{D}^*$ that minimizes the total cost for a storage of size S and then prove the result.

We expand the term $V_{\mathcal{D}}(S) - V_{\mathcal{D}}(0)$ below. For the purpose of simplicity of illustration, we expand assuming the random variables $W_t, p_t, D_t(p_t)$ are i.i.d random variables in the time dimension. Note that our proof in no way relies on the fact that random variables are in fact i.i.d. Let $Y = D^H - W$.

Assume that the prices are in discrete increments and the probability of the highest price be q_H.

$$V_{\mathcal{D}}(S) - V_{\mathcal{D}}(0) = cS$$
$$- \int_X f_X(x)dx \left[\int_Y p_H \left[y^+ - (y-x)^+\right] q_H f_Y(y)dy \right.$$
$$\left. + \sum_{p_t \neq p_H} \int_W p_t \left[w - \left(w - \frac{S-x}{\rho} \right)^+ \right] P(p = p_t) f_W(w)dw \right].$$

Rewriting the second term and eliminate the third term,

$$V_{\mathcal{D}}(S) - V_{\mathcal{D}}(0) \geq cS - \int_X \int_0^x q_H p_H P(Y \geq y)dy \, g_X(x)dx,$$

where $f_X(x) = P(X = 0)\ \delta(x) + g_X(x)\ \mathbb{I}(0 < x \leq S)$. Here, $\delta(x)$ equals 1 if $x = 0$ and 0 otherwise and $\mathbb{I}(0 < x \leq S)$ equals 1 if $0 < x \leq S$ and 0 otherwise.

Observe that $\int_0^x P(Y \geq y)dy \leq \int_0^S P(Y \geq y)dy \ \forall \ \mathcal{D}$. Substituting this we get,

$$V_{\mathcal{D}}(S) - V_{\mathcal{D}}(0) \geq cS - q_H p_H \int_0^S g_X(x)dx \int_0^S P(Y \geq y)dy$$
$$= V_{\mathcal{D}^*}(S) - V_{\mathcal{D}^*}(0).$$

where $\mathcal{D}^*$ has the following structure:

- p_t is i.i.d and is p_H with probability q_H and 0 otherwise.
- Y_t is i.i.d and is S with probability α and $\frac{-S}{\rho}$ otherwise.
- W_t is i.i.d and is greater than or equal to S in all periods.

These three assumptions implies that the steady state distribution of X is αq_H when $X = 0$ and $1 - \alpha q_H$ when $X = S$. Substituting this we get,

$$\begin{aligned} V_{\mathcal{D}^*}(S) - V_{\mathcal{D}^*}(0) &= cS - p_H\ q_H(1 - q_H\alpha)\ \alpha S \\ &\geq cS - \frac{p_H}{4}S \quad \left(\text{when } q_H\alpha = \frac{1}{2}\right) \end{aligned}$$

So, we have identified a distribution $\mathcal{D}^*$ with $q_H\alpha = \frac{1}{2}$ that minimizes $V_{\mathcal{D}}(S) - V_{\mathcal{D}}(0)$ i.e.,

$$\min_{\{\mathcal{D} | \mathcal{S}^*_{\mathcal{D}} = \mathcal{S}\}} V_{\mathcal{D}}(S) - V_{\mathcal{D}}(0) = cS - \frac{p_H}{4}S.$$

Substituting this in Eq. (8), proves the theorem. ■

This theorem shows that under the balancing policy any investment in storage is profitable only if the ratio of the amortized capital cost of storage to the highest price of energy is less than $\frac{1}{4}$. This is the first theoretical (tight) upper bound on the cost of storage and is independent of any assumptions on the distribution of uncertainties, even for constant prices.

We do a rough back of the envelop calculation to understand typical values of $\frac{c}{p}$ ratio for existing storage technologies. Capital cost of different storage technologies is roughly \$100-500/kWh depending on the type of technology with a lifecycle of about 2000-10,000 cycles respectively [13],[14]. Suppose the efficiency of storage is about 75%. Then the per-cycle cost of useful storage is in the order of 6.7 cents/kWh. Price of electricity is in the order of 10-20 cents/kWh. Then the $\frac{c}{p}$ ratio corresponds to about 0.17-0.33 (assuming that every period corresponds to at most half a cycle). This indicates that we are close to the break-even point for many technologies. This independently proves that our 0.25 bound is not necessary a loose bound.

