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An experimental analysis of fluctuating temperature measurements using hot-wires at different overheats

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Abstract. Adjacent parallel hot-wire anemometers at different temperatures have sometimes been used to measure fluctuating temperatures in turbulent flows. This work presents an extensive experimental comparison of temperatures measured with a parallel-wire probe to temperatures simultaneously measured with a standard cold-wire probe. The results show the parallel-wire probe to work well in low intensity flows with temperature signals which are not too small. However, the parallel-wire probe temperature measurements are not accurate for high turbulence intensities or for small temperature signals, and in general the cold-wire system is probably to be preferred.

1 Motivations

The cold-wire temperature transducer, often used for the measurement of turbulent temperature fluctuations, has several inherent limitations. One is its small diameter and resultant fragility. Another is its comparatively low frequency response, for all but very small diameter sensors. In principle, both of these difficulties could be overcome by using a pair of constant-temperature hot wires to measure temperature and velocity. Hot wires are generally five to ten times larger in diameter, which makes them far more rugged, and their usable frequency responses may easily reach 10 kHz. Even higher bandwidths may be obtained from hot wires if loss of spatial resolution is not important.

The output voltage of a hot wire varies with the wire temperature. If two sensors at different constant temperatures are situated parallel to each other, so that they each experience the same flow speed and temperature, then their two differing output voltages may be used to solve the voltage response equations for the flow speed and temperature. This is the principle of the parallel-wire probe. The sensors of such probes are generally confined to relatively low overheats so as to maintain a high temperature sensitivity.

The idea of a parallel-wire probe is not new, and the use of varied temperature sensitivities to separate temperature and velocity signals is almost as old as the hot-wire itself. Corrsin suggested the concept in the forties (Corrsin 1949) and, in the following years, he made several applications of it. In 1949 and 1950, Corrsin and Uberoi carefully tested the combined temperature and velocity response of a hot-wire, and, by using several different values of the wire temperature, they obtained measurements of temperature-velocity correlations in a heated jet. A similar principle was employed by Mills et al. (1957) to determine the temperature fluctuation spectrum in nonisothermal grid turbulence. The technique of mixing probe sensitivities has been employed with checkered success many times in the intervening years, and dozens of papers have been written on the subject of hot-wire temperature response (an exhaustive survey is given by Freymuth 1982). In addition, the dual-overheat probe has been 'reinvented' at least once (Sakao 1973).

A recent application of the mixed sensitivity probe was made by Blair and Bennett (1984). They employed an array of constant temperature hot-wire sensors to measure fluctuating velocity and temperature in a nonisothermal boundary layer. Their work showed that, for carefully selected overheat ratios (1.2 and 1.5) and closely spaced sensors (0.35 mm separation), good agreement was obtained with the data of other investigators.

The present authors were led to attempt parallel-wire probe measurements in the course of a study of turbulence in a strong thermally stratified air flow. The probe appeared to offer the advantages of durability, high bandwidth, and (as pointed out by Blair and Bennett 1984) could be driven with commercial CTA bridges. In a preliminary work, we found that we were unable to reproduce Blair and Bennett's reported accuracy when we attempted to apply the parallel-wire probe to fluctuating temperature measurements in a heated round turbulent jet. This led us to examine critically the limitations on the performance of the parallel-wire probe under various flow conditions. That examination is the subject of this study.

Our results show that the parallel-wire probe is inaccurate for either small temperature signals or high turbulence intensities. In light of the associated complexities of calibrating the parallel-wire probe, we conclude that the cold-wire probe will be more appropriate in almost any experimental situation.

2 Calibration and operation

The voltage response of a hot wire in nonisothermal flow may be shown to be (e.g. Lienhard 1988)

$$E_0^2 = (T_w - T)(A\, T_f^{0.84} + B\, U^{0.45}) \tag{1}$$

for T_w the wire temperature, T the flow temperature, U the flow speed, $T_f = (T_w + T)/2$ the film temperature (°K), and A and B constants independent of temperature. The film temperature factor reflects variable properties effects and the $T_w - T$ factor represents the heat transfer driving force.

