V. *On the Nature of the Molecular Forces which regulate the Constitution of the Luminiferous Ether.* *By* S. Earnshaw, M.A. *of St. John's College, Cambridge.*

[Read *March* 18, 1839.]

There are already before the world by various authors several Memoirs, which, collaterally or incidentally, embrace the subject of the present communication. There is observable in them, however, much disagreement of results, which seems chiefly to arise from the extreme length and complexity of the investigations by which those results are obtained; to avoid which, as much as possible, their authors are compelled to adopt means of simplification, which we cannot always be certain *à priori* are sufficiently approximative. In the following pages the subject will be found to be treated in a manner perfectly new and direct, and, it is hoped also, satisfactory, inasmuch as the analytical operations employed are brief and simple, involving no principles of a difficult or doubtful character.

The authors to which I have just alluded have generally adopted, as a most extensive means of simplification, symmetrical arrangements of the particles of the ethereal medium. This may be necessary and even allowable in some cases: but as it has never been shewn that such arrangements actually *do exist* in Nature, nor even that they *can exist* in Nature, I have been careful to confine myself to the investigation of properties which are independent of arrangement, or rather, which do not involve the hypothesis of a peculiar arrangement.

I may also remark that the investigations which follow are in other respects of a very general character. For, in this attempt to discover the laws of molecular action of the ether, amongst the experimental properties, assumed as the basis of analytical investigation, are, I believe, none which are *peculiar* to the luminiferous ether. I think it probable, that most terrestrial bodies possess in a greater or less degree of perfection the properties here assumed: and consequently, the title of this paper might have been made more comprehensive. It might, perhaps, not improperly be, "*An Investigation of the Nature of the Molecular Forces, which regulate the Internal Constitution of Bodies.*" This might, however, be disputed, and therefore in the investigations I have referred only to the luminiferous ether. Nevertheless, that the reader may more easily judge what degree of claim the following pages have to that general character which is here ascribed to them, I shall, in as few words as possible, introduce a statement of the experimental assumptions, and the results respectively derived from them.

I. It is assumed that the ether consists of detached particles; each of which is in a position of equilibrium, and when slightly disturbed is capable of *vibrating in any direction*. (Many solid as well as aerial bodies transmit sound, which is generally supposed to imply the existence of the same properties in them as are here assumed to be true of the ether.)

The most curious and perhaps least expected result of this assumption is, *that the molecular forces which regulate the vibrations of the ether do not vary according to Newton's law of universal gravitation:* and it is not a little remarkable, that a force, whether attractive or repulsive, varying according to this law, is the only one which *cannot possibly actuate* the particles of a *vibrating* medium.

II. It is next assumed, that the motion of a vibrating particle is more affected by the influence of the particles which are near to it than of those which are more remote. (This is certainly true of many other substances besides the ether.) The result which is sought to be derived from this assumption is, *that the molecular forces which regulate the vibra-*

tion of the particles are REPULSIVE, *and vary according to an inverse power of the distance greater than 2.*

III. It is lastly assumed, that the ether exists (or at least is capable of existing) as one mass held together by the *attraction* of its elementary molecules. This assumption is necessary, in order that the dispersion of the medium which would naturally result from the *repulsive* forces which regulate the vibration of its particles, may be thereby prevented.

The result which is derived from this necessary assumption is, *that each particle exerts* (in addition to the repulsive force before mentioned) *an attractive force, which varies according to Newton's Law of universal gravitation.*

By reversing the problem, I have been able to shew, that though Newton's law is the only one which cannot enable the particles to *vibrate,* yet it is the only law of force which can enable them to constitute and maintain themselves a *permanent* medium, without endangering, or in any way affecting their *vibrating* or luminiferous property.

I have on these grounds not hesitated to express my opinion, *that the particles of the luminiferous ether are each endued with two forces of distinct characters and uses; one attractive, to preserve themselves a permanent medium, varying inversely as the square of the distance; and the other repulsive, to which is due their luminiferous property, varying in a higher inverse ratio of the distance than the square.*

A SYSTEM OF DETACHED PARTICLES.

