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Synthesis of precision serial flexure systems using freedom and constraint topologies (FACT)

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ABSTRACT

In this paper we introduce the principles necessary to synthesize the complete body of serial flexure system concepts, which satisfy desired design requirements using Freedom and Constraint Topologies (FACT). FACT utilizes a comprehensive library of geometric shapes that represent regions were constraints may be placed for synthesizing flexure systems that possess designer-specified degrees of freedom (DOFs). Prior to the theory of this paper, FACT was limited to the synthesis of parallel flexure systems only. The ability to synthesize serial flexure systems is important because serial flexure systems (i) may possess DOFs not accessible to parallel flexure systems, (ii) exhibit larger ranges of motion, and (iii) enable cancellation of parasitic errors. Geometric shapes that represent motions only accessible to serial flexure systems have been derived and added to the existing body of FACT shapes initially intended for parallel flexure synthesis only. Systematic rules and guidelines have been created that help designers use these shapes to generate every parallel and serial flexure concept that satisfies the desired functional requirements. We demonstrate how to use these shapes to utilize or avoid underconstraint in serial flexure synthesis. A serial flexure system is designed that interfaces the lead screw of a lathe to the carriage that it drives as a case study to demonstrate the theory of this paper.

1. Introduction

Flexure systems are important precision machine elements because they (i) are capable of performing with sub-nanometer repeatability and resolution, (ii) may be designed to possess complex kinematics and dynamics, (iii) are easily miniaturized, (iv) are relatively easy to fabricate and maintain, and (v) tend to be lower cost than their competitors. Flexure systems may be divided into three main categories—parallel, serial, and hybrid. Parallel flexure systems consist of a single rigid stage that is connected directly to ground by non-conjugated flexible elements. Serial flexure systems consist of a consecutive chain of parallel flexure system modules that are nested or stacked together. Hybrid flexure systems consist of a combination of parallel and serial flexure systems. Fig. 1 shows an example of a parallel, serial, and hybrid flexure system. Although the principles of this paper may be applied to the synthesis of hybrid flexure systems as well, the focus of this paper is primarily on the synthesis of serial flexure systems. For a detailed description of how FACT may be applied to the synthesis of parallel flexure systems see Hopkins [1–3].

Serial flexure systems are generally more difficult to design than parallel flexure systems. When designing a parallel flexure system, it is difficult to keep track of (i) the relative, three-dimensional orientations of the flexural constraints, (ii) the orientation of the permitted motions, and (iii) the three-dimensional relationships between each constraint and the permissible motions. When designing a serial flexure system, however, the preceding becomes more difficult as extra stages are stacked on top of each other. The reason for this is that the kinematics, elastomechanics, and dynamics of each stage are influenced by the kinematics, elastomechanics, and dynamics of the other stages in the serial chain.

The ability to synthesize serial flexure systems is important to precision engineers because serial flexure systems possess advantages not had by parallel flexure systems. Some of these advantages include the following:

(1) Serial flexure systems are capable of possessing DOFs that are not possible for parallel flexure systems to possess. A flexure system, for instance, that permits only three independent translations, i.e., x–y–z, is only possible for a serial flexure system. The reason for this fact is that the moment a single flexible constraint is connected directly from a stage to ground, the stage looses one of its three translations along the constraint's axis. Parallel flexure modules must therefore be stacked in series to achieve three translations.

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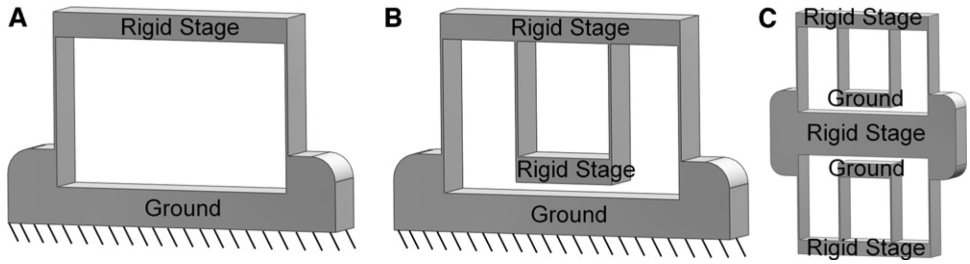

Fig. 1. Example of a parallel (A), serial (B), and hybrid (C) flexure system.

(2) Serial flexure systems may be designed to possess larger ranges of motion than same-size parallel flexure systems. Note that the serial flexure system from Fig. 1B is the same size as the parallel flexure system from Fig. 1A, but the serial flexure system's stage is capable of moving twice the distance as the parallel flexure system's stage.
(3) Serial flexure systems may be designed to reduce parasitic errors. The parallel flexure system's stage from Fig. 1A does not possess a perfect translational DOF. The stage follows an arc-like path over its range of motion. The stage of the serial flexure system from Fig. 1B moves along a "straighter" path because the vertical components of the opposing arc motions of the two rigid stages cancel.

Others have attempted to tackle the challenges of synthesizing complex flexure systems. The three most common approaches for designing flexible mechanisms include constraint-based design (CBD), the pseudo-rigid body model (PRBM), and topological synthesis. Constraint-based designers have developed rules of thumb for synthesizing flexure systems using well known flexural elements as building blocks [4–6]. Unfortunately, the principles of CBD are typically acquired after years of apprenticeship and experience. Constraint-based designers rely on good motion visualization and pattern recognition skills. As such, most precision flexure systems designed using CBD are limited to simple, intuitive designs that possess kinematics, which are possible to visualize. Pseudo-rigid body modeling models compliant mechanisms as analogies to rigid-link mechanisms [7–9]. The rigid analog is then modeled using pre-existing rigid mechanism theory and the principle of virtual work to ascertain its kinematic and elastomechanic properties. The PRB model has been used to design precision elastic mechanisms [10,11] and many consumer products. The primary aim of PRBM is to model, rather than synthesize, and so it is not ideally suited to solve the problems that are the target of this paper. Topological synthesis is based upon computer algorithms that examine a starting shape for a compliant mechanism and then determine how to add/subtract material in order to create concepts that satisfy performance specifications [12,13]. In this method, the computer makes the design decisions that determine the layout of the rigid and flexible elements. This approach is effective for rapid synthesis of non-precision compliant mechanisms for robotics, MEMS and aeronautics. Unfortunately, topology synthesis is not readily applied to solve most precision flexure design problems. The reason for this is that the knowledge needed to integrate the specialized precision design rules with topological synthesis does not currently exist. Furthermore, many of the concepts generated using topological synthesis are difficult to fabricate and implement because the designer's common sense has no influence on what the computer generates. The approach introduced in this paper is, therefore, most suited for the design of precision flexure systems of any complexity.

In this paper we demonstrate how a specialized set of mathematically based geometric shapes, like the one shown in Fig. 2, may be used to optimally synthesize serial flexure systems for a given set of specifications—kinematics, range, load capacity, thermal stability, etc. The geometric shapes used to design such systems – called freedom and constraint spaces – embody the rigorous mathematics of screw theory and help designers visually identify all the possible regions where constraints may be located/oriented in order to achieve the system's desired degrees of freedom (DOFs). In this way, novice or experienced designers may rapidly consider all the possible flexure concepts before selecting the concept that best satisfies the system's functional requirements.

