INSIGHTS AND ADVANCES IN THE ANALYSIS OF STRUCTURES

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ABSTRACT: The objective in this presentation is to survey our recent research accomplishments to advance the state of computational simulations in engineering and the sciences. We present our novel insights and recent advances in the analysis of transient and wave propagation problems, the simulation of large strain conditions of shells, the prediction of more accurate solutions using low-order elements, and the coupling of structures to general fluid flows and electromagnetic effects. At the conference, applications are given in the analysis of traditional structures and in the modeling of nano-structures, specifically, proteins and DNA assemblies. Also, major challenges are outlined for further advances in the field.

1 INTRODUCTION

Although the analysis of solids and structures has been much researched during the recent decades, and finite element methods are nowadays used abundantly, there are many analysis areas where further advances are still much needed. These areas primarily pertain to the solution of dynamic, nonlinear, and multiphysics problems. The objective in this paper is to briefly present some advances that we have recently accomplished. Due to space limitations, we mention only our books and papers, and refer the reader to the many references given therein.

When considering research achievements in the field, it is important to realize the philosophy that a research group adheres to in its research on computational methods. Our philosophy – as pursued for about 40 years now – is to focus on the development of methods that are general, reliable and efficient, and advance the current state of the art as practiced in industry and the sciences (Bathe 1996, Bathe 2009, Bathe 2012a).

The ultimate test as to whether a computational scheme is of value is, indeed, whether the scheme, once published, is used widely in industry and scientific investigations. Since there is a keen interest in engineering and the sciences to solve ever more complex and difficult physical problems, significant new, reliable and effective analysis procedures are quickly adopted.

Our research focus is on the conception of novel and general methods, their mathematical analyses and their testing to establish the generality, reliability and efficiency.

A finite element method is *general* if it is applicable to many varied problems in a certain problem area; for example, a general shell element can be used for all shell problems described by a general mathematical model like the 'basic shell model' identified in references (Chapelle & Bathe 2000, 2011).

A finite element method is *reliable* and *efficient* if identified as such; for example, a finite element discretization is reliable and efficient if the ellipticity and inf-sup conditions are satisfied without the use of any artificial factors, and the scheme shows optimal convergence at a low computational cost (Bathe 1996, Chapelle & Bathe 2011).

In the next sections, we present our recent developments, according to the above research aims, regarding the analysis of wave propagations, large strains in shells, prediction of more accurate solutions when using low-order elements, and the simulation of fluid-structure interactions including electro-magnetic effects. These schemes are widely applicable in traditional areas of analysis and also in novel areas, like the analysis of proteins and DNA structures.

2 INSIGHTS AND ADVANCES

We briefly focus in the following sections on some of the research we have pursued.

2.1 *Wave propagation problems*

The accurate analysis of transient wave propagations and the accurate solution of time harmonic problems at high frequencies have remained a significant challenge. The essential difficulties are that the waves need to be captured accurately by the mesh and the time integration scheme used. If high frequency waves are to be simulated, very fine meshes of conventional finite elements may be needed. However, even with such very fine meshes, to calculate a transient response accurately, the numerical dispersion and dissipation of waves due to the spatial and temporal discretizations must be small. To address these solution difficulties, spectral methods, spectral element methods, and spectral finite element methods have been proposed, for the solution of specific problems.

We have developed a finite element method ‘enriched for wave propagation problems’ (Ham & Bathe 2012). In this method the standard low-order Lagrangian finite element interpolations are simply enriched with harmonic functions, governed, as usual, by nodal degrees of freedom. An important point is that the usual fundamental theory of finite element methods is applicable.

