Demonstration of the Nontrivial Boundary Dependence of the Casimir Force

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(Received 9 February 1999)

The Casimir force between an aluminum-coated plate with small sinusoidal corrugations and a large sphere was measured for surface separations between 0.1 and 0.9 μm using an atomic force microscope. The measured force shows significant deviation from the perturbative theory. The measured Casimir force between the same sphere and flat plate shows good agreement with the same theory in the limit of zero amplitude of corrugation. These together demonstrate the nontrivial boundary dependence of the Casimir force. [S0031-9007(99)09229-7]

PACS numbers: 12.20.Fv, 61.16.Ch

Casimir [1] predicted an attractive force between two neutral metal plates. The force results from the alteration by the metal boundaries of the zero point electromagnetic energy $E = \sum_n^\infty (1/2)\hbar\omega_n$, where $\hbar\omega_n$ is the photon energy in each allowed photon mode n [1–4]. Initially the Casimir force was thought to be a simple extension of the van der Waals (vdW) force which is an attractive force between two neutral molecules [2]. Lifshitz [5] generalized the vdW force between two extended bodies as the force between fluctuating dipoles induced by the zero point electromagnetic fields and obtained the same result as Casimir for two perfectly reflecting flat plates. However, it was realized that the Casimir force is a strong function of geometry and that between two halves of thin metal spherical shells is repulsive [6]. The sign and value of the Casimir force becomes even more interesting for complex topologies such as encountered with a torus [4]. Thus the Casimir force explores the dependence of the vacuum fluctuations on the geometry of the boundary. The Casimir force has been demonstrated between two flat plates [7] and a large sphere and a flat plate [8] and its value shown to be in agreement with the theory to an average deviation of 1% [9]. For dielectric bodies the resulting force has been measured with reasonable agreement to the theory [10]. Here we report the first experimental demonstration of the nontrivial boundary dependence by measuring the Casimir force between a large sphere and plate with periodic uniaxial sinusoidal corrugations (PUSC) for surface separations between 0.1 and 0.9 μm using an atomic force microscope (AFM). The amplitude of the corrugation is only 59.4 nm and is much smaller than the separation. Yet the measured force shows significant deviations from a perturbative theory which takes into account the small periodic corrugation of the plate in the surface separation (the results of the theory correspond to the trivial boundary dependence). Such a deviation can be expected due to the changes in zero point photon modes from diffraction off the periodic corrugation. We also compare the measured Casimir force between the same sphere and identically coated flat plate and show that it agrees well with the same theory in the limit of zero amplitude of corrugation. The results together demonstrate the nontrivial boundary dependence of the Casimir force. The boundary dependence of the Casimir force can be easily obscured by errors in the measurement of the surface separation [4]. To eliminate this ambiguity we use the electrostatic force to determine the exact surface separation and establish procedures for consistent comparison to theory.

The regularized zero point energy per unit area given two parallel plates of infinite conductivity a distance z apart is given by [2–5]

$$U(z) = -\frac{\pi^2\hbar c}{720}\,\frac{1}{z^3}\,. \tag{1}$$

This results in a Casimir force per unit area $F/A = -\partial U/\partial z = -(\pi^2\hbar c/240)(1/z^4)$. A sinusoidal corrugation (period $= \lambda$) of one plate leads to the modification of U resulting in an averaged regularized energy per unit area

$$\left\langle U\left(z + z_0 + A\sin\frac{2\pi x}{\lambda}\right)\right\rangle = -\frac{\pi^2\hbar c}{720}\,\frac{1}{(z + z_0)^3} \times \sum_m C_m\left(\frac{A}{z + z_0}\right)^m, \tag{2}$$

where $\langle\,\rangle$ stands for average over the size L of the plate, z is the surface separation measured from contact of the two surfaces, and A is the amplitude of the sinusoidal corrugation. z_0 is the mean surface separation after contact due to the periodic corrugation and the stochastic roughness of the metal coating. The origin for the measurement of z_0 is taken such that the mean oscillation of the corrugation is zero. In the above $\lambda \ll L$, and $z + z_0 > A$ have been used. It can be observed in Eq. (2) that an exact and independent determination of z_0 is necessary for a valid comparison to a theory. The nonzero even power coefficients in Eq. (2) are $C_0 = 1$, $C_2 = 3$, $C_4 = 45/8$, $C_6 = 35/4, \ldots$. Equation (2) can also be obtained by using the regularized additive vdW type approach in Ref. [11] under the same limiting conditions. Geometric approaches can also be used [12]. Field theoretic methods can also be used and have shown PUSC surfaces to be rich in vacuum interactions [13].