The process that utilizes these geometric shapes is called Freedom and Constraint Topologies (FACT) [1–3,14–16]. For this paper, design principles and best-practices have been created and integrated into the FACT design process, which enable the synthesis of

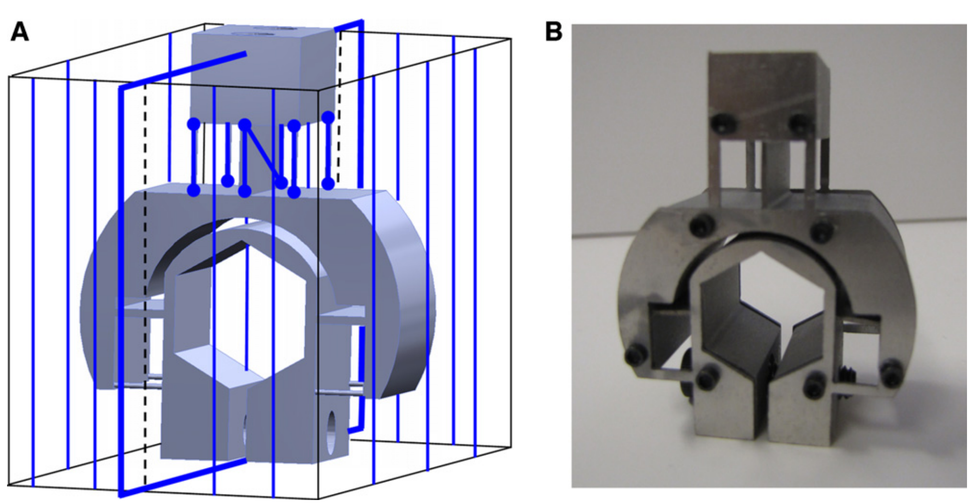

Fig. 2. Example of a geometric shape (A) from which designers may select constraints for synthesizing a serial flexure system (B).

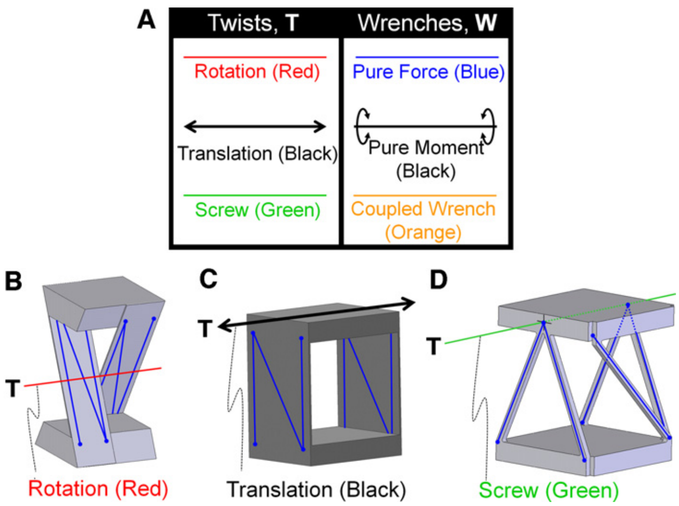

Fig. 3. Twist and wrench types (A), rotation twist (B), translation twist (C), and screw twist (D).

serial flexure systems. These rules guide designers in identifying serial concepts that (i) possess symmetry, (ii) are thermally stable, (iii) utilize or avoid over and underconstraint, and (iv) possesses the desired system kinematic, stiffness, and dynamic characteristics.

This paper applies to small-motion kinematics only. This assumption is fitting as we are designing precision flexure systems that move small amounts compared to their overall size. Despite this assumption, however, this paper does provide designers guidelines for synthesizing serial flexure systems that achieve their desired motions over a large/finite range due to symmetry or parasitic error cancellation.

2. Fundamental principles

2.1. Modeling DOFs and flexible constraints

To understand flexure synthesis, it is necessary to first understand how to model DOFs and the flexible constraints that enable them. All small motions may be modeled as a 6×1 vector called a twist, **T** [17–20]. A twist may be visualized as a line about, or along, which a stage rotates and/or translates. There are three types of twists as shown in Fig. 3A—rotations, translations, and screws. In this paper rotations are represented as red lines, translations are represented as black arrows, and screws are represented as green lines. If a flexure possessed a rotational DOF like the one shown in Fig. 3B, the flexure's stage could freely rotate about the axis of the red line. If a flexure possessed a translational DOF like the one shown in Fig. 3C, the flexure's stage could freely translate along the axis of the black arrow. If a flexure possessed a screw DOF like the one shown in Fig. 3D, the flexure's stage could freely translate along the axis of the green screw line as the stage simultaneously rotates about the same axis according to the pitch of the screw. The pitch of a screw is defined as the ratio of the stage's translation to its rotation.

All flexible constraints may be modeled using a 6×1 vector called a wrench, **W** [17–20]. A wrench may be visualized as a line along or about which a force and/or moment act. There are three types of wrenches as shown in Fig. 3A—pure forces, pure moments, and coupled force and moment wrenches. In this paper pure forces are represented as blue lines, pure moments are represented as black lines with circular arrows, and coupled force and moment wrenches are represented as orange lines. It is important to note that only pure force wrenches (blue lines) model flexible constraints. If a flexible constraint is long and slender, like those shown in Fig. 3D, a single pure force wrench oriented along the constraint's axis accurately models the constraint. If the flexible constraint is a thin blade flexure, like those shown in Fig. 3B and C, pure force wrenches that lie on the plane of the blade flexure may accurately model the constraint. At least three pure force wrenches that (i) lie on the plane of the blade flexure, (ii) are not parallel, and (iii) do not intersect at a common point are necessary to accurately model a blade flexure.

2.2. Freedom and constraint spaces

The parallel flexure systems shown in Fig. 3 are single DOF systems. Consider the 2 DOF parallel flexure system shown in Fig. 4. This flexure system is capable of guiding two independent rotational DOFs as shown in Fig. 4A and B. If these two independent rotations are simultaneously actuated and the relative ratio of their

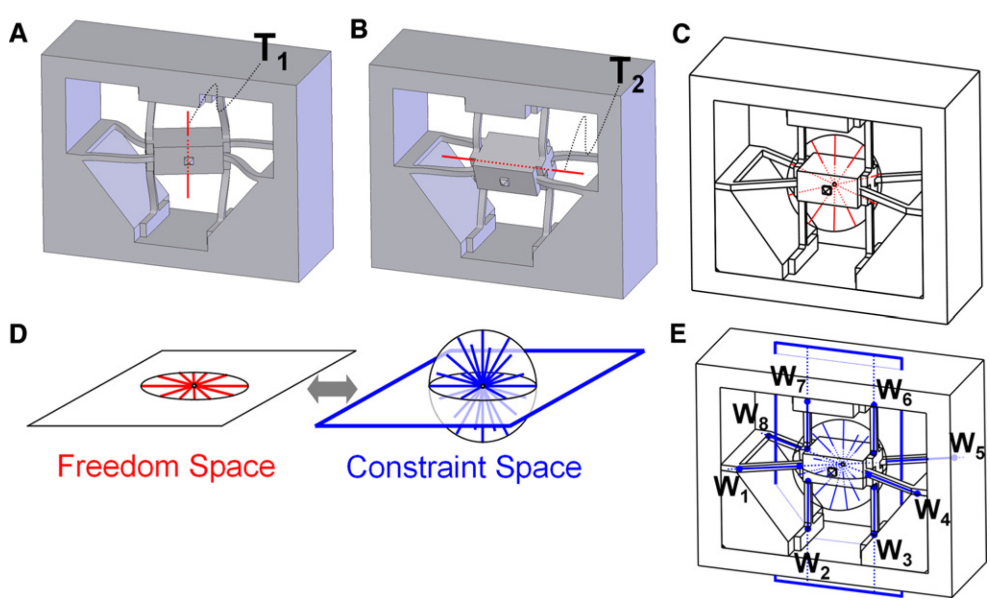

Fig. 4. Example with two rotational DOFs (A) and (B). The system's freedom space is a disk of rotations (C). Freedom and constraint space pair (D). Every constraint lies in the constraint space (E).