Hot wires may easily be calibrated for temperature changes when a cold wire is used as the temperature standard. In the present case, the temperature is also to be derived from the hot-wire signal, and it is convenient to simplify the form of the voltage response equation so that the flow temperature may be found without iteration. This simplification consists of expanding the film temperature in a Taylor series about some reference temperature, T_{ref}:

$$(T_w + T)^{0.84} \simeq T_{ref}^{0.84}\left(0.16 + 0.84\left(\frac{T_w + T}{T_{ref}}\right)\right) \tag{2}$$

This expansion is valid for variations of T less than about 85 °C at typical overheats. By making this substitution into Eq. (1), absorbing the factor of $2^{-0.84}$ into A, and collecting terms, we obtain

$$E_0^2 = C_1 + C_2 T + C_3 U^{0.45} + C_4 T^2 + C_5 T\, U^{0.45} \tag{3}$$

where the constants C_i are given by

$$C_1 = A\, T_w (0.16\, T_{ref}^{0.84} + 0.84\, T_w\, T_{ref}^{-0.16}) = -T_w (C_4\, T_w + C_2) \tag{4}$$

$$C_2 = -0.16\, A\, T_{ref}^{0.84} \tag{5}$$

$$C_3 = B\, T_w \tag{6}$$

$$C_4 = -0.84\, A\, T_{ref}^{-0.16} \tag{7}$$

$$C_5 = -B \tag{8}$$

The D.C. offset C_1 is not actually an independent constant. However, if all the C_i are fit to a set of (E_0, U, T) calibration data, then C_1 can be treated as an independent constant, creating an additional degree of freedom in the response equation. Equation (3) is linear in the constants C_i; thus, the C_i may be found directly by a least-squares fit to a set of calibration data.

For low-intensity turbulence, the response Eq. (3) may be fully linearized to express the fluctuating voltage, e, in terms of the fluctuating speed, u, and temperature, θ, as

$$e = K_u u + K_\theta \theta \tag{9}$$

with the K's given in terms of the mean speed, U, the mean temperature, T, and the mean voltage, E_0:

$$K_u = \frac{(C_3 + C_5 T)}{4.44\, E_0\, U^{0.55}} \tag{10}$$

$$K_\theta = \frac{(C_2 + 2\, C_4 T + C_5\, U^{0.45})}{2\, E_0} \tag{11}$$

Table 1. Typical parallel-wire calibration coefficients

Channel	C_1	C_2	C_3	C_4	C_5
1	4.025	0.00979	5.525	-4.462×10^{-5}	-0.01355
2	-2.374	0.04781	4.975	-1.185×10^{-4}	-0.01428

Each wire in the probe satifies this equation; the K's are known from the calibration. By elimination of e between the two equations, u and θ may be found.

The mean speed and temperature must be found using the full equation, not the linearized version. If we denote the five coefficients of Eq. (3) for wire one as C_i and the coefficients for wire two as B_i, then extensive algebra shows the mean temperature to be a root of

$$a\,T^3 + b\,T^2 + c\,T + d = 0 \tag{12}$$

with

$$a = (C_5 B_4 - B_5 C_4) \tag{13}$$

$$b = (B_2 C_5 + C_3 B_4) - (C_2 B_5 + B_3 C_4) \tag{14}$$

$$c = ((E_1^2 - C_1) B_5 - B_3 C_2 - (E_2^2 - B_1) C_5 + C_3 B_2) \tag{15}$$

$$d = (E_1^2 - C_1) B_3 - (E_2^2 - B_1) C_3 \tag{16}$$

After finding the physical root of this cubic, the mean speed may be calculated as

$$U^{0.45} = \left(\frac{E_1^2 - C_1 - (C_2 + C_4\, T)\, T}{C_3 + C_5\, T}\right) \tag{17}$$

These equations are solved using the values of E_i obtained by averaging over all records taken at the point measured.

Typical values of the calibration coefficients are given in Table 1. These values were obtained for a probe using overheat ratios of 1.5 and 1.2, respectively. From the definitions following Eq. (3), we may check these values for consistency. The coefficients C_5 are of the correct sign and magnitude, by comparison to values of B obtained in other calibrations of this equipment. The coefficients C_3 are positive, as required, and the ratio $-C_3/C_5$ gives plausible, although significantly inaccurate, values of T_w (407.7 °K, 348.4 °K). The coefficients C_1, C_2, and C_4 are more problematical. While each of these coefficients is of the correct magnitude, and the C_2's and C_4's have the correct signs, the coefficients C_1 do not correctly satisfy the relationship with C_2 and C_4 given by Eq. (4): for wire 1, the error is about 200%. Note that the condition (4) was not a constraint of the least-squares fit. Additionally, the values of T_{ref} implied by these constants are uncomfortably large (1152 °K, 2118 °K).