For two-dimensional solutions, the basic displacement interpolations for a typical solution variable $u(r,s)$ are

$$
\begin{aligned}
u(r,s) &= \\
&\sum_{\alpha} h_{\alpha}(r,s) \\
&\left[\begin{array}{l}
U_{(\alpha,0,0)} + \sum_{k_x=1}^{n}\left\{\cos\left(\frac{2\pi k_x x}{\Lambda_x}\right)U^{C}_{(\alpha,k_x,0)} + \sin\left(\frac{2\pi k_x x}{\Lambda_x}\right)U^{S}_{(\alpha,k_x,0)}\right\} \\
+\sum_{k_y=1}^{m}\left\{\cos\left(\frac{2\pi k_y y}{\Lambda_y}\right)U^{C}_{(\alpha,0,k_y)} + \sin\left(\frac{2\pi k_y y}{\Lambda_y}\right)U^{S}_{(\alpha,0,k_y)}\right\} \\
+\sum_{k_x=1}^{n}\sum_{k_y=1}^{m}\left\{\begin{array}{l}\cos\left(\frac{2\pi k_x x}{\Lambda_x}+\frac{2\pi k_y y}{\Lambda_y}\right)U^{C+}_{(\alpha,k_x,k_y)} + \sin\left(\frac{2\pi k_x x}{\Lambda_x}+\frac{2\pi k_y y}{\Lambda_y}\right)U^{S+}_{(\alpha,k_x,k_y)} \\
+\cos\left(\frac{2\pi k_x x}{\Lambda_x}-\frac{2\pi k_y y}{\Lambda_y}\right)U^{C-}_{(\alpha,k_x,k_y)} + \sin\left(\frac{2\pi k_x x}{\Lambda_x}-\frac{2\pi k_y y}{\Lambda_y}\right)U^{S-}_{(\alpha,k_x,k_y)}\end{array}\right\}
\end{array}\right]
\end{aligned}
$$

where the $U_{(\alpha,k_x,k_y)}$ with superscripts are the nodal degrees of freedom, α is the local element node, with h_{α} the conventional finite element interpolation function, and the S, C, and + and – are used in the superscripts to correspond to the harmonic expressions. The interpolations for one- and three-dimensional analyses directly follow from the above equation. Here, the two fundamental wavelengths Λ_x and Λ_y, typically equal to twice the element lengths, and the wave cut-off numbers n and m with $1 \le k_x \le n$, $1 \le k_y \le m$, typically $n, m \le 3$, need to be chosen by the analyst.

These functions can directly be used to solve time harmonic and transient problems. Figure 1 shows the finite element solution of the Helmholtz equation corresponding to a 2D planar pressure wave

$$
\begin{aligned}
&\nabla^2 P + k^2 P = 0 && \text{in } V \\
&\frac{\partial P}{\partial n} = g(x,y) && \text{on } S_f \\
&\lim_{r\to\infty}\sqrt{r}\left(\frac{\partial P}{\partial r} - ikP\right) = 0
\end{aligned}
$$

Here good accuracy is seen, although a very coarse mesh is used (Ham & Bathe 2012).

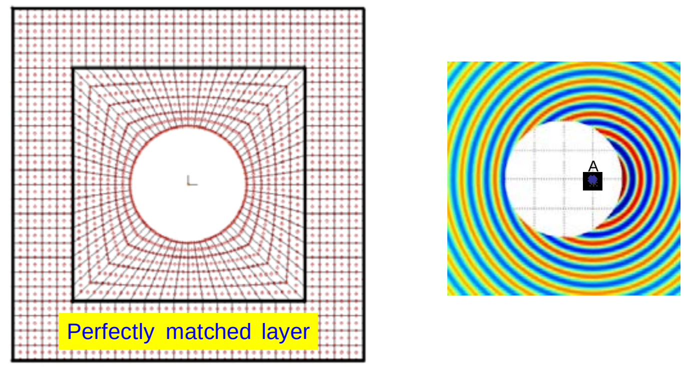


Figure 1. Solution of Helmholtz problem; 9-node elements and n=m=2; S_f = surface of the cylinder; point A is the pole

For transient analysis also an effective time integration scheme needs to be employed. Considering implicit schemes, we use the Bathe method (available in ADINA) which shows good cut-off characteristics of waves that can spatially not be resolved (Bathe 2007, Bathe & Noh 2012, Noh, Ham & Bathe, In Press).

