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Synthesis of multi-degree of freedom, parallel flexure system concepts via freedom and constraint topology (FACT). Part II: Practice

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ABSTRACT

In Part II of this paper we demonstrate how to use freedom and constraint topology (FACT) to synthesize concepts for the multi-degree of freedom, parallel precision flexure systems that fall within the scope of Part I. Several examples are provided to demonstrate how the Principle of Complementary Topologies and geometric entities from Part I are (i) relevant to flexure system characteristics, (ii) used to visualize the possible layout of flexure constraints to achieve a desired motion and (iii) used to select redundant constraints. A synthesis process is presented, and then used to visualize and construct a flexure system concept with the requisite kinematic characteristics and redundant constraints that provide increased stiffness, load capacity, and symmetry. The output of the process is a flexure concept that would then be modeled and refined by existing modeling and analysis methods.

1. Introduction

The intent of Part II of this paper is to demonstrate how one uses the principles from Part I [1] to generate flexure system concepts. In essence, Part I of this paper focused on the left side of Fig. 8 from Part I. Part II focuses upon synthesis, the right side of Fig. 8. A multi-step synthesis process is provided and augmented by additional principles that are relevant to the synthesis of flexure system concepts.

A brief review of other processes/methods is in order. Blanding created a formal base of exact constraint principles for use in the design of mechanical devices, including flexures [2], ca. 1999. Hale [3] augmented these principles for specific use with precision flexure systems. The contributions of Maxwell (covered in Part I), Blanding and Hale constitute the core of what is called constraint-based design. The fundamental premise of CBD is that all motions of a rigid body are determined by the position and orientation of the constraints, i.e. the topology of constraints, which act upon the body. In CBD, a designer arranges flexural and rigid elements into a geometric layout that endows a device with the ability to permit and forbid motions in specified directions. CBD has been practiced by using a combination of visualization techniques, experience and rules of thumb. It is currently the primary synthesis method used to engineer precision flexure systems.

In the later 20th century a new branch of research within the mechanism/robotics community focused upon large and non-linear motions of flexure-based mechanisms – compliant mechanisms. Notable advances include pseudo-rigid body modeling and topological synthesis.

Pseudo-rigid body modeling (PRBM) models compliant mechanisms (CMs) as analogies to rigid-link mechanisms [4,5]. The rigid analog is then modeled using pre-existing rigid mechanism theory and the principle of virtual work to ascertain its kinematic and elastomechanic properties. The PRB model has been used to design precision elastic mechanisms [6,7] and many consumer products. The primary aim of PRBM is to model rather than synthesize and so it is not ideally suited to solve the problems that are the target of this paper.

Topological synthesis is based upon computer algorithms that examine a starting shape for a compliant mechanism and then determines how to add/subtract material in order to create concepts that satisfy performance specifications [8–10]. In this method, the computer makes the design decisions that determine the layout of the rigid and flexible elements. This approach is highly effective for the rapid synthesis of unique, non-precision compliant mechanism concepts in applications such as robotics, MEMS and aeronautics. Unfortunately, topological synthesis is not readily applied to solve most precision flexure design problems. This is due in part to the specialized precision machine design knowledge

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that is required to satisfy the demanding requirements of precision machinery.

For example, precision machine designers must manage and understand the evolution of their designs so that they may (i) implement precision engineering-specific principles such as symmetry and design for manufacturing, (ii) understand how the mechanism works so that they may trouble-shoot the mechanism and (iii) decouple the contributions of certain flexural elements in order to minimize systematic errors that are caused by fabrication, assembly, actuator and sine errors. Given the preceding, it is difficult to justify the removal of the designer from the evolution of a precision flexure system's design. It is also important to note that topological synthesis will at times produce designs that are not readily fabricated in elegant, minimalistic forms that enable low-cost designs and/or easy integration within precision systems.

The preceding methods/processes are not compatible for use with the principles of FACT. This paper provides the knowledge that enables the use of FACT principles to generate concepts. Section 2 starts with intuitive examples that demonstrate how FACT may be used to generate multiple concepts. Sections 3 and 4 will cover two different examples. The formal design steps/process will be covered later in Section 5.