1. *If* V *denote the sum of the quotients formed by dividing each attracting body by its distance from the attracted body; then* $V = C$ *is the equation of a surface at any point of which if the attracted body be placed, it will begin to move in the direction of a normal.*

For, let f, g, h be the co-ordinates of the attracted body; F, G, H the attractions of the whole system upon it parallel to the co-ordinate axes, then

$$F = d_f V, G = d_g V, H = d_h V.$$

But the equation of the tangent plane at that point of the proposed surface where the attracted body is placed, satisfies the differential equation,

$$0 = d_f V . df + d_g V . dg + d_h V . dh;$$

$$\therefore \ 0 = F . df + G . dg + H . dh.$$

This equation shews that the resolved part of the attractive force is zero, in the direction of the tangent plane; and therefore the whole attraction is in the direction of the normal.

2. For the sake of brevity, I shall denominate the surface $V = C$, the *parametric* surface passing through the point f, g, h.

Different points in space may have corresponding different parametric surfaces; any one may be found by assigning the proper value to C. Their equations differ only in the value of the constant C, which, for this reason, I shall call the parameter.

If any parametric surface pass through an attracting particle, its parameter will be infinite, because at that point V is infinite; in which case the proposition will fail. The proposition is true of repulsive forces, or if some of the particles exert repulsive and some attractive forces; but when the forces are all attractive, V can neither be evanescent nor negative: since, however, it is infinite when the attracted particle touches any one of the attracting particles, and is not infinite in other positions, there must be some intermediate positions which make V a minimum, and there may be positions in which V is a maximum.

3. *The parametric surfaces which pass through points indefinitely near to a point of neutral attraction, are in general similar concentric*

hyperboloids of one and two sheets, the common centre of which is the point of neutral attraction. Certain points, however, have the asymptotic surface for their characteristic surface.

Let fgh be the co-ordinates of K, the point of neutral attraction, and $f+x$, $g+y$, $h+z$, the co-ordinates of P, a point very near to K. Let the value of V at K be C', and at P, C. Then the equations to the respective surfaces are

$$C' = V, \text{ and } C = V';$$

where V' is the same function of $f+x$, $g+y$, $h+z$ that V is of f, g, h.

$$\therefore\ C = V + d_fV.x + d_gV.y + d_hV.z + d_f^2V.\frac{x^2}{2} + d_g^2V.\frac{y^2}{2}$$

$$+ d_h^2V.\frac{z^2}{2} + d_fd_gV.xy + d_fd_hV.xz + d_gd_hV.yz + \&c.$$

But because K is a point of neutral attraction, $d_fV = 0$, $d_gV = 0$, $d_hV = 0$, and

$$\therefore\ 2(C - C') = d_f^2V.x^2 + d_g^2V.y^2 + d_h^2V.z^2 + 2d_fd_gU.xy + \&c.$$

This, neglecting terms above the second order, being the general equation of surfaces of the second order which have a centre, by transposing the co-ordinate axes so as to coincide with the principal axes of the surface, the terms containing xy, yz, xz will disappear, leaving only

$$2(C - C') = d_f^2V.x^2 + d_g^2V.y^2 + d_h^2V.z^2,$$

which for indefinitely small values of x, y, z may be regarded as the equation of the parametric surface. It must be remembered that the coefficients of x^2, y^2, z^2 are subject to the condition,

$$0 = d_f^2V + d_g^2V + d_h^2V;$$

and because at least one of these coefficients will be negative, and one positive, the equation is that of an hyperboloid.

4. That parametric surface which contains a point of neutral attraction will be a cone, which is asymptotic to all the hyperbolic parametric surfaces belonging to the other points.

For, the parameter of the surface passing through K, the point of neutral attraction is C', and therefore the equation of it is

$$0 = d_f^2 V . x^2 + d_g^2 V . y^2 + d_h^2 V . z^2,$$

which is the asymptote of the surfaces included in the equation,

$$2(C - C') = d_f^2 V . x^2 + d_g^2 V . y^2 + d_h^2 V . z^2.$$

6. If the position of equilibrium be such, that only one of the quantities $d_f^2 V$, $d_g^2 V$, $d_h^2 V$ is negative, as for instance, $d_f^2 V$; then the axis of the asymptotic cone will coincide with the axis of x; and all points within this cone will have hyperboloids of one sheet for their parametric surfaces, and their parameters will be less than C'. The points without this cone will have hyperboloids of two sheets for their parametric surfaces, and their parameters will be greater than C'.

If the position of equilibrium be such, that two of the quantities $d_f^2 V$, $d_g^2 V$, $d_h^2 V$ are negative, as for instance, $d_g^2 V$ and $d_h^2 V$, the axis of the asymptotic cone and the parametric surfaces will be as in the last case; but the parameters of points within the cone will be greater than C', and of points without it, less than C'.