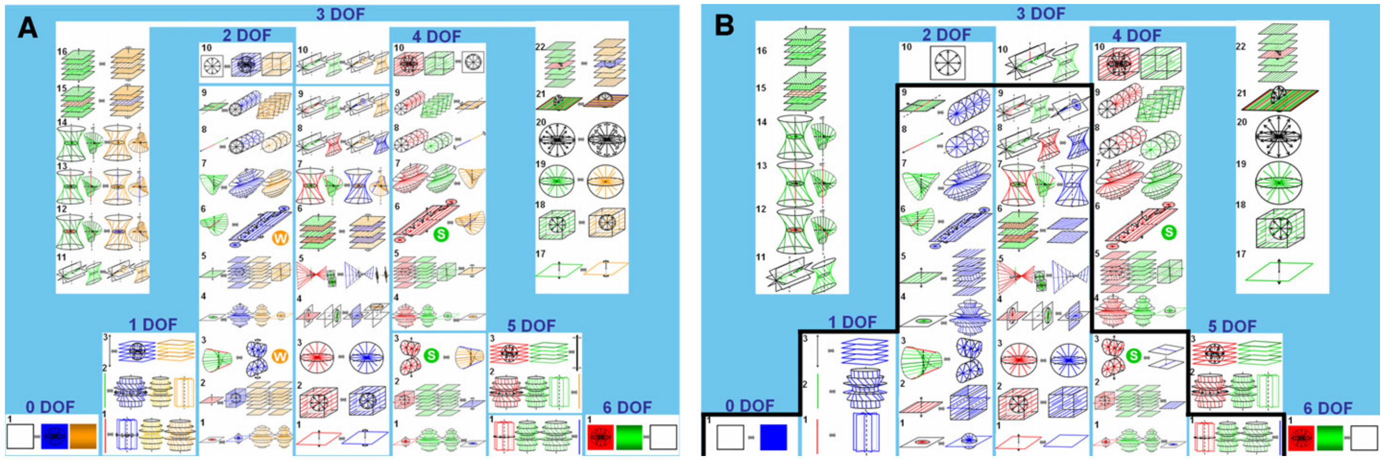

Fig. 5. Symmetric FACT chart with all twist and wrench types (A). Practical FACT chart for flexure synthesis (B).

rotations is controlled, the resulting rotation line may be any of the red lines within the disk or pencil shown in Fig. 4C. This disk is the system's freedom space [1–3,14–16]. Freedom space is the geometric shape that visually represents a system's kinematics, i.e., all the twists the flexure permits.

All freedom spaces uniquely link to a complementary constraint space [1–3,14–16]. Constraint space is the geometric shape that visually represents the regions where the flexible constraints belong for enabling the system's desired DOFs. The freedom and constraint space pair of the 2 DOF parallel flexure system of our example is shown in Fig. 4D. The disk-shaped freedom space of red rotation lines uniquely links to a constraint space that consists of every pure force wrench that lies on the disk's plane and every pure force wrench that intersects the disk's central point. These wrenches are represented by the plane and the sphere of blue lines shown on the right side of Fig. 4D. Note that the parallel flexure system's eight slender flexible constraints all belong inside this constraint space as shown in Fig. 4E. The pure force wrenches $\mathbf{W_1}$, $\mathbf{W_4}$, $\mathbf{W_5}$, and $\mathbf{W_8}$ belong to the sphere and the pure force wrenches $\mathbf{W_2}$, $\mathbf{W_3}$, $\mathbf{W_6}$, and $\mathbf{W_7}$ belong to the plane of the constraint space. Every parallel flexure system that possesses the 2 DOFs shown in Fig. 4A and B will be constrained by flexible constraints that lie within the constraint space of Fig. 4E. Note, however, that not every combination of flexible constraints that lie within this space will produce the desired DOFs. If, for instance, constraints were selected only from the sphere of the constraint space, the stage would possess extra DOFs. To exclusively achieve the two desired DOFs, four independent pure force wrenches must be selected from within the constraint space. Geometric shapes called sub-constraint spaces have been created for helping designers consider every way independent pure force wrenches may be selected from within a system's constraint space to assure the desired system kinematics and to control constraint redundancy. For more detail on selecting appropriate flexible constraints from within a particular constraint space see Hopkins [1–3].

The content of this section is fundamental to the FACT synthesis process. If a designer (i) were able to identify the freedom space that represents a desired set of DOFs and (ii) knew to which complementary constraint space that freedom space uniquely links, he/she would be able to use that constraint space to conceptualize every parallel flexure system that possessed those DOFs.

3. FACT chart

Interestingly, there are only 50 freedom and constraint space pairs called types. These types are described in detail and derived in Hopkins [3,21], and are shown in Fig. 5. All claims that pertain to the comprehensive nature of this chart stem from the derivations found in these references. The chart from this figure contains a lot of information that the reader is not yet expected to fully understand at this point in the paper. Notice, however, that all the types belong to one of seven columns. Each column pertains to the number of DOFs the type's freedom space possesses. Within each column, the freedom and constraint space pairs are labeled with type numbers. The freedom space of each type is shown to the left of a small, grey, double-sided arrow in the middle of each column and the constraint space of the same type is shown to the right of the same arrow. The freedom and constraint spaces shown in Fig. 5A include all types of twists and wrenches defined in Fig. 3A. The chart from Fig. 5A is a visual representation of all screw systems [22–35] with their complementary spaces. Although much work has been done to classify and complete screw systems [36–41], this chart provides a comprehensive classification of screw systems that is suited for flexure synthesis. Note that the spaces within the chart are symmetric about the 3 DOF column. This means that the freedom spaces within the n DOF column are identical to the constraint spaces within the $6-n$ DOF column. The proof for this symmetry is found in Hopkins [3].

Recall from Section 2.1 that flexible constraints may only be modeled using pure force wrenches. Thus the chart from Fig. 5 is only useful to flexure designers if the constraint spaces (i) contain only pure force wrenches (blue lines), and (ii) possess enough independent, pure force wrenches to produce the correct complementary freedom space. In other words, if a system's freedom space possesses n DOFs, its constraint space must possess $6-n$ independent pure force wrenches or it is not a useful constraint space. Thus, the practical FACT chart that contains only these useful constraint spaces is shown in Fig. 5B. The thick black line that separates the types with constraint spaces from the types without constraint spaces is called the "parallel pyramid". Parallel flexure systems may only possess freedom spaces that lie within the parallel pyramid. All flexure systems that possess freedom spaces that lie outside of this pyramid may only be synthesized by stacking parallel flexure modules in series from constraint spaces within the parallel pyramid. The reason for this fact is that the freedom spaces that lie outside of the pyramid do not have constraint spaces from which the designer may select constraints. Note that the freedom and constraint space pair shown in Fig. 4D is the first type at the bottom of the column labeled 2 DOF in Fig. 5B and that the pair lies within the parallel pyramid. Note also that the freedom space that contains only three pure translations (sphere of black arrows) is the Type 20 in the column labeled 3 DOF and lies outside of this pyra-

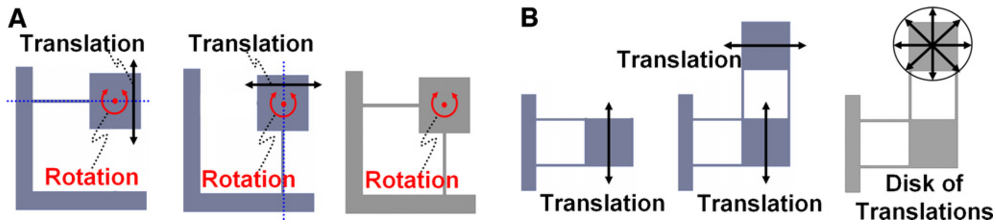

Fig. 6. Flexural elements in parallel (A) and in series (B).

mid. This freedom space is only possible for a serial flexure system to possess. The completion and presentation of the screw systems suited for synthesizing parallel and serial flexure systems are major contributions of this paper.

4. Serial synthesis principles

This section discusses the principles necessary to synthesize serial flexure systems using geometric shapes as tools. The principles that govern underconstraint and kinematic equivalence are also established.

4.1. Parallel and serial constraint fundamentals

According to constraint-based design, flexures connected in parallel retain the DOFs that the individual constraints share in common, whereas flexures connected in series will possess the DOFs of the individual constraints combined [4]. Each constraint shown in the left and center images of Fig. 6A permits the two-dimensional stage to possess a rotational DOF and a translational DOF that is perpendicular to the axis of the constraint. If these constraints are combined in parallel, the stage retains only the common rotational DOF shown on the right side of Fig. 6A. If the translational DOF parallel flexure module shown in Fig. 6B is stacked in series with itself, not only does the final stage inherit each module's translational DOF, but it also inherits every combination of those translational DOFs. The stage shown in Fig. 6B may, therefore, move with any translation in the plane of the flexure as represented by the disk of arrows. In other words, when a parallel flexure system that possesses the Type 3 freedom space from the 1 DOF column of Fig. 5B is stacked in series with another parallel flexure system that possesses the same freedom space oriented in a different direction, the resulting serial flexure system's stage possesses the Type 10 freedom space from the 2 DOF column of Fig. 5B.

