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Mechanical Response of Vacuum

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A path integral formulation is developed for the *dynamic* Casimir effect. It allows us to study arbitrary deformations in *space and time* of the perfectly reflecting (conducting) boundaries of a cavity. The mechanical response of the intervening vacuum is calculated to linear order in the frequency–wave-vector plane. For a single corrugated plate we find a correction to mass at low frequencies, and an effective shear viscosity at high frequencies, both anisotropic. For two plates there is resonant dissipation for *all frequencies* greater than the lowest optical mode of the cavity. [S0031-9007(97)03143-8]

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The standard Casimir effect [1] is a macroscopic manifestation of quantum fluctuations of vacuum. The modified boundary conditions of the electromagnetic field in the space between two parallel conducting plates change zero point vacuum fluctuations, resulting in an attractive force between the plates, which has recently been experimentally measured to high precision [2]. Thus, by observing the mechanical force between macroscopic bodies, it is, in principle, possible to gain information about the behavior of the quantum vacuum. Although less well known than its static counterpart, the dynamical Casimir effect, describing the force and radiation from moving mirrors has also garnered much attention [3–9]. This is partly due to connections to Hawking and Unruh effects (radiation from black holes and accelerating bodies, respectively), suggesting a deeper link between quantum mechanics, relativity, and cosmology [10].

The creation of photons by moving mirrors was first obtained by Moore [3] for a 1-dimensional cavity. Fulling and Davies [4] demonstrated that there is a corresponding force, even for a single mirror, which depends on the third time derivative of its displacement. These computations take advantage of conformal symmetries of the 1+1 dimensional space-time, and cannot be easily generalized to higher dimension. Furthermore, the calculated force has causality problems reminiscent of the radiation reaction forces in classical electron theory [5]. It has been shown that this problem is an artifact of the unphysical assumption of perfect reflectivity of the mirror, and can be resolved by considering realistic frequency dependent reflection and transmission from the mirror [5].

Another approach to the problem starts with the fluctuations in the force on a single plate. The fluctuation-dissipation theorem is then used to obtain the mechanical response function [6], whose imaginary part is related to the dissipation. This method does not have any causality problems, and can also be extended to higher dimensions. (The force in 1+3 dimensional space-time depends on the fifth power of the motional frequency.) The emission of photons by a perfect cavity, and the observability of this energy, has been studied by different approaches [7–9]. The most promising candidate is the resonant production of photons when the mirrors vibrate at the optical resonance frequency of the cavity [10]. A review, and more extensive references, are found in Ref. [11]. More recently, the radiation due to vacuum fluctuations of a collapsing bubble has been proposed as a possible explanation for the intriguing phenomenon of sonoluminescence [12,13].

In this Letter we present a path integral formulation, applicable to all dimensions, for the problem of perfectly reflecting mirrors that undergo arbitrary dynamic deformations [14,15]. We calculate the frequency–wave vector dependent mechanical response function, defined as the ratio between the induced force and the deformation field, in the linear regime. From the response function we extract a plethora of interesting results, some of which we list here for the specific example of lateral vibrations of uniaxially corrugated plates: (1) A single plate with corrugations of wave number $\mathbf{k}$, vibrating at frequencies $\omega \ll ck$, obtains *anisotropic* corrections to its mass. (2) For $\omega \gg ck$, there is dissipation due to a frequency dependent anisotropic shear viscosity. (3) A second plate at a separation H modifies the mass renormalization by a function of kH, but does not change the dissipation for frequencies $\omega^2 < (ck)^2 + (\pi c/H)^2$. (4) For all frequencies higher than this first optical normal mode of the cavity, the mechanical response is infinite, implying that such modes cannot be excited by any finite external force. This is intimately connected to the resonant particle creation reported in the literature [7–10]. (5) A phase angle θ between two similarly corrugated parallel plates results in Josephson–like effects: There is a static force proportional to $\sin(\theta)$, while a uniform relative velocity results in an oscillating force. (6) We calculate a (minute) correction to the velocity of capillary waves on the surface of mercury due to a small change in its surface tension.