7. If the molecular forces are all repulsive, then the sign of V will be changed: but the parametric surfaces will be hyperboloids, as before.

8. If the position of equilibrium be such, that $d_f^2 V = 0$, $d_g^2 V = 0$, and $d_h^2 V = 0$, then $d_f V$, $d_g V$, $d_h V$, *i.e.* the attractions F, G, H, being also evanescent, the particle is unattracted in every direction, at least for small displacements from the position of equilibrium. An example of this is afforded in the case of a particle placed within a spherical or ellipsoidal surface, composed of attracting or repelling par-

ticles. If the position of equilibrium be such, that one of them, as $d_f^2 V$, is evanescent, then, F being evanescent also, for small displacements parallel to the axis of x, the particle will be unattracted. An example of this is afforded in the case of a particle placed within a hollow elliptic or circular cylinder of indefinite length. The displacement of particles placed in such positions as those here considered would not bring into action any forces of restoration; on which account the particles would not vibrate. It is evident, therefore, that the phenomena of light and sound are not due to the motions of particles placed in such positions: and as the purpose of this paper is to examine the constitution of media supposed to be capable of transmitting light, a phenomenon due to *vibration*, we shall, in what follows, always suppose that none of the quantities $d_f^2 V$, $d_g^2 V$, $d_h^2 V$ are evanescent: under which supposition also, they cannot be equal, since their sum $= 0$.

9. Since the force which urges a displaced particle acts in the direction of a normal to the parametric surface in which the particle is at any moment situated, there are in general only three directions in which a particle can be displaced, so that the force called into play may act in the direction of the displacement. These directions coincide with the principal axes of the parametric hyperboloids. The exceptions to this are, when the constitution of the system is such, that two of the three quantities, $d_f^2 V$, $d_g^2 V$, $d_h^2 V$, are equal; in which case the asymptotic cone has a circular base, and the exterior parametric hyperboloid becomes the hyperboloid of revolution of one sheet: and since the normals to this surface, corresponding to points in that principal section which is perpendicular to the axis of revolution, all pass through the centre, the force of restitution will always act in the line of displacement, when the particle is disturbed in any direction in this plane. This is the only exception.

10. It is very important to remark, that since the parametric surfaces cannot be spherical in any case, the constitution of a medium, composed of detached attractive particles, can never be such that the force of restitution called into play by a disturbance in *any* direction

shall act in the line of displacement. Hence those media which are distinguished as *uncrystallized*, cannot consist of detached particles which either attract or repel each other, with forces varying inversely as the square of the distance; because it is assumed as a characteristic property of such media, that the forces of restitution act always in the direction of displacement.

11. *To find the force of restitution, when a particle is slightly disturbed from its position of equilibrium.*

Let F', G', H' be the resolved parts of the force of restitution parallel to the co-ordinate axis upon the particle at P; then F' is the same function of $f+x$, $g+y$, $h+z$, that F is of f, g, h, and therefore

$$F' = F + d_f F \,.\, x + d_g F \,.\, y + d_h F \,.\, z + \ldots$$

$$\text{or, } F' = d_f V + d_f^2 V \,.\, x + d_f d_g V \,.\, y + d_f d_h V \,.\, z + \ldots$$

$$= d_f^2 V \,.\, x + \text{terms involving } x^2,\ y^2,\ z^2,\ xy, \text{ \&c.}$$

because $d_f V = 0$, $d_f d_g V = 0$, $d_f d_h V = 0$. (Art. 4).

Similarly, $G' = d_g^2 V \,.\, y + \ldots$

and $H' = d_h^2 V \,.\, z + \ldots$

Hence, if the system consisted of fixed particles, the particle P only being moveable, the equations for P's motion would be

$$\left.\begin{aligned} d_t^2 x &= d_f^2 V \,.\, x \\ d_t^2 y &= d_g^2 V \,.\, y \\ d_t^2 z &= d_h^2 V \,.\, z \end{aligned}\right\} \text{ very nearly.}$$

It is remarkable, that $\dfrac{d_t^2 x}{x} + \dfrac{d_t^2 y}{y} + \dfrac{d_t^2 z}{z} = 0$.

12. From this investigation it appears, that the force of restitution parallel to any one co-ordinate axis depends only upon its displacement

parallel to the same axis. We may therefore consider the effect of each component of the displacement separately.

It appears from the equations just obtained, that $-d_f^2 V$, $-d_g^2 V$, $-d_h^2 V$ are the *absolute* forces of *restitution*.