Experimentally it is hard to configure two parallel plates uniformly separated by distances less than a micron. So the preference is to replace one of the plates by a metal sphere of radius R where $R \gg z$. For such a geometry the Casimir force can be calculated by use of the proximity force theorem [14,15] to be

$$F_c^0(z + z_0) = 2\pi R\langle U\rangle . \tag{3}$$

The finite conductivity of the metal leads to a correction which for a given metal plasma frequency ω_p is [16,17]

$$F_c(z + z_0) = F_c^0(z + z_0)\left[1 - 4\frac{c}{(z + z_0)\omega_p} + \frac{72}{5}\left(\frac{c}{(z + z_0)\omega_p}\right)^2\right]. \tag{4}$$

There are also corrections due to the finite temperature [18] which can be neglected for the surface separations reported here. There is also a correction due to the stochastic roughness of the metal coating [11]. In this work the rms stochastic roughness amplitude is much less than the amplitude of the periodic sinusoidal modulation of the surface.

We use a standard atomic force microscope to measure the force between a metallized sphere and the corrugated plate at a pressure below 50 mTorr and at room temperature. The experimental approach is similar to that reported in Ref. [9]. A schematic diagram of the experiment is shown in Fig. 1. Polystyrene spheres of $200 \pm 4\ \mu$m diameter were mounted on the tip of 320 μm long cantilevers with Ag epoxy. In order to implement this experiment a diffraction grating with a uniaxial sinusoidal corrugation of period $\lambda = 1.1\ \mu$m and an amplitude of 90 nm was used as the PUSC surface. The radius R of the sphere is much greater than the periodicity and the surface separation z. A 7.5×7.5 mm^2 piece of the PUSC plate was used. A 10 mm diameter optically polished sapphire plate was used to represent the flat plate. The cantilever (with sphere), corrugated plate, and flat plate were then coated with 250 nm of Al (measured with AFM) in a thermal evaporator. Aluminum is used because of its high reflectivity for wavelengths of interest (sphere-plate separations > 100 nm) and good representation in terms of a plasma wavelength $\lambda_p = 100$ nm [19]. All surfaces are then coated with a 8 nm layer (measured with AFM) of 60% Au/40% Pd. This formed a uniform nonreactive and conductive top layer which is necessary to reduce any space charge effects due to patch oxidation of the Al coating. Transparencies greater than 90% were measured for $\lambda < 300$ nm for the Au/Pd coatings. The sphere diameter was measured using the scanning electron microscope (SEM) to be $194.6 \pm 0.5\ \mu$m (the SEM was cross calibrated with the AFM using AFM standards). The amplitude of the corrugation after the metal coating was measured to be 59.4 ± 2.5 nm using the AFM. The average stochastic roughness amplitude of the metallized surfaces was measured using an AFM to be 4.7 nm. The mean roughness amplitude of the metallized sphere bottom was measured using the SEM to 5 nm. However the occasional presence of Al crystals with height 10–30 nm prevents *a priori* determination of surface separation.

