Entropy Generation Minimization for Energy-Efficient Desalination

Analogies between desalination systems and heat exchangers

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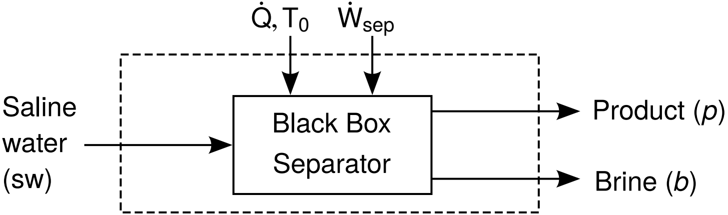
Cost of desalinated water

- Energy + CAPEX + OPEX make-up levelized cost of desalinated water
- Energy adds > 30% to cost of desalinated **seawater**
- Energy cost is higher at very high salinity (e.g., oil/gas waste water)
- Energy cost is small factor for **brackish** water

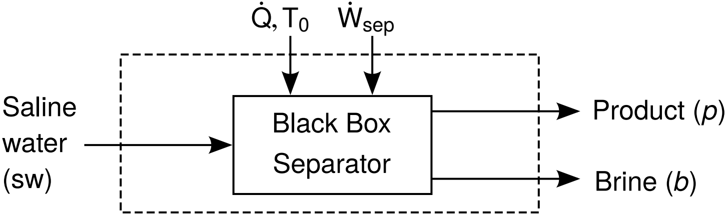
Desalination plant control volume



By eliminating $\dot{Q}$ between the first and second laws

$$\dot{W}_{sep} = \dot{m}_p(h - T_0 s)_p + \dot{m}_b(h - T_0 s)_b - \dot{m}_{sw}(h - T_0 s)_{sw} + T_0 \dot{S}_{gen}$$

Desalination plant control volume



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When $\dot{S}_{gen} = 0$:

- $\dot{W}_{sep}$ is a **thermodynamic property** of the end states.
- $\dot{W}_{sep}$ is minimized if the entering and leaving streams are at the dead state pressure and temperature, T_0 and p_0.

Entropy generation by transport processes (W/K·m^3)

- Heat transfer

$$\sigma = \nabla \frac{1}{T} \cdot \mathbf{J}_Q + \frac{1}{T} \mathbf{E} \cdot \mathbf{j} - \frac{1}{T} \sum_k \nabla_T \mu_k \cdot \mathbf{J}_k$$

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σ is product of flux vectors $\mathbf{J}_i$ and driving force vectors $\mathbf{X}_i$

$$\sigma = \sum_i \mathbf{X}_i\cdot\mathbf{J}_i$$

The driving forces are gradients that cause fluxes of heat, mass, and current.

Equipartitioning of entropy generation

The fluxes are an approximately linear function of the driving forces:

$$\mathbf{J}_i = \sum_k L_{ki} \mathbf{X}_k$$

For heat flux,

$$\mathbf{J}_Q = L_{QQ} \nabla(1/T) = -L_{QQ} T^{-2} \nabla T = -k \nabla T$$

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So, σ varies as square of temperature & concentration gradients:

$$\sigma = \sum_{i,k} \mathbf{X}_i L_{ki} \mathbf{X}_k$$

For heat flux,

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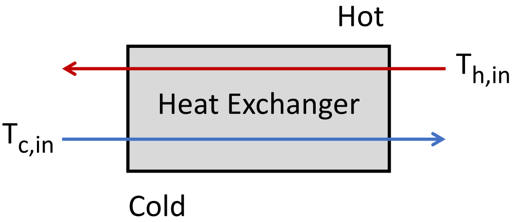
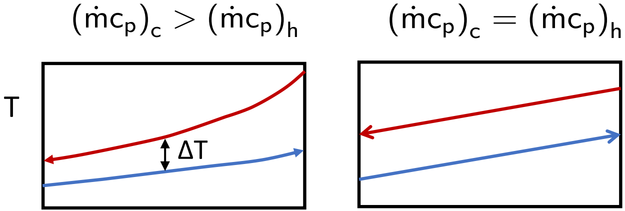
Lowering spatial/temporal variance of driving forces minimizes overall entropy generation (Tondeur & Kvaalen, 1987; Johannessen et al., 2005).

