TUTORIAL 1: FLYING BUTTRESS

setting up the parametric construction

1. Open Cabri Geometry II *Plus*.

Hints: - turn the Help *function on (press F1) for explanations on the commands and buttons*
- start a file by showing the axes (to have a horizontal / vertical reference)

2. Turn the axes on.

3. Place point 1 anywhere on the drawing area (this point will control the position of your construction on the screen).

Hint: if you type the name of the element you just created immediately after creation you give it a label. You can always label it afterwards.

4. Draw a line through point 1, parallel to the x-axis.

5. Hide the axes again

6. Draw point 2 on the parallel line through point 1.
(hover over the object you want to attach a point. It will show its name and color. notice that if you try to move point 2 it will only slide along the line)

creating the geometry of the flying buttress

We will create a parametric section of a generic flying buttress.

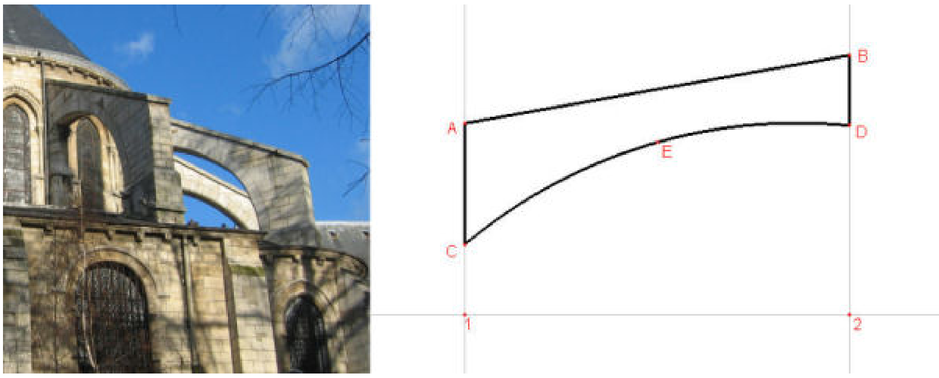


fig.1: Example of flying buttresses at Eglise Saint Germain-des-Prés in Paris, and a generic flying buttress geometry. (picture courtesy of M. Nikolinakou)

7. Draw two lines perpendicular to horizontal through points 1 and 2.

8. Create points A, B, C and D on perpendicular lines as shown in fig.1.
(we want point C never to be above point A ⇨ construct C on a ray starting from point A and going downwards)

> Show Axes

> Point

> Label

> Parallel Line

> Point on Object

> Perpendicular Line

> Ray

9. Draw the segments AB, AC, BD and CD.

10. Construct the perpendicular bisector between points 1 and 2.

11. Draw a segment between the intersections of this bisector and the segments AB and CD. Construct point E on this segment.

12. Draw an arc through the 3 points C, E and D.

> Segment

> Perpendicular Bisector

> Intersection

> Arc

Result: We now have a parametric drawing of a typical flying buttress (fig.1). By dragging points A to E, you can change its geometry.

Hint: to create clear constructions, you want to hide your construction lines and points and give color and thickness to your drawings.

> Color...

> Thick...

construction of the thrust line

The construction for the thrust line is using graphic statics.