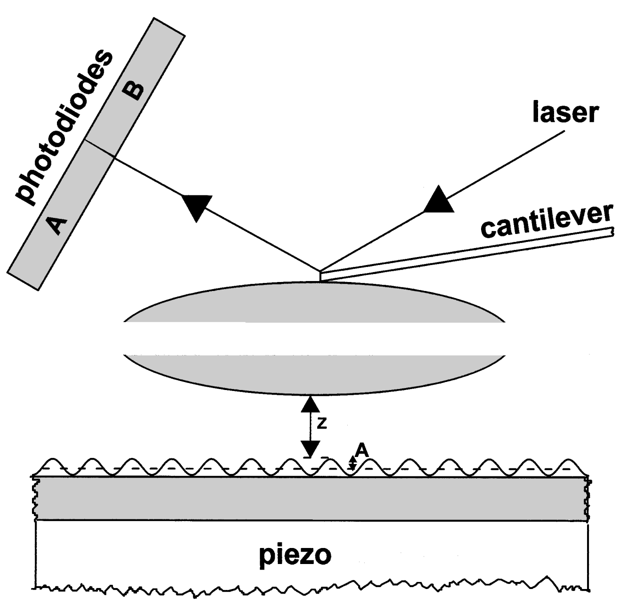

FIG. 1. Schematic diagram of the experimental setup. The picture is not to scale.

In Fig. 1, a force on the sphere would result in a cantilever deflection leading to the deviation of the laser beam and a difference signal between photodiodes A and B. This force and the corresponding cantilever deflection are related by Hooke's law: $F = k\Delta z$, where k is the force constant and Δz is the cantilever deflection. The piezo extension with applied voltage was calibrated with height standards and its hysteresis was measured. The corrections due to the piezo hysteresis and cantilever deflection were applied as reported in Ref. [9] to the sphere-plate separations in all collected data. As reported in Ref. [9] the cantilever is calibrated by measuring the electrostatic force between the flat plate and sphere for surface separation $>2\ \mu$m. The average of all measured k is 0.021 ± 0.001 nN/m.

The electrostatic force between the sphere and the PUSC surface is given by

$$F_e = \frac{-\pi R\varepsilon_0}{z + z_0}(V_1 - V_2)^2 \sum_{m=0}^{\infty} D_m\left(\frac{A}{z + z_0}\right)^m, \tag{5}$$

where as before z is the distance between the surfaces measured from contact and as before z_0 is the true average separation on contact of the two surfaces due to the periodic corrugation and stochastic roughness of the aluminum coating. The nonzero even power coefficients in Eq. (5) are $D_0 = 1$, $D_2 = 1/2$, $D_4 = 3/8$, $D_6 = 5/16, \ldots$. V_1 and V_2 are voltages on the corrugated plate and sphere, respectively. The above expression is obtained in a manner analogous to Eq. (3) from Eqs. (1) and (2), by starting from the electrostatic energy between parallel flat plates.

Next the residual potential of the grounded sphere was measured. The sphere is grounded and the electrostatic force between the sphere and the corrugated plate was measured for four different voltages and five different surface separations $z \gg A$. With Eq. (5), from the difference in force for voltages $+V_1$ and $-V_1$ applied to the corrugated plate, we can measure the residual potential on the grounded sphere V_2 as 14.9 mV. This residual potential is a contact potential that arises from the different materials used to fabricate the sphere and the corrugated plate.

To measure the Casimir force between the sphere and the corrugated plate they are both grounded together with the AFM. The plate is then moved towards the sphere in 3.6 nm steps and the corresponding photodiode difference signal was measured. The signal obtained for a typical scan is shown in Fig. 2. Here "0" separation stands for contact of the sphere and corrugated plate surfaces, i.e., $z = 0$. It does not taken into account z_0. Region 1 can be used to subtract the minor ($<1\%$) experimental systematic due to scattered laser light without biasing the results in region 2. In region 2 (absolute separations between contact and 450 nm) the Casimir force is the dominant characteristic far exceeding all systematic errors (the electrostatic force is $<2\%$ of the peak Casimir force). Region 3 is the flexing of the cantilever resulting from the continued extension of the piezo after contact of the two surfaces.