Identifying which freedom space results by combining the freedom spaces of various parallel flexure systems stacked in series can be difficult. Consider the serial flexure system shown in Fig. 7. This system consists of two parallel flexure systems stacked in series. If the rigid stage labeled 3 were grounded, the freedom space of the rigid stage labeled 2 would be the Type 2 freedom space from the 2 DOF column of Fig. 5B. This freedom space, shown in Fig. 7A, consists of a plane of parallel red rotation lines and a perpendicular translation depicted as a black arrow. If stage 3 were grounded, therefore, the flexible constraints of the first parallel flexure system would permit stage 2 to rotate about any line on the plane shown in Fig. 7A and translate in the direction normal to this plane. If, however, the rigid stage labeled 2 were grounded, the freedom space of the rigid stage labeled 1 would be the same freedom space as before but rotated 90 degrees as shown in Fig. 7B. In other words, if stage 2 were grounded, the flexible constraints of the second parallel flexure system would permit stage 1 to rotate about any line on the plane shown in Fig. 7B and translate in the direction normal to this plane. Suppose we now grounded the rigid stage labeled 3 but left the rigid stages labeled 2 and 1 free to move. If we wished to find the new freedom space of stage 1, we would expect a freedom space that inherits the rotation lines and translation arrows from both freedom spaces shown in Fig. 7A and B as well as all the motions that would result from combining these two freedom spaces just as the two translation arrows combined to form the disk of arrows from Fig. 6B. This new freedom space, shown in Fig. 7C, contains every red rotation line that lies in an infinite stack of disks or pencils that lie on planes that are parallel to the face of the flexure (only two of these red disks are shown in the figure). The central point of each disk may be intersected by a single dashed line that is normal to the planes of the disks. A disk of translational arrows that point in directions perpendicular to this dashed line also exists in the freedom space and is shown in Fig. 7C. This freedom space is the Type 8 freedom space from the 4 DOF column of Fig. 5B. Note from the right side of this freedom space in Fig. 5B that green screw lines would also exist, which are collinear with the red rotation lines in the disks. These screw lines were not shown in Fig. 7C to avoid visual clutter. They result from the fact that the stage may rotate around the axes of each red rotation line while simultaneously translating along their axes in the direction of the black arrows. If it is difficult to visualize how the freedom space shown in Fig. 7C results from the combination of the freedom spaces shown in Fig. 7A and B, a mathematical approach can be used to generate the space. The freedom space of Fig. 7C results from the linear combination of the independent twists that describe the freedom spaces of Fig. 7A and B. We will discuss these principles in greater detail in later sections and will demonstrate how these principles apply to serial flexure system synthesis.

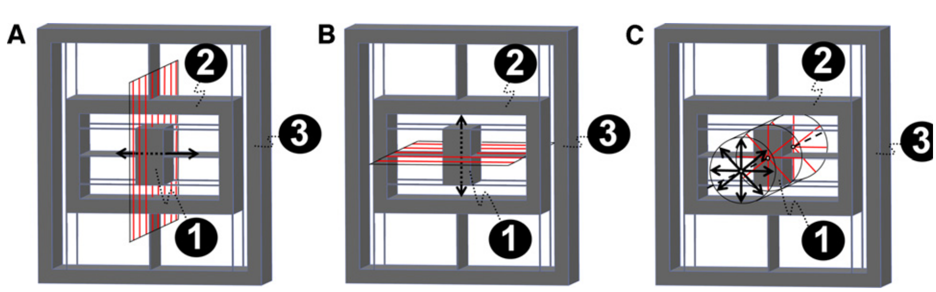

Fig. 7. The freedom spaces (A) and (B) of each parallel flexure module combine to form the freedom space (C) of the serial flexure system.

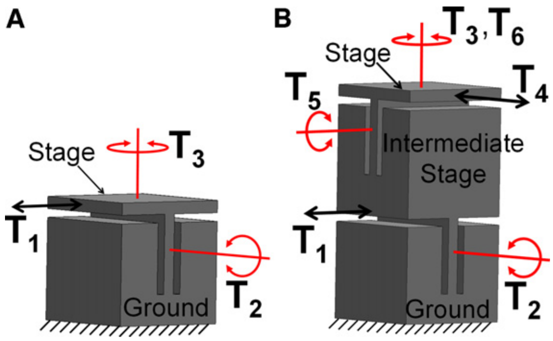

Fig. 8. 3 DOF parallel flexure system (A). 5 DOF stacked serial flexure system (B).

4.2. Underconstraint

Consider the 3 DOF parallel flexure system constrained by the flexure blade shown in Fig. 8A. The system's stage is permitted to move with a translation and two rotations as depicted by the three twists that are shown in the figure. If this parallel flexure module were stacked on top of itself as shown in Fig. 8B, one might expect the new stage to possess 6 DOFs—3 from each module. Although the stage inherits the DOFs of both parallel modules as shown in Fig. 8B, the stage only possesses 5 DOFs because the rotations labeled $\mathbf{T_3}$ and $\mathbf{T_6}$ are redundant meaning that both parallel modules possess the same DOF. When a serial flexure system possesses one or more redundant DOFs, the system will possess intermediate stages that are underconstrained. Such intermediate stages still possess the redundant DOFs even when the ground and stage of the serial flexure system are held fixed. Consider, for example, holding the stage and ground of the serial flexure system from Fig. 8B fixed. The intermediate stage labeled in the figure would remain free to rotate about the axis of the redundant rotation labeled $\mathbf{T_3}$ and $\mathbf{T_6}$. We will later show how geometric shapes may be used in conjunction with the principles of this section to avoid or utilize underconstraint in the design of serial flexure systems.

Advantages and challenges are associated with flexure systems that possess underconstrained elements. Parallel modules stacked in series like those shown in Figs. 1B and 8B achieve a greater range of motion than the individual parallel modules alone because each module contributes to the full stroke of the final stage. A properly underconstrained system may, therefore, markedly increase a flexure's stroke to size ratio. Unfortunately, underconstrained flexure systems generally have poor dynamic characteristics. Reducing the mass of the intermediate stages and stiffening the flexible elements that connect them together helps mitigate this problem.

4.3. Intermediate spaces

Consider the planar constraint space of the parallel flexure module from Fig. 8A. This constraint space is shown with a different orientation in Fig. 9A. Note that the constraint lines (i.e., pure force wrench lines) of the flexure blade shown in Fig. 9B lay on the plane of the system's constraint space. The constraint space belongs to the first type in the 3 DOF column of Fig. 5B. The constraint space's complementary freedom space, shown in Fig. 9A, consists of all rotational lines that lie on the plane of the constraint space as well as a translation perpendicular to this plane. Notice from Fig. 9B that the three DOF twists shown in Fig. 8A belong to the freedom space of the system. If the parallel flexure module is stacked in series with itself as shown in Fig. 9C, the resulting freedom space of the serial stage not only possesses twists from the planar freedom spaces of each individual parallel flexure module, but it also possesses the twists that result from the combination of these motions according to the principles discussed in Section 4.1. The freedom space

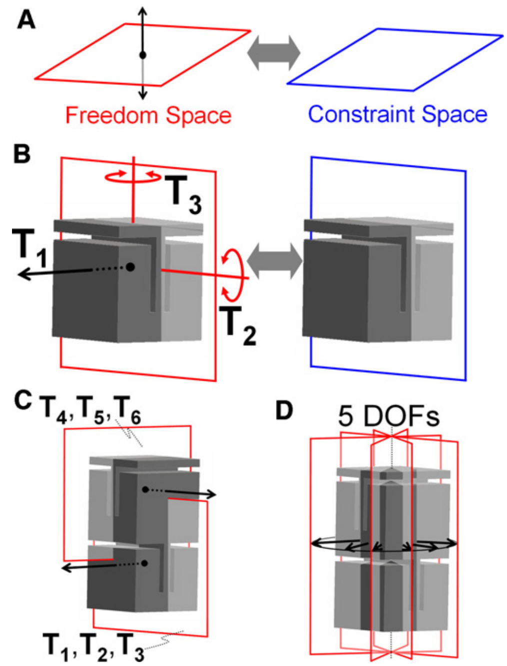

Fig. 9. 3 DOF Type 1 freedom and constraint spaces (A). Spaces imposed on the 3 DOF parallel flexure module (B). Intermediate freedom spaces (C) sum together to produce the 5 DOF freedom space of the serial flexure system (D).