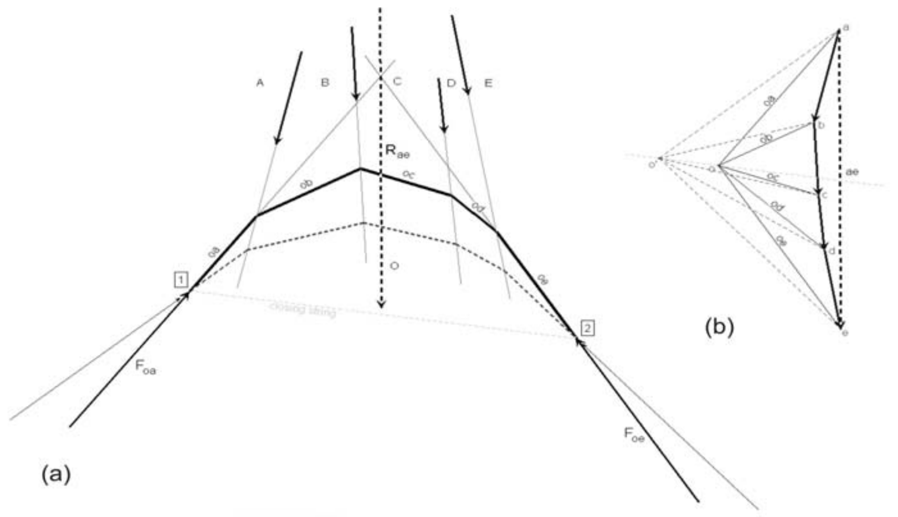


fig.3: Graphic statics is a method that allows the construction of funicular shapes (only tension or compression) for a certain set of loads (a), using Bow's Notation and a force polygon (b) that gives the magnitude of the forces of the segments in the funicular polygon (After Zalewski and Allen, 1998).

For an introduction to Graphic Statics, we refer to the book "Shaping Structures" by Allen & Zalweski and the website "Active Statics" by Greenwold & Allen.

We will now add this construction to our parametric drawing.

Hint: when dividing a structure in pieces for a graphical analysis, it is best to make cuts that represent the real structure the best, e.g. horizontal slices in a buttress (cf. Tutorial 2: Buttress) or radial slices in a semi-circular arch (cf. Tutorial 3: Arch on spreading supports).

To simplify the construction and because the error is small for a flying buttress, we will take vertical cuts.

13. Decide on how many divisions you want. The more pieces will -of course- result in a smoother thrust line. Let's take 10 pieces...

14. Measure the distance between points 1 and 2.


> Distance...

15. Use the calculator function to divide the distance and drag your result to the drawing plane.
(the result will be updated when the original distance is altered)


> Calculate

16. Use the compass function to transfer the measurement and draw vertical lines through the intersections as shown in fig.2.


> Compass

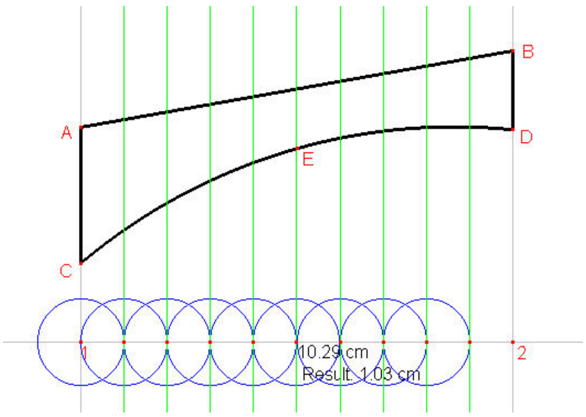


fig.2: Using compass function to transfer measurements.

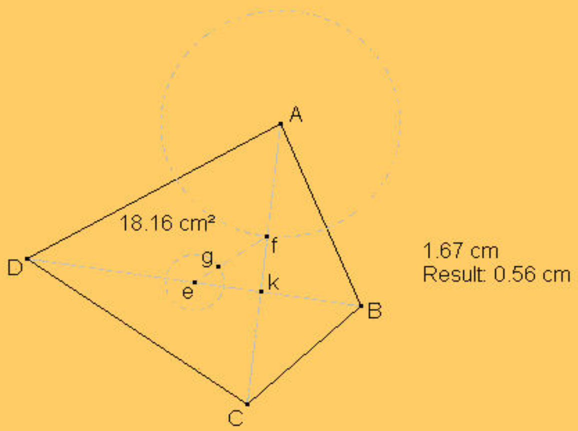
17. Make a macro to find the centroid and the area of the pieces.

Making the *centroid.mac* macro

This macro takes 4 points to construct a quadrilateral, find its centroid and returns its area.

1. Start a new file to make the macro.

construction to find the centroid (cf fig.)



2. Place the 4 vertices A, B, C and D and draw the quadrilateral using the Polygon function.


> Polygon

3. Connect the diagonals AC and BD.
4. Find the midpoint e of the segment BD.
5. Use the compass to find point f so that
   |Af | = |Ck|
6. Now find the centroid g using the measuring tool, calculator and compass, so that
   |ge| = 1/3.|fe|
7. Find the area of the quadrilateral.
8. Save this file to keep the construction.