Balancing a counterflow heat exchanger





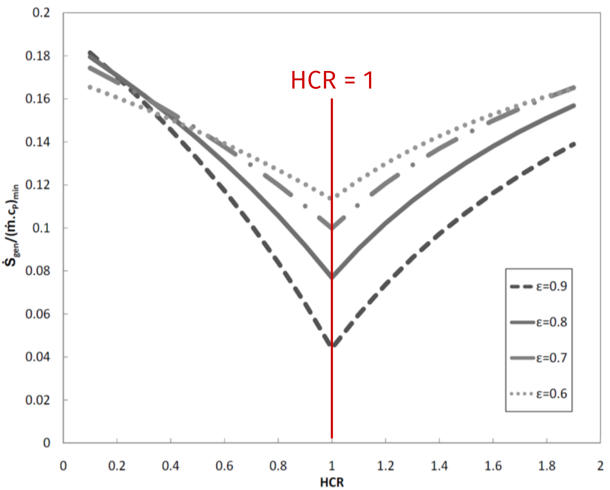
Entropy generation minimization by balancing

Counterflow heat exchanger at fixed effectiveness

$$\frac{\dot{S}_{\text{gen}}}{(\dot{m}c_p)_{\text{min}}} = f\left(T_{\text{h,in}}/T_{\text{c,in}}, \text{HCR}, \varepsilon\right)$$

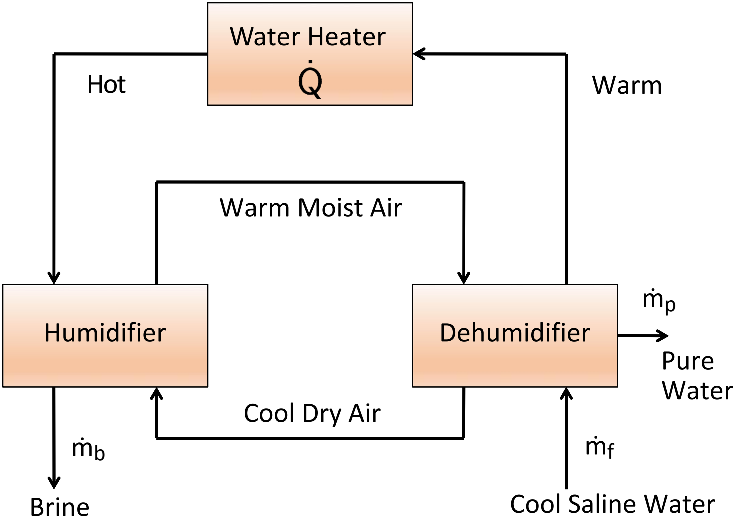
Heat capacity rate ratio

$$\text{HCR} \equiv \frac{(\dot{m}c_p)_{\text{cold}}}{(\dot{m}c_p)_{\text{hot}}}$$



