Black boxes: Hypothesis testing via indirect perceptual evidence

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Abstract

Studies of children's causal learning typically provide learners with clear evidence for direct causal relations, e.g., a machine that activates when a toy is placed upon it. But causal systems in the real world often present indirect perceptual evidence generated by interactions between hidden variables: Consider a child trying to figure out what's inside a box by shaking it. We propose that effective learning and exploration depend on being able to interpret evidence through the lens of intuitive theories – theories of both the physical world and one's own perceptual apparatus – to imagine how one's actions might change the state of the world and what kinds of changes would be most perceptually discriminable. We present three studies exploring these capacities in young children, and suggest how they could support powerful and sophisticated inferences about hidden causes.

In science, much of the evidence we get for causal relationships is indirect: we cannot observe the activation of neurons or the presence of microorganisms directly so we develop sophisticated technologies to detect neural activation from blood flow, or the presence of microorganisms from the amount of dissolved oxygen. This kind of inference is not restricted to scientific practice. Even in everyday life, there are many cases where we have to infer unobserved causes from indirect evidence. We see a curtain move and infer the cat behind it, we hear dripping and worry that the faucet is leaking. These are very ordinary examples, ones that might be accessible even to small children, but they suggest an extraordinary capacity: the ability to reason backwards from new evidence to probable unobserved causes of the data.

While there is a wealth of research on children's causal reasoning, the majority of studies has focused on childrens inferences about direct causal relationships: observed blocks that do (or do not) light up machines, levers that do (or do not) activate toys, etc (Gopnik, Sobel, Schulz, & Glymour, 2001; Gopnik & Sobel, 2000). Moreover, although there is some work outlining childrens ability to infer unobserved causes from patterns of evidence (Kushnir & Gopnik, 2005; Schulz & Sommerville, 2006; Schulz, Goodman, Tenenbaum, & Jenkins, 2008), in such cases children were asked about arbitrary causes and what is at stake is simply childrens ability to detect conditions under which a latent variable might explain the pattern of evidence. Here we ask a different question: Do children's intuitive theories support these inferences in cases where the mapping from causes to effects is unspecified and complex?

One way of investigating unfamiliar scenarios is to take actions designed to reduce uncertainty. We can pull back the curtain, we can use more powerful tools as they're developed. Cook, Goodman, and Schulz (2011) demonstrated that children preferentially select more informative interventions when exploring a new causal domain; and there is more generally a rich literature on active learning and information selection in adult human cognition (Oaksford & Chater, 1994; Tsividis, Gershman, Tenenbaum, & Schulz, 2014).

We aim to investigate a complementary question. People (scientists or otherwise) are never able to directly observe data that most informs the questions they actually want answered, for good reasons: the equipment is too expensive, the cat (or is it?) runs away as we approach. What scientists (and other people too) actually do is more pragmatic. We know our tools, eyes, ears, or electron microscopes; and we know how they sense and transform the objects we're interested in studying. Answering questions thus requires us to select not only the right tests, but *data* that would yield informative results given the right test.

We are especially interested in what learners decide to investigate in situations where evidence is richly perceptual and interpretation involves intuitive theories of the physical world, because these are closest to the ways young children most naturally explore their surroundings. Our focus here is on physical interactions between objects that generate sound, and on what we need to know, both about these physical events and our abilities to perceive them, in order to learn about objects from the sounds they produce. We take sound as a case study because children have expectations about how objects interact to produce auditory signals, because these interactions produce nontrivial evidence (so that there is clearly some work to be done in interpretation), and finally because of the intuitive, everyday quality of perceptual inferences.

A rubber ball shaken inside a closed box will sound different than a glass marble. Two rubber balls will sound different from either; five glass marbles will sound more different still. A novel object that looks pointy or feels hard to the touch will sound different from one that looks rounded or feels soft. How do we know these things? One might think that these examples can be explained as consequences of cross-modal perception and memory. We have experienced seeing a faucet drip while also hearing it. We have all shaken a wrapped present to find out what might be inside. However, we do not always have access to information in multiple modalities, and in these cases we are still able to reason about the causes of various sounds. Indeed, the combinatorial explosion that accompanies real-world perception suggests that reasoning from indirect evidence requires sophisticated and subtle capabilities which are far more powerful than those commonly assumed.

