NO. 21615

National Committee for Fluid Mechanics Films

FILM NOTES

for

MAGNETOHYDRODYNAMICS*

By

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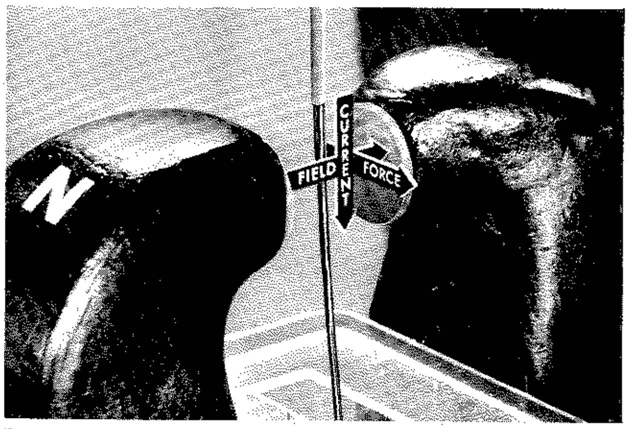
Introduction

Magnetohydrodynamics concerns fluid motion under magnetic fields. Fluids are not, in general, magnetic but if they can conduct electric currents **j**, these can interact with a magnetic field **B** to produce forces **j** × **B** per unit volume which profoundly affect the motion. Figure 1 shows a simple example of the effect of the **j** × **B** force.



1. A vertical mercury jet carrying a vertical electric current j in a horizontal magnetic field B is deflected by the j × B force.

2. Solar flares.

The most abundant conducting fluid in the universe is ionized gas, or plasma, because all stars are composed of it. Plasmas can support phenomena besides those which are described by the magnetohydrodynamic approximation, which treats the fluid as a continuum.

Magnetic fields are common in cosmic phenomena, and Fig. 2 shows the strong effect of magnetic fields on solar flares. Plasmas also occur terrestrially, but on earth we also have conducting liquids, either electrolytic or metallic.

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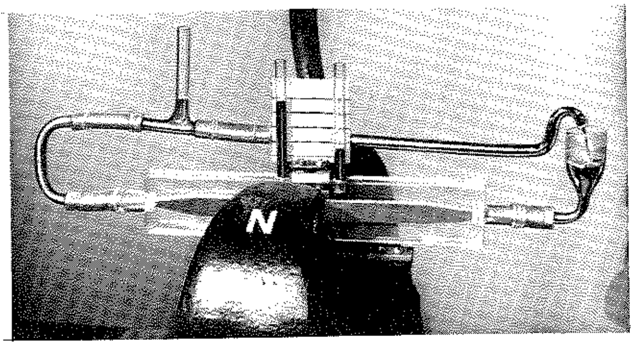
Magnetic Pumping

The $\mathbf{j} \times \mathbf{B}$ forces can be made to pump a conducting fluid. In Fig. 3, a vertical current is being passed through mercury in a duct in the horizontal field of a magnet.

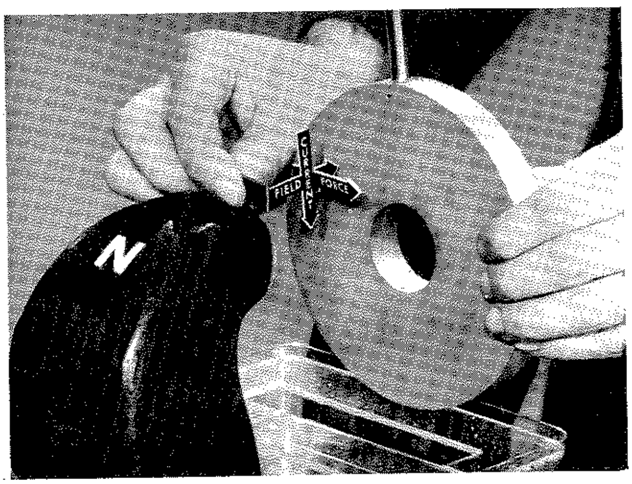


3. A simple electromagnetic pump.

The $\mathbf{j} \times \mathbf{B}$ acts to the left and pumps the mercury clockwise round the circuit, as is revealed by the "mercury-fall" in the right-hand side of the picture. The manometer above the magnet reveals the pressure rise across the pump.

Induced Currents

In the pump the current is *imposed,* but currents may be *induced* by motion of the conductor itself. In Fig. 4

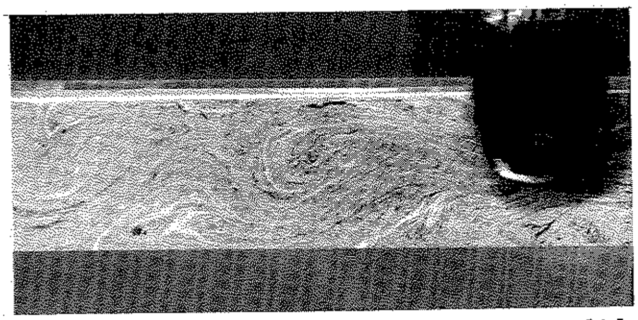


4. When the aluminum ring moves into the magnetic field, induced currents produce a $\mathbf{j} \times \mathbf{B}$ force.

a ring entering a magnetic field links an increasing amount of magnetic flux. A current is induced around the ring, and $\mathbf{j} \times \mathbf{B}$ force strongly opposes the motion.

