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Lower-bound Analysis of Unreinforced Masonry Vaults

Philippe Block and John Ochsendorf

Medieval vault builders explored three-dimensional equilibrium, creating complex forms carefully balanced in compression. The structural properties of these sophisticated forms are still poorly understood because of a lack of appropriate analysis methods, i.e. methods relating stability and form. Thrust-Line Analysis is a powerful graphical method to explore and understand the range of lower-bound equilibrium solutions of compression-only systems, such as unreinforced masonry structures. It visualises the relative stability of these structures and suggests possible collapse mechanisms [1]. However, it is primarily a two-dimensional technique. In order to analyse three-dimensional structures using the same intuitive methods, the analyst has to slice the structures up, hence reducing them to a combination of two-dimensional problems. This process obviously does not capture the full three-dimensional behaviour of the structure and can rely heavily on the chosen discretisation. These limitations have been the main reasons why thrust-line analysis has not been used as much for the assessment of complex 3-D structures.

This paper presents *Thrust-Network Analysis*, a new fully three-dimensional method developed at MIT, as a 3D extension to thrust-line analysis [2]. It clearly visualises possible three-dimensional force networks within the vaults geometry, representing the possible paths of the compression forces. Using the reciprocal relationship between geometry and forces [3], the forces in the system are represented very graphical at all time. Variations in load paths and geometry can easily be explored. Using linear optimization, results are produced within seconds to quickly compare the influence of different parameters and assumptions. The methodology uses existing 3-d drawing software as input and output resulting in an interactive tool with a visual representation of results and force distributions. Analysis proceeds from an accurate 3-d model of the vault without the need for abstraction or simplification.

The viability of the proposed method is demonstrated through two case studies, highlighting its multiple applications. First, an analysis of a series of groin and quadripartite rib vaults investigates the interrelationship between different parameters and the range of possible equilibrium solutions of these vaults. Variables influencing the range of vault thrust include the influence of the web geometry, the boundary conditions, the role and effect of cross-ribs, the difference between alternative assumptions for the internal force patterns, the presence of cracks and other pathologies, and the impact of fill above the haunches. A second case study looks specifically at the fan vaults of King's College Chapel at Cambridge University, England. Using thrust-network analysis, these thin vaults with complex geometries are analysed. Different load path distributions are compared and the importance of the main ribs and the weight of the bosses are considered.

This paper introduces a fully three-dimensional computational method for obtaining lower-bound solutions on the thrust behaviour of unreinforced masonry vaults with complex geometries. Through several examples, it demonstrates its potential as a powerful tool for understanding, visualising and exploring the equilibrium of compression-only structures.

References:

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