Figure 2 gives the relative wave speed errors when traditional 2-node elements are used to solve a 1D wave propagation problem, here the CFL number is defined as CFL = $c_0\Delta t/\Delta x$. The key point to note is that for CFL=1.0, the wave speed error is very small for reasonable spatial discretizations – hence the dispersion error is small for those waves that can be represented spatially – and waves of lengths less than 3 elements are ‘cut-out’ of the response.

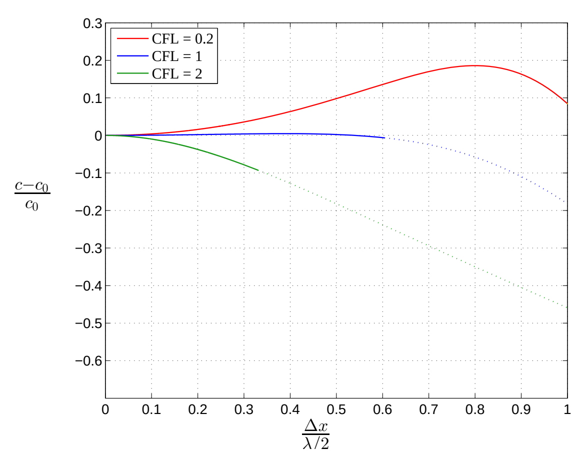


Figure 2. Relative wave speed errors of the Bathe method for various CFL numbers; results for discarded wave modes are given by dotted lines

The corresponding behavior of the widely-used trapezoidal rule of the Newmark method is given in Figure 3, here the property of cutting out the high-frequency modes is not present.

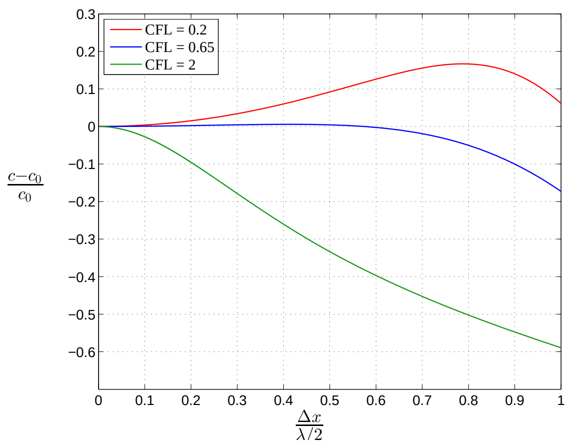


Figure 3. Relative wave speed errors of the trapezoidal rule for various CFL numbers

Figure 4 describes an illustrative 1D problem solved using standard 2-node elements. The response is given in Figure 5 using the Bathe method with CFL=1.0 and the trapezoidal rule with CFL=0.65, these being the optimal numbers to use. Less oscillations are seen when using the Bathe method. This better response prediction is due to cutting out those waves that cannot be accurately captured.