B. Tradeoffs of optimal storage size and gain with system parameters

In this section, we are interested in studying the impact of pricing, cost and efficiency of storage and distributional parameters that characterize the uncertainty in demand and wind on the optimal storage size and its gain. Our focus will be on understanding the nature of the trend and its relative impact. So, for simplicity of our study we assume that the net uncertainty, $Y_t = D_t^H - W_t$, is a uniform i.i.d distribution with mean m and width u (i.e., variance is $\frac{u^2}{12}$). Although the uniform and i.i.d assumption may seem restrictive, it is an assumption on the net-gap or error in the system and hence not too bad. All our results extend to the case of the i.i.d Gaussian random variable with 0 mean using certain scaling arguments with an application of the central limit theorem. In the interest of limited space, we do not expand on the details of this analysis

We consider a base case with constant prices and no conversion losses of storage where we derive the optimal storage and the percentage gain in closed form. We then study extensions of the base case to see the impact of pricing and the effect of conversion losses. Unfortunately, in both these extensions, it is not easy to derive the optimal storage in closed form but computationally these steps are easy to replicate and we use MATLAB to do so. For our study, we only restrict to the case when the balancing policy is optimal.

a) Constant prices and no conversion losses (base case): Consider the case of constant prices for electricity and no conversion losses i.e., $\rho = 1$. We only outline the approach of finding the optimal storage in the interest of limited space. Under the balancing policy, we first derive the steady state distribution, $f_X(x)$, for storage of size S using the following equation:

$$\int_0^S f_X(y)\left[(1 - F_Y(y))\delta(x) + f_Y(y-x)\mathbb{I}(0 < x < S) \right.$$
$$\left. + F_Y(y-S)\delta(x-S)\right]dy = f_X(x),$$

where $\delta(x)$ is a dirac-delta function that is 1 when the $x = 0$ and 0 otherwise and $\mathbb{I}(0 < x < S)$ is the unit function which is 1 if $0 < x < S$ and 0 otherwise. Since Y has a uniform distribution, $F_Y(y) = \frac{y - m + \frac{u}{2}}{u} \ \forall \ m - \frac{u}{2} \leq y \leq m + \frac{u}{2}$. Substituting $x = 0, x = S$ and x s.t $0 < x < S$, in the integral equation, we get $f_X(x)$ as follows:

$$f_X(x) = \frac{m + \frac{u}{2} - E[X]}{u}\delta(x) + \frac{1}{u}\mathbb{I}(0 < x < S)$$
$$+ \frac{E[X] - (S + m - \frac{u}{2})}{u}\delta(x - S),$$

where $E[X] = \frac{-S(S+2m-u)}{2(u-S)}$. We use this to estimate the objective function $V(S)$ as follows:

$$\frac{p}{4u^2}\left[-\frac{S^3}{3} - uS(u-S) + \frac{4m^2uS}{u-S} + (2m+u)^2\frac{u}{2}\right] + cS.$$