Our approach is a natural generalization of the path integral methods developed by Li and Kardar [16] to study fluctuation-induced interactions between deformed manifolds embedded in a correlated fluid. Such interactions result from *thermal fluctuations* of the fluid. These methods are readily generalized to zero point quantum fluctuations of a field, taking advantage of the path integral quantization formalism. Since in the Euclidian path integral formulation the space and time coordinates are equivalent, deformations of the boundaries in space and time appear on the same footing. As is usual, we simplify the problem by considering a scalar field ϕ (in place of the electromagnetic vector potential [17]) with the action

$$S = \frac{1}{2}\int d^dX\,\partial_\mu\phi(X)\partial_\mu\phi(X), \tag{1}$$

where summation over $\mu = 1,\ldots,d$ is implicit. Following a Wick rotation, imaginary time appears as another coordinate $X_d = ict$ in the d–dimensional space-time. We would like to quantize the field subject to the constraints of its vanishing on a set of n manifolds (objects) defined by $X = X_\alpha(y_\alpha)$, where y_α parametrize the αth manifold. Following Ref. [16], we implement the constraints using delta functions, and write the partition function as

$$\mathcal{Z} = \int \mathcal{D}\phi(X)\prod_{\alpha=1}^{n}\prod_{y_\alpha}\delta[\phi(X_\alpha(y_\alpha))]\exp\left\{-\frac{1}{\hbar}S[\phi]\right\}. \tag{2}$$

The delta functions are next represented by integrals over Lagrange multiplier fields. Performing the Gaussian integrations over $\phi(X)$ then leads to an effective action for the Lagrange multipliers which is again Gaussian [16]. Evaluating $\mathcal{Z}$ is thus reduced to calculating the logarithm of the determinant of a kernel. Since the Lagrange multipliers are defined on a set of manifolds with nontrivial geometry, this calculation is generally complicated. To be specific, we focus on two parallel 2D plates embedded in 3+1 space-time, and separated by an average distance H along the x_3 direction. Deformations of the plates are parametrized by the height functions $h_1(\mathbf{x},t)$ and $h_2(\mathbf{x},t)$, where $\mathbf{x} \equiv (x_1,x_2)$ denotes the two lateral space coordinates while t is the time variable. Following Ref. [16], $\ln\mathcal{Z}$ is calculated by a perturbative series in powers of the height functions. The resulting expression for the effective action (in real time), defined by $S_{\rm eff} \equiv -i\hbar\ln\mathcal{Z}$, after eliminating h independent terms, is

$$\begin{aligned} S_{\rm eff} = \frac{\hbar c}{2}\int \frac{d\omega\, d^2\mathbf{q}}{(2\pi)^3}\{&A_+(q,\omega)[|h_1(\mathbf{q},\omega)|^2 + |h_2(\mathbf{q},\omega)|^2] \\ &- A_-(q,\omega)[h_1(\mathbf{q},\omega)h_2(-\mathbf{q},-\omega) \\ &+ h_1(-\mathbf{q},-\omega)h_2(\mathbf{q},\omega)]\} + O(h^3). \end{aligned} \tag{3}$$

The kernels $A_\pm(q,\omega)$ that are closely related to the mechanical response of the system (see below) are functions of the separation H, but depend on $\mathbf{q}$ and ω only through the combination $Q^2 = q^2 - \omega^2/c^2$. The closed forms for these kernels involve cumbersome integrals, and are not very illuminating. Instead of exhibiting these formulas, we shall describe their behavior in various regions of the parameter space. In the limit $H \to \infty$, $A_-^\infty(q,\omega) = 0$, and

$$A_+^\infty(q,\omega) = \begin{cases} -\frac{1}{360\pi^2c^5}(c^2q^2 - \omega^2)^{5/2}, & \text{for } \omega < cq, \\ i\,\frac{\mathrm{sgn}\,\omega}{360\pi^2c^5}(\omega^2 - c^2q^2)^{5/2}, & \text{for } \omega > cq, \end{cases} \tag{4}$$

where $\mathrm{sgn}(\omega)$ is the sign function. While the effective action is real for $Q^2 > 0$, it becomes purely imaginary for $Q^2 < 0$. The latter signifies dissipation of energy [6], presumably by generation of photons [9]. It agrees precisely with the results obtained previously [6] for the special case of flat mirrors ($\mathbf{q} = 0$). (Note that dissipation is already present for a single mirror.)