of the serial flexure system's stage, therefore, contains (i) every rotational line that lies on the planes that intersect the dotted line shown in Fig. 9D and (ii) every translation that is perpendicular to the same line. Note that the two planar freedom spaces of the serially stacked parallel flexure modules lie within this freedom space. These two freedom spaces that combine to form the serial flexure system's freedom space are called intermediate freedom spaces. These spaces each contain three independent twists that are labeled in Fig. 9C. If the three independent twists from one of these planar freedom spaces were simultaneously actuated with different magnitudes, their combined effect would behave as other twists that lie within the same planar freedom space. If, however, the three independent twists from each of the two planar freedom spaces, labeled in Fig. 9C as $\mathbf{T_1}$ through $\mathbf{T_6}$, were combined and simultaneously actuated with different magnitudes, they would behave as other twists that lie within the full freedom space of Fig. 9D. If Gaussian elimination were performed with these six twists, only five of them would be shown to be independent. It is not surprising therefore, that the combined freedom space shown in Fig. 9D belongs to the first type in the 5 DOF column of the chart from Fig. 5B because the system possesses only five DOFs.

The intermediate freedom spaces from Fig. 9C reveal why this serial flexure system possesses an underconstrained element. The reason is that each intermediate freedom space individually possesses three DOFs, whereas the combined freedom space of the system only possesses five DOFs leaving one DOF to be redundant. If, therefore, the sum of the number of DOFs of each intermediate freedom space is more than the number of DOFs of the system's freedom space, the serial flexure system will possess underconstrained elements.

4.4. Kinematic equivalence

The freedom and constraint spaces of the serial flexure chain from Fig. 9D are shown as Type 1 in the 5 DOF column of the chart from Fig. 5B and again in Fig. 10A. Note that the freedom space not only contains rotation lines and translation arrows, but it also

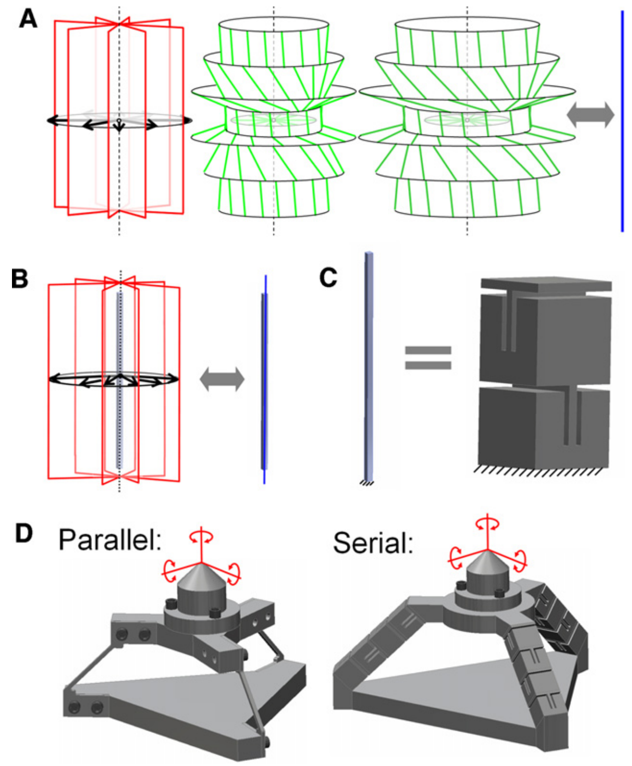

Fig. 10. 5 DOF Type 1 freedom and constraint space (A). Spaces imposed on the 5 DOF wire flexure (B). Wire flexure and serial flexure chain are kinematically equivalent (C). Two kinematically equivalent flexure systems (D).

contains screws. The shapes that describe the locations and orientations of these screws are described in detail in Hopkins [21]. The constraint space of the system is a pure force wrench line that is collinear with the line of intersection of the planes of rotations from the freedom space. Note that a single wire flexure also possesses the same freedom and constraint spaces as shown in Fig. 10B. A wire flexure is a long, slender flexible constraint. The wire flexure and the serial flexure chain from Fig. 10C are said to be kinematically equivalent because they possess the same freedom space. The parallel and hybrid flexure systems shown in Fig. 10D are also kinematically equivalent because the wire flexures of the parallel flexure system were replaced with the kinematically equivalent flexure chains of the hybrid flexure system. Although kinematically equivalent flexure elements may be substituted without changing a system's DOFs, the system's elastomechanics, dynamics, and manufacturability may be altered to satisfy desired design requirements.

5. FACT design process

This section describes the 6 steps of the FACT design process shown in Fig. 11 that is used to synthesize parallel and serial precision flexure systems. A lead screw serial flexure system will be designed using FACT.

Step (1): identify desired motions: The designer must first recognize which DOFs the system should possess and which directions should be constrained.

Step (2): identify freedom space: The designer must then identify the freedom space that embodies the DOFs that were specified in Step (1). This freedom space will belong to the column from Fig. 5B that pertains to the number of DOFs the system should possess.

Step (3): parallel or serial: The designer must then decide whether to synthesize a parallel or a serial flexure system. If the freedom space identified in Step (2) does not belong inside the parallel pyramid from Fig. 5B, then the designer must synthesize a serial flexure system to achieve the desired DOFs. If, however, the freedom space does belong inside the parallel pyramid, then parallel or serial concepts exist that achieve the desired DOFs.

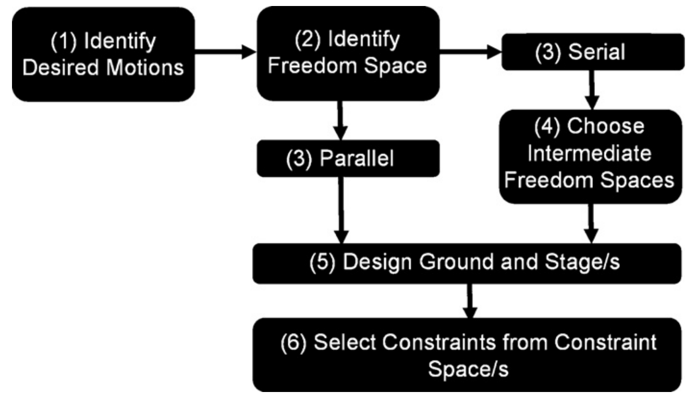

Fig. 11. Steps of the FACT design process for parallel and serial flexure systems.

Step (4): choose intermediate freedom spaces: If the designer chooses to synthesize a serial flexure system, intermediate freedom spaces must be selected. The intermediate freedom spaces must exist in the chart of Fig. 5B to the left of the column that contains the selected freedom space because the intermediate freedom spaces must exist within the freedom space. The intermediate freedom spaces must also belong within the parallel pyramid. Designers may select any number of viable intermediate freedom spaces as long as the total number of independent twists from all of the selected intermediate freedom spaces combined equals the number of DOFs within the freedom space selected in Step (2). An intermediate freedom space may be selected multiple times. The number of intermediate freedom spaces determines the number of rigid stages or parallel flexure modules the flexure system will possess. The fewer the stages, the less complex the design, the easier to fabricate and assemble, and the better will be the dynamic characteristics. Serial flexure systems require a minimum of two intermediate freedom spaces. Intermediate spaces that possess orthogonal features generally produce designs that are easily fabricated. Appendix A provides a complete list of every freedom space's intermediate freedom spaces.

Step (5): design ground and stage(s): The ground and rigid stages must then be designed. If the designer chose to synthesize a parallel flexure system, only one stage should be designed. If the designer chose to synthesize a serial flexure system, the number of stages should equal the number of intermediate freedom spaces that were selected from the previous step. The rigid stages should be far enough away from each other that they do not collide as they move.