Equation (3) with a least-squares fit to the coefficients lacks full physical significance and must be viewed as an empirical law which correctly reproduces the calibration data, if not the physics of the probe. We have, however, repeated the calibration using the full King's Law, Eq. (1). The physically correct C_i's which result are numerically similar to those found by least-squares and give exactly the same statistical and spectral results as the least-squares C_i's.

3 Experimental comparison to the cold wire

A series of experiments were conducted in which a parallel-wire probe was situated adjacent to a cold-wire probe. The probes were calibrated simultaneously in the laminar outlet of a heated jet over a range of speeds and temperatures (the velocity/temperature calibrations required 3–4 h to complete). They were then used to obtain simultaneous measurements of temperature statistics and spectra in a heated turbulent jet and in stratified grid turbulence.

The parallel-wire probe used in this study was a modified TSI model 1244. Operating conditions were chosen to be similar to those of Blair and Bennett. The separation of the wires was 0.35 mm and the wire lengths were 1.25 mm. The wires were set to overheat ratios of 1.2 and 1.5 operated with a pair of DISA model 55M10 constant temperature anemometer bridges. The cold-wire probe was a DISA model 55P31, driven with a recently-designed cold-wire bridge (Haugdahl and Lienhard 1988). The cold-wire sensor was 1 μm in diamter and 0.4 mm long. It was located parallel to the other sensors with a separation of about 1 mm.

The calibration standard for the mean speed was a Pitot tube with an MKS Baratron differential pressure transducer (models #398HD and #170-6C) and for the mean temperature was an Omega Electronics three-wire platinum resistance thermometer with an Analogic PRT bridge. Signals were offset and amplified using precision buck and gain amplifiers (UCSD model 107) bringing them to the full range of the 14 bit analog-to-digital converter (Datastream Systems). Data were digitally tape-recorded using an LSI 11/23 microcomputer. Statistical and spectral analyses were performed subsequently.

At each point measured, time series were digitally recorded in records of 4096 samples per channel. For the jet experiments, the sample rate was 10,000 samples per second per channel (with low pass filtering at 4000 Hz), and 200 records were taken for each point. For the low speed grid data, the sample rate was 2500 samples per second per channel (with low pass filtering at 1000 Hz), again taking 200 records; for the high speed grid data, the sample rate was increased to 14,400 samples per second per channel (low pass filtered at 6000 Hz). For the low frequency grid and jet data, the sample rate was 250 samples per second per channel (filtered at 100 Hz), and 100 records were taken.

3.1 Biplane grid experiments

The parallel-wire and cold-wire probes were used to measure the fluctuating temperature, θ, in strongly stratified grid turbulence under several conditions. Data were taken at a downstream position $x/M = 33$ for a 2.54 cm grid and a temperature gradient of 200 °C/m. Details of this flow are given by Lienhard (1988).

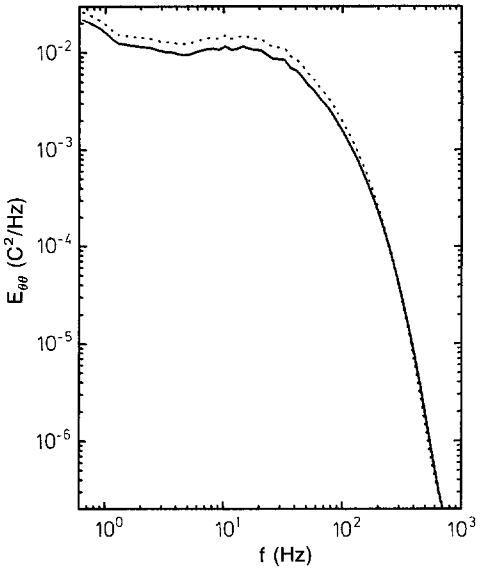

Fig. 1. Temperature spectra for grid turbulence, low bandwidth; —— parallel-wire probe, ···· cold-wire probe

In the first case studied, the mean speed was 2.4 m/s, a turbulence of fairly low frequency bandwidth ($R_\lambda \approx 25$). The cold wire measured an rms temperature of $\theta' = 1.34$ °C while the parallel-wire probe obtained $\theta' = 1.19$ °C, a 13% difference. The mean temperatures obtained were in reasonable agreement; the cold wire reported $T = 48.29$ °C and the parallel-wire probe reported $T = 47.75$ °C, a difference of 0.5 °C. The temperature spectra at this point are shown in Fig. 1. The spectra have the same shape for both temperature probes. For frequencies less than about 100 Hz, the cold-wire spectrum is about 10% greater than the parallel-wire spectrum, which is consistent with the statistical data. At high frequencies, the greater frequency response of the parallel-wire probe becomes apparent as the cold-wire spectrum drops off more rapidly.