The Distribution of Magnetic Force and Its Effect on the Fluid's Vorticity

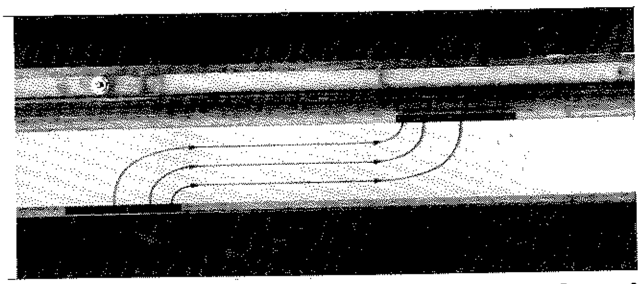
In a solid the distribution of $\mathbf{j} \times \mathbf{B}$ does not matter so much as the *overall* force, because the rigid solid sustains any force distribution. But a *fluid* subjected to $\mathbf{j} \times \mathbf{B}$ forces cannot in general sustain them without being continuously deformed. Figure 5 shows a top view of mercury being stirred by forces due to the cur-



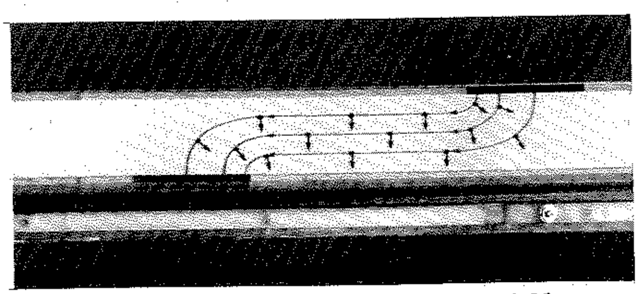
5. A mercury surface stirred by a moving magnet which is seen leaving the picture to the right. The other pole is under the trough, so the magnetic field is normal to the surface. The magnet poles do not touch the mercury.

rents induced by a moving magnet which provides a vertical field. Powder floating on the mercury reveals its motion. The effect of $\mathbf{j} \times \mathbf{B}$ forces on a liquid is more subtle than on a solid. We must examine how the force affects each individual fluid element.

A transparent electrolyte enables dye lines to be used to reveal the motion. Copper sulphate solution is



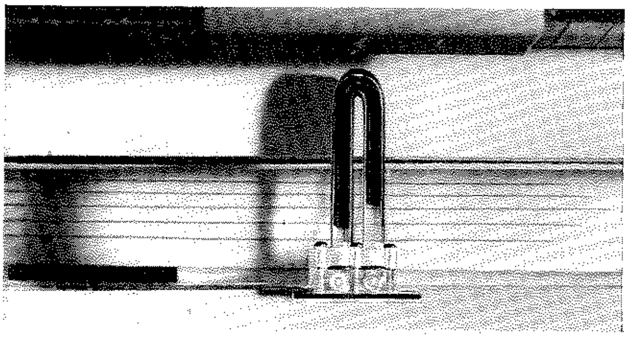
6a. Current distribution between electrodes at edges of flat duct containing electrolyte. The magnetic field is uniform and normal to the paper.



6b. $\mathbf{j} \times \mathbf{B}$ distribution in uniform magnetic field.

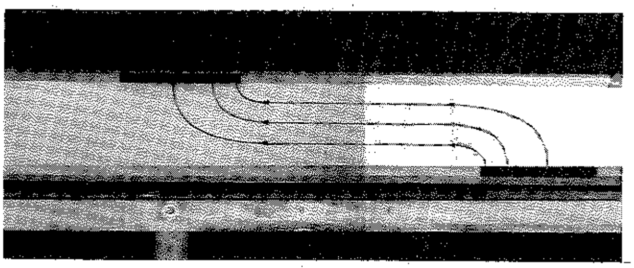
at rest in a flat box when the two-dimensional current shown in Fig. 6a is passed between the electrodes. If a uniform magnetic field is applied in the direction of viewing, dye lines in the fluid show that it barely moves, though the $\mathbf{j} \times \mathbf{B}$ distribution is complicated (Fig. 6b) and causes pressure changes which may be detected by the manometer shown in Fig. 6c.

Here $\mathbf{j} \times \mathbf{B}$ is being balanced by pressure gradients, i.e. $\mathbf{j} \times \mathbf{B} = \text{grad } p$ ($p =$ pressure). But curl grad $\equiv 0$ and so curl $\mathbf{j} \times \mathbf{B} = 0$, i.e. the magnetic field is an *irrotational* vector, incapable of making fluid elements spin, even though the force is far from uniform.

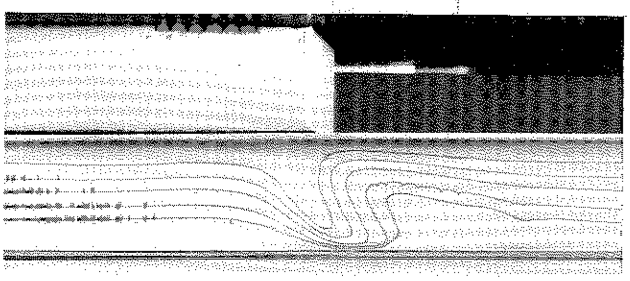


6c. Dye lines in the duct. The upper fluid in the manometer is also dyed. The manometer shows the pressure difference across the duct.