Step (6): select constraints from constraint space(s): If the designer is synthesizing a parallel flexure system, he/she must select constraints from the constraint space of the system's freedom space. These constraints must connect the ground to the rigid stage. If the designer is synthesizing a serial flexure system, constraints from the constraint space of the first intermediate freedom space must be selected such that they connect the ground to the first intermediate rigid stage. Then constraints from the constraint space of the second intermediate freedom space must be selected such that they connect the first intermediate rigid stage to the second intermediate rigid stage. This process continues until constraints have been selected from the constraint spaces of every intermediate freedom space and all the stages have been stacked together to form the full serial flexure chain.

It is important that a suitable number of non-redundant constraints are selected from each freedom space's constraint space such that the rigid stage possesses the desired DOFs. If a freedom space contains n DOFs, $6-n$ non-redundant constraints should

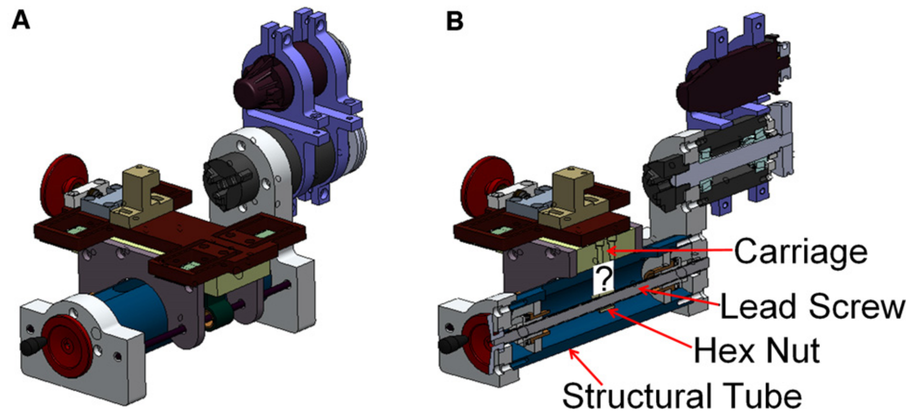

Fig. 12. Desktop lathe (A) and cross-section (B).

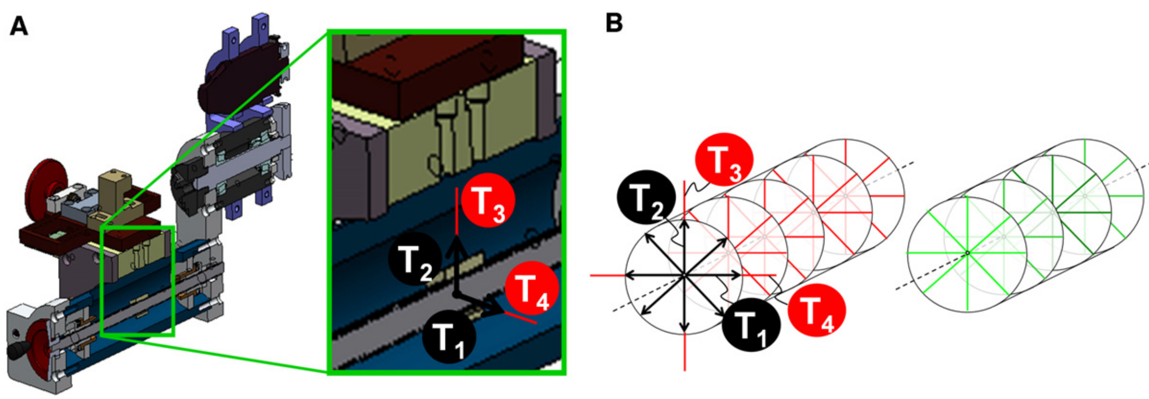

Fig. 13. The desired motions that should be imparted on the hex nut by the flexure system (A) and the freedom space of those desired motions (B).

be selected from the freedom space's complementary constraint space. Not all constraints selected from the constraint space will be non-redundant. The designer could select $6-n$ constraints from the constraint space and then apply Gaussian elimination to ensure that the wrenches of the constraints selected are all independent. Or the designer could use the constraint space's sub-constraint spaces to ensure constraint independence. Every constraint space possesses a certain number of sub-constraint spaces that guide the designer in selecting independent constraints. Every sub-constraint space for any constraint space is derived and described in Hopkins [3].

5.1. Lead screw flexure case study

In this section, a serial flexure system will be designed using the steps of the FACT design process. Consider the desktop lathe shown in Fig. 12A. A cross-section of the lathe is shown in Fig. 12B. A mechanism must be designed that transforms the lead screw's rotation into the carriage's translation. If the hex nut on the lead screw were rigidly attached to the carriage, the carriage would stick and slip as it translates along the bearing rails because multiple elements of the system would be constraining the same DOFs. A flexure system should be designed, therefore, that stiffly links the hex nut to the carriage in certain directions while allowing for compliance in other directions. In this way, imperfections in the lead screw's geometry and alignment are accommodated.

Step (1): identify desired motions: The flexure system should be stiff along the axis of the lead screw such that the carriage simultaneously translates with the hex nut. The rotational motion about the lead screw's axis should also be stiff such that the friction between the hex nut and threads of the lead screw is overcome as the lead screw rotates. This avoids errors that manifest as backlash. The other two translations and two rotations perpendicular to the axis of the lead screw, shown in Fig. 13A, should be as compliant as possible to accommodate for imperfections in the lead screw's straightness and alignment. The four DOFs shown in Fig. 13A are, therefore, the desired motions of the flexure system.

Step (2): identify freedom space: The freedom space that contains these desired motions belongs to the 4 DOF column of the chart from Fig. 5B and is Type 8. This freedom space and the desired motions are shown in Fig. 13B. This freedom space was described in detail in Section 4.1 and depicted earlier in Fig. 7C. The system's screws were not shown in Fig. 7C but are shown on the right side of Fig. 13B as green disks. The two dashed lines shown in Fig. 13B are collinear.

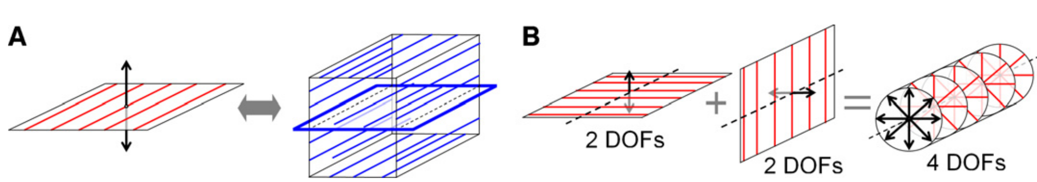

Fig. 14. Freedom and constraint space of 2 DOF Type 2 (A). Two intermediate freedom spaces sum together to generate the freedom space of the system (B).

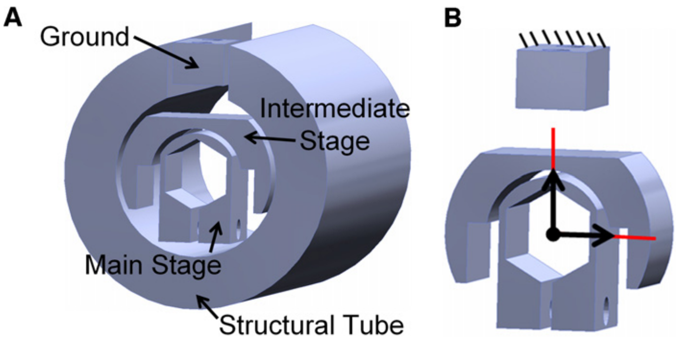

Fig. 15. Geometric constraints shape the ground and stage designs (A). The ground and stages shown with the desired DOFs (B).

Step (3): parallel or serial? Note that the freedom space of 4 DOF Type 8 from the chart of Fig. 5B lies outside the parallel pyramid and has no complementary constraint space. In this case, therefore, the designer has no choice but to synthesize a serial flexure system as only a serial flexure system is capable of achieving the desired DOFs.