In the presence of a second plate (i.e., for finite H), the parameter space of the kernels subdivides into three different regions as depicted in Fig. 1. In region I ($Q^2 > 0$ for any H), the kernels are finite and real, and hence there is no dissipation. In region IIa, where $-\pi^2/H^2 \le Q^2 < 0$, the H-independent part of A_+ is imaginary, while the H-dependent parts of both kernels are real and finite. (This is also the case at the boundary $Q^2 = -\pi^2/H^2$.) The dissipation in this regime is simply the sum of what would have been observed if the individual plates were decoupled, and unrelated to the separation H.

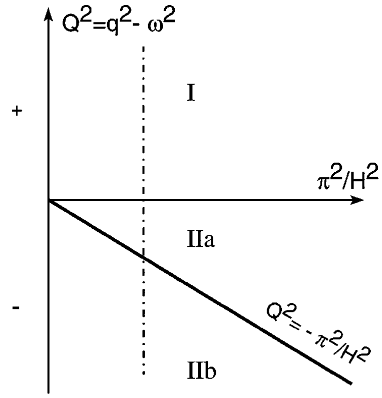

FIG. 1. Different regions of the $(\mathbf{q}, \omega)$ plane.

By contrast, in region IIb, where $Q^2 < -\pi^2/H^2$, both kernels diverge with infinite real and imaginary parts [18]. This H-dependent divergence extends all the way to the negative Q^2 axis, where it is switched off by a $1/H^5$ prefactor.

As a concrete example, let us examine the lateral vibration of plates with fixed roughness, such as two corrugated mirrors. The motion of the plates enters through the time dependences $h_1(\mathbf{x}, t) = h_1(\mathbf{x} - \mathbf{r}(t))$ and $h_2(\mathbf{x}, t) = h_2(\mathbf{x})$; i.e., the first plate undergoes lateral motion described by $\mathbf{r}(t)$, while the second plate is stationary. The lateral force exerted on the first plate is obtained from $f_i(t) = \delta S_{\mathrm{eff}}/\delta r_i(t)$. Within linear response, it is given by

$$f_i(\omega) = \chi_{ij}(\omega) r_j(\omega) + f_i^0(\omega), \tag{5}$$

where the "mechanical response tensor" is

$$\chi_{ij}(\omega) = \hbar c \int \frac{d^2q}{(2\pi)^2}\, q_i q_j \bigg\{ [A_+(q,\omega) - A_+(q,0)] |h_1(\mathbf{q})|^2 + \frac{1}{2} A_-(q,0) [h_1(\mathbf{q}) h_2(-\mathbf{q}) + h_1(-\mathbf{q}) h_2(\mathbf{q})] \bigg\}, \tag{6}$$

and there is a residual force

$$f_i^0(\omega) = -\frac{\hbar c}{2}\, 2\pi \delta(\omega) \int \frac{d^2q}{(2\pi)^2}\, i q_i A_-(q,0) \times [h_1(\mathbf{q}) h_2(-\mathbf{q}) - h_1(-\mathbf{q}) h_2(\mathbf{q})]. \tag{7}$$

For a single corrugated plate with a deformation $h(\mathbf{x}) = d \cos \mathbf{k} \cdot \mathbf{x}$, we can easily calculate the response tensor using the explicit formulas in Eq. (4). In the limit of $\omega \ll ck$, expanding the result in powers of ω gives $\chi_{ij} = \delta m_{ij} \omega^2 + O(\omega^4)$, where $\delta m_{ij} = A\hbar d^2 k^3 \mathbf{k}_i \mathbf{k}_j / 288\pi^2 c$ can be regarded as corrections to the mass of the plate. (Cut-off dependent mass corrections also appear, as in Ref. [11].) Note that these mass corrections are *anisotropic* with $\delta m_{\parallel} = A\hbar k^5 d^2/288\pi^2 c$ and $\delta m_{\perp} = 0$. Parallel and perpendicular components are defined with respect to $\mathbf{k}$, and A denotes the area of the plates. The mass correction is inherently very small: For a macroscopic sample with $d \approx \lambda = 2\pi/k \approx 1$ mm, density ≈ 15 g/cm^3, and thickness $t \approx 1$ mm, we find $\delta m/m \sim 10^{-34}$. Even for deformations of a microscopic sample of atomic dimensions (close to the limits of the applicability of our continuum representations of the boundaries), $\delta m/m$ can only be reduced to about 10^{-10}. While the actual changes in mass are immeasurably small, the hope is that its *anisotropy* may be more accessible, say, by comparing oscillation frequencies of a plate in two orthogonal directions.