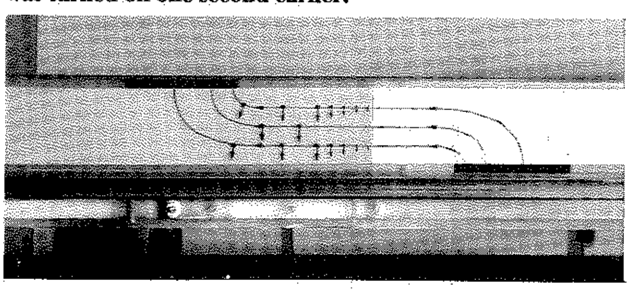
If the same $\mathbf{j}$, $\mathbf{B}$ and $\mathbf{j} \times \mathbf{B}$ distributions* are applied with the fluid in motion at a fixed flow rate along the duct, the flow pattern is quite unchanged when the current is turned on. Again the pressure distribution is able to balance the $\mathbf{j} \times \mathbf{B}$ forces because these are still irrotational.



7a. Current distribution crossing the edge of the magnetic field. The field normal to the paper is uniform in the darker region on the left and falls to zero in the right-hand region.



7b. Distorted dyelines in the fluid, which was at rest with the dyelines straight when the current shown in Fig. 7a was turned on one second earlier.



7c. $\mathbf{j} \times \mathbf{B}$ distribution, which is rotational because of the falling off of the magnetic field.

These experiments are very different if repeated at the edge of the magnetic field. When the current flows through fluid at rest between the electrodes across the edge of the field as in Fig. 7a, the fluid is stirred. Figure 7b shows its state after one second. The $\mathbf{j} \times \mathbf{B}$ force now spins the fluid elements, i.e. it *creates vorticity,* which is counterclockwise here. Figure 7c shows the new $\mathbf{j} \times \mathbf{B}$ distribution with curl $\mathbf{j} \times \mathbf{B} \neq 0$ in the edge region. But curl grad $p \equiv 0$ and so grad $p \neq \mathbf{j} \times \mathbf{B}$ here. The pressure gradient now cannot balance $\mathbf{j} \times \mathbf{B}$, and the fluid cannot stay still.

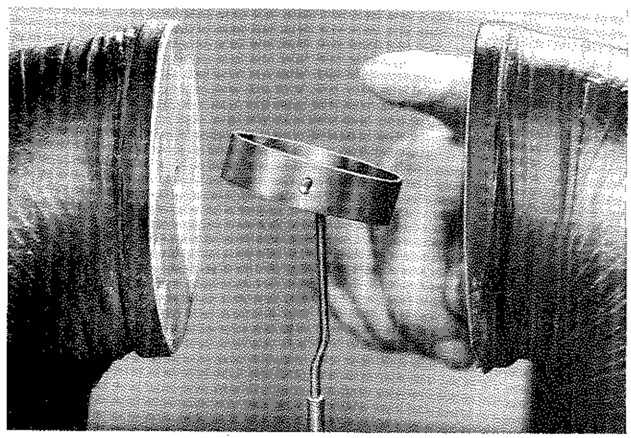
If the fluid moving along the duct encounters the same $\mathbf{j} \times \mathbf{B}$ forces in the edge region, they give it counterclockwise vorticity with the result that the velocity profile is deformed.

In this electrolyte experiment, vorticity is generated by *imposed* currents, but in the mercury experiment shown in Fig. 5 it is generated by *induced* currents.

To sum up this section: to understand the effect of $\mathbf{j} \times \mathbf{B}$ on a fluid, we must consider also the unknown pressure field which can balance *irrotational* forces. Only to the extent that the $\mathbf{j} \times \mathbf{B}$ force is *rotational*, i.e. tending to alter the fluids vorticity, can it elude the pressure gradient and affect the motion. Thus the discussion must be in terms of vorticity.

Vorticity Suppression

If a metal loop is spun about a diameter perpendicular to a magnetic field, it links a changing magnetic flux (Fig. 8). As a result, induced currents produce $\mathbf{j} \times \mathbf{B}$ forces which damp out the motion. For small in-

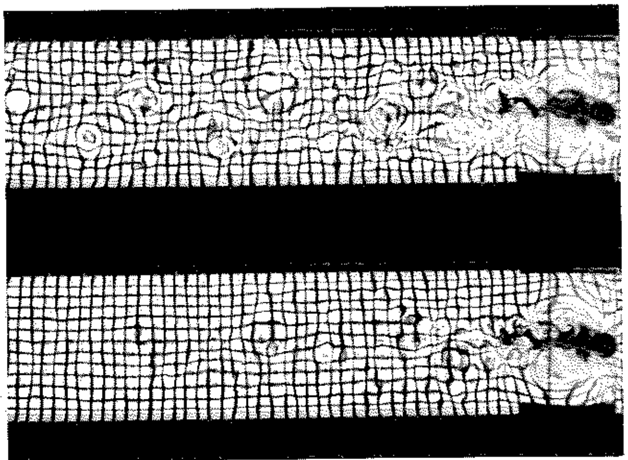


8. Metal loop in a magnetic field. When the loop is spun about an axis perpendicular to the paper, a clockwise torque acts to oppose its motion.

