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National Committee for Fluid Mechanics Films

FILM NOTES
for
STRATIFIED FLOW*

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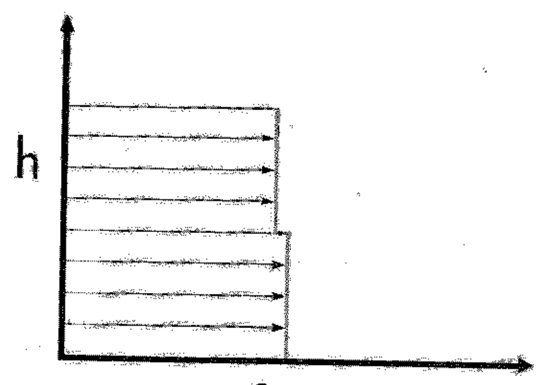
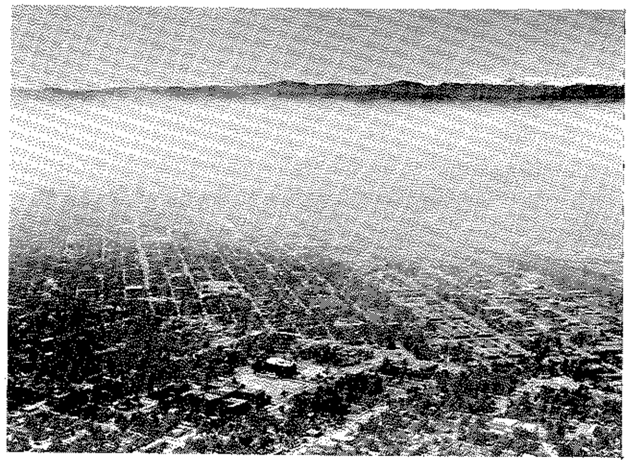
Introduction

A stratified fluid is a fluid with density variations in the vertical direction. One example is a system of two superimposed fluids in a channel with the lighter fluid on top. In this case the density changes abruptly with height, as illustrated in Fig. 1. Layered systems of stratified fluids occur, for instance, where warm water lies above cold water or fresh water above salt water. An abrupt change also occurs at the interface between air and water. Both air and water are fluids, and together they may be thought of as a stratified fluid system. Frequently, however, density varies continuously, as in the oceans and atmosphere. Density variations profoundly affect the motion of water and air. Wave phenomena in air flow over mountains and the occurrence of smog (Fig. 2) are examples of stratification effects in the atmosphere.



1. Density-height curve in a two-fluid system.



2. Smog. (Courtesy U.S. Public Health Service.)

In the film we use laboratory demonstrations to illustrate the basic phenomena in stratified fluids. We emphasize fluid systems in which the density decreases with height. When such a system is disturbed, gravity

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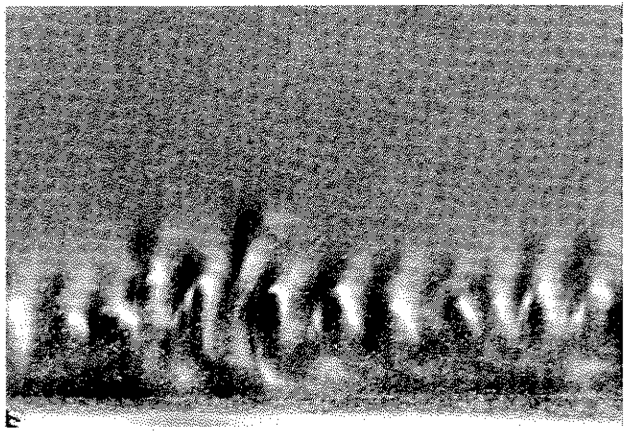


3. **Convective motions in a fluid heated from below.**

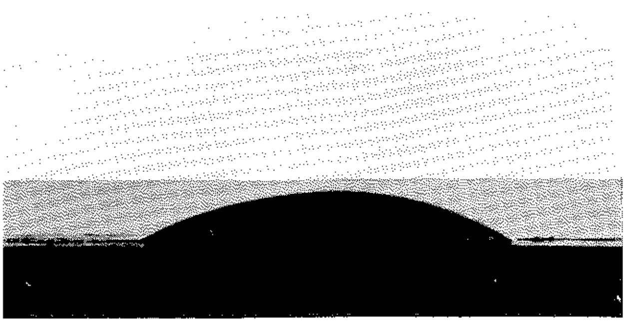
waves result, but gravity and friction eventually restore undisturbed conditions, and the system is judged to be stable. If the density increases with height, however, the fluid tends to be unstable. Figure 3 illustrates the cellular type of motion that results when a fluid is heated in its lower portions to produce an unstable distribution.