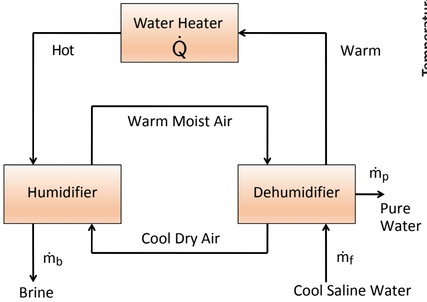
Humidification-dehumidification

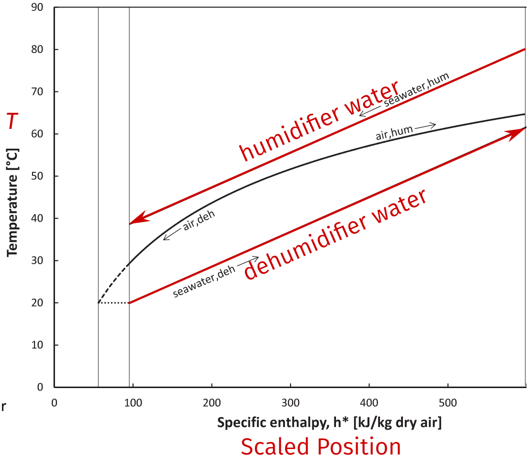
Open water, closed air, HDH cycle



Entropy generation

$$\sigma = k\left(\frac{\nabla T}{T}\right)^2 + \frac{\rho^2 R}{M_a M_w w_a w_w c}(\nabla w_w)^2$$





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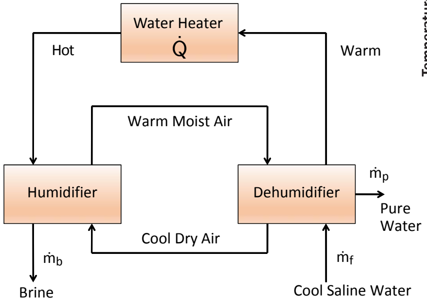
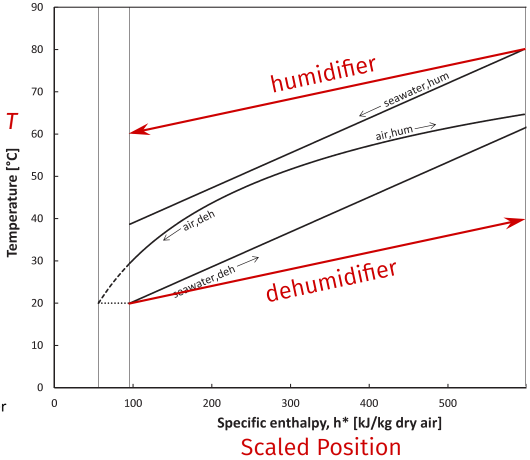