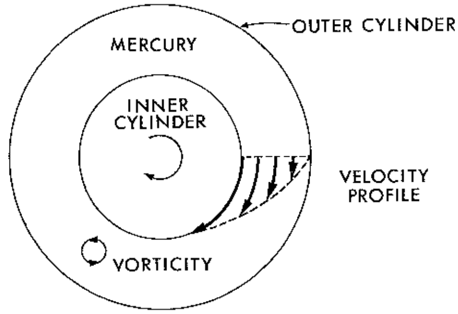
clinations of the plane of the loop to the field, the opposing torque is proportional to angular velocity. But if the spindle is *parallel* to the field, there is no change of flux linked and no damping of the motion.

Similarly, vorticity of fluid elements about axes perpendicular to a magnetic field should be suppressed by

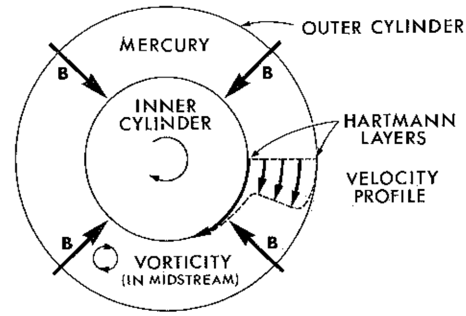
* Electrolytic conductivity is so low that the current distribution is not affected by the fluid motion, because the induced e.m.f.'s are far smaller than the ohmic potential differences.



9. **Damping of vorticity by a transverse magnetic field. The vortices in the mercury in the wake of the towed paddle decay more quickly when a transverse B-field is present (lower picture) than without (upper picture).**



10. **Plan view of cylinders and velocity profile without magnetic effects.**



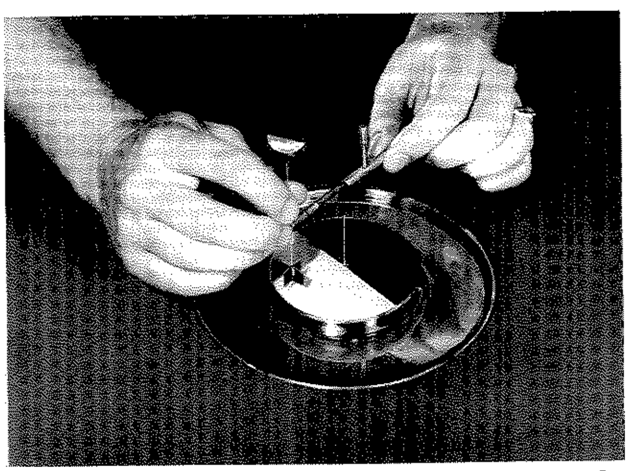
11. **Velocity profile between cylinders, with the radial field turned on.**

induced $\mathbf{j} \times \mathbf{B}$ forces.* Figure 9 shows two top views of the surface of mercury in a shallow trough. In the lower picture the arrows indicate the magnetic field. The mercury surface is being used to reflect an illuminated pattern. A moving paddle is shedding vortices in its wake. These vortices, with vertical vorticity, are visible because they create dimples which distort the reflection. Figure 9 presents a comparison between cases without the magnetic field (upper picture) and with the field (lower picture). The vortices are seen to decay much more swiftly when the field is present. A stronger field prevents the vortices ever being shed. So $\mathbf{j} \times \mathbf{B}$ forces can suppress vorticity as well as generate it.

Vorticity Redistribution

As another example† of rotational $\mathbf{j} \times \mathbf{B}$ forces altering vorticity, consider the circular motion of mercury between two concentric cylinders, the outer one fixed, the inner one rotating about its vertical axis. In the absence of magnetic effects and secondary flow, the fluid velocity falls off monotonically from its value at the inner cylinder to zero at the outer cylinder, and the vorticity is opposed to the rotation of the inner cylinder (Fig. 10). Now add a radial magnetic field between the cylinders and consider an imaginary loop initially lying in a plane through the axis. If the loop moves with the fluid, it rotates in such a way as to start linking magnetic flux. Thus induced currents must produce forces which change the motion so that the loop moves without linking magnetic flux, i.e. it stays in planes through the axis and the mercury rotates like a solid with uniform vorticity and velocity proportional to radius, as in Fig. 11. The vorticity is now in the *same* sense as that of the inner cylinder. The uniform angular velocity of the fluid becomes exactly one-fifth of that of the inner cylinder when the field comes on, a consequence of the ratio of the cylinders' radii being 2:1 (see the Heiser-Shercliff reference).