We follow in a research tradition that highlights how people use mental models and theories about the world to generate predictions, which are used to interpret new data (Tenenbaum, Kemp, Griffiths, & Goodman, 2011). But we suggest that learners also have intuitive models of the psychophysical properties of evidence - we know what, given the constraints of our senses and our world knowledge, we can know via perception. In particular, children and adults know when patterns of perceptual data are more or less discriminable, and they use this knowledge to guide their exploration of the world.

In the experiments to follow, we test children's ability to make indirect perceptual inferences; to use these judgments when interpreting ambiguous evidence; and to exploit their "intuitive psychophysics" to predict and evaluate the quality of data, even before encountering it.

In a preliminary investigation, we considered how learners interpret auditory cues to determine their (unseen) causes. We examined whether 3- to 6-year-olds can determine the *kind* of hidden object that generates the sound they heard, whether they can infer the *number* of identical hidden objects that produce the percept, and we discuss several additional experiments that control for low-level confounds but are also of independent interest. Study 1 probed children's use of indirect acoustic cues to reason about what's *not* perceived: can children evaluate the diagnosticity (or discriminability) of evidence when inferring hidden causes? Specifically, we tested whether participants choose more discriminable evidence over less, both when evaluating differences in kind and in number of objects. Finally, Study 2 investigated whether children can make these judgments predictively, even without observing the effects of hidden causes. We hypothesize that learners can imagine possible patterns of evidence and choose the most distinctive data set, without needing any perceptual access to distinguishing evidence.

Box-Shaking Game

We used a simple paradigm which we will call the "box-shaking game". The basic schema is as follows. Children were shown two objects (or two sets of objects) and told that each would be hidden in a box and shaken several times, and that they'd be asked to choose which box had each object based on the sounds they heard (they also saw the way the box was shaken). We hid everyday objects like bean bags and plastic balls, and also sets of marbles.

(a) Object Identity

In the first version of the box-shaking game, the experimenter showed children (N = 16; mean: 3 years, 11 months) a pair of boxes, identical except for color. The objects that we hid and shook were a bean bag and a plastic ball; the ball was weighted to match the mass of the bean bag. The experimenter produced the objects to be shaken and briefly described their properties. Children chose which object they preferred and then the experimenter then explained the task: "I'm going to put each one of these things in a different box, and then shake each box! Then we'll listen and try to figure out which box has your favorite thing in it. Do you want to help me figure out which box has your favorite thing in it?" The objects and box were then hidden by an occluding screen, after which each object was hidden in a different box and the occluder finally removed. After reminding participants of the task, each box was shaken for several seconds (five shakes). The experimenter then asked, "Which box has your favorite thing in it?" In this task, all of the children chose the box that contained their preferred ball.

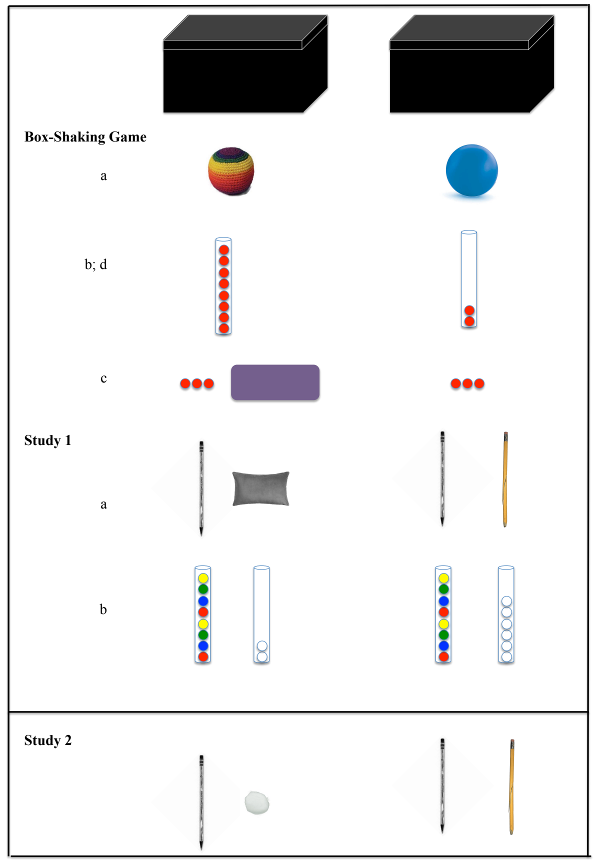

Figure 1: Illustration of methods. In the preliminary box-shaking game, we placed the objects on the left and right sides in different boxes and shook them. Children identified (a) their preferred object; (b) the box with fewer or more marbles; (c) the box with the blanket; and (d) the box with more marbles under a different box manipulation that controlled for loudness. In Study 1(a-b), we placed one object (target or distractor) from each side (left=Unambiguous, right=Ambiguous in figure) in a different box, then shook each box. The target objects were shiny pencils and colorful marbles, and the other objects were distractors. Children picked a box to find a target object. In Study 2, we asked children to choose which box to shake before receiving any data, requiring them to imagine which stimulus set (left or right) was more discriminable.