2. The use of FACT to generate multiple concepts

2.1. Multiple concepts for a flexure system that permits one rotation

Here we wish to synthesize multiple concepts that may only rotate about a specific axis. This motion is associated with Case 5, Type 1 defined in Appendix A of Part I of this paper [1]. The relevant freedom topology, shown in Fig. 1A, is the same as the freedom space in this case. The complementary constraint space is shown in Fig. 1B. The constraint space consists of constraint lines from planes wherein each plane intersects a common line as shown in Fig. 1B. For a one DOF flexure system, Eq. (1) indicates that one should select five non-redundant constraints from the constraint space.

$$R = 6 - C \tag{1}$$

From Section 3.4 of Part I of this paper [1], we know that we may select a suitable number of redundant constraints as long as they reside within the constraint space. As noted in Section 3.4, there are many ways to select a set of five independent constraints plus redundant constraints. Each set of constraints forms a constraint topology that represents a flexure system concept.

Fig. 1C–E, F–H and I–K shows three possible ways to select the constraints. Note that several blade flexures are used. Blade flexures emulate two parallel constraints, which bound a third constraint that runs across the diagonal of the flexure blade [2]. The use of multiple blade flexures leads to redundant constraints in all of the concepts. Fig. 1D, G and J shows physical embodiments of the concepts. Fig. 1E, H and K contrasts the deformed and non-deformed states of the flexure system.

3. Multiple concepts for a flexure system that permits one rotation and one translation

Here we wish to synthesize multiple concepts that possess a freedom topology that consists of a translation that is orthogonal to an axis of rotation. These DOFs belong to Case 4, Type 2 defined in Part I of this paper [1]. The associated freedom space is shown in Fig. 2A and its complementary constraint space is shown in Fig. 2B. The freedom space consists of two freedom sets: (1) a hoop that represents the translation and (2) a plane of parallel freedom lines that are orthogonal to the translation. The complementary constraint space consists of two constraint sets: (1) a box that represents every constraint line that is parallel to a specific direction and (2) every line that lies on a plane. For a two DOF flexure system, Eq. (1) indicates that four non-redundant constraints should be selected from the constraint space. Again, there are many ways to select a set of four independent constraints plus redundant constraints.

Fig. 2C–F, G–J, K–N shows three possible ways to pick four independent constraints, an exploded view of the physical embodiments of the constraints, and contrasts the deformed and non-deformed states of the flexure system during independent translational and rotational motions.

4. Sub-constraint spaces

This section discusses how to select constraints from the desired constraint space such that (i) the constraints selected are independent and (ii) enough independent constraints are selected to achieve the desired system kinematics. Sub-constraint spaces were developed to help designers better visualize (a) the different ways in which independent constraints may be selected from a constraint space and (b) how to select these constraints. There is no general rule that may be used to assign sub-constraint spaces for each constraint space. The constraint spaces for several of the 26 cases in Part I of this paper [1] consist of many sub-constraint spaces, too many to describe within this paper. A full accounting of sub-constraint spaces is provided by Hopkins [11]. The intent of this paper is to introduce the concept and then show how they may be used in the synthesis process.

4.1. Example: sub-constraint spaces for Case 4, Type 2

Sub-constraint spaces always lie within the constraint space to which they are assigned. An example that is familiar, the example from Section 3, will be used to clarify the concept of sub-constraint spaces. The approach taken in this sub-section is to use logic to (i) identify the sub-constraint spaces and (ii) show how to select the constraints within each. The later point is key. Each sub-constraint space is accompanied by brief instructions that provide guidance on the selection of the independent constraints. A full accounting of sub-constraint spaces and complementary instructions are given by Hopkins [11].

Consider the constraint space of Case 4, Type 2 which is shown in Fig. 3. This constraint space consists of two constraint sets:

(1) a box that contains every constraint line that is parallel to a given direction and
(2) a plane that contains every constraint line that exists on that plane.

It is not possible to select more than four independent constraints from this space, but it is possible to make a mistake and select four constraints from this space that are not independent. Sub-constraint spaces and complementary instructions guide the user in selecting the number of lines that are independent. We have predetermined these spaces' instructions in order to eliminate the need for a designer to 'reason out' how the independent constraints could be selected. For the sake of this example/explanation, we will 'reason out' the solution in this sub-section, but the reader should understand that there is no need to do so during the design process, all solutions are given by Hopkins [11].