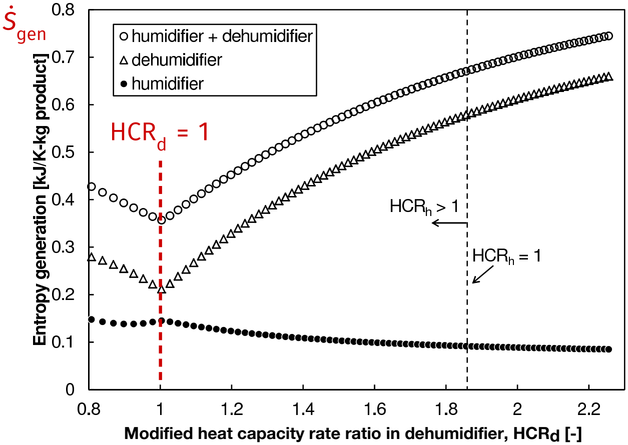
Humidification-dehumidification

Entropy generation minimization thru balancing: $HCR_d = 1$

Modified heat capacity rate ratio

$$HCR = \frac{\Delta \dot{H}_{max,\ cold}}{\Delta \dot{H}_{max,\ hot}}$$

accounts for latent heat of water vapor

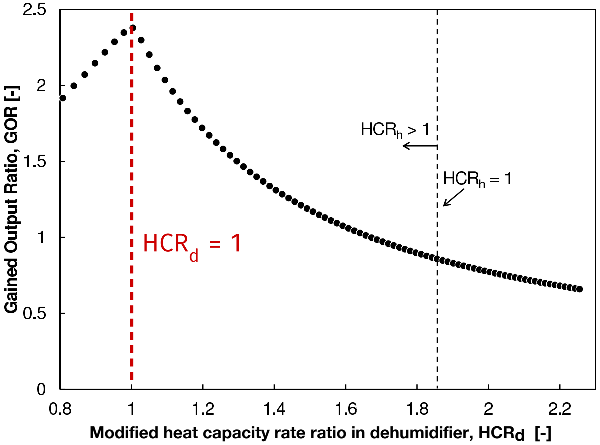


Humidification-dehumidification

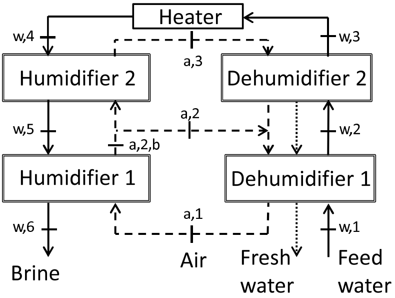
Energy efficiency maximized at HCR of minimum entropy generation

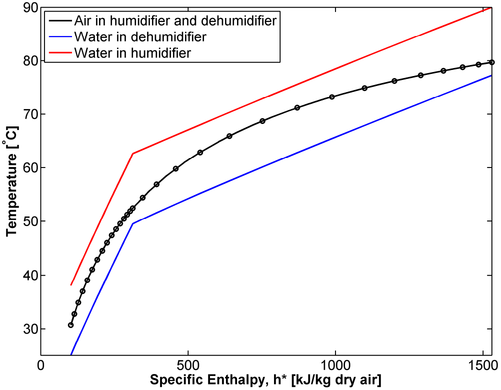
Gained output ratio

$$GOR = \frac{h_{fg}\dot{m}_p}{\dot{Q}}$$



Balancing HDH with a single extraction



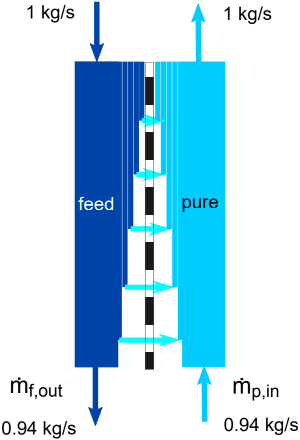


Humidification-dehumidification plant

HDH balancing technology is patented, licensed, and commercialized

Membrane distillation cycle balancing

Direct-contact MD



Membrane distillation cycle balancing

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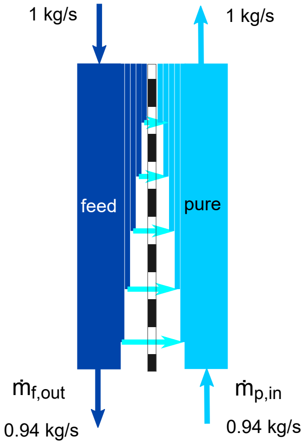
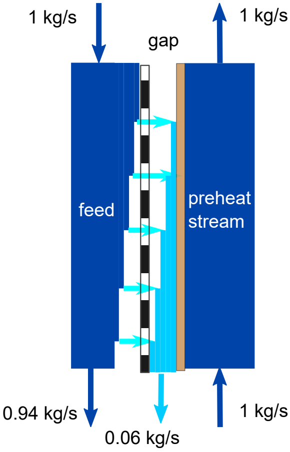
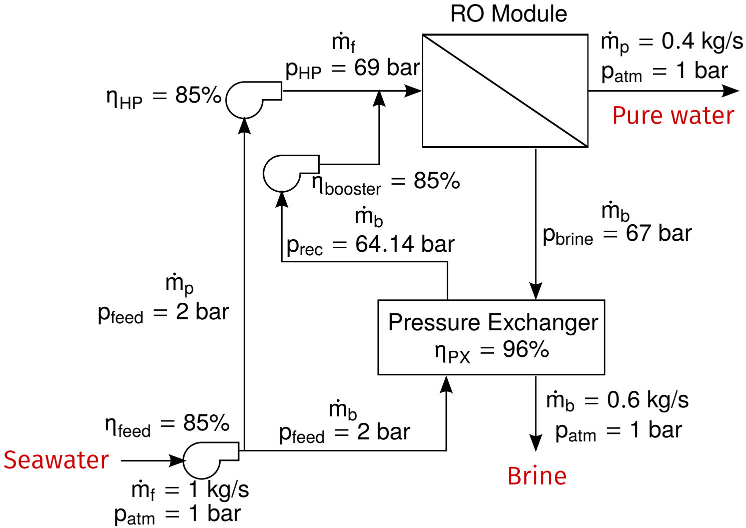