In our examination of fluids that are stably stratified in the vertical, we concentrate on flows over obstacles. Such flows reveal most of the fundamental phenomena that occur in stratified fluids. Moreover, such flows frequently occur in nature.

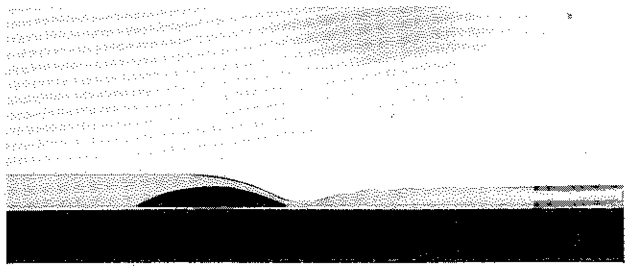
Surface Waves Produced by Flows over Obstacles

Figures 4-8 show water flow over a barrier in a channel. The flow relative to the obstacle is produced by moving it through the water. When the camera moves with the obstacle, we see a uniform flow from the left over a stationary barrier.

Phenomena in water flows depend on the dimensions of the obstacle relative to the depth and on the approach Froude number. A local Froude number F may be defined at any section as $F = U/\sqrt{gh}$, where U is the fluid velocity at the section, h is the local fluid depth, and g is the gravitational acceleration. Since $\sqrt{gh}$ is the speed of the fastest small-amplitude gravity



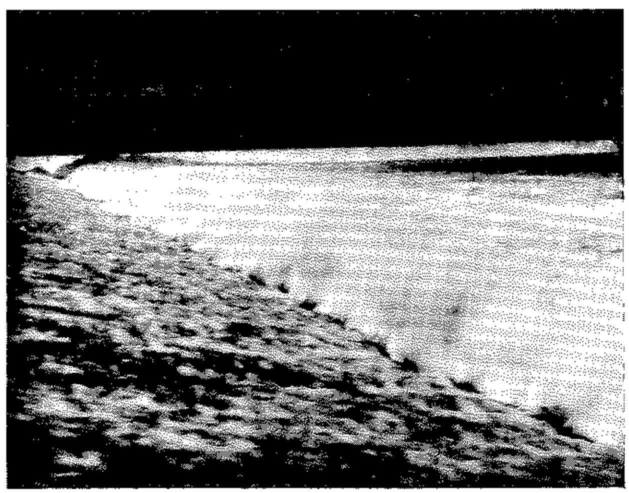
4. **Subcritical flow of water over an obstacle; the local Froude number is less than one everywhere.**



5. **Hydraulic jump at a water surface. The approach Froude number is less than one.**

wave, the Froude number compares the fluid speed to this wave speed. Where the Froude number is *less than one,* the flow is called *subcritical.* Where it is *greater than one,* the flow is called *supercritical.*

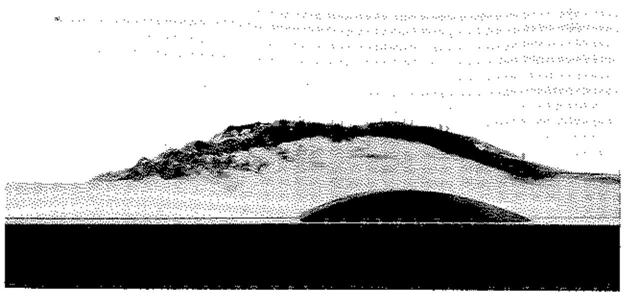
We first illustrate the change of flow patterns as the Froude number based on the approach conditions is increased from low to high values. Figure 4 shows a very low approach Froude number. The flow is subcritical everywhere. The free surface dips down slightly over the obstacle. In Fig. 5 the approach Froude number is higher, although still subcritical. When started from rest the upstream and downstream levels



