Flow Instabilities

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Introduction

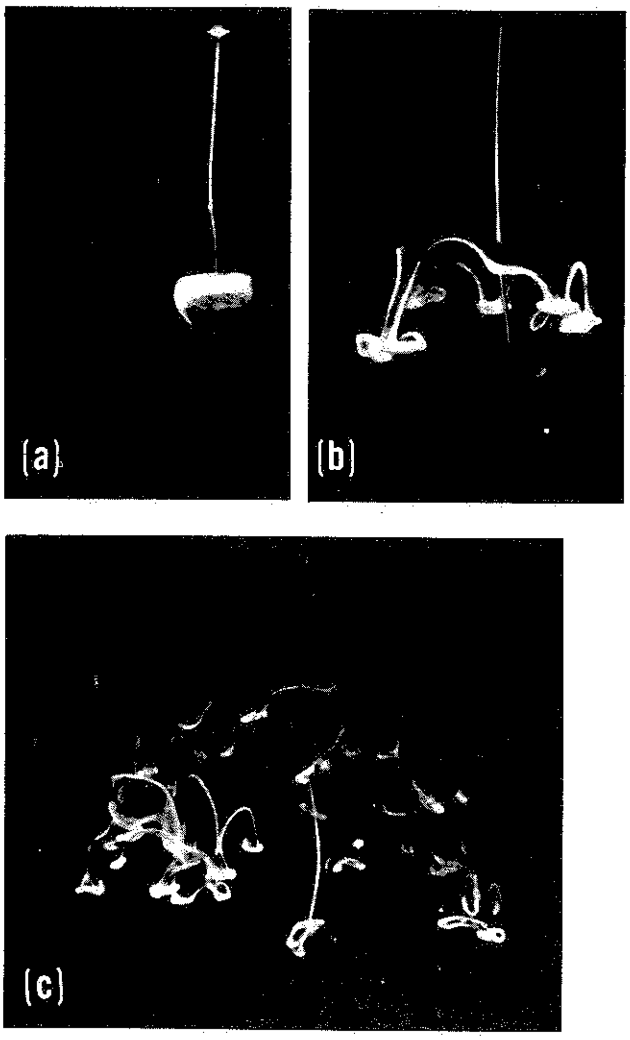
Flow Instabilities play important roles in all branches of fluid mechanics.

The purpose of this film is to show several examples of flow instabilities and to point out some of their common features. The concepts of a critical condition and frequency selective amplification are illustrated in the context of an experimental investigation of a particular flow instability — namely, wind-generated waves.

These notes, in addition to serving as a reminder of the film, give some information on the techniques used in designing and performing the experiments, as well as some references for further study.

Three Examples of Flow Instability

A spectacular but poorly understood instability involving an interplay among viscous, surface tension, and buoyancy effects is illustrated in Fig. 1. A drop of