Air-Gap MD



Single-pass reverse osmosis system



Entropy generation in RO

To find $\dot{S}_{\text{gen}}$ from water transport through membrane, $\nabla_T \mu_w$ is required:

$$
\begin{aligned}
\nabla_T \mu_w &= \nabla_T\left(\bar{g}_w + RT \ln a_w\right) \\
&= \bar{v}_w \nabla_T\left(p - \Pi_w\right)
\end{aligned}
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Entropy generation per unit membrane area:

$$\begin{aligned}\dot{S}''_{\text{gen}} &= \int_0^L \sigma\, dx = \int_0^L \left[\nabla \frac{1}{T} \cdot \mathbf{J}_Q - \mathbf{J}_w \cdot \frac{\bar{v}_w}{T} \nabla_T\left(p - \Pi_w\right)\right] dx\\ &= J_Q\left(\frac{1}{T_L} - \frac{1}{T_0}\right) + \frac{\bar{v}_w J_w}{T}\left(\Delta p - \Delta\Pi_w\right) \approx \frac{\bar{v}_w J_w}{T}\left(\Delta p - \Delta\Pi_w\right)\end{aligned}$$

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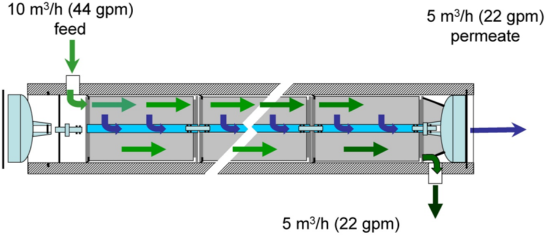
Water flux from solution-diffusion model, for A the membrane permeability:

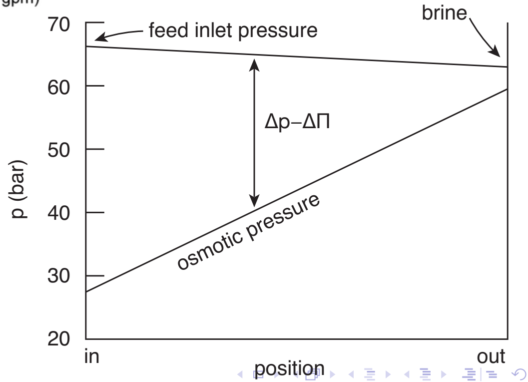
$$J_w = A\left(\Delta p - \Delta\Pi_w\right)$$

Thus,

$$\boxed{\dot{S}''_{\text{gen}} = \frac{\bar{v}_w A}{T}\left(\Delta p - \Delta\Pi_w\right)^2}$$

Pressure variation in single-pass RO

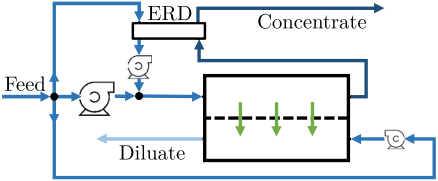




Entropy generation

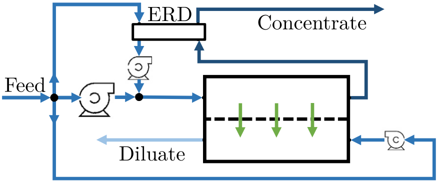
$$\dot{S}''_{\mathrm{gen}} = \frac{\bar{v}_w A}{T} \left(\Delta p - \Delta \Pi_w\right)^2$$