Figure 4. 1D bar impact problem, initial velocity = 0

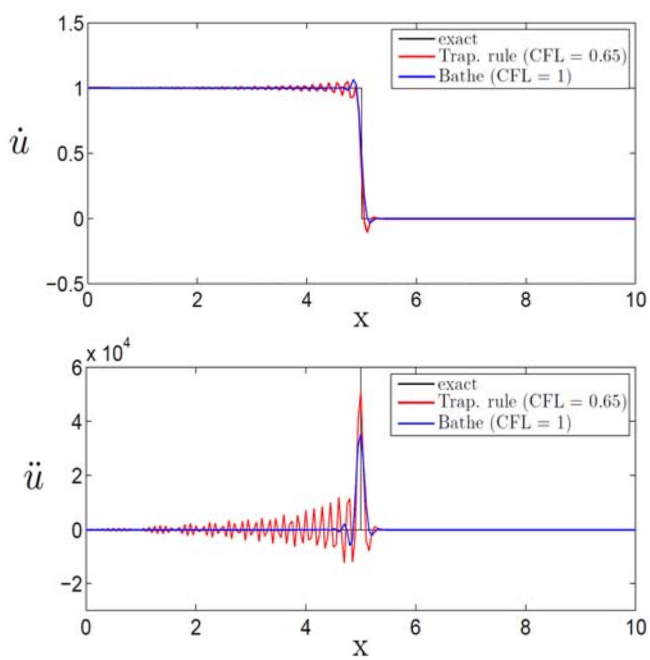


Figure 5. 1D bar impact problem, 200 2-node elements

For further solutions, see Ham & Bathe 2012, Bathe & Noh 2012, Noh, Ham & Bathe, In Press, and Kazancı & Bathe 2012. Also, using the Bathe method, an added benefit is that the load at the half-step can be selected for optimal accuracy, like for rapidly varying loads, the mean value of the loads at times t and $t+\Delta t$ is best used. An explicit time integration scheme with the property of 'cutting out' undesirable frequencies, as in the Bathe method, is given in Noh & Bathe 201x.

2.2 *3D-shell elements for large strain analysis*

Significant research efforts over many decades have focused on the analysis of shells, but there are still many outstanding challenges in the field (Chapelle & Bathe 2011); one such is the analysis of large strain behaviors. The 3- and 4-node MITC 3D-shell elements build upon the classical MITC shell elements but include important three-dimensional effects (Kim & Bathe 2008, Sussman & Bathe, In Press). The elements (available in ADINA) can be used to model very large deformations with large elastic or plastic strains. They are employed like the conventional MITC shell elements with 5 or 6 degrees of freedom at each midsurface node and additional nodal degrees of freedom to represent through-the-thickness straining and warping of the transverse fibers. In the formulations, MITC interpolations are used on the transverse fiber motions to preserve the volume and prevent shear locking, and in incompressible analysis the u/p formulation is employed (Chapelle & Bathe 2011, Sussman & Bathe, In Press).

Figure 6 shows schematically a 4-node element. The important point is that at each node two control vectors are used to describe the deformations. To eliminate shear locking, the motions of the transverse fibers are described by a tying rule that represents an extension of the rule used for the classical MITC shell elements (Chapelle & Bathe 2011), namely we employ

$$\begin{bmatrix} \dfrac{\partial\, {}^t x_1}{\partial r_1} & \dfrac{\partial\, {}^t x_2}{\partial r_1} & \dfrac{\partial\, {}^t x_3}{\partial r_1} \\ \dfrac{\partial\, {}^t x_1}{\partial r_2} & \dfrac{\partial\, {}^t x_2}{\partial r_2} & \dfrac{\partial\, {}^t x_3}{\partial r_2} \\ \left(\dfrac{\partial\, {}^t \mathbf{x}}{\partial r_1}\times\dfrac{\partial\, {}^t \mathbf{x}}{\partial r_2}\right)_1 & \left(\dfrac{\partial\, {}^t \mathbf{x}}{\partial r_1}\times\dfrac{\partial\, {}^t \mathbf{x}}{\partial r_2}\right)_2 & \left(\dfrac{\partial\, {}^t \mathbf{x}}{\partial r_1}\times\dfrac{\partial\, {}^t \mathbf{x}}{\partial r_2}\right)_3 \end{bmatrix} \begin{bmatrix} \left(\dfrac{\partial\, {}^t x_1}{\partial r_3}\right)^{AS} \\ \left(\dfrac{\partial\, {}^t x_2}{\partial r_3}\right)^{AS} \\ \left(\dfrac{\partial\, {}^t x_3}{\partial r_3}\right)^{AS} \end{bmatrix} = \begin{bmatrix} {}^t_r C_{13}^{AS} \\ {}^t_r C_{23}^{AS} \\ \det {}^t_r \mathbf{X}^{DI} \end{bmatrix}$$

where ${}^t x_i,\ i=1,2,3$ are the components of ${}^t\mathbf{x}$, $\partial\, {}^t\mathbf{x}/\partial r_i$ are the covariant base vectors, ${}^t_r C_{ij}$ are the components of the Cauchy-Green deformation tensor referred to the isoparametric description,