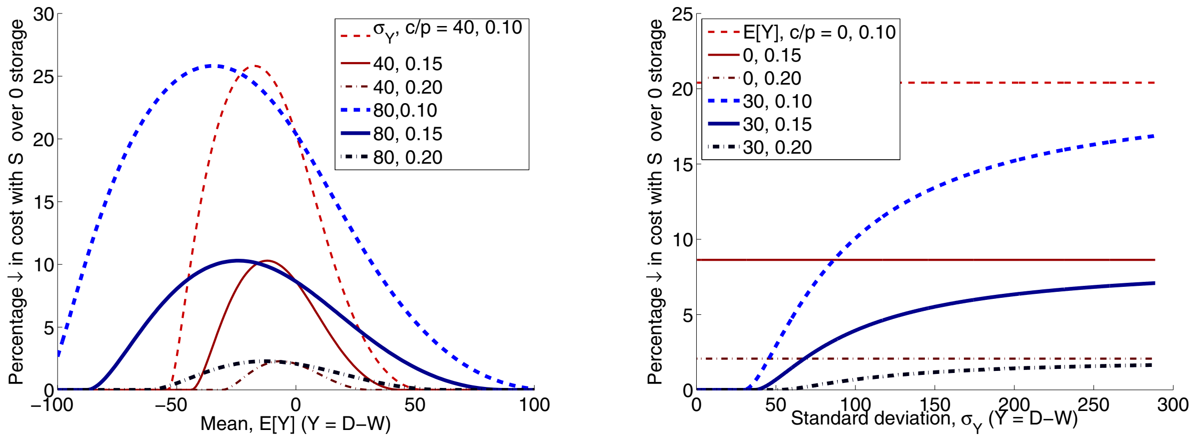

Fig. 1: Percentage decrease in cost with optimal storage investment: trends with respect to mean and standard deviation.

A solution such that $V'(S) = 0$ and $V''(S) < 0$ is the optimal storage, S^*, to the problem. We get,

$$S^* = \max\left\{0, u\left[1 - \sqrt{\frac{2c}{p}\left(1 + \sqrt{1 + \frac{m^2p^2}{u^2c^2}}\right)}\right]\right\}.$$

We can immediately make the following observation about the size of optimal storage

Observation 5.2: $S^* > 0$ if and only if $\frac{c}{p} < \frac{1}{4} - \left(\frac{m}{u}\right)^2$. This observation strengthens the result of Theorem 5.1 for the case of the uniform distribution in the following ways: (a) the uniform distribution with 0 mean is a class of distribution for which the $\frac{1}{4}$ bound is tight; (b) the bound is smaller for larger $\frac{m}{u}$ values; and, (c) it proves converse of the theorem as well.

Observation 5.3: For a given u and $\frac{c}{p}$, S^* is maximum at 0 mean decreases in the $O(\sqrt{m})$ symmetrically around 0.

Observation 5.4: For a given m and u with $m << u$, S^* decreases in the $O\left(\sqrt{\frac{c}{p}}\right)$.

Observation 5.5: For a given m and $\frac{c}{p}$ ratio with $m << u$, S^* increases linearly with respect to u, the standard deviation whenever $S^* > 0$.

In the above observations we restrict to the region when $m << u$, i.e., m much smaller than u. In many situations this is exactly the region of interest as the net gap, Y_t, can be viewed as the net error in balancing which usually has a small mean but a larger standard deviation.

For understanding the variations of gain from the optimal storage size S^*, we plot the percentage decrease in cost, $\frac{V(0)-V(S^*)}{V(0)}$ with respect to m and u for different values of $\frac{c}{p}$ in Fig. 1. With respect to m and $\frac{c}{p}$, we observe that the gain from storage decreases rapidly (has a square effect) for larger values of $|m|$ and $\frac{c}{p}$. Note that the peak gain is not at 0 mean because we are plotting the percentage gain and not the absolute gain. The absolute gain peaks at 0 mean. The gain is non-decreasing in the standard deviation and asymptotically approaches the gain at 0 mean.

b) Impact of peak and off-peak pricing: In this part, we want to study if and how differential pricing impacts optimal storage size and its gain and whether uncertainty in the differential pricing plays a role and how. In order to answer these questions, we consider two types of simple pricing schemes for our analysis. In the first scheme, prices are i.i.d with peak price p_H with probability q and off-peak price $p_L = 0$ with probability $(1-q)$. In the second scheme, the peak price is always followed by the off-peak price. The schemes differ in the uncertainty associated with the next period price and can be compared at $q = 0.5$. We refer to the latter scheme as the time-of-use (TOU) pricing scheme.

For this analysis, we need to make assumptions on the distribution of W_t. For simplicity, we assume that D_t^H is a constant so that W continues to be i.i.d uniform. We make no assumption on the D_t^L and hence demand may very well be elastic.

In Fig. 2, we plot the optimal storage size and its gain with respect to the probability of the high price, q, for the i.i.d pricing scheme. In the same plot we also mark the corresponding values for TOU pricing scheme when $q = 0.5$. First, comparing $q = 1$ and any other value of q for the i.i.d pricing scheme, we observe that differential pricing can sometimes increase the value from storage but not always. This is because of the tradeoff between how much energy is actually needed at the peak price on average to support excess demand but also how often it can be obtained at the lower price. This tradeoff results in a unique probability above and below which the value from optimal storage decreases for a fixed price differential.