<u>making of the macro</u>

9. Push the macro button and pick the initial objects, i.e. the 4 points.
10. Define what you want to output, i.e. the quadrilateral, the centroid and the area.
11. Make the macro by pushing the define macro button. If it works, the "Define Macro" window will pop up. Check the save to file option. Save it as e.g. *centroid.mac*.
(You can create your own button and add a Help file to describe what your macro does…)

<u>Reference centroid construction</u>

Wolfe, W.S. (1921) "Graphical analysis: a handbook on graphic statics." New York, McGraw-Hill Book Company.

> Midpoint

> Initial Object(s)

> Final Object(s)

18. If not already available after opening the flying buttress file, open the *centroid.mac* file to load the macro you just created.

19. Use the macro to find all the centroids of all the pieces and draw vertical lines through the centroids (cf. fig.3).

> Centroid Quadrilateral

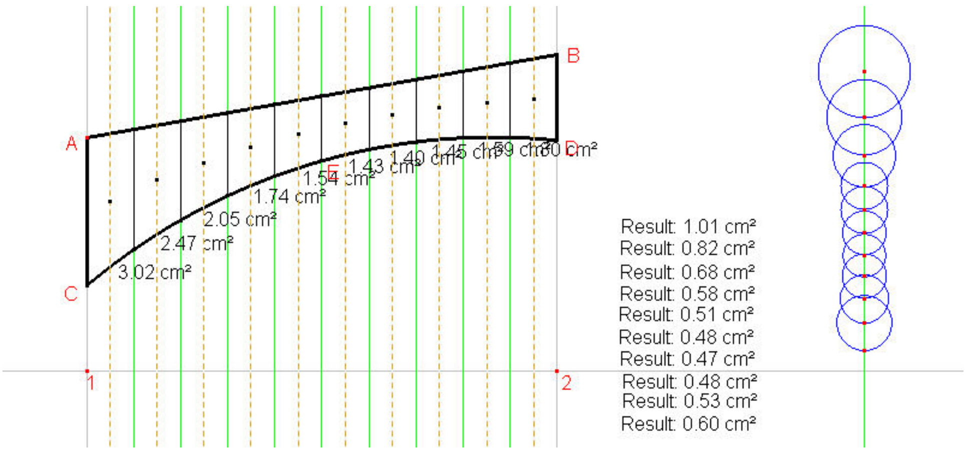


fig.3: Force action lines through centroids and load line.

20. Add a point and a vertical line randomly to start the load.

21. Scale (if too big for the drawing) the areas (representative for the weight of each piece) and transfer them to the load line using the calculator and compass (fig.3).

Hints: - think ahead of how you want to control/change the thrust line, because this will influence the construction method
- a thrust line always needs 3 element to define its position, e.g. 3 points, 2 points and 1 direction or 1 point and 2 directions.

Let's construct the thrust line in two different ways:

Option a: controlled by a point and the "pole" of the funicular polygon

a.1. Draw point o (arbitrary), the pole of the funicular polygon to the left of the load line

a.2. Draw the rays by connecting the pole to the scaled vectors on the load line with segments (fig.4).

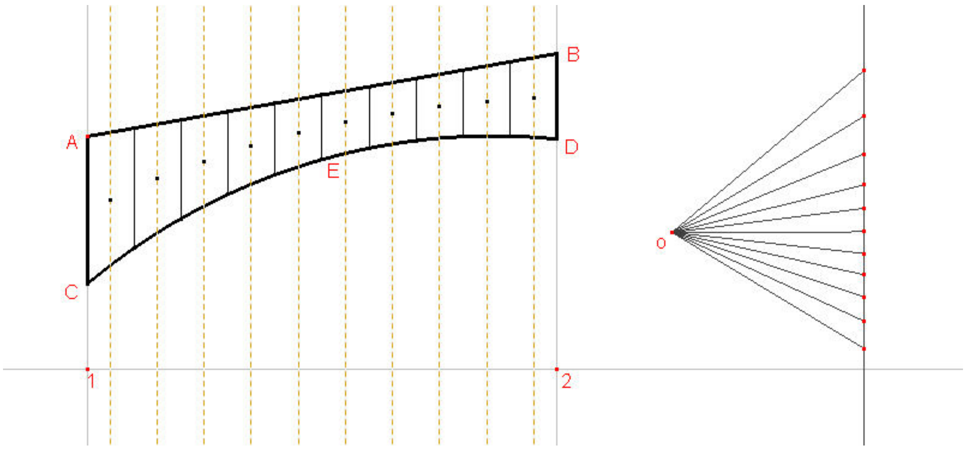


fig.4: Funicular polygon with arbitrarily chosen pole o.

a.3. Pick a starting point for the thrust line, i on segment AC.

a.4. Draw through point i a line parallel to the first ray (top one) of the funicular polygon.

a.5.Through the intersection point of this line and the vertical through the centroid of the first piece, draw a line parallel to the second ray.

a.6. Now, repeat this process until the end of the element (fig.5).