(b) Number of Objects

We next tested whether preschoolers can perceive the quantity of hidden objects from indirect evidence. Children listened to two boxes shaken, one with 2 marbles and one with 8 marbles, and were asked to identify which box had a few

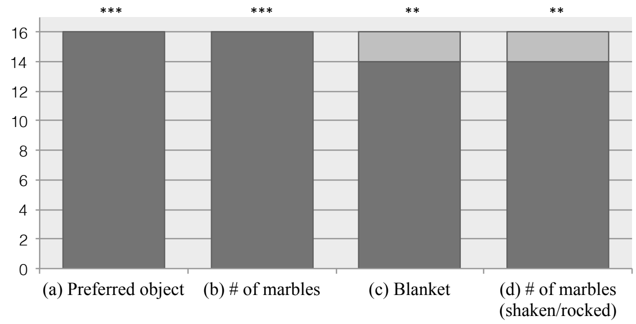

Figure 2: Results of preliminary box-shaking task (figure label corresponds to experiment heading) - number of children choosing the indicated box. (a) Box containing preferred object. (b) Box containing more or less marbles. (c) Box containing felt (blanket). (d) Box containing more marbles (rolling/shaking contrast). $** = p < .01$, $*** = p < .001$ via binomial test

(or many) marbles. Children, (N = 16, mean: 3 years, 11 months) were shown two identical black boxes with detachable lids (preloaded with 2 and 8 marbles) along with a set of 2 marbles and a set of 8 marbles each placed in a translucent cylindrical tube. We also used a puppet, "Bunny", who expressed a preference for either a few (2) marbles or a lot (8) of marbles, counterbalanced across participants. Next, the experimenter showed the children the two closed boxes and explained that one box had a few marbles and one box had a lot of marbles. The experimenter shook each box for approximate 5 seconds before asking, "Which box does Bunny want to open?" All sixteen children identified the correct box based on Bunny's preference.

(c) Level

Children could be performing the number discrimination task by using a simple heuristic: higher sound level means more objects. This rule indeed holds for the case we've considered above. But this rule fails in many cases, for example when a piece of soft material is added to a box (dampening the sound of other objects). The next experiment conducted as part of the box-shaking games rules out simple heuristic-based accounts, and also demonstrates the productivity and flexibility of these perceptual judgments.

To test whether children are able to reason flexibly about the contents of boxes from indirect evidence, we first asked children to identify which of two boxes had a felt blanket inside it, along with other objects. The felt dampened the sound; if children succeed nonetheless, we can conclude that a level heuristic does not account for participant's judgments. Participants (N = 16; mean: 4 years, 10 months) chose the correct box more often than chance ($p < .01$ by two-tailed binomial test). This result suggests that the ability to make judgments about quantity based on indirect evidence does not depend on cross-modal mapping between more items in the visual domain and a louder sound in the auditory domain.

(d) Diverse Cues

All of the previous experiments in the box-shaking game used the same physical manipulation, namely up-and-down box-shaking, for all contrasts. Is it possible that this simplifies children's task, by allowing them to focus in on some single dimension of sound (number of collisions, for example)? To address this hypothesis, we again asked children to identify which of the boxes contained more marbles, but shook only the box with 2 marbles and gently rocked the box with 8 marbles, producing sounds very different in quality. Children (N =16, mean: 4 years, 11 months) interpreted the perceptual results of these different manipulations and recovered information about the quantity of marbles inside the boxes in this case as well. Gentle rocking and vigorous shaking produce very different sounds even with equal numbers of marbles, but 14 of the 16 children successfully discounted the physical motion to choose the box with more marbles ($p < .01$ by two-tailed binomial test). The perception of numerosity from sound may not therefore be attributed to simple heuristics – it must incorporate some intuitive knowledge of the physics of objects.