Next, comparing the two pricing schemes when $q = 0.5$, we observe that even though the optimal storage size from TOU pricing is larger than i.i.d pricing the gain from their respective optimal storage sizes is higher for i.i.d pricing over the TOU pricing. The larger storage size for TOU pricing is natural because uncertainty results in stocking less on average. And since the optimal storage sizes are not very different from each other, there is a gain from uncertainty in

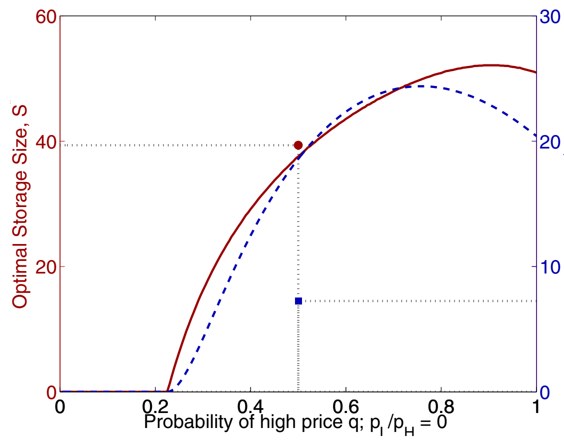

Fig. 2: Impact of uncertainty in i.i.d pricing on optimal storage size and gain. For $q = 0.5$, we plot the corresponding values for TOU pricing. $E[Y] = 0$, $\sigma_Y = 40$, $\frac{c}{p} = 0.1$.

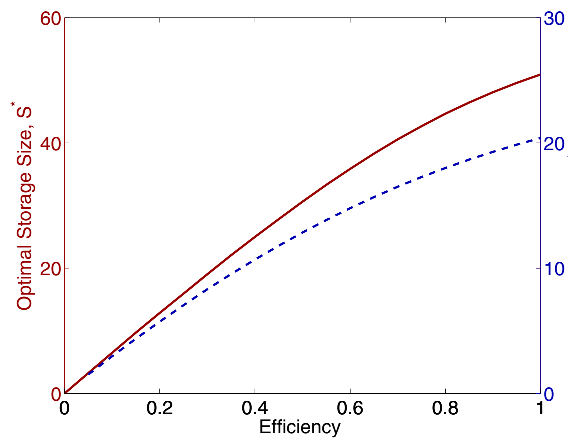

Fig. 3: Variation in the optimal storage size and gain with the efficiency of storage. $E[Y] = 0$, $\sigma_Y = 40$, $\frac{c}{p} = 0.1$.

differential pricing.

c) Impact of conversion losses: To study the impact of conversion losses, we incorporate this to the base model. In Fig. 3, we plot the variation in optimal storage size and the percentage gain with efficiency. Note that in our experiments we have the same amortized cost of storage at all efficiency levels. This is because we defined S as the size of useful storage and hence the amortized cost corresponds to cost of useful storage and hence it is already normalized between two storage devices of different efficiencies. Observe that the optimal storage size as well as the percentage gain increases with efficiency. This should not be surprising because for two storages of the same size, conversion losses only decrease the value of storage. And since, both scenarios have the same cost, the optimal storage size and its value decrease.

VI. Conclusions

In our work, we formulate the optimal storage investment problem for a renewable generator that aims to satisfy a local demand. We propose a balancing control to manage storage and find the optimal storage size under this control. We show that this control is optimal under simple pricing schemes. We also provide a theoretical tight upper bound of $\frac{1}{4}$ on the cost of storage to the price of energy for the storage to be profitable. For a uniform i.i.d distribution of the net gap between demand and renewable generation, we show that the optimal storage size and gain increases with variance, storage efficiency and level of differential pricing and decreases with the absolute value of the mean. We also see that uncertainty in differential pricing can increase the gain from storage depending on the level of uncertainty.

In our current and future work we will investigate the optimal management and investment in storage under more general pricing schemes with elastic demand. We are also working on real-world data from renewable generators and specific storage technologies to study the true cost benefit analysis of storage investment.

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