Step (4): choose intermediate freedom spaces: The designer must now identify which freedom spaces exist within the freedom space shown in Fig. 13B that also lie within the parallel pyramid. These freedom spaces will be the intermediate freedom spaces that the designer may use to synthesize the serial flexure system. Every freedom space maps to a comprehensive list of intermediate freedom space options from which the designer may choose. If a designer is not familiar enough with the freedom spaces in the chart of Fig. 5B to visually identify all of the intermediate freedom space options for a given freedom space, Appendix A contains a complete list of all of them. Note that according to Appendix A, the intermediate freedom space options that exist within the Type 8 freedom space of the 4 DOF column of Fig. 5B include Types 4 and 5 in the 3 DOF column, Types 1 through 9 in the 2 DOF column, and Types 1 through 3 in the 1 DOF column. Ideally, only two intermediate freedom spaces should be selected so that the serial flexure system will possess the fewest number of possible stages/conjugated elements. If the designer wishes to avoid under-constraint, the sum of the number of DOFs from each intermediate freedom space should equal four as the system's freedom space consists of four DOFs. Many options would satisfy these requirements and would generate viable flexure concepts, but for the purposes of this case study, the Type 2 freedom space from the 2 DOF column will be selected twice because it is less complex than most of the other options and because we are familiar with this freedom space from the example of Section 4.1. This intermediate freedom space and its complementary constraint space are shown in Fig. 14A. The intermediate freedom space consists of every parallel rotation line on a plane and a translation that is perpendicular to that plane. The constraint space consists of every pure force wrench line that lies on the same plane and every pure force wrench line that is parallel to the rotation lines represented by the box of parallel lines. The pure force wrench lines that lie on the plane (highlighted with thick blue lines) run in all directions on the surface. The planes of the two intermediate freedom spaces are oriented at 90° angles with respect to each other as shown in Fig. 14B.

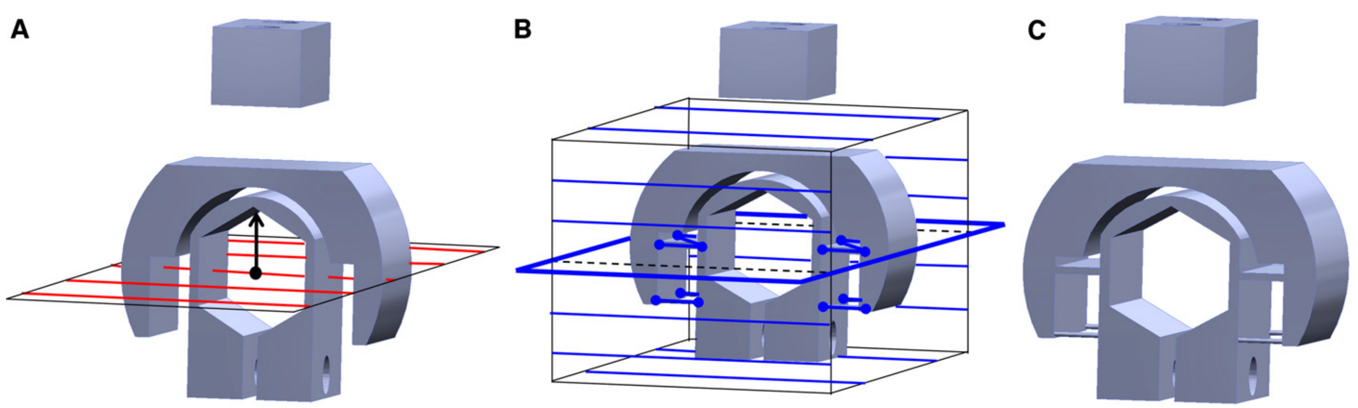

Fig. 16. Orientation of the first intermediate freedom space (A) and its complementary constraint space (B). Flexible constraints selected from the constraint space connect the main stage to the intermediate stage (C).

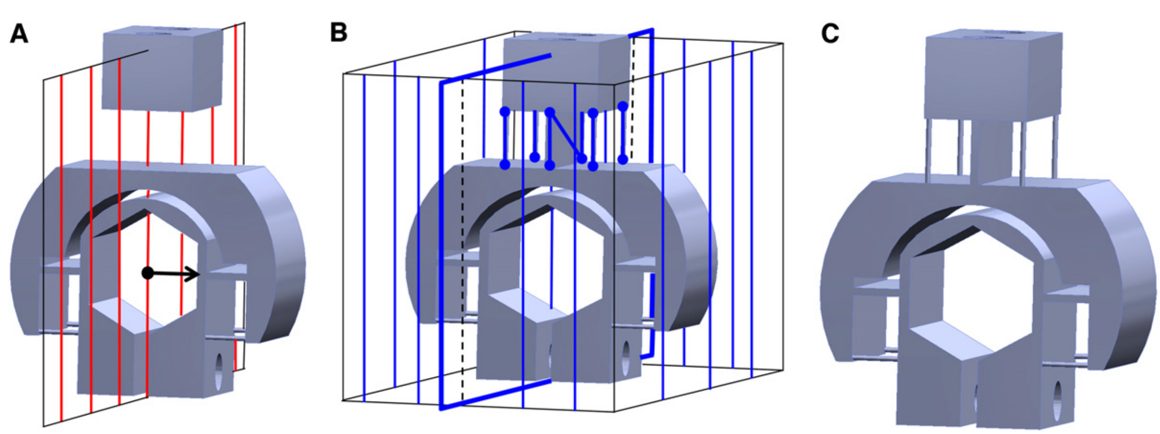

Fig. 17. Orientation of the second intermediate freedom space (A) and its complementary constraint space (B). Flexible constraints selected from the constraint space connect the intermediate stage to the ground (C).

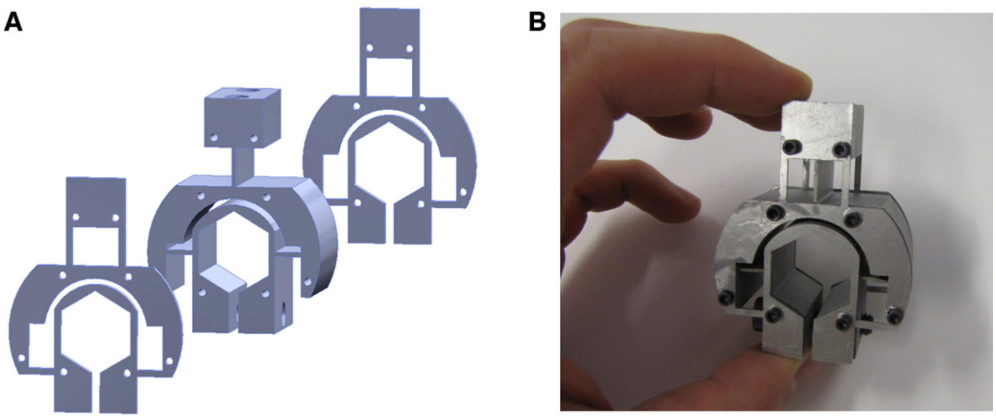

Fig. 18. The flexure consists of three planar pieces (A). The final lead screw flexure (B).

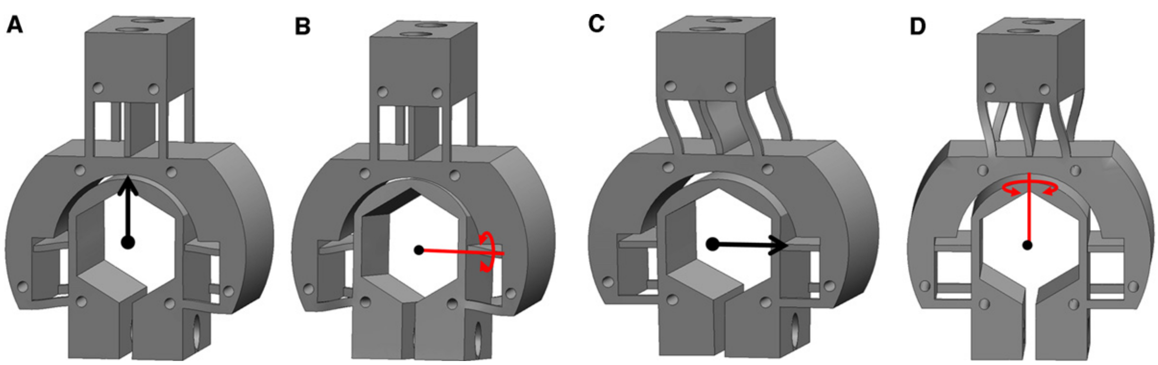

Fig. 19. The four DOFs of the lead screw flexure (A)–(D).