The cold-wire temperature spectra were systematically greater than the hot-wire temperature spectra for all data; this suggests that the parallel-wire probe's static calibration does not correctly imply its dynamic sensitivity. Whether that is a generic feature of the parallel-wire probe or a shortcoming of the calibration remains unclear at present. Heat conduction to the prong supports of the sensor wire has been found to produce small reductions of the dynamic temperature sensitivity of both hot wires and cold wires below the value obtained by static temperature calibrations such as used here (Paranthoen et al. 1982, 1983). Estimates of the magnitude of the end conduction effect for the sensors involved, based on the models of the cited works, show it to be roughly the same for both the hot-wire and cold-wire probes used in these experiments. However, sufficient uncertainty exists in these models that a significant difference in the dynamic responses of the two probes cannot be ruled out.

As a further test of the comparative frequency responses of the two probes, the same measurements were repeated with the mean speed increased to 8.7 m/s, giving a signal of much greater frequency content ($R_\lambda \approx 50$). The mean temper-

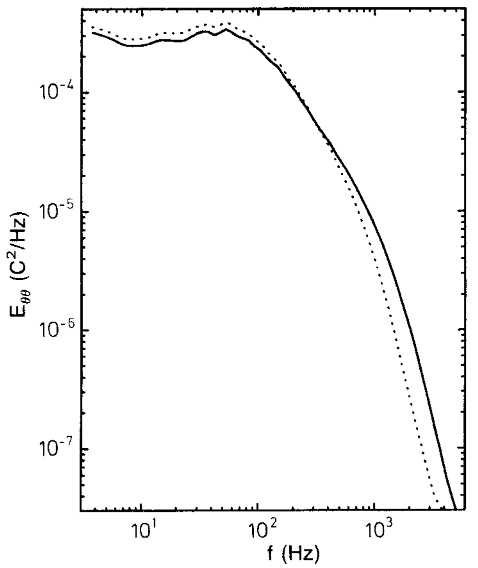

Fig. 2. Temperature spectra for grid turbulence, high bandwidth; —— parallel-wire probe, ···· cold-wire probe

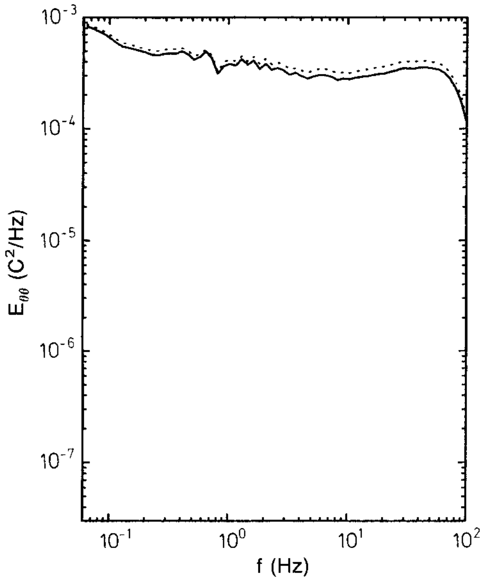

Fig. 3. Temperature spectra for grid turbulence, high bandwidth, low frequencies; —— parallel-wire probe, ···· cold-wire probe

ature obtained with the cold wire was 36.51 °C, and with the hot-wires it was 37.98 °C, a difference of 1.5 °C. The rms temperatures were 0.381 °C and 0.384 °C; these are closer because the variance producing eddies extend to higher frequencies where the declining frequency response of the cold wire brings the sensitivities of the two probes closer together.

The spectra for this case are shown in Fig. 2. These curves clearly illustrate that the hot-wire probe possesses much better frequency response than the cold-wire probe. The cold-wire spectrum is meaningless above 2 or 3 kHz, owing to the low-pass filtering of its thermal inertia. We made no attempt to compensate the cold-wire signal electronically or digitally for thermal inertia, although such compensation can be used to extend the frequency bandwidth of cold-wire sensors; e.g., Hopkins et al. (1988) added a decade to the frequency response of thermocouples using a compensation procedure. This point is discussed further in the conclusions.