Now we describe the use of the electrostatic force between the sphere and the corrugated plate to arrive at an independent and consistent measurement of z_0, the average surface separation on contact of the two surfaces. This is done immediately following the Casimir force measurement without breaking the vacuum and no lateral movement of the surfaces. The corrugated plate is connected to a dc voltage supply (calibrated against voltage standards) while the sphere remains grounded. The applied voltage V_1 in Eq. (5) is so chosen that the electrostatic force is >20 times the Casimir force. The open squares in Fig. 3 represent the measured total force for an applied voltage of 0.566 V as a function of distance. The force results from a sum of the electrostatic force represented by Eq. (5) and the Casimir force ($<5\%$) of Eq. (4). The solid line which is a best χ^2 fit for the data in Fig. 3 results in a $z_0 = 134.5$ nm. The experiment is repeated for other voltages between 0.4–0.7 V leading to an average value of $z_0 = 132 \pm 5$ nm. Given the 8 nm Au/Pd coating on each surface this would correspond to an average surface separation $132 \pm 5 + 8 + 8 = 148 \pm 5$ nm for the case of the Casimir force measurement.

The electrostatically determined value of z_0 can now be used to apply the systematic error corrections to the force curve of Fig. 2. Except for the independent determination of z_0 done here, the corrections are applied in a manner similar to Ref. [9]. Here the force curve in region 1 is fit to a function: $F = F_c(z + 148) + F_e(z + 132) + Cz$. The first term is the Casimir force contribution to the total force in region 1. The second term represents the electrostatic force between the sphere and corrugated plate as given by Eq. (5). The third term C represents the linear coupling of scattered light from the moving plate into the diodes and corresponds to a force <1 pN ($<1\%$ effect). Here again the difference in z_0 in the electrostatic term and the Casimir force is due to the 8 nm Au/Pd coating on each surface. The value of C is determined by minimizing the χ^2. The value of C determined in region 1 and the electrostatic force corresponding to $V_2 = 14.9$ mV and $V_1 = 0$ in Eq. (5) is used to subtract the systematic

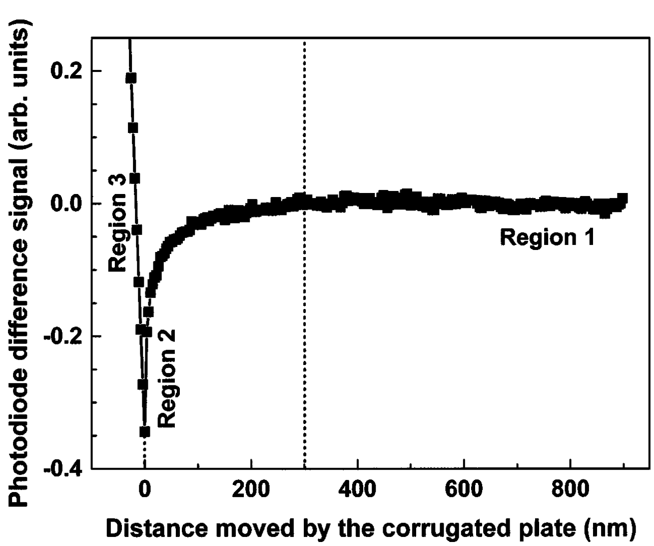

FIG. 2. A typical force curve as a function of the distance moved by the plate. The "0" distance stands for point of contact and does not take into account the amplitude of the corrugation and the roughness of the metallic coating.

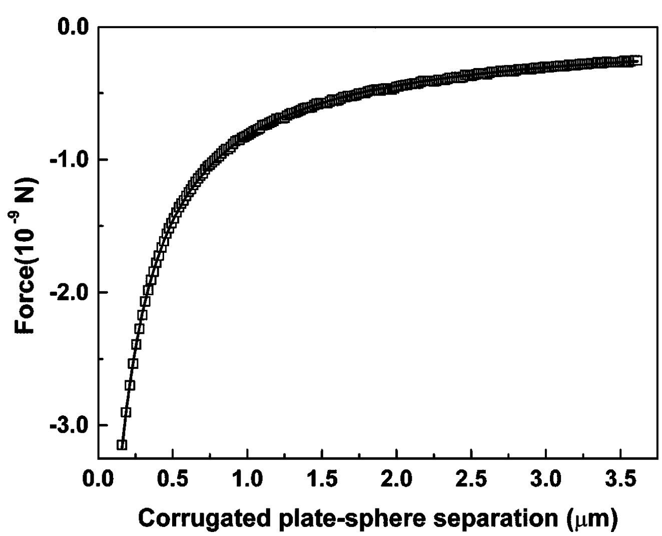

FIG. 3. Open squares are the measured force as a function of distance for a voltage of +0.566 V applied to the corrugated plate. The sphere was grounded. The solid line is the best χ^2 fit using the electrostatic force of Eq. (5) and the Casimir force of Eq. (4) resulting in a surface separation on contact of $z_0 = 134.5$ nm.

