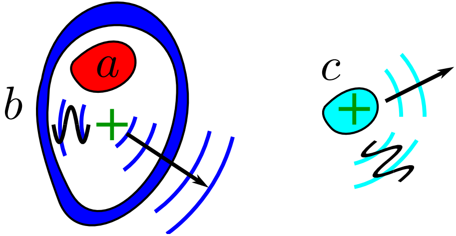
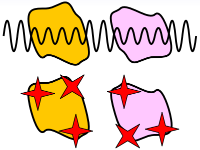
General formula



$$\mathcal{E} = \frac{\hbar c}{2\pi} \int_0^\infty d\kappa \, \ln\det \begin{pmatrix} \mathbb{F}_a^{-1} & \mathbb{X}_{ab} & \mathbb{X}_{ac} \\ \mathbb{X}_{ba} & \mathbb{F}_b^{-1} & \mathbb{X}_{bc} \\ \mathbb{X}_{ca} & \mathbb{X}_{cb} & \mathbb{F}_c^{-1} \end{pmatrix} \begin{pmatrix} \mathbb{F}_a & 0 & 0 \\ 0 & \mathbb{F}_b & 0 \\ 0 & 0 & \mathbb{F}_c \end{pmatrix}$$

Rahi, Emig, Graham, Jaffe, Kardar, PRD (2009)

$$\int \mathcal{D}A\, e^{-\int d^3x\, \mathbf{E}(\cdots)\mathbf{E}^* + \frac{1}{\kappa^2}\mathbf{E}\mathbb{V}(ic\kappa,\mathbf{x})\mathbf{E}^*}$$



$$\sim \int \mathcal{D}\mathbf{J}|_{\mathrm{obj}}\, e^{-\int d^3x d^3x'\, \mathbf{J}\, \underbrace{\left[\mathbb{G}_0(ic\kappa,\mathbf{x},\mathbf{x}') + \frac{\delta(\mathbf{x}-\mathbf{x}')}{\mathbb{V}(ic\kappa,\mathbf{x})}\right]}_{\mathbb{T}^{-1}(ic\kappa,\mathbf{x},\mathbf{x}')}\mathbf{J}^*}$$

$$\sim \det{}^{-1}\left(\begin{array}{c|c|c} \langle \mathbf{x}_a|\mathbb{T}_a^{-1}|\mathbf{x}'_a\rangle & \langle \mathbf{x}_a|\mathbb{G}_0|\mathbf{x}'_b\rangle & \cdots \\ \hline \langle \mathbf{x}_b|\mathbb{G}_0|\mathbf{x}'_a\rangle & \langle \mathbf{x}_b|\mathbb{T}_b^{-1}|\mathbf{x}'_b\rangle & \cdots \\ \hline \cdots & \cdots & \cdots \end{array}\right)$$

$$\mathbb{G}_{0,ij} \sim \sum_{\alpha\beta} |\mathbf{E}_\alpha\rangle \mathbb{X}_{ij,\alpha\beta}\langle \mathbf{E}_\beta| \qquad \mathbb{F}_{i,\alpha\beta} \sim \langle \mathbf{E}_\alpha|\mathbb{T}_i|\mathbf{E}_\beta\rangle$$

Result:

$$\mathcal{E} = \frac{\hbar c}{2\pi}\int_0^\infty d\kappa \ln\det \begin{pmatrix} \mathbb{F}_a^{-1} & \mathbb{X}_{ab} & \cdots \\ \mathbb{X}_{ba} & \mathbb{F}_b^{-1} & \cdots \\ \cdots & \cdots & \cdots \end{pmatrix} \begin{pmatrix} \mathbb{F}_a & 0 & \cdots \\ 0 & \mathbb{F}_b & \cdots \\ \cdots & \cdots & \cdots \end{pmatrix}$$

Scattering

Wave equation

$$\left(\boldsymbol{\nabla} \times \boldsymbol{\nabla} \times + \mathbb{V}_i(\omega, \mathbf{x})\right) |\mathbf{E}^{\text{tot}}\rangle = \omega^2 |\mathbf{E}^{\text{tot}}\rangle$$

Lippmann-Schwinger:

$$|\mathbf{E}^{\text{tot}}\rangle = |\mathbf{E}^{\text{hom}}\rangle - \mathbb{G}_0 \underbrace{\left(\mathbb{V}_i \frac{\mathbb{I}}{\mathbb{I} + \mathbb{G}_0 \mathbb{V}_i}\right)}_{\mathbb{T}_i} |\mathbf{E}^{\text{hom}}\rangle$$

$$|\mathbf{E}^{\text{tot}}_{lmP}\rangle = |\mathbf{E}^{\text{reg}}_{lmP}\rangle + \sum_{l'm'P'} |\mathbf{E}^{\text{out}}_{l'm'P'}\rangle \underbrace{\left(-C \langle \mathbf{E}^{\text{reg}}_{l'm'P'} | \mathbb{T}_i | \mathbf{E}^{\text{reg}}_{lmP} \rangle\right)}_{\mathbb{F}_{i,l'm'P',lmP}}$$