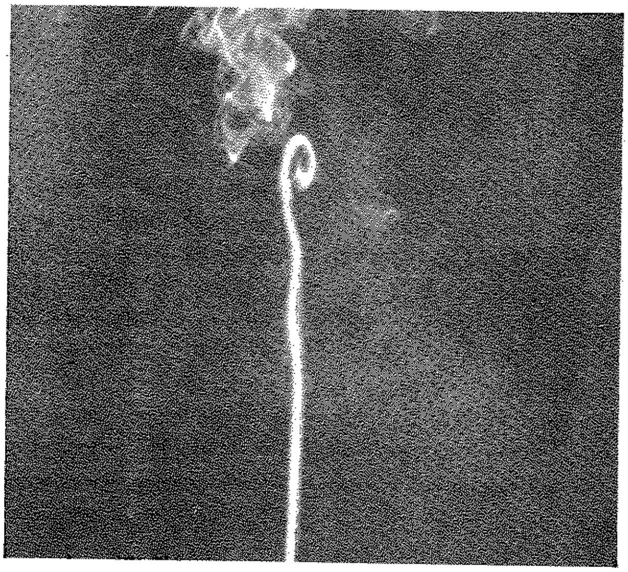


1. A drop of light cream falling into cold water creates an unstable flow pattern.

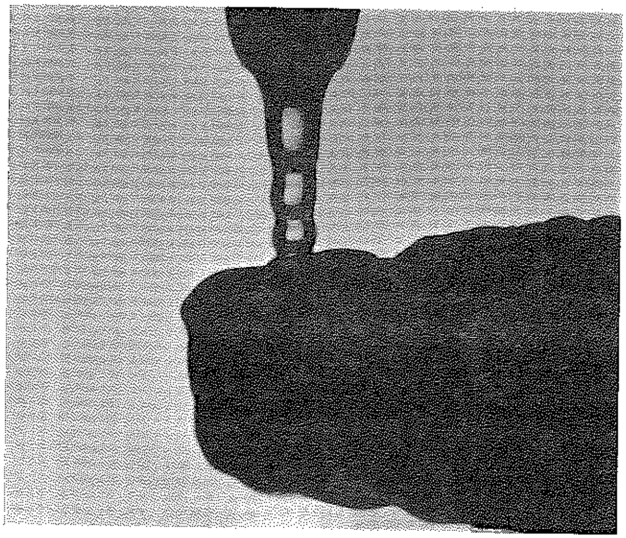
light cream is released from an eyedropper, one millimeter above a water surface. A vortex ring forms (Fig. 1a) then develops an instability and breaks up into smaller vortex rings (Fig. 1b). Each of the smaller rings goes through the same cycle, and a cascade of rings develops (Fig. 1c). Light cream (half cream, half milk) and ice water work quite well as fluids.

A familiar flow instability occurs in the buoyant smoke plume of a smoldering taper (Fig. 2). The smoke plume is steady near its source. Higher up, waves occur, and the plume becomes unstable and then turbulent. In a very still room, or if the plume is protected from room currents by glass walls, the plume may rise quite far before becoming turbulent.

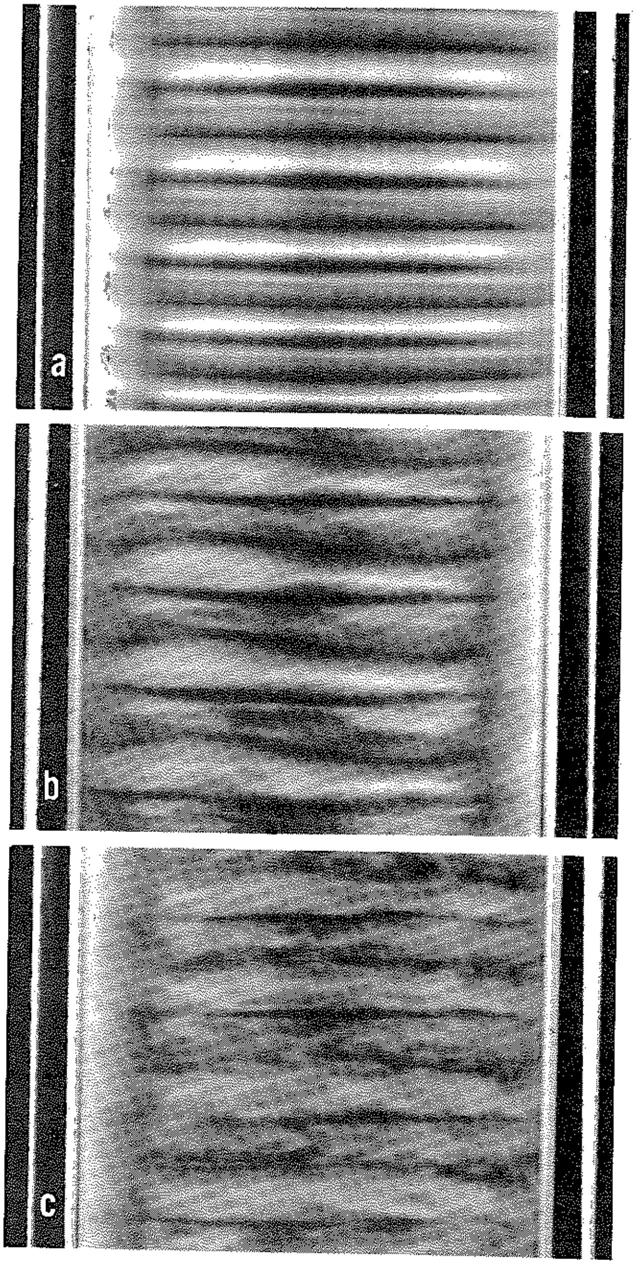
A third instability, easily seen by using a kitchen faucet, is a low-speed capillary jet of water approaching an obstacle (Fig. 3). The jet may be stable and smooth, or unstable and appear to buckle, depending upon its length. In Fig. 3 the jet is short, and capillary waves occur along its edge. This instability has been treated theoretically (Ref. 1, Article III, Ch. XII).