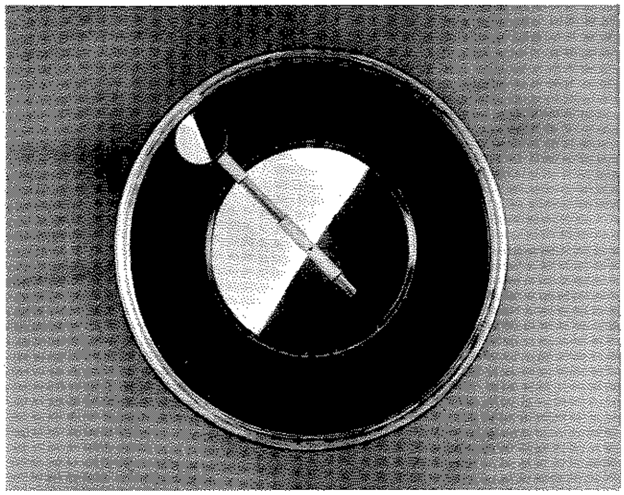
The mercury is now "slipping" over both cylinders, the slip taking place across Hartman boundary layers (see Fig. 11) containing intense vorticity of opposite



12a. **The vorticity indicator, which is free to rotate at the end of its restraining arm, about to be inserted into the mercury. The annular mercury surface is visible round the central, rotatable cylinder, which is painted black and white.**

* This is a slightly oversimplified argument, because vorticity is the sum of the angular velocities of perpendicular planes in a fluid element, but closer investigation does not invalidate the argument in the cases chosen.

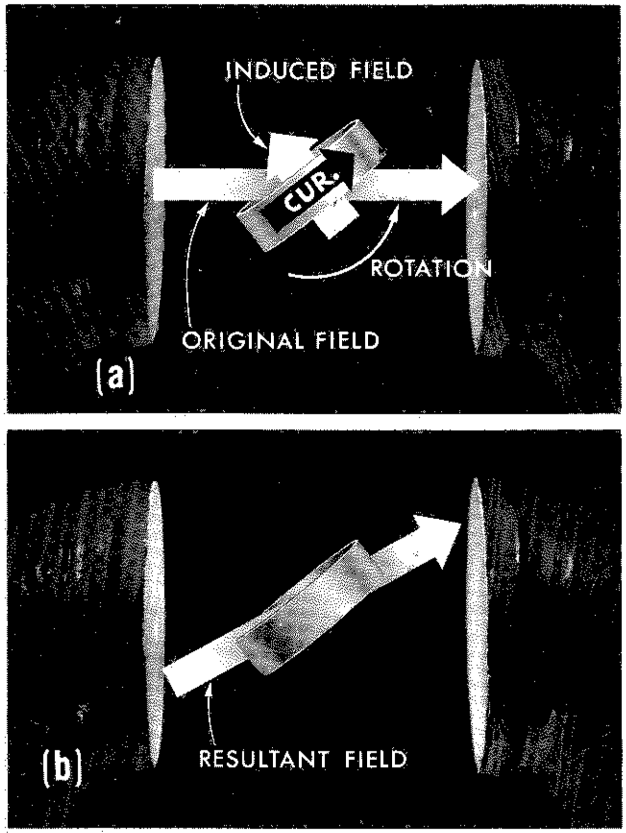
† This experiment is described and analyzed more fully by Heiser and Shercliff reference.



12b. Top view of the mercury and cylinders, with vorticity indicator in use.

and has been ignored. When the fields due to the currents induced in the fluid are too big to be ignored, the nature of magnetohydrodynamics changes drastically.

Consider a conducting loop rotating so that its plane becomes increasingly inclined to a magnetic field. Figure 14 shows how the field due to the induced current



14. Currents induced in the rotating loop perturb the field so as to reduce the change of flux linked. (a) Field components. (b) Resultant field.

makes the total field less inclined to the loop than before. Thus the field is perturbed so that the rise of flux linked by the loop is reduced.

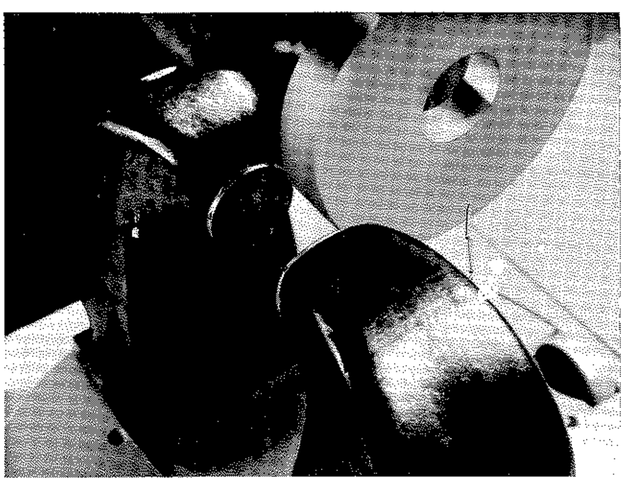
sign. The *total* vorticity is unaltered by the magnetic field and the $\mathbf{j} \times \mathbf{B}$ forces have merely *redistributed* it.

The behavior of the vorticity outside the boundary layers when the field comes on is revealed by the vorticity indicator shown in Figs. 12a and 12b. The cruciform paddles take a correct mean for the fluid's angular velocity, which is indicated by the black-and-white disc. The freely rotating arm keeps the indicator vertical in midstream.

Perturbation of the Magnetic Field

So far we have considered only half of magnetohydrodynamics — the effect of the magnetic field on the flow. The currents in the fluid must also affect the field to some extent. Figure 13 shows the experiment in which a conducting loop swings through a magnet gap. The current induced in the loop affects the field, as is revealed by the twitching of the iron nails on the side of the magnet pole.