If four constraints are chosen from the box and no constraints are chosen from the plane, at least one of these constraints must

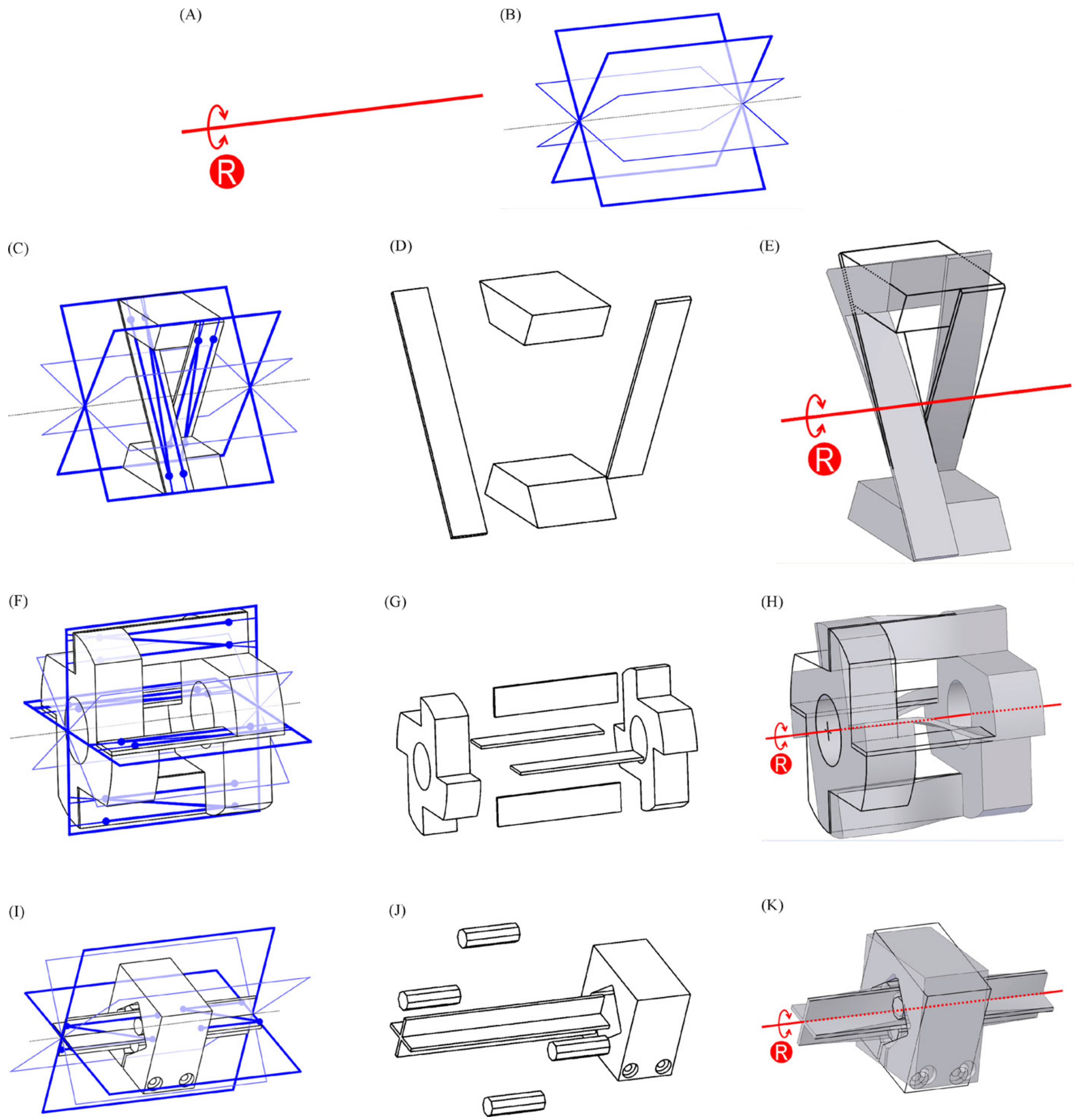

Fig. 1. (A) The freedom space for a rotational DOF and (B) the constraint space that yields several viable topologies/embodiments – (C), (F), and (I) – of constraints. FEA post-processing of the components – (D), (G), and (J) – of each embodiment contrasts the respective deformed and non-deformed states – (E), (H), and (K) – of each flexure system. The lines in (A), (E), (H), and (K) are red to denote rotational freedom lines. The lines in (B), (C), (F), and (I) are blue to denote constraint lines. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of the article.)

be redundant; this is not a valid sub-constraint space and so the designer would not want to select four constraints only from the box. If no constraints are chosen from the box and all four constraints are chosen from the plane, then at least one of these constraints will be redundant; this is also not a valid sub-constraint space.

Suppose one chose three constraints from the box and one constraint from the plane. If the constraint chosen from the plane is parallel to the constraint lines in the box, the set of four lines will not be independent. The designer must, therefore, be instructed to select a constraint from the plane that is not parallel to the constraint lines in the box. The designer must also be instructed to select three constraints from the box that do not all lie on the same plane. This first sub-constraint space is shown in Fig. 4. Note also that it does not matter if the designer selects constraints from the box that also lie on the plane.

