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**IMPLEMENTATION OF THE ENRICHED
SUBSPACE ITERATION METHOD**

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Module ESSPACE is an implementation of the enriched subspace iteration method [1] for the solution of the smallest eigenvalues and corresponding eigenvectors of the generalized symmetric eigenproblem $\mathbf{K}\phi = \lambda\mathbf{M}\phi$. The argument variables and use of the subroutine are described using comment lines in the module. The module is written in Fortran free-format and requires a set of subroutines: STARTV, DECOMP, REDBAK, MULT, JACOBI and SCHECK.

An example of running the codes

The plane frame structure, shown in Figure 1, is considered as an example problem to show how to run the module ESSPACE. We provide a driving program “ESSPACE_MAIN” with some input data files. The binary files “frame2d_ex.stiff” and “frame2d_ex.mass” are the corresponding stiffness matrix data and mass matrix data, respectively, and the formatted file “frame2d_ex.in” provides the required argument variables. In the each binary file, the matrix data is stored sequentially as follows: order of the matrix in single precision, number of elements below skyline of the matrix in single precision, array storing the addresses of diagonal elements of the matrix in single precision and array storing the elements of the matrix in compacted form [2] in double precision. You can, of course, modify the program “ESSPACE_MAIN” depending on the format that the data is stored in.

In order to run the program, please do the following:

1. If the GNU fortran is not installed, install it by using “sudo apt-get install gfortran” for Linux and by downloading and running this [link](#) for Windows.
2. Extract ESUBSPACE.zip to any location on your computer.
3. Run compile.sh for Linux and compile.bat for Windows (compiling and linking) and then you will have an executable file ESSPACE_MAIN.
4. Run ESSPACE_MAIN and type “frame2d_ex”

Table 1 shows the convergence history of the enriched method and the basic method. The enriched method requires 7 iterations (actually 8 iterations because we perform a single basic subspace iteration when establishing the starting vectors) while the basic method requires 19 iterations.

References

- [1] Ki-Tae Kim and Klaus-Jürgen Bathe. The Bathe subspace iteration method enriched by turning vectors. *Computers & Structures*, 186:11–21, 2017.
- [2] Klaus-Jürgen Bathe. *Finite Element Procedures*. Klaus-Jürgen Bathe, Watertown, MA, second edition, 2014.

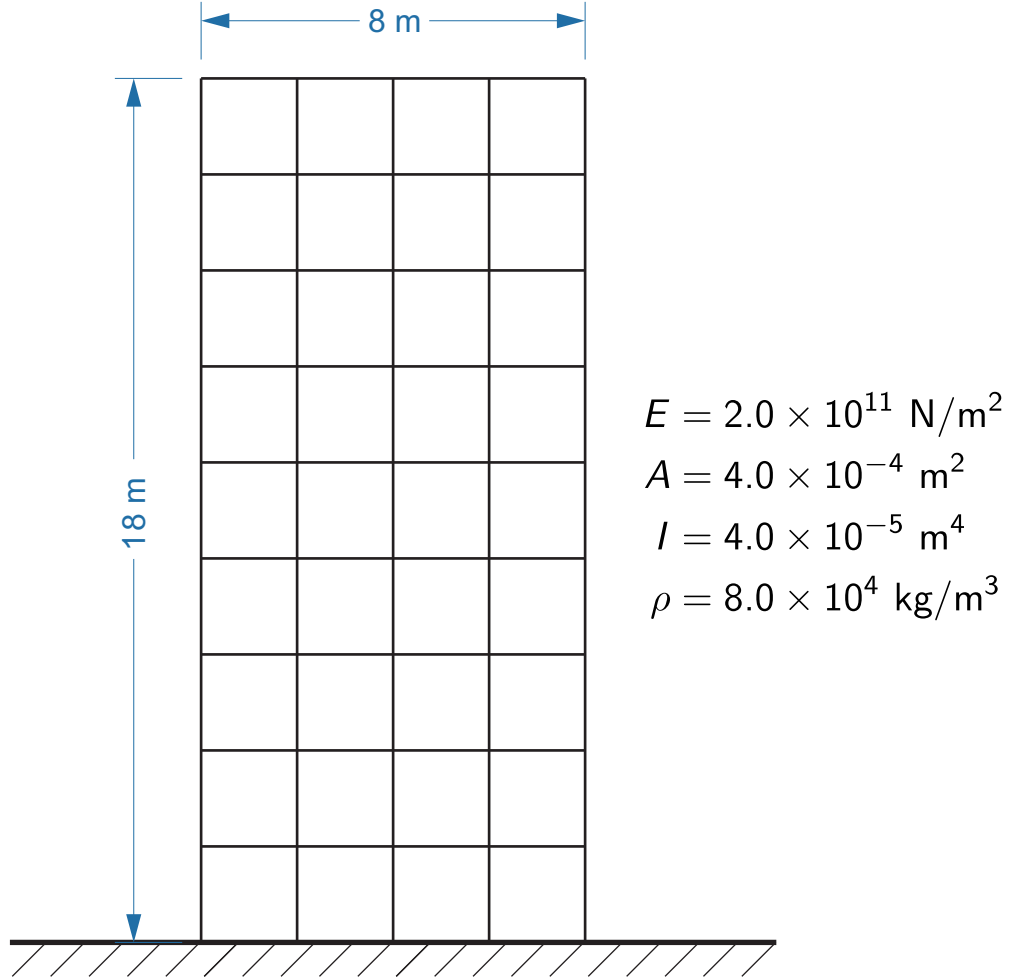


Figure 1: A plane frame structure; all the frame members have the same material properties, Young's modulus E , sectional area A , second moment of area I and mass density ρ , the values of which are shown in the figure, with the length $L = 2 \text{ m}$.

Table 1: Number of converged vectors after each iteration when using the enriched method and the basic method

Iteration number	Cumulative number of converged vectors	
	Enriched method	Basic method
1	0	0
2	3	0
3	7	0
4	14	1
5	20	3
6	22	4
7	25	5
8		8
9		10
10		12
11		14
12		16
13		19
14		19
15		22
16		23
17		23
18		25
19		26