$\det {}_r^t\mathbf{X}^{DI}$ is the determinant of the deformation gradient directly calculated, the superscript t denotes at time t, and the superscript AS denotes "assumed".

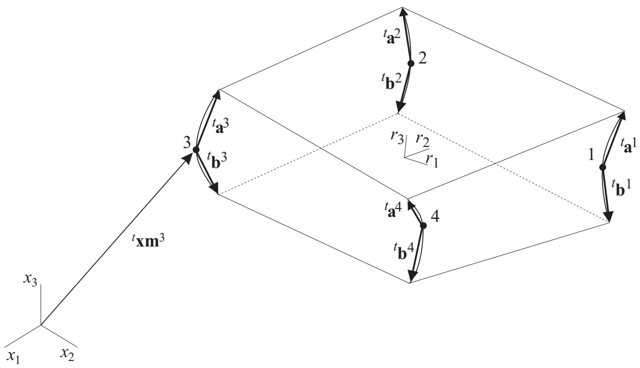
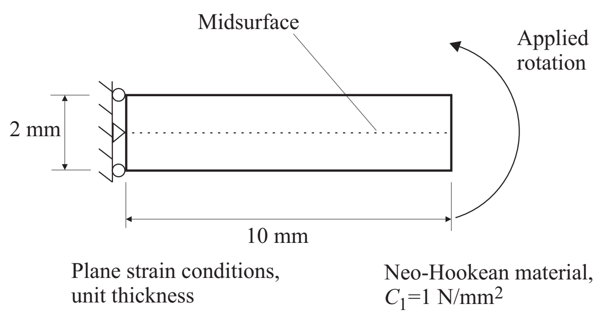
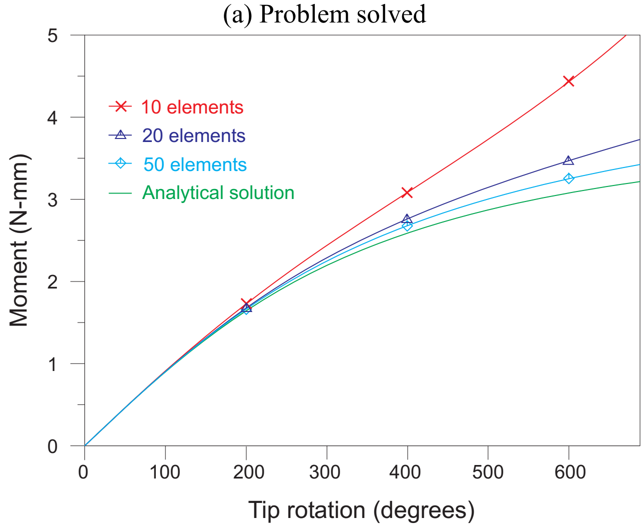


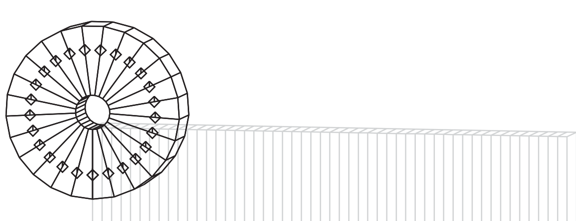
Figure 6. Nodes and control vectors of the 4-node 3D-shell element