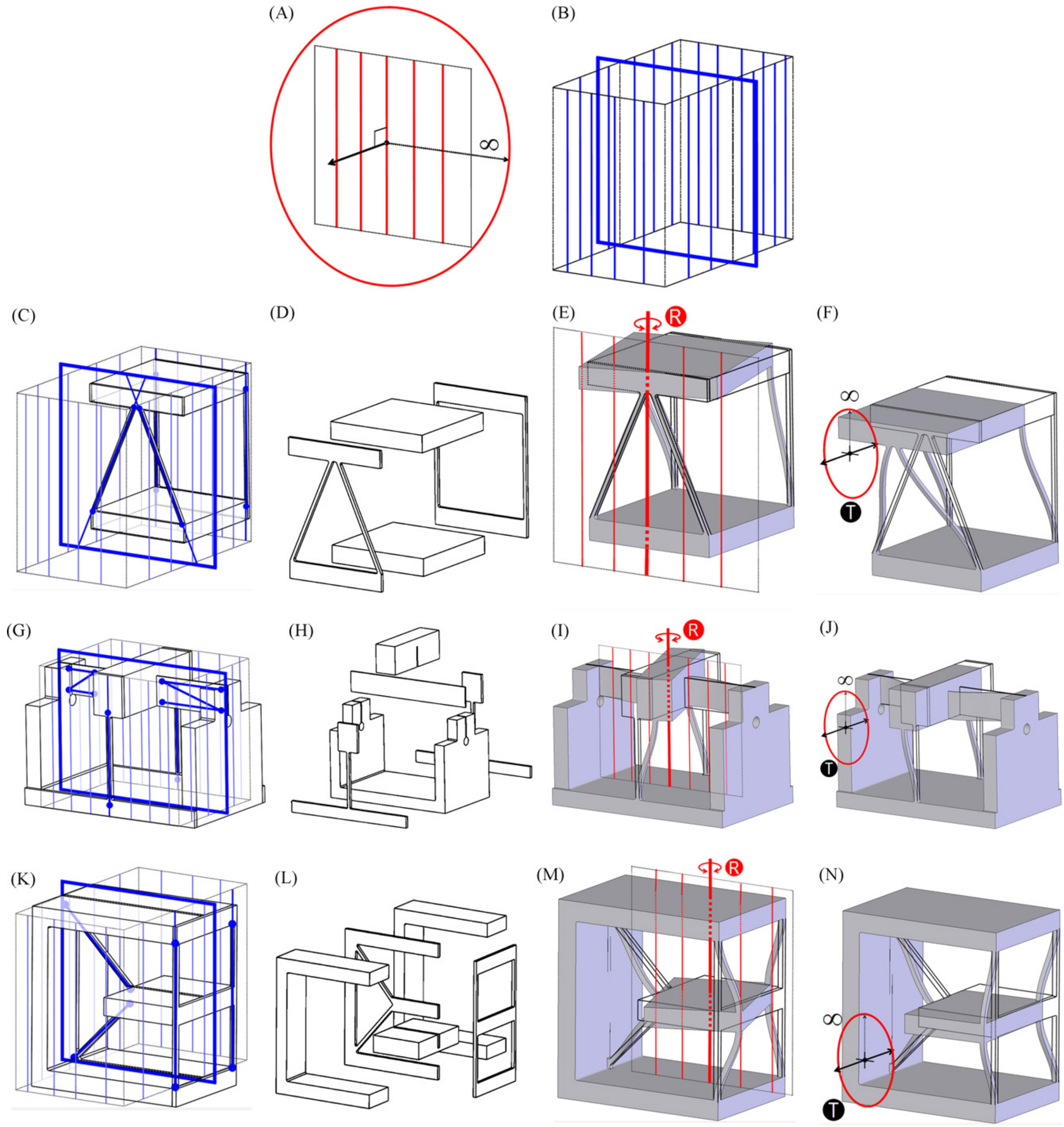

Fig. 2. (A) The freedom space for orthogonal rotational and translation motions and (B) the constraint space that yields several viable topologies/embodiments – (C), (G), and (K). FEA post-processing of the components – (D), (H), and (L) – of each embodiment contrast the respective deformed and non-deformed states for the rotational motion – (E), (I), and (M) – and the translation motion– (F), (J), and (N) – of each flexure system. The lines in (A), (E), (F), (I), (J), (M), and (N) are red to denote rotational freedom lines. The lines in (B), (C), (G), and (K) are blue to denote constraint lines. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of the article.)