Field perturbation is weak in the experiments so far,



13. The currents induced in the metal loop affect the field and cause the iron nails (on the right of the nearer magnetic pole) to move.

If the loop stops rotating, its resistance causes the induced current to decay and the field relaxes back to its original form in a "magnetic relaxation time" L/R that depends on the inductance L and the resistance R of the loop. It gets *longer* if we reduce the resistance. The extent to which a rotating loop perturbs the original field depends on how L/R compares with the time for a revolution.

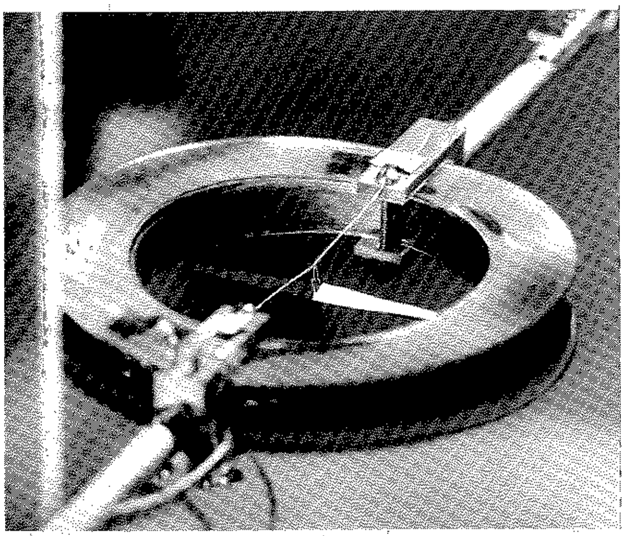
When L/R is small, the field continually relaxes back to its original form and is hardly perturbed. Then the magnetic forces on the loop are *dissipative*; there is a torque only when the loop is moving, inducing the current.

But when L/R is large because the conductivity is high, the field is deformed strongly, and when L/R is very large, there is virtually no change of flux linked.

With perfect conductivity, the induced e.m.f. becomes negligible and spontaneous currents alter the field so that the flux linked never changes.

Modeling Perfect Conductivity by the Use of a Feedback

We may show how a perfectly conducting loop behaves by energizing a coil from a power source to which feedback is applied from a search coil in such a way that the flux linked is constant.* Figure 15 shows such a coil, pivoted in a horizontal magnetic field.



15. Artificial "perfectly-conducting" loop perturbing a magnetic field. When the coil is tilted slightly and released, it oscillates about the pivot. The compass needle in the center shows the direction of the magnetic field; it oscillates with the coil, showing that the magnetic field lines are distorted so that the coil never links any flux.

When the coil is disturbed, it oscillates as though subject to an elastic, torsional restraint. There cannot be dissipation when there is "perfect" conductivity. The torque is now proportional (for small angles) to the tilt, *not* to angular velocity (as in the low conductivity case). The time for an oscillation is much less than the magnetic relaxation time and the magnetic forces act in a pseudo-elastic, non-dissipative manner. There are corresponding effects in a fluid conductor.

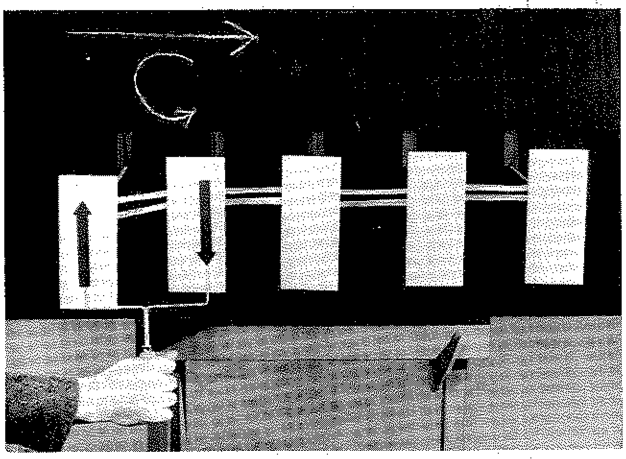
High Conductivity Behavior in Fluids; the Alfvén Wave

For any loop drawn in the fluid, the magnetic relaxation time is of order $\mu_0\sigma l^2$, which is big if σ, the conductivity, is large or l, the scale, is large (as it is in astrophysics).

In the experiment shown in Fig. 9, $\mu_0\sigma l^2$ for loops drawn in the vortices is short compared with their rotation time, and the magnetic force is dissipative, not elastic.

To see the effect of "elastic" $\mathbf{j} \times \mathbf{B}$ forces in a highly

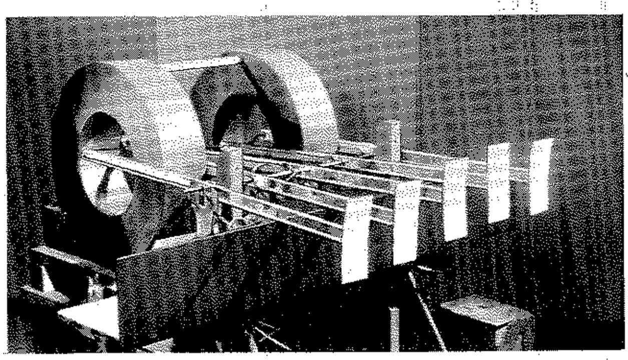
* For further details of this experiment see reference 2.