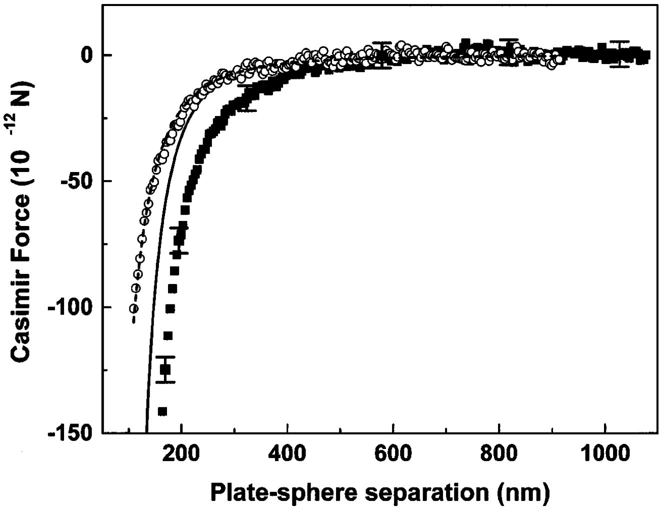

FIG. 4. The solid squares are the measured Casimir force as a function of corrugated plate-sphere surface separation. The solid line is the theoretical Casimir force of Eq. (4), with no adjustable parameters. The open circles are the measured Casimir force for a flat plate and the same sphere. The dashed line is the theoretical Casimir force for a flat plate obtained by setting $A = 0$ in Eq. (4), with no adjustable parameters.

errors from the force curve in regions 1 and 2 to obtain the measured Casimir force as $F_{c\text{-}m} = F_m - F_e - Cz$ where F_m is the measured total force. Thus the measured Casimir force from region 2 has no adjustable parameters.

The experiment is repeated for 15 scans and the average Casimir force measured is shown as solid squares in Fig. 4. The height of the squares represents the experimental uncertainty at each data point and the error bars represent the range of data. The theoretical curve given by Eq. (4) with $z_0 = 148$ nm (determined from the electrostatic result) and no adjustable parameters is shown as a solid line in the same figure. Significant deviation between the measured force and the perturbative theory can be observed. Even allowing z_0 to be completely adjustable by matching theory and experiment at the largest forces will not reconcile the two for surface separations between 200–500 nm.

The experiment and analysis were repeated for the same sphere and an identically coated flat plate. The average measured Casimir force from 15 scans is shown as open circles. The theoretical Casimir force due to a flat plate and sphere obtained by setting $A = 0$ in Eq. (4) shows good agreement with the experiment. The electrostatically measured surface separation on contact of 49 ± 4 nm and $8 + 8 = 16$ nm Au/Pd coating leading to $z_0 = 65$ nm was used in the theory. The inclusion of the stochastic roughness of amplitude 5 nm will lead to 1% changes. All of the above experiments have been repeated with many different sets of spheres, flat plates, and PUSC plates with the same results.

In conclusion, the measured Casimir force between a large sphere and a plate with small amplitude periodic corrugations is significantly different from that predicted by a perturbative theory which accounts only for changes in separation between surfaces. Such a deviation from theory is to be expected due to the diffractive effects associated with the corrugated surface. The same theory in the limit of zero amplitude of corrugation is in good agreement with the measured Casimir force between the same sphere and an identically coated flat plate. These two results taken together demonstrate the nontrivial boundary dependence of the Casimir force.

Discussions with V. M. Mostepanenko, G. L. Klimchitskaya, M. Kardar, and R. Golestanian are acknowledged.

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