In a similar way, one may reason the existence of the remaining sub-constraint spaces that are shown in Figs. 5–8.

4.2. Relationship of sub-constraint spaces to flexure system design

For N sub-constraint spaces, there are N ways to select combinations of constraints so that they will be independent. The final form of a flexure system design is largely determined by the sub-constraint space that governs the layout of its constraints. For example, concepts from one sub-constraint space may be inherently easier to fabricate, to make symmetric and therefore thermally stable, or to fit within a given space constraint. Flexure systems that come from this sub-constraint space would possess these characteristics while concepts from other sub-constraint space may not.

5. FACT synthesis process steps

The FACT synthesis process consists of six steps that are shown in Fig. 9:

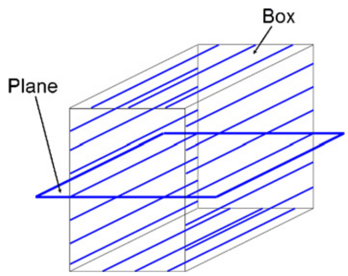

Fig. 3. Constraint space (blue) that contains four non-redundant constraints. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of the article.)

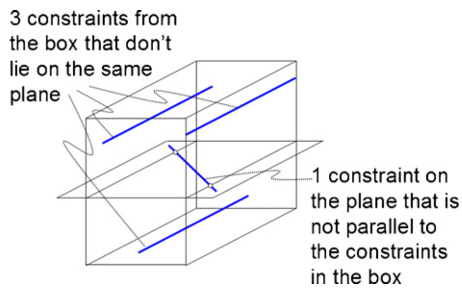

Fig. 4. First sub-constraint space (blue) for the constraint space shown in Fig. 3. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of the article.)

Step 1: determine the stage's geometry.
Step 2: specify the desired motions, e.g. the freedom topology.
Step 3: select the best freedom and constraint space type.
Step 4: select the desired sub-constraint space.
Step 5: select non-redundant constraints.
Step 6: select redundant constraints.

The process will be demonstrated in the context of a practical flexure design problem.

Flexure system design characteristics: We wish to design a two-axis probe that permits yaw and pitch rotations such that the probe may trace out a spherical cavity in a surface. This type of flexure stage finds use in instruments and equipment that scan an optic's focal point or the motion of a probe tip over a spherical/curved surface. At this point, we are only interested in the early stage synthesis of concepts that would provide the desired kinematics. After this process is complete, conventional methods may be used to model and refine the final geometry.

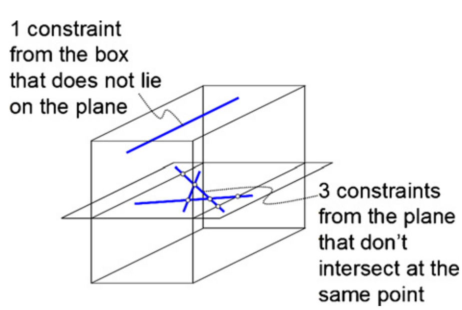

Fig. 5. Second sub-constraint space (blue) for the constraint space shown in Fig. 3. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of the article.)

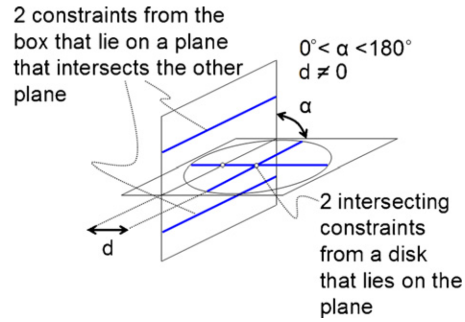

Fig. 6. Third sub-constraint space (blue) for the constraint space shown in Fig. 3. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of the article.)

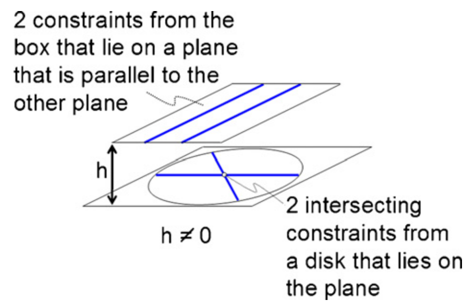

Fig. 7. Fourth sub-constraint space (blue) for the constraint space shown in Fig. 3. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of the article.)