Since one at least of the quantities $d_f^2 V$, $d_g^2 V$, $d_h^2 V$ is negative, and one at least positive, there will be at least one principal axis parallel to which a disturbed particle can vibrate, and at least one parallel to which a disturbed particle *cannot* vibrate. Suppose for instance, that $d_f^2 V$ is positive and $d_h^2 V$ negative, then the first equation $d_t^2 x = d_f^2 V . x$ takes the form

$$d_t^2 x = a^2 x,$$

the integral of which is

$$x = C' e^{at} + C'' e^{-at};$$

a result which shews that x must increase continually with t. The motion in this direction will therefore be one of *translation*.

But for that part of the displacement which is parallel to the axis of z, the equation of motion is

$$d_t^2 z = -\gamma^2 z.$$

The integral of which is

$$z = A \cos (\gamma t + B),$$

which denotes vibration.

13. If the constitution, or arrangement of the particles, of the medium is such that $d_g^2 V$ is positive, the motion parallel to y will be one of translation; and consequently there will only be one line in which a particle can be displaced, so that its motion may be vibratory.

14. If the constitution of the medium be such that $d_g^2 V$ is negative, the motion parallel to y will be vibratory; and therefore if the particle be displaced in any direction in the plane yz, it will continue to vibrate in that plane, describing an elliptic orbit.

15. It appears then that at the most, the equilibrium can only be stable in one *plane;* and that the medium may be so constituted that the equilibrium shall be stable only in one *line.* The character of instability, which in the preceding articles we have shewn necessarily attaches to a medium constituted of particles placed at finite intervals, and *attracting* each other with forces varying as $\frac{1}{D^2}$, cannot be removed by supposing the particles to *repel* each other with forces varying according to the same law. The equation $d_f^2 V + d_g^2 V + d_h^2 V = 0$, from which the instability arises, holds equally for attraction and repulsion.

It may be observed also that the instability cannot be removed by *arrangement;* for though the values of $d_f^2 V$, $d_g^2 V$, $d_h^2 V$ depend upon the arrangement of the particles, the fact that one at least must be positive and one negative depends only upon the equation $d_f^2 V + d_g^2 V + d_h^2 V = 0$, which is true for every arrangement. And consequently, whether the particles be arranged in cubical forms, or in any other manner, there will always exist a direction of instability.

It is therefore certain, that the medium in which luminiferous waves are transmitted to our eyes is not constituted of such particles. The coincidence of numerical results, derived from the hypothesis of a medium of such particles, with experiment, only shews that numerical results are no certain test of theory, when limited to a few cases only.

16. It has been noticed, that the instability of a system depends upon the equation $d_f^2 V + d_g^2 V + d_h^2 V = 0$. With the ordinary law of attraction it always holds good. If, however, the force of molecular attraction be assumed to vary as $\frac{1}{D^n}$, and we write

$$V \text{ for } \Sigma \frac{\left(\frac{m}{r^{n-1}}\right)}{n-1},$$

we shall find

$$d_f^2 V + d_g^2 V + d_h^2 V = (n-2)\, \Sigma \left(\frac{m}{r^{n+1}}\right) \;\ldots\ldots\; (1).$$

By an investigation precisely similar to that in Art. 11, we find

$$\left.\begin{aligned} F' &= d_f^2 V \,.\, x + \ldots \\ G' &= d_g^2 V \,.\, y + \ldots \\ H' &= d_h^2 V \,.\, z + \ldots \end{aligned}\right\} \ldots\ldots (2).$$

Now, since $\Sigma \left(\frac{m}{r^{n+1}}\right)$ is necessarily positive, one at least of the quantities, $d_f^2 V$, $d_g^2 V$, $d_h^2 V$, in equation (1) is necessarily positive for all values of n equal to or greater than 2; and consequently, one at least of the equations (2) must necessarily denote *translation*. And this is true whatever be the *arrangement* of the particles.

But when n is less than 2 the right-hand member of (1) is negative, in which case it is possible that all the equations (2) may denote *vibration*.

Hence, if the luminiferous ether consist of detached particles which *attract* each other with forces varying as $\frac{1}{D^n}$, n must be less than 2.

In a similar manner it may be shewn, that if the ether consist of repulsive particles, n must be greater than 2.

It must be remarked, however, that although these conditions with regard to the value of n should be satisfied by the law of attraction of the particles, yet their arrangement must be such as shall make $d_f^2 V$, $d_g^2 V$, $d_h^2 V$ all negative for every particle in the system, otherwise it will be unstable and incapable of transmitting light.