The results of the box-shaking game, in its various forms, established that children can test hypotheses using evidence that is very different in kind than that typically used in developmental studies, in several ways. First, the perceptual data that children received were natural sounds, which are interesting because they seem both harder and easier to interpret - real perceptual data is ambiguous and variable, unlike preferred developmental stimuli. However, real data is strongly tied to real-world processes, so it may be easier to interpret than the arbitrary causal associations found in other work. Second, as we didn't preview to participants the sounds produced by our box-shaking game, their success implies that they must have access to task-relevant memories (what each object sounds like when shaken) or to the means to generate such data (we suggest via mental simulation).

Study 1

In the box-shaking game, we demonstrated that children readily draw inferences about physical properties of hidden objects based on indirect, perceptual evidence. We next considered whether children intuitively grasp the relative ambiguity of perceptual data, and how this knowledge informs their decisions. Suppose you have a black box that contains either a shiny pencil or a normal pencil. If you shake the box, you would hear the sound of a hard, light object hitting the walls, but you wouldn't know exactly what was inside the box. But suppose you have another box that contains either a shiny pencil or a cotton ball, and you hear the same sound; you would be certain that the box contained the pencil. If you had a choice between these two boxes in front of you (knowing what could be inside and hearing, in both cases, a hard light object) and you wanted the shiny pencil, which box would you choose to open? Intuitively, of course, you would choose the box that had either the shiny pencil or the cotton ball. But to judge that this box would give you a better chance of getting the shiny pencil, you must understand what each object would sound like when shaken inside a box, and you must reason both about what you heard and about what you didn't. You must also compare the probability of obtaining the desired object from each box. In Study 1, we put children in exactly this situation to investigate whether they understand that what they hear can be more or less informative depending on what they didn't perceive.

Experiment 1(a): Object Kind

Participants We tested twenty-four preschoolers (mean: 4 years, 7 months; range: 3 years, 1 month - 6 years, 2 months) visiting the children's museum.

Materials We used the same two boxes from our preliminary investigations, two pencils with holographic coating as target stimuli, and one standard pencil and a small, cotton-filled fabric cushion as distractor stimuli.

Procedure The setting and instructions were similar to those described earlier. Children were first shown two pairs of objects, each of which consisted of a target and a distractor stimulus. The target stimulus (i.e., shiny pencil) was identical across both pairs, and was more desirable than either distractor. The distractor in one pair (Ambiguous pair) was an object that would sound very similar to the target when shaken inside a box (i.e., a standard No. 2 pencil). The distractor in the other pair (Unambiguous pair) was chosen such that it would sound very different from the target (i.e., a pillow). After introducing the objects in each pair, the experimenter asked children what their favorite object is in each pair. In order for the task to measure children's sensitivity to ambiguity, it was important that children have a preference for the target object in both pairs; we inferred learners' sensitivity to ambiguity by presenting evidence that was consistent with the (identical) target objects being in both boxes, and if children preferred one of the distractor objects, they might simply choose the box it might be in. Therefore, we dropped and replaced children who preferred the distractor object in either pair. After children picked their favorite (target) objects, the experimenter then told children that he would take just one of the objects from each pair and hide it in each of the two boxes, and that they could choose a box and take its contents home. After the objects were hidden, he reminded children about what could be inside each box (e.g., "Remember, inside this box, there could be a cool shiny pencil or the pillow"). The experimenter then shook each box twice, after which children picked a box and collected its contents.

Results and Discussion Before the objects were hidden, the experimenter asked children which object in each pair was their favorite. Therefore, we only included and analyzed data from the 16 children who preferred the target objects (shiny pencils) in both pairs. Thirteen of sixteen children correctly selected the Unambiguous box ($p < .05$ by two-tailed binomial test). The results of this experiment suggest that children understand that the Unambiguous pair box had to contain the target object (because if it didn't, a different sound would have been produced), whereas the Ambiguous box may have contained either the target or distractor object (the shiny pencil or the yellow pencil).

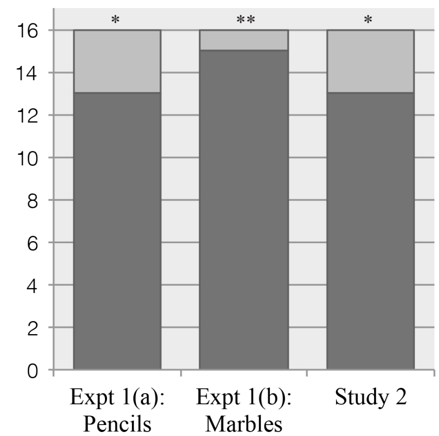

Figure 3: Results of Studies 1 & 2 - children's preference for Unambiguous boxes containing (Expt 1(a)) pencils and (Expt 1(b)) marbles, and their predictive judgments (Study 2).