For $\omega \gg ck$ the response function is imaginary, and we define a frequency dependent effective shear viscosity by $\chi_{ij}(\omega) = -i\omega\eta_{ij}(\omega)$. This viscosity is also anisotropic, with $\eta_{\parallel}(\omega) = \hbar A k^2 d^2 \omega^4/720\pi^2 c^4$, and $\eta_{\perp}(\omega) = 0$. Note that the dissipation is proportional to the fifth time derivative of displacement, and there is no dissipation for a uniformly accelerating plate. However, a freely oscillating plate will undergo a damping of its motion. The characteristic decay time for a plate of mass M is $\tau \approx 2M/\eta$. For the macroscopic plate of the previous paragraph, vibrating at a frequency of $\omega \approx 2ck$ (in the 10^{12} Hz range), the decay time is enormous, $\tau \sim 10^{18}$ s. However, since the decay time scales as the fifth power of the dimension, it can be reduced to 10^{-12} s, for plates of an order of 10 atoms. However, the required frequencies in this case (in the 10^{18} Hz range) are very large. Also note that, for the linearized forms to remain valid in this high frequency regime, we must require very small amplitudes, so that the typical velocities involved, $v \sim r_0\omega$, are smaller than the speed of light. These difficulties can be somewhat overcome by considering resonant dissipation in the presence of a second plate.

With two plates at an average distance H, the results are qualitatively the same for frequencies less than the natural resonance of the resulting cavity. There is a renormalization of mass in region I, and dissipation appears in region IIa, of Fig. 1. However, the mass renormalization at low frequencies ($\omega \ll ck$) is now a function of both k and H, with a crossover from the single plate behavior for $kH \sim 1$. In the limit of $kH \ll 1$, we obtain $\delta m_{\parallel} = \hbar A B k^2 d^2/48cH^3$ and $\delta m_{\perp} = 0$, with $B = -0.453$. Compared to the single plate, there is an enhancement by a factor of $(kH)^{-3}$ in $\delta m_{\parallel}$. The effective dissipation in region IIa is simply the sum of those due to individual plates, and contains no H dependence.

There are additional interesting phenomena resulting from resonances. We find that both real and imaginary parts of $A_{\pm}(q,\omega)$ diverge for $\omega^2/c^2 > q^2 + \pi^2/H^2$. In the example of corrugated plates, we replace q by k to obtain a continuous spectrum of frequencies with diverging dissipation. Related effects have been reported in the literature for 1+1 dimensions [7–10], but occurring at a *discrete* set of frequencies $\omega_n = n\pi c/H$ with integer $n \geq 2$. These resonances occur when the frequency of the external perturbation matches the natural normal modes of the cavity, thus exciting quanta of such modes.

In one space dimension, such modes are characterized by a discrete set of wave vectors that are integer multiples of π/H. The restriction to $n \geq 2$ is a consequence of quantum electrodynamics being a "free" theory (quadratic action): Only two-photon states can be excited subject to conservation of energy. Thus the sum of the frequencies of the two photons should add up to the external frequency [9].

In higher dimensions, the appropriate parameter is the combination $\omega^2/c^2 - q^2$. From the perspective of the excited photons, conservation of momentum requires that their two momenta add up to q, while energy conservation restricts the sum of their frequencies to ω. The in-plane momentum q, introduces a continuous degree of freedom: The resonance condition can now be satisfied for a continuous spectrum, in analogy with optical resonators. In Ref. [9], the lowest resonance frequency is found to be $2\pi c/H$ which seems to contradict our prediction. However, the absence of $\omega_1 = \pi c/H$ in 1+1 D is due to a vanishing prefactor [9], which is also present in our calculations. However, in exploring the continuous frequency spectrum in higher dimensions, this single point is easily bypassed, and there is a divergence for all frequencies satisfying $\omega^2/c^2 > q^2 + \pi^2/H^2$, where the inequality holds in its strict sense.