These intermediate freedom spaces combined generate the freedom space of the system because the total number of independent twists from both intermediate freedom spaces is four, which is the number of DOFs of the system's freedom space. Note also that the system will not possess any underconstrained elements because the sum of the DOFs of both intermediate freedom spaces is also four.

Step (5): design ground and stage(s): The flexure system should be grounded to the carriage. The system should consist of two rigid stages because two intermediate freedom spaces were selected. The main stage that possesses the desired DOFs should clamp around the hex nut. Finally, the entire system should fit within the structural tube of the lathe shown in Fig. 12B and again in Fig. 15A. Some possible ground and stage designs that satisfy these conditions are shown in Fig. 15B.

Step (6): select constraints from constraint space/s: The first intermediate freedom space from Fig. 14B is shown superimposed on the system in Fig. 16A. The complementary constraint space of this intermediate freedom space is shown in Fig. 16B. Two flexure blades and four wire flexures are selected from the constraint space. These flexible elements shown in Fig. 16C connect the main stage to the intermediate stage.

The second intermediate freedom space from Fig. 14B is shown superimposed on the system in Fig. 17A. The complementary constraint space of this intermediate freedom space is shown in Fig. 17B. A flexure blade and four wire flexures are selected from the constraint space. These flexible elements, shown in Fig. 17C, connect the intermediate stage to the ground.

The final lead screw flexure shown in Fig. 18 may be fabricated by cutting three planar pieces with a waterjet or wire EDM machine. Using FEA, images of the four desired DOFs are shown in Fig. 19. These motions are much more compliant than the translation and rotation about the lead screw's axis. Note also, if the ground and main stage are held fixed, the intermediate stage will be fully constrained because the system is not underconstrained.

6. Conclusions

In this paper we demonstrated how FACT may be applied to the synthesis of serial flexure systems. More specifically, we described how a comprehensive library of geometric shapes may be used to stack parallel flexure modules in series to achieve a desired set of DOFs. Principles of underconstraint were described and incorporated into the FACT design process. A lead screw flexure was synthesized as a case study.

Acknowledgement

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Appendix A.

This appendix lists all of the possible intermediate freedom spaces that could be selected from within each freedom space in the chart from Fig. 5B for synthesizing serial flexure systems. Note also that any freedom space can be its own intermediate freedom space.

0 DOF Type 1 – None
1 DOF Type 1 – None
1 DOF Type 2 – None
1 DOF Type 3 – None
2 DOF Type 1 – 1 DOF Type 1
2 DOF Type 2 – 1 DOF Types 1, 3
2 DOF Type 3 – 1 DOF Types 1, 2
2 DOF Type 4 – 1 DOF Type 2
2 DOF Type 5 – 1 DOF Types 2, 3
2 DOF Type 6 – 1 DOF Types 1, 2
2 DOF Type 7 – 1 DOF Type 2
2 DOF Type 8 – 1 DOF Types 1, 2, 3
2 DOF Type 9 – 1 DOF Types 1, 2, 3
2 DOF Type 10 – 1 DOF Type 3
3 DOF Type 1 – 1 DOF Types 1, 3; 2 DOF Types 1, 2
3 DOF Type 2 – 1 DOF Types 1, 3; 2 DOF Type 2
3 DOF Type 3 – 1 DOF Type 1; 2 DOF Type 1
3 DOF Type 4 – 1 DOF Types 1, 2, 3; 2 DOF Types 1, 2, 3, 6, 7, 8, 9
3 DOF Type 5 – 1 DOF Types 1, 2, 3; 2 DOF Types 3 through 9
3 DOF Type 6 – 1 DOF Types 1, 2, 3; 2 DOF Types 2, 3, 5
3 DOF Type 7 – 1 DOF Types 1, 2; 2 DOF Types 3, 4, 6, 7
3 DOF Type 8 – 1 DOF Types 1, 2; 2 DOF Types 3, 4, 6, 7
3 DOF Type 9 – 1 DOF Types 1, 2; 2 DOF Types 1, 3, 6, 7
3 DOF Type 10 – 1 DOF Types 1, 2; 2 DOF Types 4, 6, 7
3 DOF Type 11 – 1 DOF Type 2; 2 DOF Types 4, 7
3 DOF Type 12 – 1 DOF Types 1, 2; 2 DOF Types 1, 6, 7
3 DOF Type 13 – 1 DOF Types 1, 2; 2 DOF Types 4, 6, 7
3 DOF Type 14 – 1 DOF Type 2; 2 DOF Types 4, 7
3 DOF Type 15 – 1 DOF Types 1, 2, 3; 2 DOF Types 2, 5, 6
3 DOF Type 16 – 1 DOF Types 2, 3; 2 DOF Types 5, 7
3 DOF Type 17 – 1 DOF Types 2, 3; 2 DOF Types 4, 5
3 DOF Type 18 – 1 DOF Types 2, 3; 2 DOF Type 5
3 DOF Type 19 – 1 DOF Type 2; 2 DOF Type 4
3 DOF Type 20 – 1 DOF Type 3
3 DOF Type 21 – 1 DOF Types 1, 2, 3; 2 DOF Types 2, 5, 8, 9
3 DOF Type 22 – 1 DOF Types 1, 2, 3; 2 DOF Types 2, 5, 9
4 DOF Type 1 – 1 DOF Types 1, 2, 3; 2 DOF Types 1, 2, 3, 6, 7, 8, 9; 3 DOF Types 1, 3, 4, 9
4 DOF Type 2 – 1 DOF Types 1, 2, 3; 2 DOF Types 1, 2, 3, 5, 6, 7, 8, 9; 3 DOF Types 1, 2, 4, 6
4 DOF Type 3 – 1 DOF Types 1, 2, 3; 2 DOF Types 1 through 9; 3 DOF Types 4 through 9
4 DOF Type 4 – 1 DOF Types 1, 2, 3; 2 DOF Types 3 through 9; 3 DOF Types 5, 7, 8
4 DOF Type 5 – 1 DOF Types 1, 2, 3; 2 DOF Types 2 through 9; 3 DOF Types 5, 6
4 DOF Type 6 – 1 DOF Types 1, 2, 3; 2 DOF Types 1 through 9; 3 DOF Types 4, 5, 7, 8, 9
4 DOF Type 7 – 1 DOF Types 1, 2, 3; 2 DOF Types 3 through 9; 3 DOF Types 5, 8
4 DOF Type 8 – 1 DOF Types 1, 2, 3; 2 DOF Types 1 through 9; 3 DOF Types 4, 5
4 DOF Type 9 – 1 DOF Types 1, 2, 3; 2 DOF Types 1 through 9; 3 DOF Types 4, 5
4 DOF Type 10 – 1 DOF Types 1, 2, 3; 2 DOF Types 2, 5, 8, 9; 3 DOF Type 2
5 DOF Type 1 – 1 DOF Types 1, 2, 3; 2 DOF Types 1 through 9; 3 DOF Types 1 through 9; 4 DOF Types 1, 2, 3
5 DOF Type 2 – 1 DOF Types 1, 2, 3; 2 DOF Types 1 through 9; 3 DOF Types 4 through 9; 4 DOF Type 3
5 DOF Type 3 – 1 DOF Types 1, 2, 3; 2 DOF Types 1 through 9; 3 DOF Types 1, 2, 4, 5, 6; 4 DOF Types 2
6 DOF Type 1 – 1 DOF Types 1, 2, 3; 2 DOF Types 1 through 9; 3 DOF Types 1 through 9; 4 DOF Types 1, 2, 3; 5 DOF Type 1

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