(a) Problem solved



(b) Response of cantilever



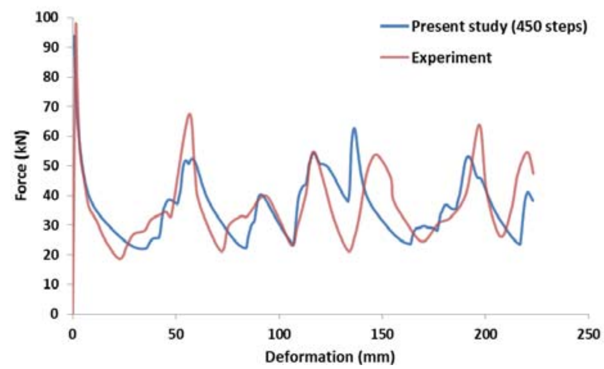
(c) Configuration after 720 degree rotation

Figure 7. Large strain analysis of thick plane strain cantilever

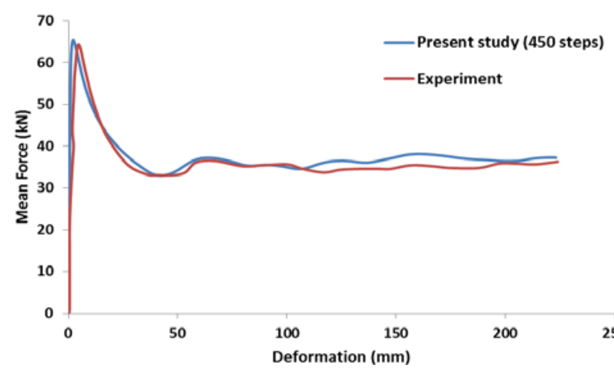
Figure 7 shows a large strain solution obtained with the MITC4 3D-shell element. Here the large strains in the structure result into a significant downward shifting of the midsurface nodes during the response.

Figure 8 shows some analysis results obtained in a slow crush analysis of a square tube using the MITC4 3D-shell element and the Bathe time integration scheme; for details and also crash solution results see Kazancı & Bathe 2012.

(a)



(b)



(c)

Figure 8. Quasi-static crushing of a square-section tube, length of tube is 310 mm; (a) experimental and computed results in final configuration; (b) force - displacement curves; (c) mean crushing force - displacement curves

2.3 *Procedures for solution improvements*

As well known, the 3-node triangular and 4-node quadrilateral elements in 2D solutions and 4-node tetrahedral and 8-node brick elements in 3D solutions lead to well-conditioned small bandwidth assembled matrices, they are robust but are not effective in predicting stresses accurately. Very fine meshes are needed in practice. Hence, it is of much interest to increase the order of stress convergence of these elements.

We have developed schemes to improve the predictions of stresses (Payen & Bathe 2012) and, both, displacements and stresses (Kim & Bathe 2013). To establish more accurate stresses from calculated displacements, we use two projection equations over the patch of elements that contains the element, for which improved stresses are sought, and those surrounding it (Payen & Bathe 2012)

$$\sum_{m=1}^{N_P}\left(\int_{V^{(m)}} \delta\underline{\overline{\tau}}^{(m)T}\left\{\underline{\tau}^{(m)}-\underline{\tau}_h^{(m)}\right\}dV\right)=0 \quad \forall\ \underline{\overline{\tau}}^{(m)}\in \overline{\mathbf{V}}^{\tau}$$

$$\sum_{m=1}^{N_P}\left(\int_{V^{(m)}} \delta\underline{\zeta}^{(m)T}\left\{div\left[\underline{\tau}^{(m)}\right]+\underline{f}^{B}\right\}dV\right)=0$$