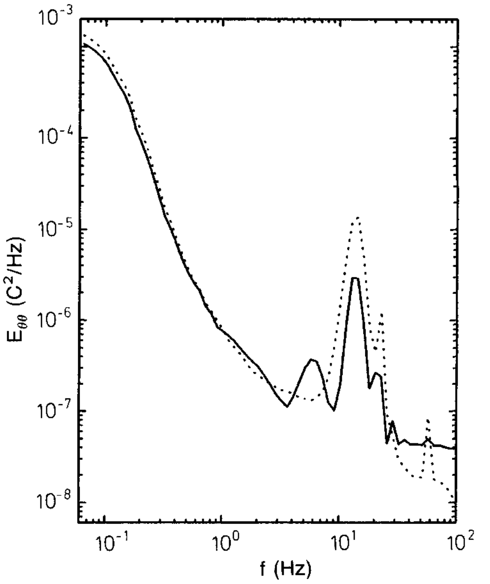

Fig. 4. Temperature spectra in an isothermal laminar flow; —— parallel-wire probe, ···· cold-wire probe

Figure 3 shows the low frequency portions of the spectra measured under the same conditions. The difference in sensitivities persists to the lowest frequencies, but decreases as the frequency decreases. This decrease may be associated with the fact that, in principle, the static (zero frequency) calibrations of the two temperature probes are identical.

3.2 Round jet experiments

Experiments were also performed with a 1.6 cm diameter heated jet. The first test made was to measure θ at the laminar outlet of the jet ($U = 5.8$ m/s). This case is essentially isothermal, with some small level of upstream noise, and thus provides a good indication of the real noise level of each temperature measuring system. In this case, the cold wire reported $\theta' = 0.0289$ °C and the parallel-wire probe reported $\theta' = 0.0247$ °C, a difference of 15%. The measured spectra, shown for low frequencies in Fig. 4, agree quite well at the lowest frequencies, but are completely different at higher frequencies. It appears that the parallel-wire probe suffers from severe noise at small temperature signals. This noise is almost certainly the result of the low temperature sensitivity of the parallel-wire system and the resultant electronic and computational error.

The true utility of the jet, however, lies in the higher intensity turbulence it produces. Data were recorded at a downstream position $x/D = 25$ on the centerline, where the turbulence intensity was $u'/U \simeq 0.28$. A series of seven jet outlet temperatures were employed, giving a range of small

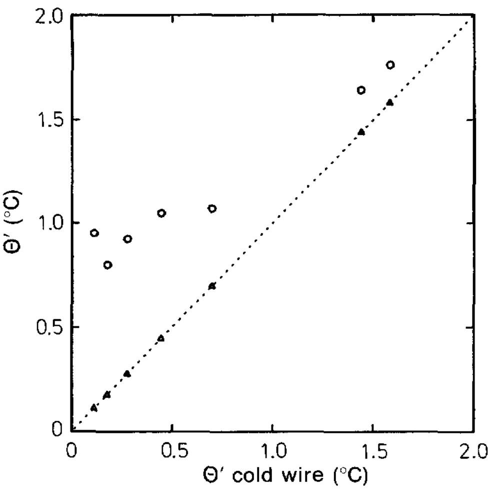

Fig. 5. Fluctuating temperature measured in a jet at different outlet temperatures; ○ parallel-wire probe, △ cold-wire probe

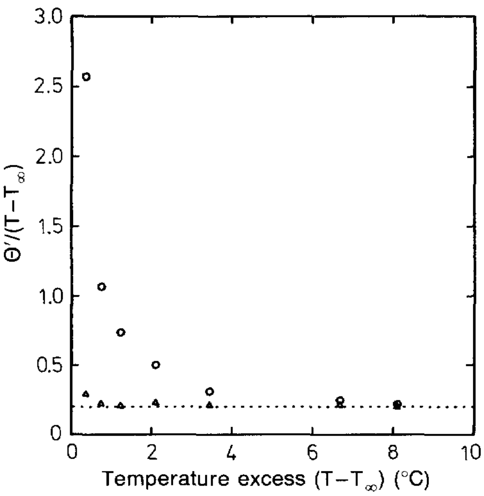

Fig. 6. Measured temperature intensity as a function of excess temperature; ○ parallel-wire probe, △ cold-wire probe

to large magnitudes of temperature signal in the turbulent jet. The jet speed at the point of measurement was held at about 3.6 m/s for all temperatures with a corresponding R_λ of about 150.