16. Schematic model of rectilinear shear motion. The curved arrow represents the vorticity, the horizontal arrow the field. The sides of white rectangular loops lying in a plane normal to the page are visible between the white cards. The vertical arrows on the left represent the magnetic torque on the first loop.

conducting fluid, consider the simplest motion that contains vorticity, namely a rectilinear, shear motion. In Fig. 16 the white rectangles represent layers of fluid with left-hand layer moving downward and counterclockwise vorticity between it and the next layer.

To understand the magnetic effects when there is an imposed horizontal field, consider rectangular loops lying between the layers, initially linking no magnetic flux. When the first layers move down, the induced currents produce elastic forces on the first loop, as shown in Fig. 16, tending to twist the loop back in line. The force on the second layer causes it to accelerate downward in turn, and then the third layer begins to feel the downward force, and so on.



17. Working model of Alfvén wave.

Figure 17 shows a working version of this apparatus,* with the rectangles mounted on pivoted rods. Flexible loops, attached to the rods, simulate perfect conductivity by means of feedback. The imposed magnetic field is again horizontal.

When the first rectangle is displaced downward, a downward motion propagates from rod to rod as a wave. In a fluid this corresponds to the propagation of

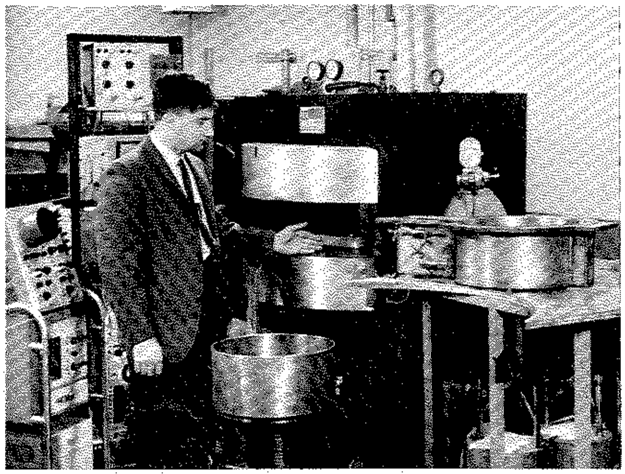
* See Melcher reference for fuller details.

the vorticity between the layers. Thus, when the rotational magnetic forces behave elastically because the conductivity is high, vorticity propagates along the field lines as a wave, known as an *Alfven wave.*

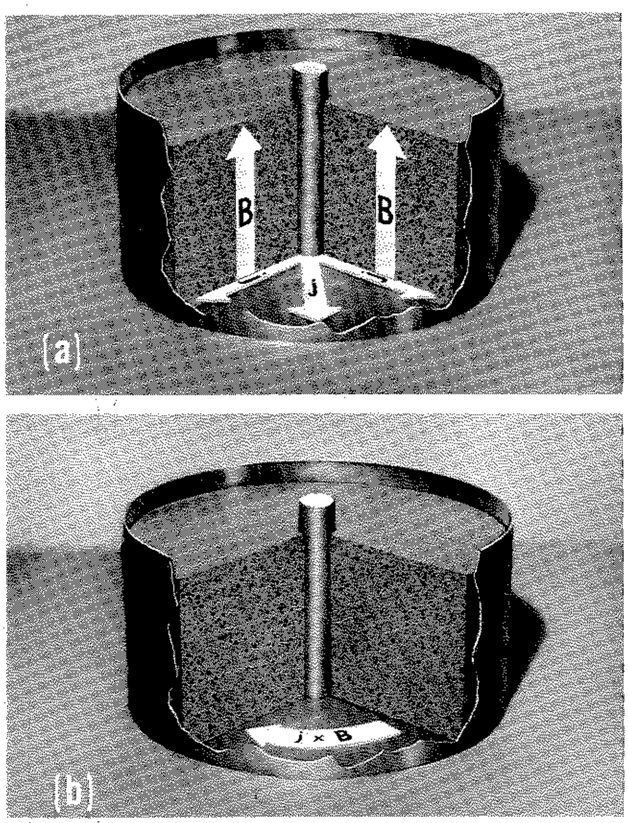
An Alfv́en Wave Experiment in Liquid Metal

Alfv́en waves can be produced in liquids if $\mu_0\sigma l^2$ is made large by choosing a highly conducting fluid such as NaK (sodium-potassium eutectic) and by making l, the scale, as large as possible, and at the same time making the characteristic time (the transit time of the waves) small by making them travel fast in a strong magnetic field.