17. If the medium be of the kind denominated uncrystallized, the vibration of a particle in any direction must be completed in the same time, in which case the arrangement must be such as simultaneously to satisfy the equations,

$$d_f^2 V = d_g^2 V = d_h^2 V = \frac{n-2}{3} \; \Sigma \left(\frac{m}{r^{n+1}}\right),$$

n being less than 2.

We shall arrive at the same result if we consider an uncrystallized medium to be such that the force of restitution acts always in the line of displacement; for in this case the parametric surface, the general equation of which is

$$2(n-1)(C-C') = d_f^2 V . x^2 + d_g^2 V . y^2 + d_h^2 V . z^2 \ldots\ldots(\text{Art. } 3).$$

must be spherical; which requires that

$$d_f^2 V = d_g^2 V = d_h^2 V.$$

18. It can be easily shewn that n must be greater than unity.

For the number of particles at the distance r from the attracted particle is proportional to r^2, and therefore

$$\frac{n-2}{3}\,\Sigma\frac{m}{r^{n+1}} \propto \Sigma\frac{r^2}{r^{n+1}},$$

$$\propto \Sigma\frac{1}{r^{n-1}};$$

hence, unless n be greater than unity, the effect of the more distant parts of the medium upon the value of $\frac{n-2}{3}\,\Sigma\frac{m}{r^{n+1}}$ will be greater than the effect of the adjacent particles. Now the time of vibration of a particle depends on the value of $d_f^2 V$, or $\frac{n-2}{3}.\Sigma\frac{m}{r^{n+1}}$; and therefore unless n be greater than unity, the parts of the medium which are more remote will exert a greater influence upon the time of vibration than those exert which are near. Now, Optical phenomena seem to indicate that the adjacent particles exercise most influence; and therefore n must be greater than 1.

19. It is probably not conformable to the simplicity of Nature, that n should be fractional; we have shewn that it must be greater than 1 and cannot be equal to 2, consequently n is greater than 2.

This result is important, as we are enabled to infer from it immediately, by the aid of (16), that

If the ethereal medium consist of detached particles, the action of which on each other is proportional to a power of the distance, that power must be greater than 2, and the force must be repulsive.

I have pleasure in remarking, that this result so far as it goes, coincides exactly with that which M. Cauchy has obtained in his "Mémoire sur la dispersion de la lumière," page 191, where from his investigations he infers respecting the mutual action of two molecules of ether, "que, *dans le voisinage du contact, cette action soit répulsive et réciproquement proportioncllc au bi-carré de la distance.*"

20. If the particles of ether exert a repulsive action upon each other, as we have just shewn must be the case, they will naturally endeavour to disperse themselves through all space, and form a medium coextensive with the boundaries of the universe. Here then a formidable difficulty presents itself to our notice. If the medium be of finite dimensions it must be enclosed in an envelope, capable of restraining the expansive energy of the whole mass of particles. The more extensive the medium the greater must be the strength of the envelope. Is it probable that the constitution of the Universe is such as to require that the whole should be enclosed in a huge vessel of inconceivable strength? This objection would in my opinion be fatal to the hypothesis of a system of detached particles, were it not for the following considerations.

Upon examining the preceding articles, it will be seen that the luminiferous ether must be such that $d_f^2 V$, $d_g^2 V$, $d_h^2 V$ are all equal and negative. Now the properties of these quantities will not be in the least affected, if we assume that the particles exert *attractive* forces as well as repulsive forces, providing the attractive forces are proportional to $\frac{1}{D^2}$. For let us suppose that

$$V = \Sigma\left(\frac{\mu}{r}\right) - \Sigma\left\{\frac{\left(\frac{m}{r^{n-1}}\right)}{n-1}\right\},$$

where μ and m are respectively the *attractive* and *repulsive* forces exerted by the same particle at the distance unity.

Then as in (16), we have

$$d_f^2 V + d_g^2 V + d_h^2 V = -(n-2)\,\Sigma\left(\frac{m}{r^{n+1}}\right);$$

$$\therefore\ d_f^2 V = d_g^2 V = d_h^2 V = -\frac{n-2}{3}\,\Sigma\left(\frac{m}{r^{n+1}}\right);$$

equations which do not contain the quantity μ.

I think it therefore not improbable, that *each particle of the luminiferous ether exerts two forces, one attractive and varying reciprocally as the square of the distance; and the other repulsive and varying inversely in a higher ratio than the square;* at any rate this supposition does away with the necessity of the envelope mentioned at the beginning of this article.