Resonant dissipation has profound consequences for motion of plates. It implies that, due to quantum fluctuations of vacuum, *components of motion with frequencies in the range of divergences cannot be generated by any finite external force*. The imaginary parts of the kernels are proportional to the total number of excited photons [9]. Exciting these degrees of motion must be accompanied by the generation of an infinite number of photons; requiring an infinite amount of energy, and thus impossible. However, as pointed out in Ref. [9], the divergence is rounded off by assuming finite reflectivity and transmissivity for the mirrors. Hence, in practice, the restriction is softened and controlled by the degree of ideality of the mirrors in the frequency region of interest.

We shall next examine the constant term in Eq. (7). For two plates corrugated at the same wavelength, with deformations $h_1(\mathbf{x}) = d_1 \cos(\mathbf{k} \cdot \mathbf{x})$ and $h_2(\mathbf{x}) = d_2 \cos(\mathbf{k} \cdot \mathbf{x} + \alpha)$, there is a (time independent) lateral force,

$$\mathbf{F}_{\mathrm{dc}} = \frac{\hbar c A}{2} A_-(k,0)\mathbf{k} d_1 d_2 \sin\alpha \,, \tag{8}$$

which tends to keep the plates 180° out of phase, i.e., mirror symmetric with respect to their midplane. The dependence on the sine of the phase mismatch is reminiscent of the dc Josephson current in superconductor junctions, the force playing a role analogous to the current in superconductor-insulator-superconductor (SIS) junctions. There is also an analog for the ac Josephson effect, with velocity (the variable conjugate to force) playing the role of voltage: Consider two corrugated plates separated at a distance H, described by $h_1(\mathbf{x},t) = d_1 \cos\{\mathbf{k} \cdot [\mathbf{x} - \mathbf{r}(t)]\}$ and $h_2(\mathbf{x},t) = d_2 \cos[\mathbf{k} \cdot \mathbf{x}]$. The resulting force at a constant velocity $[\mathbf{r}(t) = \mathbf{v}t]$,

$$\mathbf{F}_{\mathrm{ac}} = \frac{\hbar c A}{2} A_-(k,0)\mathbf{k} d_1 d_2 \sin[(\mathbf{k} \cdot \mathbf{v})t] \,, \tag{9}$$

oscillates at a frequency $\omega = \mathbf{k} \cdot \mathbf{v}$. Actually both effects are a consequence of the attractive nature of the Casimir force. It would be difficult to separate them from similar forces resulting from, say, van der Waals attractions.

As a final example, we study the capillary waves on the surface of mercury, with a conducting plate placed at a separation H above the surface. The low frequency–wave-vector expansion of the kernel due to quantum fluctuations in the intervening vacuum starts with quadratic forms q^2 and ω^2. These terms result in corrections to the (surface) mass density by $\delta\rho = \hbar B/48cH^3$, and to the surface tension by $\delta\sigma = \hbar cB/48H^3$. The latter correction is larger by a factor of $(c/c_s)^2$, and changes the velocity c_s of capillary waves by $\delta c_s/c_s^0 = \hbar cB/96\sigma H^3$, where σ is the bare surface tension of mercury. Taking $H \sim 1$ mm and $\sigma \sim 500$ dyn/cm, we find another very small correction for $\delta c_s/c_s^0 \sim 10^{-19}$.

In summary, we have developed a path integral formulation for the study of quantum fluctuations in a cavity with dynamically deforming boundaries. As opposed to previous emphasis on spectra of emitted radiation, we focus on the mechanical response of the vacuum. Most of the predicted phenomena, while quite intriguing theoretically, appear to be beyond the reach of current experiment: The most promising candidates are the anisotropy in mass and resonant dissipation. The path integral method is quite versatile, and future extensions could focus on nonlinear response, other geometries (e.g., wires), the gauged electromagnetic field, and calculations of emitted spectra using correlation functions.

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[17] Calculations with the electromagnetic field are complicated by the requirement of gauge fixing. However, the final results only change by a numerical prefactor. We have explicitly checked that we reproduce the known answer for flat plates by this method.
[18] The divergence of kernels in IIb comes from integrations over space-time. Given a cutoff L in plate size, and an associated cutoff L/c in time, the kernels diverge as $\exp[(K-2)L/H]/[K(L/H)^3]$, with $K = 2QH/\pi$. Some care is necessary in the order of limits for $(L, H) \to \infty$.