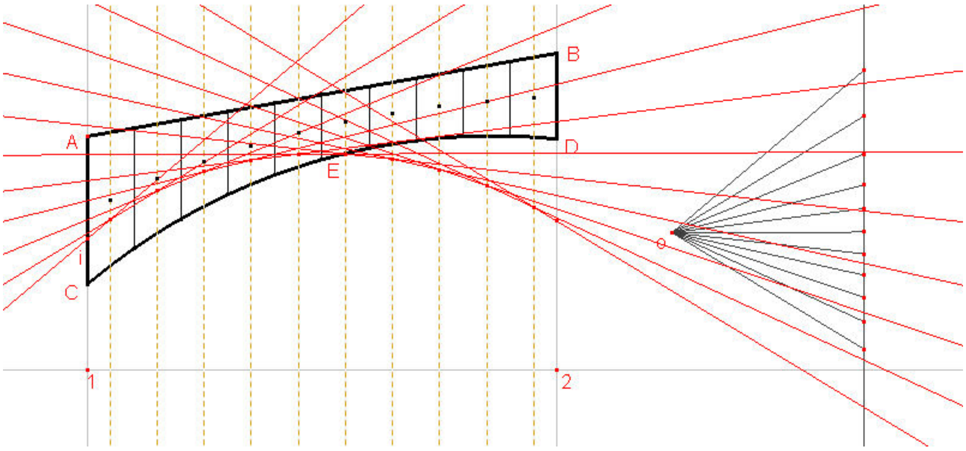


fig.5: Parallel lines to the rays construct the thrust line from the start point i.

a.7. Connect all the neighboring intersection points with segments and hide the parallel lines.

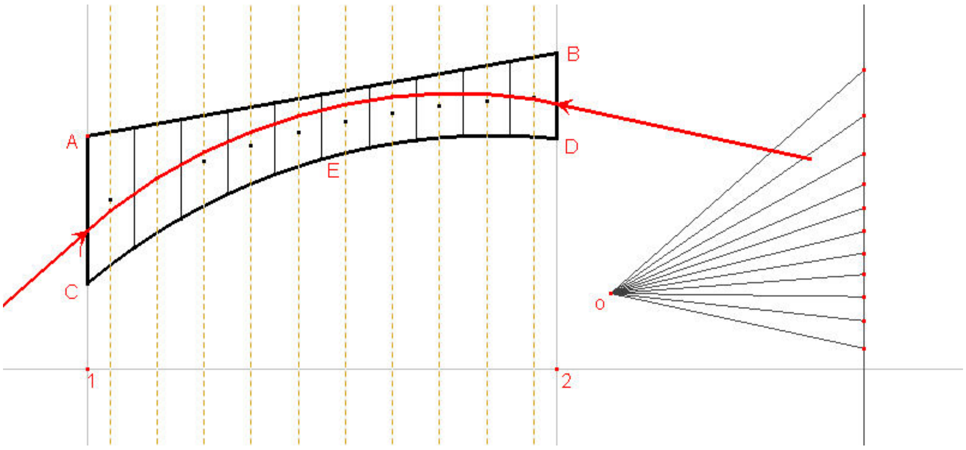


fig.6: Final model with interactive thrust line and end resultants.

a.8. Add the resultant reaction forces at the extremities. Their direction is given by the last ray and their magnitude can be read from the last rays from the funicular polygon. Use the compass function for this.

<u>Result:</u> We now have a parametric drawing of a typical flying buttress with a thrust line controlled by point i and the pole of the funicular polygon o (fig.6).