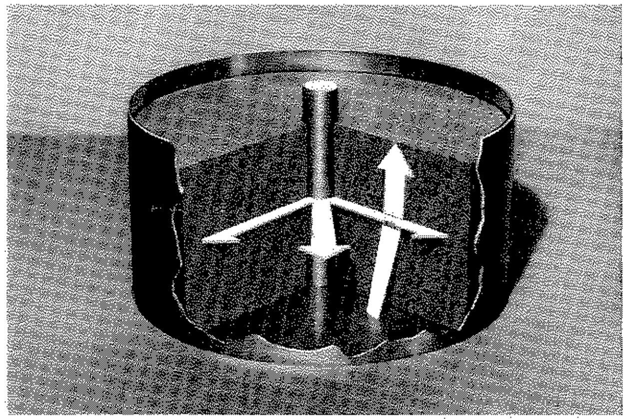
Figure 18 shows the apparatus. The stainless steel



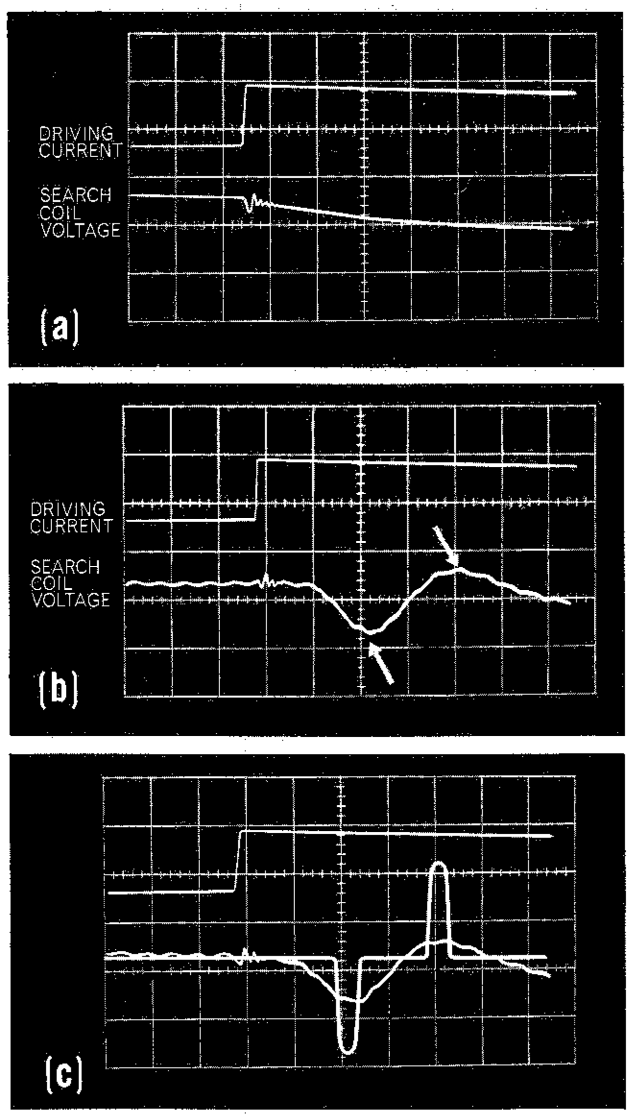
18. Experiments on Alfv́en waves in NaK. The large magnet is at the rear, the stainless-steel tank (without lid) on the right, and the outer copper electrode in the foreground. Instrumentation is on the left.



19. (a) Initial current and (b) j × B distribution in Alfven wave experiment.



20. Model of Alfv́en wave deforming magnetic field.



21. Oscilloscope traces from Alfv́en wave experiment. (a) Field off. (b) Field on. (c) Perfect conductor case.

tank on the right contains the NaK. The motion is excited by passing a current radially between the central rod and the outer wall, both made of copper (Fig. 19a). The imposed magnetic field is vertical.

The current at first flows across the bottom of the NaK and the resulting $\mathbf{j} \times \mathbf{B}$ force (Fig. 19b) makes just the bottom layer of fluid swirl. There is horizontal vorticity between the moving layer and the stationary fluid above. Immediately this vorticity propagates away along the vertical field lines as an Alfvén wave. The layer of radial current travels with it, and this deforms the magnetic field, as in Fig. 20. A search coil in the middle of the NaK detects the magnetic perturbation due to the traveling current layer, and the signal is displayed on an oscilloscope.

Figure 21 shows the results; the upper trace shows the sudden onset of the driving current, and the lower trace shows the voltage in the search coil. In Fig. 21a, the vertical magnetic field is absent, no waves occur and the lower trace shows only some stray signals, because the NaK shields the search coil itself. But in Fig. 21b, with the field on, the lower trace reveals the wave passing the search coil (the first arrow) and passing it again after reflection (the second arrow).

With a perfect conductor, the signals would be as in Fig. 21c, in which the pulse width is fixed by the finite width of the search coil. The resistance of the NaK makes the first pulse broader and weaker, and the second pulse after reflection is even broader and weaker. Nevertheless there is clear evidence of propagation of current and vorticity by the Alfvén mechanism in a real fluid.

Summary

In studying the effect of magnetic forces on a conducting liquid, it is fruitful to concentrate on their rotationality, or effect on vorticity, because then the unknown pressure gradient can be left out of consideration. When perturbation of the field by the currents is small, the magnetic force tends to be dissipative, altering the fluid's vorticity in various ways, but when the fluid conductivity is high enough for strong perturbation of the field to occur, the magnetic force is pseudo-elastic and vorticity propagates in Alfvén waves.

References

1. **Heiser, W. H., and Shercliff, J. A. (1965), "A Simple Demonstration of the Hartmann Layer."** ***J. Fluid Mech. Vol. 22, p. 701.***

2. **Melcher, J. R., and Warren, E. P. (1965), "Demonstration of Magnetic Flux Constraints and a Lumped Parameter Alfvén Wave."** ***IEEE Transactions on Education, Vol. E-8.***