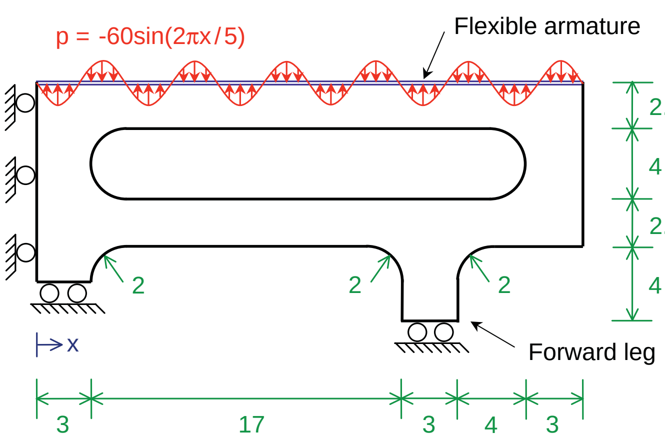
$$\forall\ \underline{\zeta}^{(m)}\in P_1$$

where P_k denotes the kth order polynomial, $\underline{\tau}^{(m)}$ represents the enhanced stresses to be calculated in P_2, $\underline{\tau}_h^{(m)}$ are the directly-calculated stresses, $\underline{\overline{\tau}}^{(m)}$ is an element in $\overline{\mathbf{V}}^{\tau}$, the subspace of P_2 in which all stress functions satisfy the equilibrium equations $div\left[\underline{\overline{\tau}}^{(m)}\right]=\underline{0}$, and N_P is the number of finite elements in the patch considered.

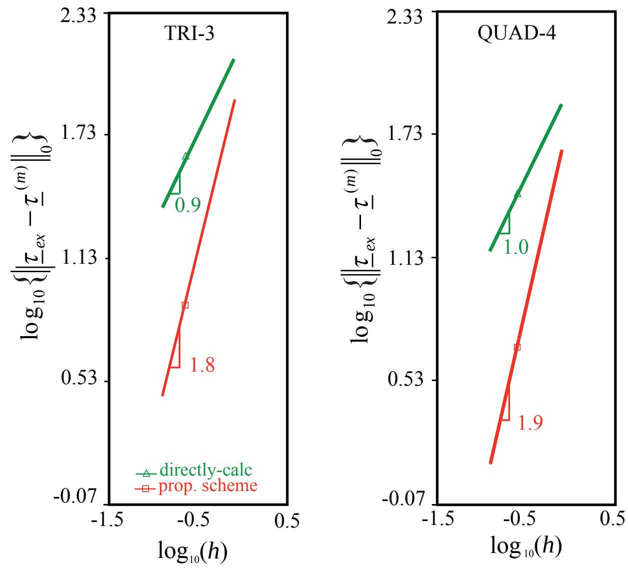
The scheme is effective, needs to only be applied in certain critical stress areas, and can be used in linear and nonlinear, static and dynamic analyses.

Figure 9 shows an actuator analyzed and the results obtained. It is seen that with the scheme used the stresses converge with almost second order, i.e. with the same order as the displacements.

This scheme only improves the stresses. We have also researched another approach in order to improve all solution variables when using 3-node 2D triangular and 4-node 3D tetrahedral elements through 'enriching the finite element method by interpolation covers' (Kim & Bathe 2013). These covers are also applied only in those regions where higher accuracy is sought in the solution.



(a) Actuator modeled in 2D



(b) Stress convergence curves for the actuator problem. Left: using 3-node element; right: 4-node element

Figure 9. Analysis of actuator subjected to Lorentz force

Considering a patch of 2D 3-node elements, the essence of the enrichment scheme is to interpolate the solution variable using

$$u_h=\sum_{i=1}^{3}h_i u_i+\sum_{i=1}^{3}\underline{\tilde{H}}_i\,\underline{\tilde{a}}_i \qquad \text{in the } element$$

where u_i is the usual nodal degree of freedom and $\underline{\tilde{a}}_i$ lists additional degrees of freedom with

$$\underline{\tilde{H}}_i=h_i\left[\overline{x}_i \quad \overline{y}_i \quad \overline{x}_i^2 \quad \overline{x}_i\overline{y}_i \quad \overline{y}_i^2 \quad \cdots \quad \overline{y}_i^p\right]$$

Here, the $(\overline{x}_i,\overline{y}_i)$ are coordinates measured from the node i, and p is the cover order used. Important considerations pertain to choosing computationally effective covers and having a well-conditioned coefficient matrix (Kim & Bathe 2013).