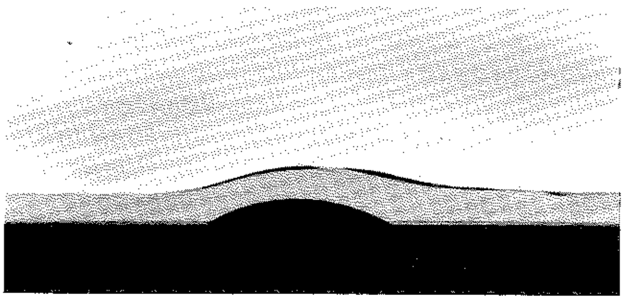
6. **Hydraulic jump at the base of a dam.**

were the same. A blocking effect sets in, however, and causes a rise in the upstream level. When the motion becomes established, as in Fig. 5, the upstream subcritical flow near the obstacle draws down as it passes over the barrier, becoming supercritical in the lee. This supercritical flow changes abruptly to subcritical flow downstream as the fluid passes through a hydraulic jump. The hydraulic jump is an important phenomenon in nature. Figure 6 shows a jump at the base of a dam.

When we increase the approach Froude number to a value somewhat greater than one, we get an upstream hydraulic jump as in Fig. 7. The flow goes from supercritical to subcritical as it passes through the



7. Upstream hydraulic jump. The upstream Froude number is greater than one.



8. Supercritical flow. The approach Froude number is greater than one. The *local* Froude number exceeds one everywhere.

jump, then accelerates to supercritical as it passes over the obstacle crest.

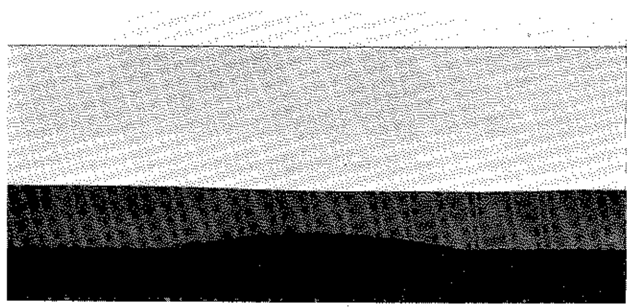
When the approach Froude number is much greater than one, the fluid swells symmetrically over the obstacle (Fig. 8), and conditions are supercritical everywhere. Since the Froude number is much greater than one at all sections, the fluid speed everywhere exceeds the speed of the fastest waves, and all disturbances are swept downstream.

Flows of a Two-layer Fluid over Obstacles

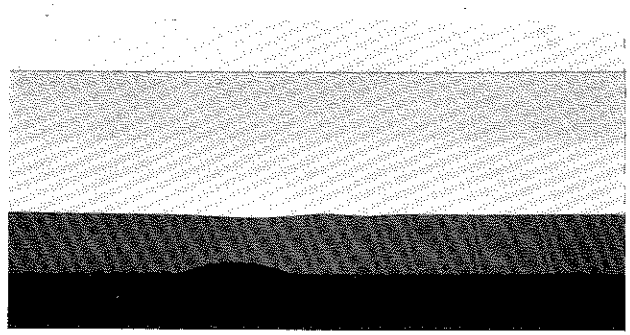
Phenomena similar to those of water flows in a channel occur in liquids with internal density variatons. For example, waves, similar to waves on a water surface, can occur at the interface of two liquids with a slight density difference. The frequencies of the interfacial waves and the water waves are both proportional to the square root of the restoring force. The restoring force for water waves is the gravitational force g. But for the internal waves the restoring force is g multiplied by the small number $\Delta\rho/\rho$, where $\Delta\rho$ is the density difference between the liquids and ρ is the average density. As a result, the frequency of internal waves is usually much smaller than that of water waves. The slow motion of internal waves is a characteristic feature of the experiments of the film.

In the flow of a two-layered fluid over obstacles, the important Froude number is the internal Froude number of the approach flow, $F_i = U \Big/ \sqrt{g \frac{\Delta\rho}{\rho} h}$, in which g is replaced by "modified gravity" $g\Delta\rho/\rho$, h is the total depth of the two fluids, and U is the common approach velocity of the two fluids. A critical value of F_i, as in water flows, corresponds to a fluid speed equal to the speed of long waves. However, the critical value is no longer one, since the wave speed also depends on the ratio of the depth of the lower fluid to the total depth. For two fluids, the flow patterns are determined by F_i, the ratio of the depths, and the height and length of the obstacle relative to the depth.