Figure 5 shows θ' measured with the parallel-wire probe as a function of θ' measured with the cold wire alone. The results show that the parallel-wire probe is not accurate for small temperatures in high intensity turbulence. At low levels of θ' the parallel-wire probe gives nonsensical temperature data. As θ' increases, the results improve. The figure also shows θ' measured with the cold wire with the hot wires on; this test confirms that temperature contamination of the cold-wire signal by the wakes of the hot wires did not occur.

The data reductions for the turbulent jet were performed using the low intensity turbulent response equations. The approximation that a 25% intensity could be considered low was tested for some of the data by repeating the reductions with the full response equations. The parallel-wire probe results were not noticeably improved by using the full equations.

Figure 6 shows the measured θ' as a function of the local temperature excess of the jet (above ambient temperature) for both probes. The data are presented as $\theta'/(T-T_\infty)$ for T the local mean jet temperature and T_∞ the temperature of the environment. For large values of the excess temperature, the ratio tends to 20% for both probes. The value of 20% compares favorably with that measured in other turbulent jet studies (Lockwood and Moneib 1980; Chevray and Tutu 1978; Becker et al. 1967). In fact, the ratio should be the same irrespective of the temperature excess, by virtue of the similarity properties of the jet flow, which are discussed by Hinze (1975). For small values of the excess temperature, however, the ratio measured with the parallel-wire probe departs strongly from the classical value, exceeding 250% for the coolest jet.

Temperature spectra measured in the jet at low temperature levels are shown in Fig. 7. To the authors' eyes, these curves seem totally unrelated. The parallel-wire probe is completely inaccurate in a flow of this type.

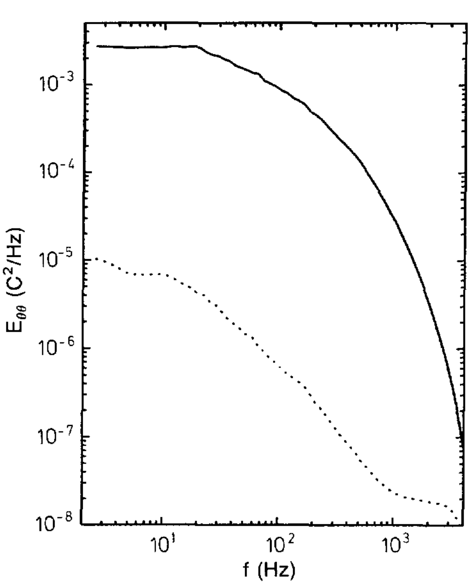

Fig. 7. Temperature spectra in a slightly heated jet; —— parallel-wire probe, ···· cold-wire probe

4 Overview of the parallel-wire probe

These experiments illustrate some severe limitations on the parallel-wire probe as a temperature transducer. The dynamic sensitivity of the probe is systematically low by about 10%, an effect unexplained at present. While it might be a feature of the present calibration procedure, the appearance of the discrepancy suggests that the probe may have some inherent skittishness which renders accurate calibration difficult.

The probe fails dramatically in high intensity flows with small temperature signals, both statistically and spectrally.

This may be the result of cross-talk between the temperature and velocity signals or the result of the finite spatial resolution of the two-wire probe. The probe should not be used in any high intensity flow without careful study of the size of temperature signal necessary to ensure accurate results.

The parallel-wire probe is reasonably accurate in the low intensity grid turbulence ($u'/U \simeq 0.02$) with strong temperature signals. This may account for Blair and Bennett's (1984) good results, since the boundary layer turbulence they studied had u'/U, $\theta'/\Delta T < 0.1$, fairly low intensities (they did not report ΔT). In such low-intensity flows, the greater frequency response of the parallel-wire probe could provide it with a significant advantage over an uncompensated cold wire.

However, cold-wire bandwidths can be enhanced by either electronic or digital compensation. Given the complexity of the calibration requirements of the hot-wire velocity-temperature system, the cold-wire system is probably to be preferred in any conceivable experimental circumstance for which fragility is not a limiting factor.

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