2.4 *Coupling of structures to fluid flows and electromagnetic effects*

Separate solutions of solids, structures, fluid flows, and electromagnetics have been obtained for decades. However, the solution of multiphysics problems involving fully coupled fluid-structure interactions with temperature and electromagnetic effects has hardly been tackled and presents special difficulties. Such fully coupled problems need to be solved, for example, in biomedical engineering, metal processing, and plasma physics.

We have developed a novel finite element scheme for the solution of the general Maxwell's equations to simulate fluid flows, solids, and structures coupled with electromagnetic effects (Bathe 2012a, Bathe, Zhang & Yan 201x). The finite elements used are similar to those we proposed for the solution of the Navier-Stokes equations (Bathe & Zhang 2009).

The basic equations we solve are, considering Faraday's law and Ampere's law with the Maxwell term, in the E-H form

$$\nabla \bullet \left(\left(p + \rho_0 / \varepsilon^* \right) \mathbf{I} - \nabla \mathbf{E} + \mathbf{I} \times \mathbf{K} \right) = \mathbf{0} \quad in\, \Omega_e$$

and

$$\nabla \bullet \left(q\mathbf{I} - \nabla \mathbf{H} - \mathbf{I} \times \mathbf{J} \right) = \mathbf{0} \quad in\, \Omega_m$$

where Ω_e and Ω_m denote the domains of electric and magnetic fields, resp., $\mathbf{K}$ and $\mathbf{J}$ are source terms for the electric and magnetic field intensities, ρ_0 is the charge density, ε^* is the effective permittivity (for static and harmonic solutions), $\mathbf{I}$ is the identity tensor, and we introduced

$$p = \nabla \bullet \mathbf{E} - \rho_0 / \varepsilon^*$$
$$q = \nabla \bullet \mathbf{H}$$

Of course, these equations specialize to specific cases by omitting certain terms, and the equations must be used with appropriate boundary and interface conditions.

However, for certain problems, a potential formulation can be more effective, in which the electric and magnetic potentials, ϕ and $\mathbf{A}$, are used

$$\mathbf{E} = -\nabla \phi - \frac{\partial \mathbf{A}}{\partial t}; \qquad \mathbf{B} = \nabla \times \mathbf{A}$$

with $\nabla \bullet \mathbf{A} = g_A$ and g_A a gauge condition. Solutions using these formulations in ADINA are shown at www.adina.com.

3 CONCLUDING REMARKS

In this paper we briefly summarized some of the research we have pursued during the recent years, with our research philosophy adhered to in these developments.

Significant advances in many areas are still to be accomplished – in particular when the actual *practice* of simulations in engineering and the sciences is focused upon. Here the reliability and efficiency of procedures is very important. Major challenges exist in solving multiphysics problems, at small and large scales, and in multiscale conditions.

In all applications, the endeavor is, in essence, to 'understand nature and try to predict the future through analysis'. This endeavor will be of increasing interest and value throughout the sciences and engineering, and the hierarchical process of analysis will be of particular value, see references (Bathe 1996, Bucalem & Bathe 2011). However, while novel areas of simulations will attract exciting attention, it is also important to continuously increase the efficiency of methods in traditional areas of analysis, like for the analysis of shells and for the solution of large finite element systems (Chapelle & Bathe 2011, Bathe 2012b).

4 ACKNOWLEDGEMENTS

I would like to acknowledge my most valuable collaboration with my students and colleagues, referred to in the references.

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