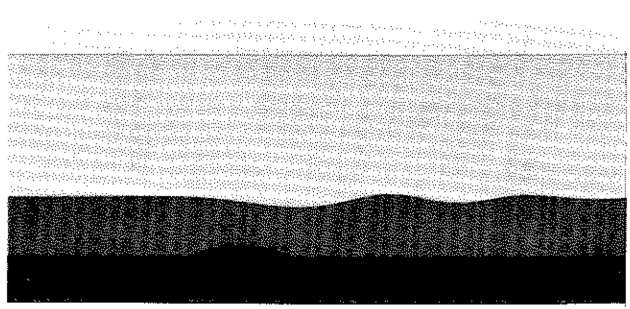
As these parameters are changed the flow patterns change. A long obstacle and a low value of the internal Froude number of the approach flow leads to subcritical flow at every section, and a slight draw-down of the interface over the barrier (Fig. 9). This long, gently-shaped barrier minimizes vertical accelerations of the fluid, and the pressure remains nearly hydrostatic, i.e., the pressure is proportional to the depth. However, for the same ratio of fluid depths and the same internal Froude number, a shorter model of the same height



9. Subcritical flow of two fluids over an obstacle. The flow condition is analogous to that in Fig. 4.

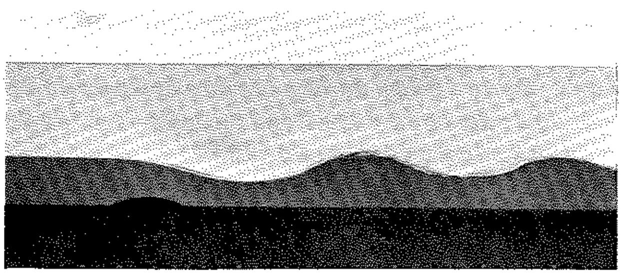


10. Weak lee waves in a two-fluid system.

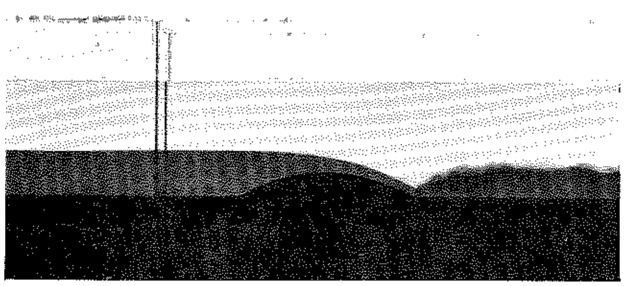


11. Strong lee waves in a two-fluid system.

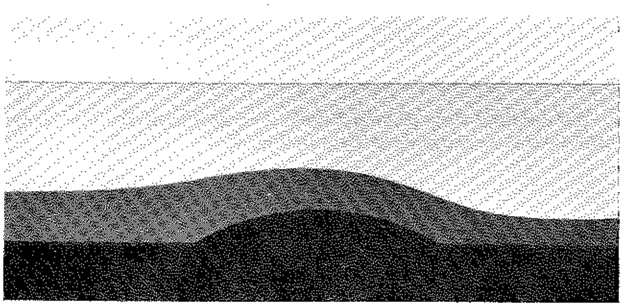
results in weak lee waves (Fig. 10), as nonhydrostatic pressures become important. As we increase the Froude number by increasing the speed, these lee waves become stronger (Fig. 11). At a still higher Froude



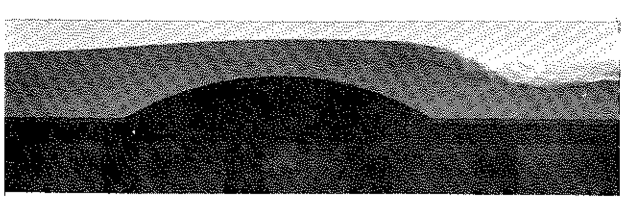
12. Breaking waves in a two-fluid system.