Experiment 1(b): Number of Objects

Participants Sixteen children (mean: 4 years, 6 months; range: 3 years, 1 month 5 years, 9 months) were recruited from the museum.

Materials We used the colored boxes described above. We used four different sets of marbles: two sets of 8 marbles, of all different colors, one set of 2 white marbles, and one set of 6 white marbles. The four sets of marbles were placed in clear tubes, with the top sealed using packing tape. We printed out cartoon versions of the marble tubes as a memory cue. Finally, the bunny puppet (Bunny) from the box-shaking game was used as a confederate.

Procedure The study began with a warm-up task, similar to the basic object-identification box-shaking game, except that after the child identified his/her preferred object, Bunny expressed a preference for the other object. After hiding and shaking each box, children were given a sticker for successfully choosing the box containing their previous choice. Next, the experimenter displayed the four tubes, filled as described above. Bunny expressed a preference for white marbles, touching the appropriate tubes and exclaiming, "White marbles! I love these white marbles!" Next, the experimenter indicated the two tubes containing 8 colorful marbles and said, "See these marbles of different colors? For this game, these are yours! You're going to try to find your marbles." Next, children were introduced to the hiding game, and told that one tube of marbles would be hidden inside each box. For the Ambiguous box, the possible contents were 6 white marbles or 8 colorful marbles; for the Unambiguous box, the possible contents were 2 white marbles or 8 colorful marbles. Before hiding the marbles, the experimenter familiarized children with pictures depicting the possible contents of the two boxes (shown in Figure 1). The experimenter then hid 8 marbles inside each box. After removing the screen, the experimenter reminded children about the boxes' possible contents by pointing to the cartoon pictures, (e.g., "Remember, in this box there could be your marbles (indicate picture of child's 8 marbles) or there could be Bunny's marbles (indicate picture of Bunny's 2 marbles (less ambiguous) or Bunny's 6 marbles (Ambiguous)." The experimenter shook both boxes twice. Finally, children were asked, "Which box do you want to open to find your marbles?"

Results and Discussion Fifteen of the sixteen children ($p < .001$ by two-tailed binomial test) chose the less ambiguous box, confirming and extending the results of Experiment 1(a). Critically, children heard identical evidence from both boxes, so they must have taken into account information other than the sounds they heard when choosing which box to open. What sort of reasoning might support these judgments? Our account is simple but sufficient. After hearing the results of shaking each box, children imagine (mentally simulate) the sound that they would have heard had the boxes contained either the target or each of the distractors.

Study 2

We demonstrated above that children are able to use knowledge of their own perceptual capabilities to prune hypotheses, distinguishing informative sets of data from less informative ones. Our results revealed that children interpret equivalent percepts differently depending on what else they might have heard. Thus children use the outcomes of mental queries to interpret real evidence. But if learners are able to imagine alternative data, they shouldn't need to hear *any* evidence; they should be able to imagine the perceptual results of each intervention on each object, and in this way design an appropriate experiment before any actual manipulation is performed.

We modified Study 1 so that children reported their choice before they heard any relevant perceptual data. Children were presented with the game and told that they could choose the set of objects (which were more or less ambiguous when shaken) to play with. We therefore tested whether children can predict which set of data they'd be able to discriminate.

Participants Another 16 children (mean: 4 years, 9 months; range: 3 years, 9 months 6 years, 0 months) were recruited from the museum. An additional 17 other participants were replaced because they preferred the distractor objects, because of experimenter error, because of language difficulties, or because they chose not to complete the study.

Materials We used the black boxes and pencils from Experiment 1(a) as well as an ordinary cotton ball.

Procedure The study began with a warmup task, identical to the object-identification game. Next, children were told they would play another hiding game where they could keep the object inside the box that they chose. As in Experiment 1(a), children were shown two sets of objects and were asked to choose their favorite object from each pair. The Ambiguous pair of objects consisted of a shiny pencil and a yellow pencil, and the Unambiguous pair consisted of a shiny pencil and a cotton ball. Once children picked their favorite objects from each pair, the experimenter said, "This game is special because you get to choose what I hide. I could hide this shiny pencil (from Ambiguous pair) in one box and this boring pencil (from Ambiguous pair) in the other box, and you'll listen to me shake each box. Or I could hide this shiny pencil (from Unambiguous pair) in one box and this piece of fuzz (cotton ball) in the other box, and you'll listen to me shake each box. Which set of things do you want me to hide?"