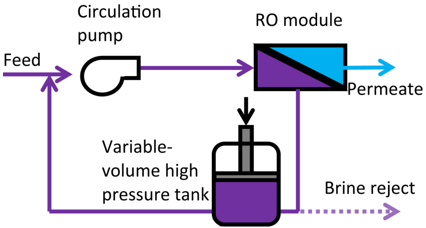
Novel RO configurations that are better balanced



- Split-feed counterflow RO (above left)

Novel RO configurations that are better balanced





- Split-feed counterflow RO (above left)
- Batch RO with pressurized tank (right top)

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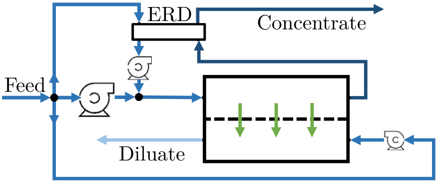
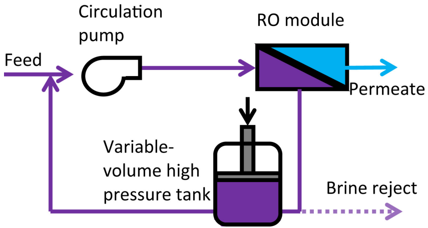
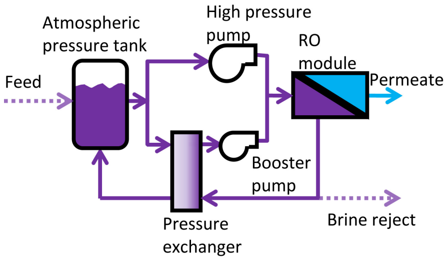




- Split-feed counterflow RO (above left)
- Batch RO with pressurized tank (right top)
- Batch RO with pressure exchanger (right bottom)



Entropy generation in balanced, counterflow HX

$\dot{Q} = UA\Delta T$

For $T >> \Delta T$:

$$\dot{S}_{gen} \approx \dot{Q}\left(\frac{\Delta T}{T_{h,in}\, T_{c,in}}\right)$$

For a given $\dot{Q} = UA\Delta T$:

- At fixed *UA*, a higher flux, more compact device has same $\dot{S}_{gen}$

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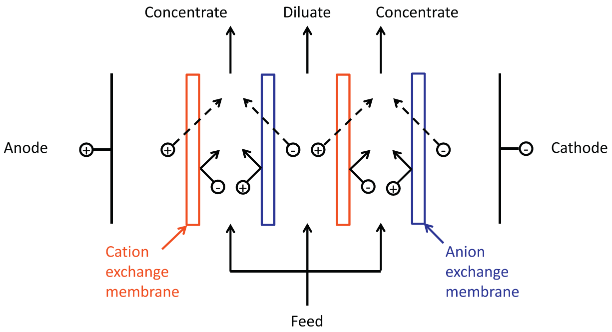
- At fixed UA, a higher flux, more compact device has same $\dot{S}_{gen}$
- Lower ΔT lowers $\dot{S}_{gen}$, which favors raising UA (e.g., use more area)
- If additional area is expensive and U cannot be raised, a higher ΔT can limit capital investment, but with higher $\dot{S}_{gen}$ and lower energy efficiency (CAPEX vs. OPEX)
  - Brackish water reverse osmosis (BWRO)
  - Electrodialysis

Electrodialysis

High membrane cost favors high average flux: concentration balancing less useful



$$\sigma = \frac{1}{T}\mathbf{E}\cdot\mathbf{j} - \frac{1}{T}\sum_k \nabla_T \mu_k \cdot \mathbf{J}_k$$

$$\dot{S}''_{\text{gen}} \approx \frac{\text{j}}{T}\left(\Delta V_{\text{cp}} - \frac{\Delta\mu_s}{F}\right) \quad \text{for} \quad \begin{cases} \Delta V_{\text{cp}} & \text{voltage diff. of one cell pair} \\ \Delta\mu_s & \text{salt chem. potential diff.} \end{cases}$$

Summary

Minimizing entropy generation minimizes desalination energy consumption

Desalination systems are like thermal power cycles in which the useful output is the work of separation.