13. Hydraulic jump in the lee of an obstacle in a two-fluid system.



14. Supercritical flow in a two-fluid system. Compare this with Fig. 8 for the water-air case.



15. Hydraulic drop in a two-fluid system. Note the increase of thickness of the upper fluid downstream.

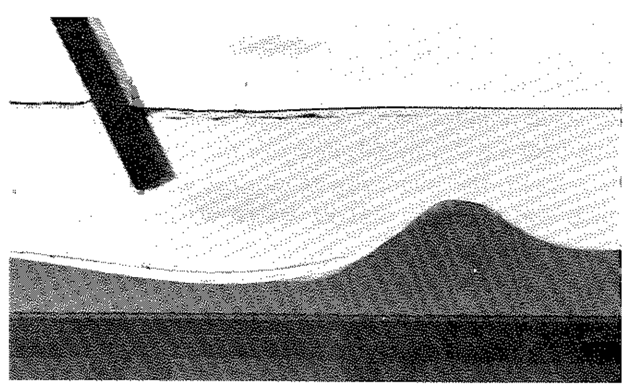
number (Fig. 12), the waves show signs of breaking, as in the undular hydraulic jump on a water surface. Further increase in Froude number only results in a lengthening of these waves.

When we increase the size of the obstacle, a strong hydraulic jump occurs downstream (Fig. 13). The upstream subcritical flow changes to supercritical as it passes over the barrier. Further downstream, it jumps to a new level, and the flow again becomes subcritical. As in water, if the approach Froude number is high enough, the flow is supercritical everywhere (Fig. 14). To this point, phenomena at the interface resemble those on a water surface, largely because there is a deep upper fluid in these cases, as is shown by the fact that when we make the lower fluid deeper than the upper, as in Fig. 15, the depth of the lower fluid abruptly decreases downstream of the obstacle in what might be called a "hydraulic drop."

A common feature of all the two-fluid experiments we have described is that the liquid-air surface remains quite undisturbed despite very large disturbances at the interface. We can explain this if we forget for a moment that there are two fluids in the channel, and notice that in all of these two-fluid experiments the *ordinary* Froude number is much less than one. In other words, conditions are very much subcritical as far as disturbances of the liquid-air interface are concerned. The free surface becomes disturbed if we move the obstacle fast enough to obtain an ordinary Froude number of order one (Fig. 16). In such cases the internal density variations are no longer important.

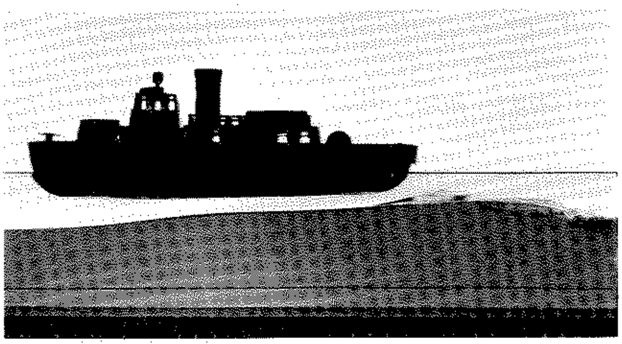
16. Free-surface disturbance of a two-fluid system.

We can also disturb a two-layer system by moving an obstacle in the upper layer (Fig. 17). This experiment has an interesting application. It was noticed



17. Disturbance of a two-fluid interface by an obstacle moving in the upper fluid.

long ago that ocean-going vessels off the coast of Norway suddenly found themselves unable to maintain their accustomed speed as they moved past the mouth of a fjord. This "dead water" phenomenon was first explained by the Swedish oceanographer Ekman. He pointed out that the fresh water from the fjords flows out in a density current over the heavier water of the ocean, and forms a two-layer stratified fluid system. At certain speeds, much of the power from the ship's engines goes into the creation of waves at the fluid interface (Fig. 18), while the liquid-air surface is rather undisturbed. If the ship moves at a higher speed, the interfacial disturbances are much weaker and the dead-water phenomenon no longer exists.



18. Disturbance of the interface of a two-fluid system by a model ship moving at the free surface.

Flows of a Continuously Stratified Fluid

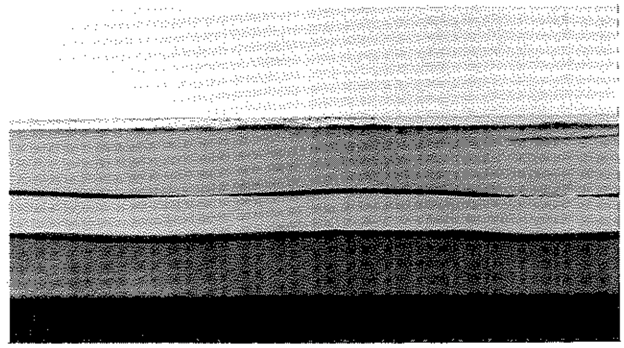
The fundamental mode of oscillation in a two-fluid system involves a simple sine wave at the interface. A three-layer system, however, has two distinct modes (Figs. 19 and 20). Indeed, one mode is added for each layer. Carried to the limit, an infinite number of infinitely thin layers has an infinity of modes. This is equivalent to a fluid system in which the density varies continuously with height, as in the atmosphere and oceans.