5.1. Step 1: determine the stage's geometry

The first step is to determine the stage's geometry, which is constrained by fabrication capabilities, allowable footprint, etc. In Step 1 of Fig. 9, a simple rectangular probe holder has been selected. It is also important to note that this geometry can later be altered if the designer decides that a different stage would be more appropriate.

5.2. Step 2: specify the desired motions of the stage (freedom topology)

The second step requires that the designer specifies the system's freedom topology, i.e. the degrees of freedom that are desired. Step 2 of Fig. 9 illustrates the two desired rotations that the probe's stage is intended to move with, yaw and pitch.

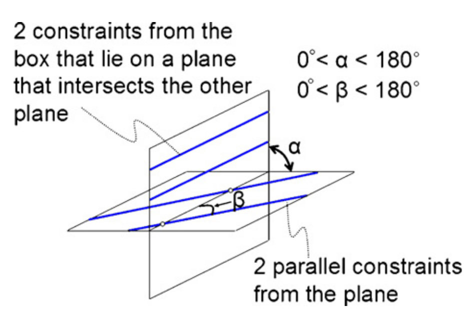

Fig. 8. Fifth sub-constraint space (blue) for the constraint space shown in Fig. 3. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of the article.)

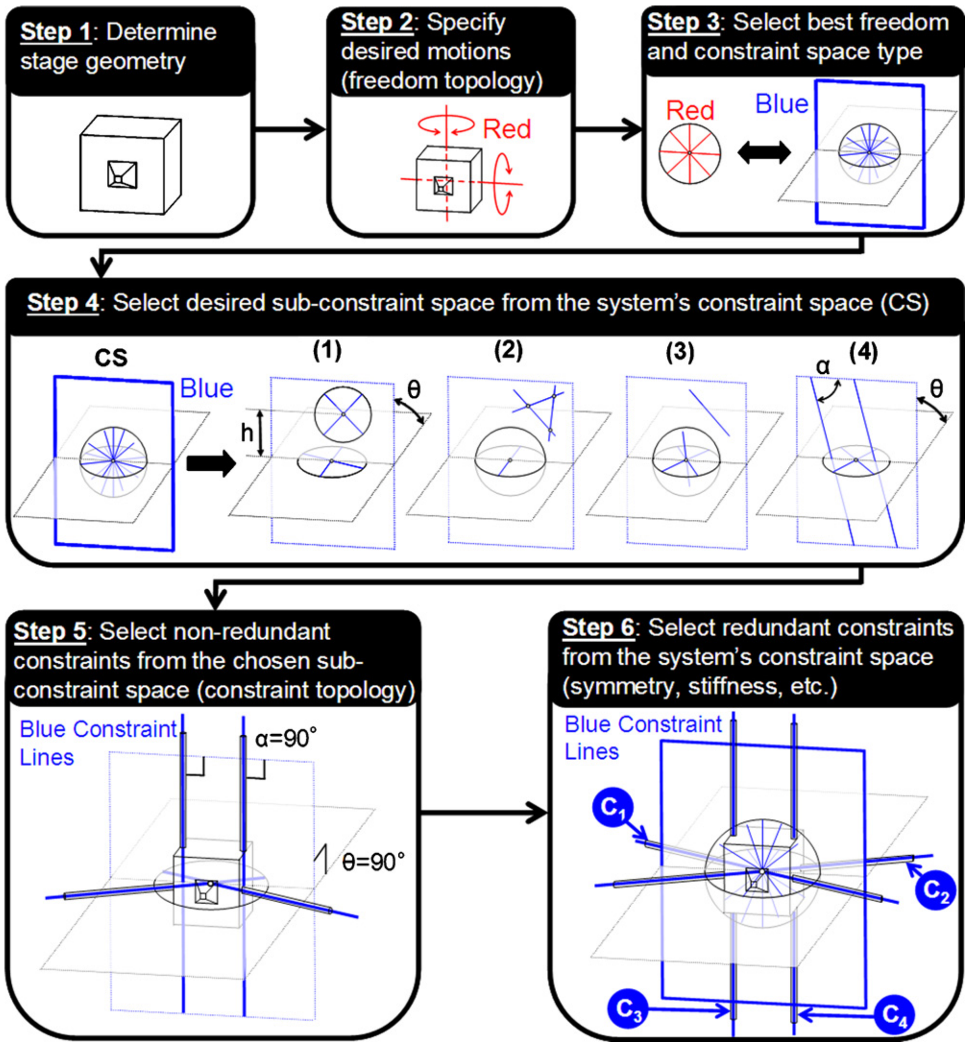

Fig. 9. FACT synthesis process applied to the design of a two-axis, i.e. yaw and pitch, flexure system. Note in Step 4, the four sub-constraint spaces have been laid out.