21. Let us now generalize the problem, and inquire for what laws of molecular force vibration is possible in the particles of ether.

Let ϕr be the law of molecular force; and assume $V = -\Sigma(m \int_r \phi r)$;

$$\therefore\ d_f^2 V + d_g^2 V + d_h^2 V = -\Sigma\left\{m\left(\frac{2\phi r}{r} + \phi' r\right)\right\},$$

$\phi' r$ for brevity denoting $d_r \phi r$.

Now one condition to be fulfilled is, that $d_f^2 V + d_g^2 V + d_h^2 V =$ a negative quantity, and consequently the law of force must be such that

$$\frac{2\phi r}{r} + \phi' r = \text{a positive quantity};$$

for all values of r from $r =$ the distance between two neighbouring particles, to $r = \infty$; let ψr be any function of r which is positive between these limits, then

$$\frac{2\phi r}{r} + \phi' r = \psi r;$$

$$\therefore\ 2r\phi r + r^2 \phi' r = r^2 \psi r,$$

$$\therefore\ r^2 \phi r = C + \int_r (r^2 \psi r),$$

$$\therefore\ \phi r = \frac{C}{r^2} + \frac{1}{r^2}\cdot \int_r (r^2 \psi r).$$

This formula contains every possible law of force: the first term shews the propriety of what we have done in the last article, and further proves, that an attractive molecular force varying inversely as the square of the distance is the only force which possesses the properties requisite for removing the difficulty there stated; or that at any rate it is the simplest and best adapted for that purpose.

Further; for a reason analogous to that assigned in (18),

$$r^2\left(\frac{2\phi r}{r} + \phi' r\right), \text{ or } r^2\psi r$$

must be a function of r, which decreases as r increases, and vanishes when r is infinite. Hence, if $\chi(r)$ be any function of r which is positive between the least and greatest limits of r for the whole medium, and which decreases as r increases and vanishes when r is infinite, then

$$\phi(r) = \frac{C}{r^2} + \frac{1}{r^2}\int_r \chi(r).$$

Every possible law of force is included in this formula; but the converse is not necessarily true, viz. that every law of force included in this formula is possible.

There may be other conditions to be satisfied, either as to the form of the arrangement of the particles, or as to their distance from each other, or as to the possibility of the medium existing in a state of finite extension, or as to other circumstances unknown to us at present which may perhaps exclude all the forms but one; which one would in that case be the actual law in the luminiferous ether. Or there may be peculiarities in the vibrations which constitute the waves of light (such as their *transversality*) which will hereafter enable us to determine the required law of mutual action of the particles.

22. Whatever be the law of molecular force of the luminiferous ether, each particle is placed in such a position when in equilibrium, that the value of V for that particle is a maximum.

Let us employ the notation of (21): then $V = -\Sigma(m\int_r \phi r)$, and

$$d_f^2 V + d_g^2 V + d_h^2 V = -\Sigma\left\{m\left(\frac{2\phi r}{r} + \phi' r\right)\right\}$$
$$= -\Sigma(m\psi r),$$

and every one of the quantities $d_f^2 V$, $d_g^2 V$, $d_h^2 V$ is negative, whether the particle of ether (the state of which we are investigating) be within a crystallized body, or in vacuo, or in an uncrystallized body.

In order that V may be a maximum, we must have fulfilled the following conditions, viz.

$$d_f V = 0, \quad d_g V = 0, \quad d_h V = 0 \;\ldots\ldots\; (1),$$

and

$$d_f^2 V, \quad d_g^2 V, \quad d_h^2 V \text{ all negative} \;\ldots\ldots\; (2),$$

$$\left.\begin{array}{l} d_f^2 V . d_g^2 V > (d_f d_g V)^2 \\ d_g^2 V . d_h^2 V > (d_g d_h V)^2 \\ d_f^2 V . d_h^2 V > (d_f d_h V)^2 \end{array}\right\} \ldots\ldots\; (3).$$

The three conditions marked (1) are fulfilled, because the particle is in equilibrium by hypothesis; we have shewn above that the three conditions (2) are fulfilled, otherwise the medium could not be luminiferous, *i. e.* its particles could not vibrate in *any* direction; and the last three conditions marked (3) are fulfilled, because the directions of the co-ordinate axes have been taken, such that $d_f d_g V = 0$, $d_g d_h V = 0$, and $d_f d_h V = 0$. Consequently V is a maximum.

S. EARNSHAW.