Results and Discussion Thirteen of sixteen children asked the experimenter to hide the Unambiguous pair ($p < .05$ by two-tailed binomial test). Recall that we proposed mental simulation as a way to know what shaken objects would sound like, had they been shaken. In light of children's successes in Study 1, we also noted that this simulation account should predict that children can make the same judgments even before hearing any relevant sounds. The results of Study 2 thus confirm this prediction, giving evidence that simulation is indeed a good candidate for explaining the remarkable perceptual achievements necessary for solving these tasks.

Discussion and Conclusions

In three studies, we investigated how children discover hidden causal structure in the world by reasoning about what they can perceive. In the preliminary box-shaking game, children made accurate inferences about hidden causes from perceptual evidence that required nontrivial interpretation, inferring the nature and quantities of objects inside a box from diverse forms of sound data. After verifying childrens ability to identify a variety of items based on only their sounds, we considered whether children can use their mental models of physics and perception to assess the discriminability of different sets of evidence. In Study 1, children distinguished evidence that was more or less likely to be informative about a box's contents, even though the evidence they heard was perceptually identical. In Study 2, we showed that children design novel informative experiments, evaluating what which data should be tested given a hypothetical tool (box shaking).

These results contribute to a long-running debate on how, and to what extent, children or adults effectively learn about the world like scientists do (Gopnik et al., 2004; Schulz, 2012; Chen & Klahr, 1999). Going back to J. S. Mill, it has often been proposed that experiments (either formal or informal) should be designed by a process of deductive reasoning, where the causal structure of the experimental setup (which variables might possibly influence each other) suggests which interventions to perform. This syntactic view (e.g. the "control of variables strategy" (Chen & Klahr, 1999)) indeed can qualitatively describe some behavioral and cognitive aspects of human learning. But it also leaves out several key issues that our work aims to address. Variables are rarely directly accessible, so we must figure out ways to get at them, whether through microscopes or sounds made by shaking objects in boxes. If learners understand the way that their instruments work, they can work backwards from measurements to causes, but this means solving a nontrivial and non-obvious problem (as children did in our box-shaking game). Also, the strength of the causal relation between variables matters. Thus knowing whether variables are confounded in a logical sense is not enough, as the result of the intervention in our study (and of interventions in the world) can produce confounded results – each box in Study 1 produced a sound that could possibly have been made by either of its possible contents. If children had been unable to reason about the evidence produced if the other object or set of marbles been inside the box instead, they would have chosen boxes at chance. Yet children overwhelmingly picked the box whose evidence, based on its possible contents, was more discriminable.

Taken together with Cook et al. (2011), the results of the current study make a strong case for children as rational experimenters. Simply put, children know when and how to carry out actions that effectively answer questions. Children learn about the world by intervening on their physical surroundings and perceiving outcomes filtered through extrinsic (the box, for example) and intrinsic (their own senses) intermediaries. Such learning is therefore contingent on knowing what sort of interventions will produce informative data, and what sort of data are informative with respect to specific interventions. Future work might investigate how an "intuitive psychophysics – a mental model of one's own perceptual capacities – guides childrens spontaneous exploration and experimentation. But what are these capacities? How do learners, children and scientists alike, know what they can perceive? We hypothesized that children can use their intuitive physical theories along with an "intuitive psychophysics" to simulate the rich perceptual evidence that would result from an intervention like shaking or rolling a box, to filter observed evidence through their own ability to interpret it, and to plan interventions guided by their understanding of these abilities. In other words, people are able to simulate and imagine the world's dynamics and their own perceptual grasp of the same.

The experimental evidence presented here points to a mostly unexplored capacity of even young learners, and we have included the makings of a potential explanatory theory. Future work aims to cash out these intuitions in a formal modeling framework encompassing both the perceptual and metacognitive aspects of the abilities demonstrated here.

Children, and probably scientists too, interpret data and evaluate candidate experiments through a process of inference to the best explanation where probabilistic inferences about one's measurement or perceptual apparatus go hand-in-hand with inferences about the world's hidden causal structure. We argue that the capacities studied here may be at the heart of how we come to know what we know.

Acknowledgments

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