5.3. Step 3: select the appropriate freedom and constraint space type

It is important to note that there are only 26 general types of motions that may be achieved with purely parallel flexure systems [11]. This means that there are unfortunately several types of motions that one cannot obtain using parallel flexure systems only. The import of this fact is that a parallel flexure designer may want a set of DOFs that are not possible to achieve. In short, if a desired freedom topology is not found within the 26 types, the corresponding motion may not be achieved.

The existence, and limitation of the 26 general types, has been rigorously proven via math and logic [11]. This concept may be intuitively understood via the following example. There is no type that describes a flexure system which possesses three independent translations, e.g. *x*–*y*–*z*. According to Blanding, once a first constraint is added, a translation has been lost along the axis of the constraint. Therefore, it is only possible for two independent translations to exist. Logic dictates that no parallel flexure system may be designed to provide independent *x*–*y*–*z* translations. An *x*–*y*–*z* flexure would, however, be possible if serially conjugated flexure elements were used. As previously explained, these types of flexure systems are beyond the scope of this paper.

We return to the third step, which is to determine the appropriate freedom and constraint space type from among the 26 types in [1]. Note the selection of a freedom space predetermines the constraint space because each freedom space maps to a unique constraint space per the Principle of Complementary Topologies. The optimal freedom and constraint space type for the two-axis probe example of this section is Case 4, Type 1. This type is shown in Step 3 of Fig. 9. The freedom space consists of a pencil of rotational freedom lines. The matching constraint space will contain several constraint topologies that solve the motion problem. Given that the generation of multiple concepts has already been demonstrated, we will only synthesize one concept here. This process could be repeated to generate other viable concepts.

5.4. Step 4: select desired sub-constraint space from the system's constraint space

For any Case "*C*", one needs to select "*C*" non-redundant constraints in order to obtain the desired freedom topology. In some cases, it is not intuitively obvious how one may select *C* number of non-redundant constraints from a given constraint space so that they are independent. The problems associated with the selection of independent constraints have been addressed via the use of sub-constraint spaces. For our example, the constraint space of Case 4, Type 1 contains four sub-constraint spaces and so there are only four different ways to select four non-redundant constraints from within this constraint space.

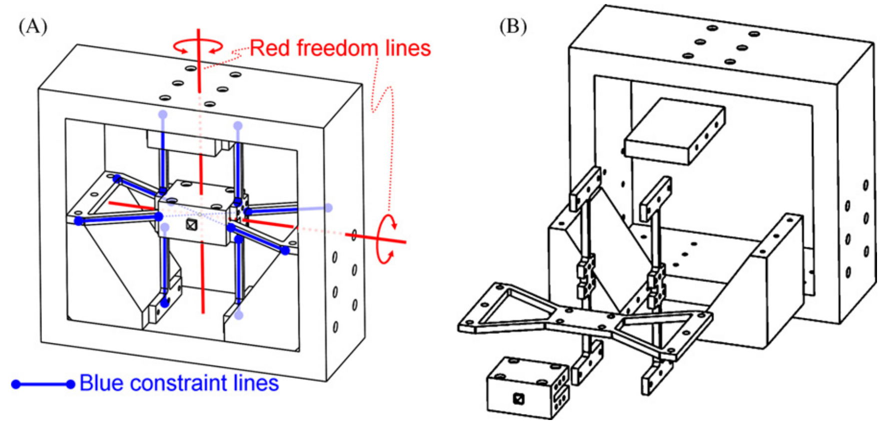

Fig. 10. A practical two-axis flexure system that is capable of yaw and pitch rotations.

5.4.1. Case 4, Type 1, Sub-1

The four sub-constraint spaces associated with Case 4, Type 1 are shown in Step 4 of Fig. 9. The first sub-constraint space consists of two pencils of constraint lines. One pencil lies within the sphere of constraint lines and the other pencil lies on the plane of constraint lines. The planes that contain the pencils intersect each other with an included angle of θ. This angle may be any value greater than zero and less than 180°. Furthermore, the distance between the center of the pencil on the plane of constraints and the intersection line of the two planes, h, must be non-zero. The designer must select two constraints from each pencil of constraint lines in order to create a set of four independent lines.

5.4.2. Case 4, Type 1, Sub-2

The second sub-constraint space, shown in Step 4 of Fig. 9, instructs the designer to select three constraints from the plane of constraints that do not all intersect at the same point (i.e. three parallel lines or three lines in a pencil on the plane). The designer is also instructed to select one constraint line from the sphere of constraint lines such that the constraint line does not lie on the plane of constraints.

