THRUST NETWORK ANALYSIS: A NEW METHODOLOGY FOR THREE-DIMENSIONAL EQUILIBRIUM

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Summary

This paper presents a new methodology for generating compression-only vaulted surfaces and networks. The method finds possible funicular solutions under gravitational loading within a defined envelope. Using projective geometry, duality theory and linear optimisation, it provides a graphical and intuitive method, adopting the same advantages of techniques like graphic statics, but offering a viable extension to fully three-dimensional problems. The proposed method is applicable for the analysis of vaulted historical structures, specifically in unreinforced masonry, as well as the design of new vaulted structures. This paper introduces the method and shows examples of applications in both fields.

Keywords: Compression-only structures; Unreinforced masonry vaults; Funicular analysis; Thrust Network Analysis; Reciprocal diagrams; Form-finding; Lower-bound analysis; Structural optimization.

1. Introduction

Medieval vault builders created complex forms carefully balanced in compression. The structural properties of these sophisticated forms are still poorly understood because of a lack of appropriate analysis methods, i.e. methods relating stability and form. Understanding the mechanics of these vaulted structures leads to new insights for both analysis and design.

Thrust Line Analysis is a powerful graphical method for calculating the range of lower-bound equilibrium solutions of compression-only systems, such as unreinforced masonry structures. It visualises the stability of these structures and suggests possible collapse mechanisms [1]. Unfortunately, thrust line analysis is primarily suitable for 2-D cases and this limitation has prevented it from being used for the assessment of complex 3-D structures. While numerical methods based on elastic solutions give one possible answer, they no longer suggest better form as was inherent to the more holistic graphical methods.

There is a real need for tools to better understand and visualise the stability of compression-only structures, such as historic unreinforced masonry structures, as well as design tools that suggest better form. Both problems are related to finding axial force structures in equilibrium acting only in compression or tension. Currently, graphic statics provides a holistic analysis and design tool for two-dimensional structures. With today's availability of powerful virtual 3-D and parametric environments, the following question arises: can a fully three-dimensional version of thrust-line analysis provide the same freedom to explore the infinite equilibrium solutions for a certain loading condition?

2. Thrust Network Analysis

The *Thrust-Network Analysis* method presented in this paper is inspired by O'Dwyer's work on funicular analysis of vaulted masonry structures [2]. It is extended by adding the concept of duality between geometry and the in-plane internal forces of networks [3,4]. Key elements in the proposed process are (1) force networks, representing possible forces in equilibrium in the structure; (2) interactive reciprocal diagrams, visualising the proportional relationship of the horizontal forces in the network and providing a high level of control to manipulate the force distributions in the system; (3) the use of envelopes defining the solution space; and (4) linear optimisation, resulting in fast computation of results.

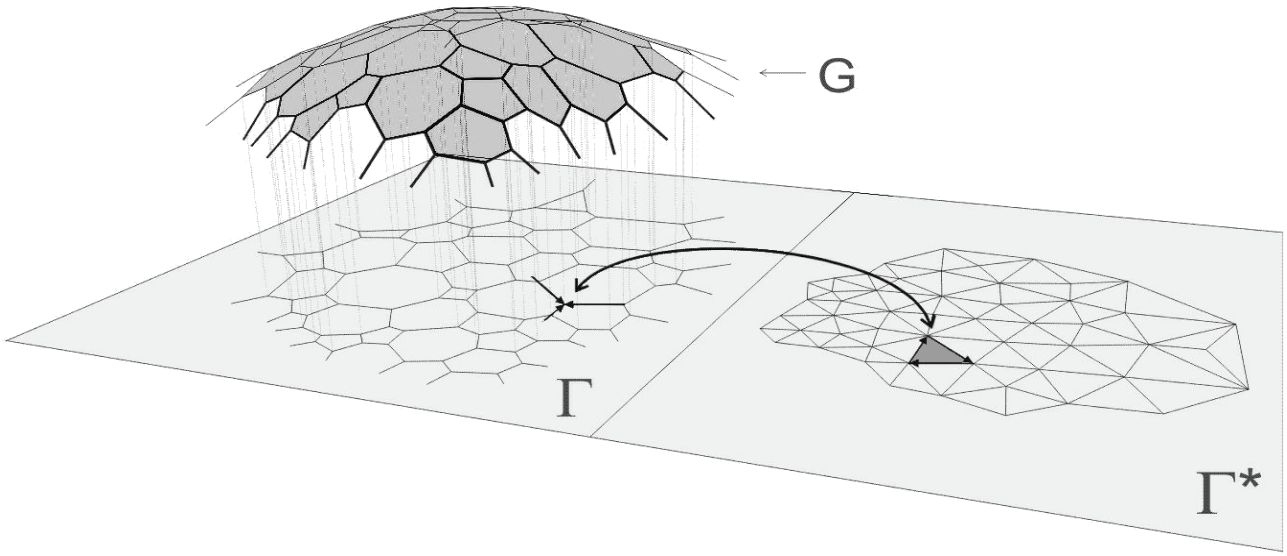


Fig. 1 Relationship between compression shell (G), its planar projection (primal grid Γ) and the reciprocal diagram (dual grid Γ^*) to determine equilibrium.

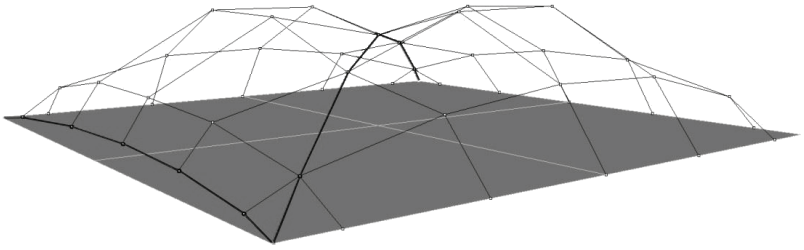
This paper introduces the Thrust Network Analysis method. It provides

- a viable three-dimensional extension for thrust-line analysis;
- a flexible, intuitive and interactive design tool for finding three-dimensional equilibrium of compression-only surfaces and systems; and
- an improved lower-bound method for the assessment of the stability of masonry vaults with complex geometries.

Key features are

- clear graphical representation of forces in the system (through the use of force diagrams, i.e. the dual grids);
- a high level of control, allowing the exploration of different possible equilibrium solutions; and
- fast solving times because of the formulation as a simple linear optimisation problem.

The Thrust Network Analysis method is applicable for the analysis of vaulted historical structures, specifically in unreinforced masonry, as well as the design of new vaulted structures. This paper shows examples of applications in both fields: the assessment of a typical vault in unreinforced masonry and a demonstration of the power of the proposed method for design.



3. Selected References

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