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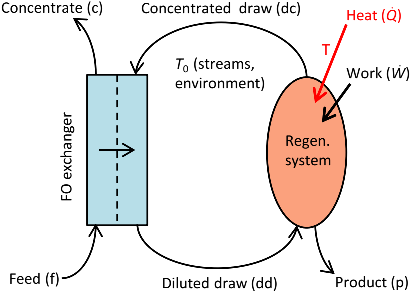
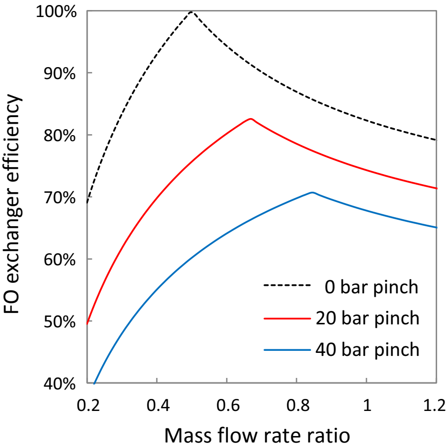
4. Balancing of concentration difference is often most significant in evaporative devices (with carrier gas).
5. In systems designed to minimize CAPEX, differences in driving force may be too large for balancing to help.

Supplementary slides

Forward osmosis mass exchanger balancing





Entropy generation

$$\dot{S}''_{gen} = \frac{\bar{v}_w A}{T} \left(\Pi_{draw} - \Pi_{feed}\right)^2$$

Entropy generation in a balanced counterflow heat exchanger

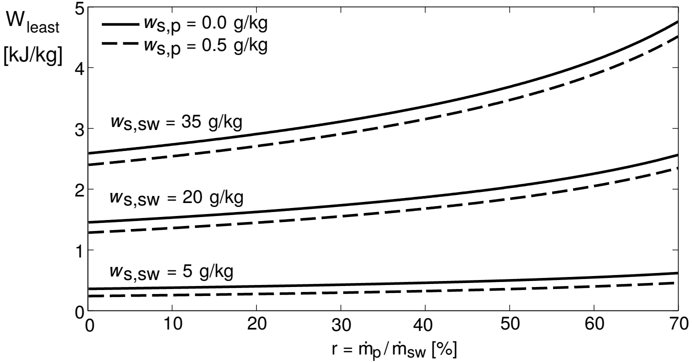
$$d\dot{S}''_{\text{gen}} = d\dot{Q}\left(\frac{1}{T_c} - \frac{1}{T_h}\right) \approx \frac{d\dot{Q}\,\Delta T}{T_c^2} = \frac{UP\Delta T^2}{T_c^2}\,dx$$

For a balanced counterflow exchanger of length L, $T_c = T_{c,\text{in}} + ax$ where the constant $a = \left(T_{c,\text{out}} - T_{c,\text{in}}\right)/L$. Integrating the local entropy generation for $\Delta T << T_{c,\text{out}}$ gives:

$$\begin{aligned}
\dot{S}_{\text{gen}} &= UP\Delta T^2 \int_0^L \frac{dx}{(T_{c,\text{in}} + ax)^2} \\
&= \frac{UP\Delta T^2}{a}\left(\frac{1}{T_{c,\text{in}}} - \frac{1}{T_{c,\text{out}}}\right) \\
&= \left(\frac{\dot{Q}\Delta T}{T_{c,\text{in}}T_{c,\text{out}}}\right) \approx \left(\frac{\dot{Q}\Delta T}{T_{c,\text{in}}T_{h,\text{in}}}\right)
\end{aligned}$$

Least work of separation

Reversible limit: $\dot{S}_{gen} = 0$



$$\eta^{II} = \frac{\dot{W}_{least}}{\dot{W}_{sep} + \dot{Q}_{sep}\left(1 - T_0/T_{sep}\right)}$$