<u>Option b: controlled by two points and the initial slope</u>

b.1. The initial steps are the same as for option a. Do step a.1. to a.6.

b.2. The funicular construction we now have is only a “Helper construction” to find the line of action R of the resultant vector of the flyer. Find the intersection point r of the first and the last ray and the vertical through that point is R (fig.7).

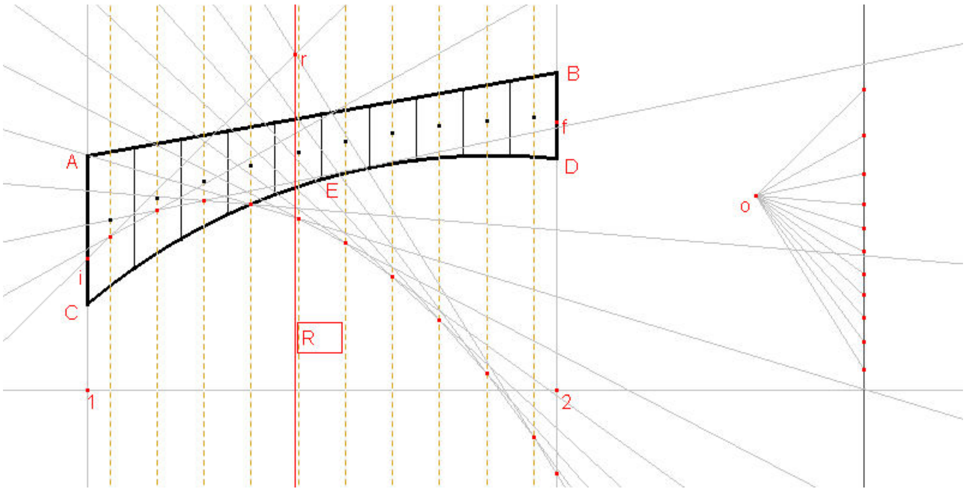


fig.7: Helper funicular construction to find the resultant of the flyer R.

b.2. Pick an end point f on the segment BD and choose an initial slope for the thrust line by drawing a line through i.

 > Line

b.3. Draw a line from f to the intersection r' of the line controlling the initial slope and the resultant R. This line is the slope at the end of the thrust line (fig.8).

b.4. Draw lines parallel lines to the initial and final slope lines through the top and the bottom point of the load line. Their intersection will be the actual pole 0.

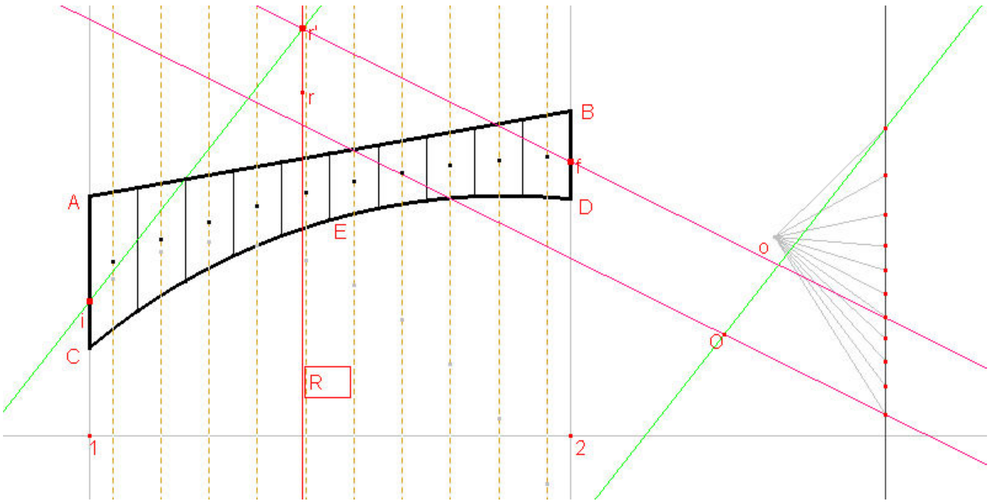


fig.8: Helper funicular construction to find the resultant of the flyer R.

b.5. Repeat step a.2., but connect to the new pole O now.

b.6. Repeat steps a.4. to a.8.

<ins>Result:</ins> We now have a parametric drawing of a typical flying buttress with a thrust line controlled by points i and f and its initial slope.

Any additional questions or suggestions, please contact Philippe Block (ph_block@mit.edu)