5.4.3. Case 4, Type 1, Sub-3

The third sub-constraint space shown in Step 4 of Fig. 9 consists of a single constraint from the plane of constraints such that it does not intersect the sphere's center point. The designer must select three constraints from the sphere that do not all lie on the same plane.

5.4.4. Case 4, Type 1, Sub-4

The fourth and final sub-constraint space, shown in Step 4 of Fig. 9, consists of a pencil of constraint lines that lies within the constraint space's sphere, and a plane of parallel constraint lines that is coincident with the constraint space's plane of constraints. The intersection angle of the plane of parallel constraint lines and the plane of the pencil occur with an included angle of θ. This angle may be any value greater than zero and less than 180°. The angle between the intersection line of these planes and the parallel constraint lines is α. This angle may be any value greater than zero and less than 180°. The designer is also instructed to select two constraints from the pencil and two constraints from the plane of parallel constraint lines.

When the goal is to generate a maximum number of concepts for a design problem, it is prudent to synthesize concepts from each sub-constraint space. Other factors, for instance symmetry, balanced stiffness and geometric constraints, affect which sub-constraints may generate concepts that provide a practical solution to some equipment design problems. For example, if the stage and ground geometries are set, it may not be possible to select constraints from a certain sub-constraint space that would link the stage to ground in a practical way. We select the fourth sub-constraint space because it is easily used to create a symmetric design if redundant constraints are added.

5.5. Step 5: select non-redundant constraints from the sub-constraint space (constraint topology)

The next step is to decide where to place the stage among the constraint lines within the sub-constraint space so that the non-redundant constraints attach the stage to ground. In this example, we position and orient the stage body within the fourth sub-constraint space as shown in Step 5 of Fig. 9. Fabrication considerations lead us to select θ and α equal to 90°. Stability considerations lead us to select two constraints from the plane of parallel constraint lines that are as far apart as possible. We select two constraints from the pencil of constraint lines in such a way that allows space for a sample to approach the point of interest, i.e. a probe, without interfering with the constraints.

It is important to note, that once Step 5 is complete, the flexure system is 'ideally' constrained and therefore the stage will move with the desired degrees of freedom. In some instances, the designer could stop at this point, having achieved his/her objective. For this example, however, we continue on to Step 6, where we will have the opportunity to add redundant flexure constraints that improve the design's symmetry, and thereby improve its thermal stability.

There may be situations where it is most practical to use certain flexure elements, e.g. blades, to generate a flexure and thereby yield many redundant constraints. In this situation, one ends up with redundant constraints rather than selecting them. For example, see Fig. 1 wherein the blades lead to many redundant constraints. It is not always possible to identify a specific constraint as "the" redundant constraint given that it is made redundant by the existence of other constraint(s). The only guidance that is possible given the preceding is that the designer must compare the existing constraints within the design, to C, the number of non-redundant constraints.

5.6. Step 6: select redundant constraints from the system's constraint space

In Step 6, the designer selects permissible redundant constraints (permissible redundant constraints do not affect the flexure system's kinematics) from within the system's constraint space. This would be done for instance if the addition of a redundant constraint would improve a flexure system's stiffness or symmetry, Permissible redundant constraints may only be selected from within the flexure system's constraint space.

In Step 6 of Fig. 9, four redundant constraints have been added to the stage from within the constraint space of Case 4, Type 1. Fig. 10A shows a more practical embodiment of the flexure system just designed. Red rotational freedom lines are shown to depict the flexure's permissible yaw and pitch DOFs. Fig. 10B shows an exploded view of this flexure.

6. Moving beyond early stage synthesis

This paper helps designers get over the first hurdle, that is to generate a conceptual representation of the design. This generation phase is the front-end of the engineering process and its outcome (concepts) is readily ported into conventional modeling/simulation methods that underlie the optimization which occurs in the next step – refinement.

7. Summary

In this paper we have shown how multiple flexure concepts may be generated via FACT. Sub-constraint spaces have (i) been defined and shown to be useful in selecting independent constraints and (ii) generating different flexure system concepts for a given motion problem. A design process was introduced and demonstrated via example. The selection of permissible redundant constraints was demonstrated in this example. We are currently implementing FACT in a computer design tool that enables designers to visualize the three dimensional aspects of the spaces, select constraints according to the synthesis process and then optimize them via the FACT design process and standard flexure modeling codes.

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