To understand experiments with continuous density stratification, we consider the parameters which govern the flow of a continuously stratified flow over an obstacle. Again, the most important nondimensional number is the internal Froude number,

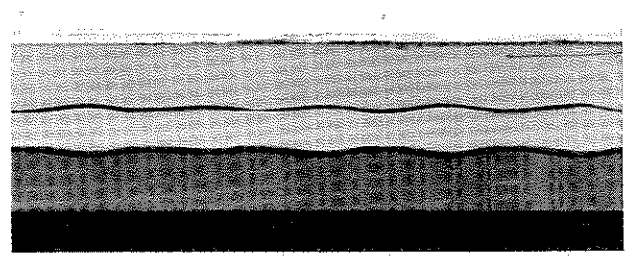
$$F_i = \frac{U}{\sqrt{g\frac{\Delta\rho}{\rho}h}},$$

where $\Delta\rho/\rho$ is now the fractional density difference from the top to the bottom of the channel, and h is the total depth. The flow patterns are influenced not only by the internal Froude number, but also by the characteristics of the obstacle relative to the depth.

Our experimental fluid is a saline solution in which the salt concentration, and therefore the density, decreases continuously with height. The tracer particles are neutrally buoyant polystyrene beads. The variation of the salt concentration, and therefore the density, is more or less linear with height.



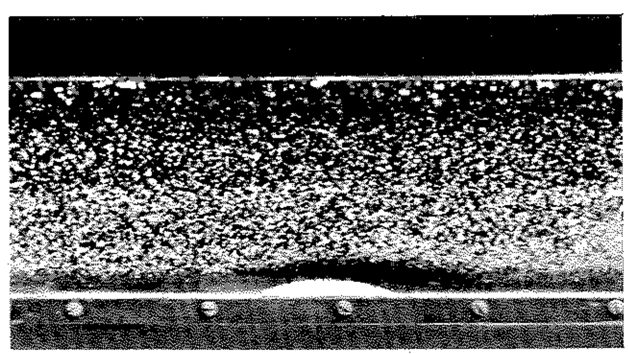
19. Symmetric disturbance of a three-layer system.



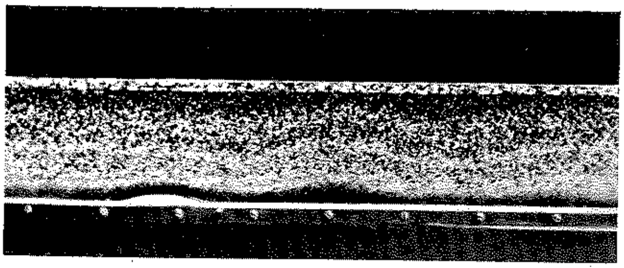
20. Antisymmetric disturbance of a three-layer system.

In Fig. 21 the motion is analogous to the supercritical flow of water. The internal Froude number is high, and the fluid simply swells over the barrier as water does at supercritical speeds.

If we drop F_i below its first critical value of π^{-1}, simple sine waves appear in the lee of the obstacle (Fig. 22). These internal waves are not possible in a homogeneous fluid. To see how they can arise in a stratified fluid, consider the vorticity equation for a frictionless, nonhomogeneous fluid,



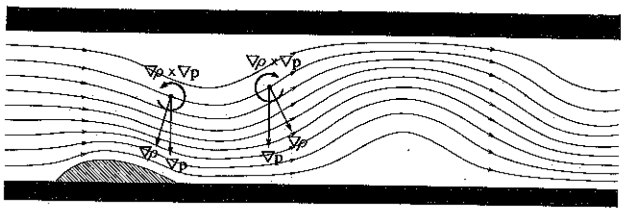
21. Supercritical flow of a continuously stratified fluid over an obstacle. The uneven distribution of tracer particles in this and subsequent pictures does not reflect the density distribution. The density distribution depends only on salt concentration and is nearly linear with height.



22. Flow of a continuously stratified fluid over an obstacle at an internal Froude number of less than $1/\pi$.

$$\frac{d\omega}{dt} = (\omega \cdot \nabla)\,\mathbf{V} + \frac{1}{\rho^2}(\nabla \rho \times \nabla p).$$

ω is the vector vorticity, $\mathbf{V}$ is the velocity, and p is the pressure. $d\omega/dt$ is the rate of increase of vorticity of a moving parcel of fluid. The last term shows that the vorticity can be generated through the interaction of the pressure and density fields. This equation can be solved under certain circumstances to yield flow patterns corresponding to the experiments we have just seen (Fig. 23). We can make use of this pattern to show the relationship between the generation of vorticity and the wave motion. The density varies from point to point in the fluid, but because the motion is steady, the density is constant along a given streamline. The density gradient $\nabla\rho$ is everywhere normal to the streamlines. Despite the motion, the constant pressure surfaces are nearly horizontal, i.e., the pressure gradient ∇p is vertical. As the fluid descends in the lee of the obstacle, the last term in the vorticity equation, $\nabla\rho \times \nabla p$, yields a vorticity generation, or fluid rotation, in a counterclockwise sense (Fig. 23). This rotation permits the descending fluid to turn upward toward its original level. On the way up, vorticity of the opposite sense is generated. The fluid can then turn back into the next phase of the wave.

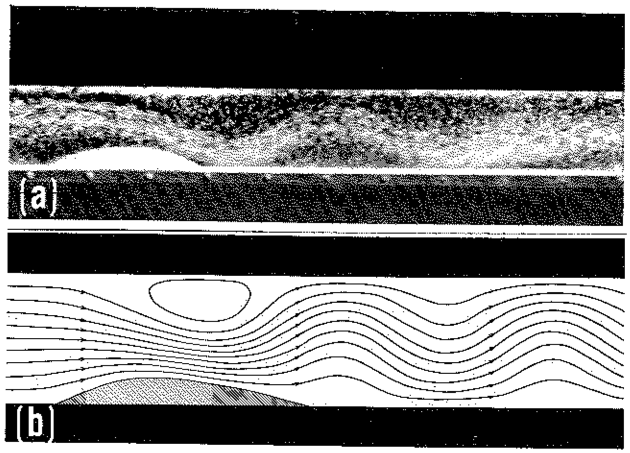


23. Theoretical flow pattern of a continuously stratified fluid over an obstacle.

Of course, these waves can also be explained from a buoyancy-force viewpoint. The descending fluid particle finds itself in a heavier environment and is forced back up as it moves along. It overshoots, then descends again, and so on.

When the barrier is small, the waves have small amplitudes, as in Fig. 22. If we increase the height of the barrier, keeping F_i the same, the amplitude of the waves gets much larger, as shown in Figure 24a.

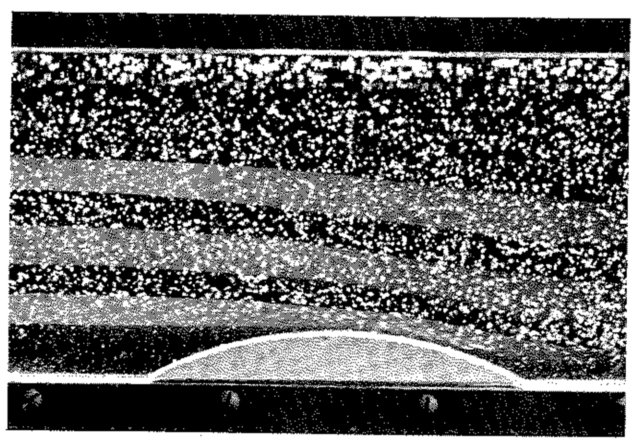
In the theoretical flow pattern (Figure 24b) corresponding to the experiment in Figure 24a, closed streamlines appear near the free surface above the obstacle. In the experiment, such regions appear to be unstable and break into turbulent eddies. The loss of energy in this turbulence causes the wave amplitude downstream to be less than that indicated by the corresponding theory.



24. Experimental (a) and theoretical (b) flow of a continuously stratified fluid over an obstacle.

In going from the experiment of Fig. 22 to that of Fig. 24, we raised the height of the model but kept F_i the same. A *decrease* of F_i reduces the length scale of the fluid motion, and, in addition, the wave structure is no longer a simple sine wave.

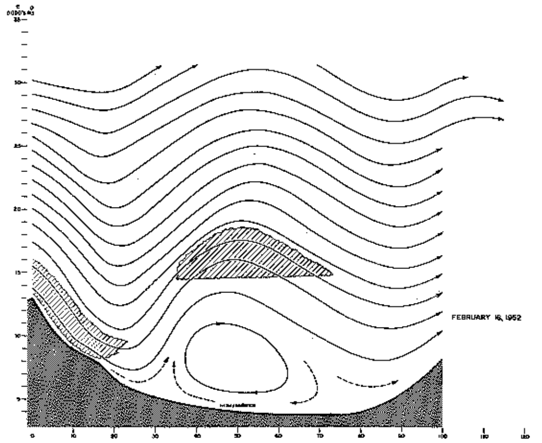
When the Froude number is low, but the obstacle is large, the flow pattern again has a small scale, but is of a very different character (Fig. 25). The approach flow consists of a number of jets moving rapidly toward the obstacle, sandwiched between layers of fluid that are stagnant with respect to the obstacle. In these



25. Jet-patterns in flow of a stratified fluid over an obstacle. Notice that the fluid is stagnant with regard to the obstacle in a number of layers, with jets sandwiched between. (The jets are delineated by the lightened areas.) The first stagnant layer extends from the bottom to the level of the obstacle crest.

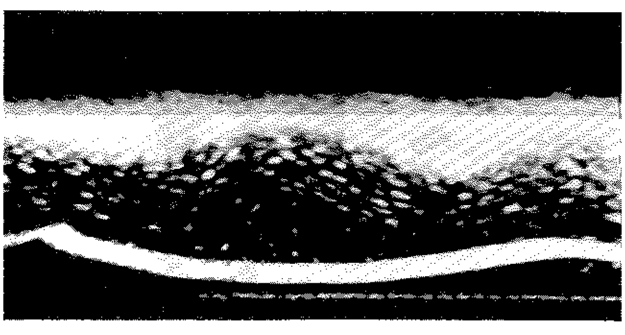
flows, there is always a stagnation layer extending from the bottom of the channel to the level of the crest. At low Froude numbers, the fluid has insufficient kinetic energy to move up over the obstacle from this region. This illustrates the tendency of a slowly moving parcel of stratified fluid to maintain its original level.

An example of a natural flow of a stratified fluid is the flow of air over a mountain ridge. Such flows have been carefully studied in the Owens Valley east of the Sierra Nevada in California. A typical streamline pattern constructed from balloon and glider observations is shown in Fig. 26.



26. Streamline pattern over the Owens Valley. The vertical scale is exaggerated in this line drawing.

We can model this flow in the laboratory by creating a model of the terrain in this region and simulating the density stratification of the atmosphere by the density variation in salt water. Figure 27 shows an experimental photograph in which the Froude number is the same as that in the atmosphere on the day the observations were made in Fig. 26. A single large wave appears over the valley. When the wind is weaker the number of waves over the valley increases. When the flow in the model was adjusted downward to give a corresponding Froude number, the same increase of wave number appeared in the experiment.



27. Flow over a model of the Sierra Nevada Mountains simulating the observed flow in Fig. 26.

Effects of Stratification on Diffusion

Density stratification has an important effect on diffusion in fluids. For example, smoke coming from a chimney diffuses turbulently if the atmosphere is not stably stratified, as seen in Fig. 28. When the lower air is stable, as in the morning or early evening, the smoke comes out and flattens into a long, thin layer (Fig. 29). Strong stratifications, or inversions as they are sometimes called, confine contaminants to the lower layers of the atmosphere, and cause many of our air-pollution problems.

28. Diffusion of smoke in conditions of neutral stability.

29. Diffusion of smoke when the atmosphere is stably stratified.

Conclusion

Thus, stratification gives rise to forces that generate internal waves, inhibit turbulent diffusion, and create strong velocity gradients and jets. These phenomena have far-reaching effects on the motion of air and water in the atmosphere, oceans, lakes, and reservoirs.

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