2. A smoke plume develops waves and then becomes turbulent.



3. A capillary jet of water shows a buckling instability.

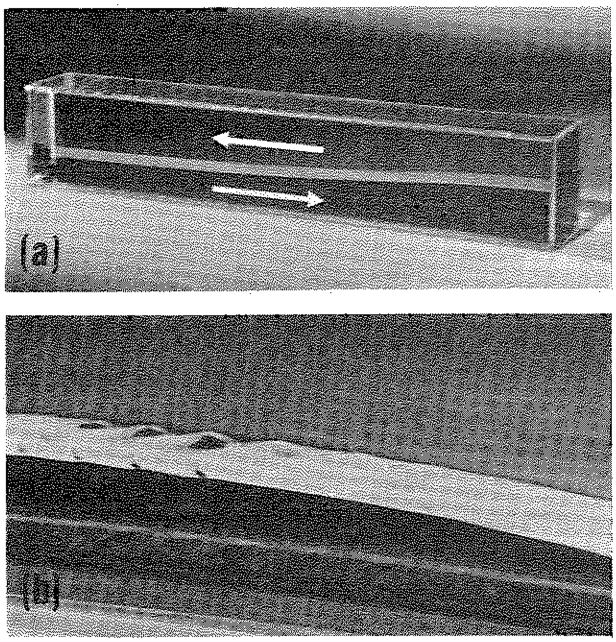


4. The flow in the circular space between two cylinders, the outer cylinder stationary, the inner rotating, is stable at low angular velocity, Ω. At higher Ω, Taylor vortices appear (a); at still higher Ω, waves develop (b); and ultimately, turbulence (c).

The Existence of a Critical Condition

The flow between concentric cylinders, produced by rotating the inner cylinder with the outer cylinder stationary (Fig. 4), has several critical speeds. At very low rotation speeds, the flow between the cylinders is a simple Couette flow and is stable. Instability, in the form of Taylor vortices (seen as bands from the side in

Fig. 4a), appears above a critical angular velocity, Ω, measured in terms of Taylor number, $\nu\,\Omega R^2/2$, where R is the radius and ν the kinematic viscosity (Ref. 2).* At a second critical Taylor number, a second instability occurs (Fig. 4b) and the Taylor vortices develop circumferential waves. At still higher Taylor numbers (Fig. 4c), transition to turbulence occurs.

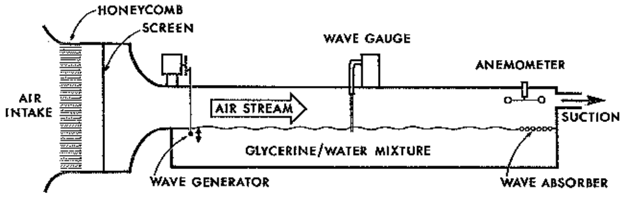


5. Two fluids of different density completely fill a tank. The shear flow created by tilting the tank (a) goes unstable and waves appear on the interface (b).

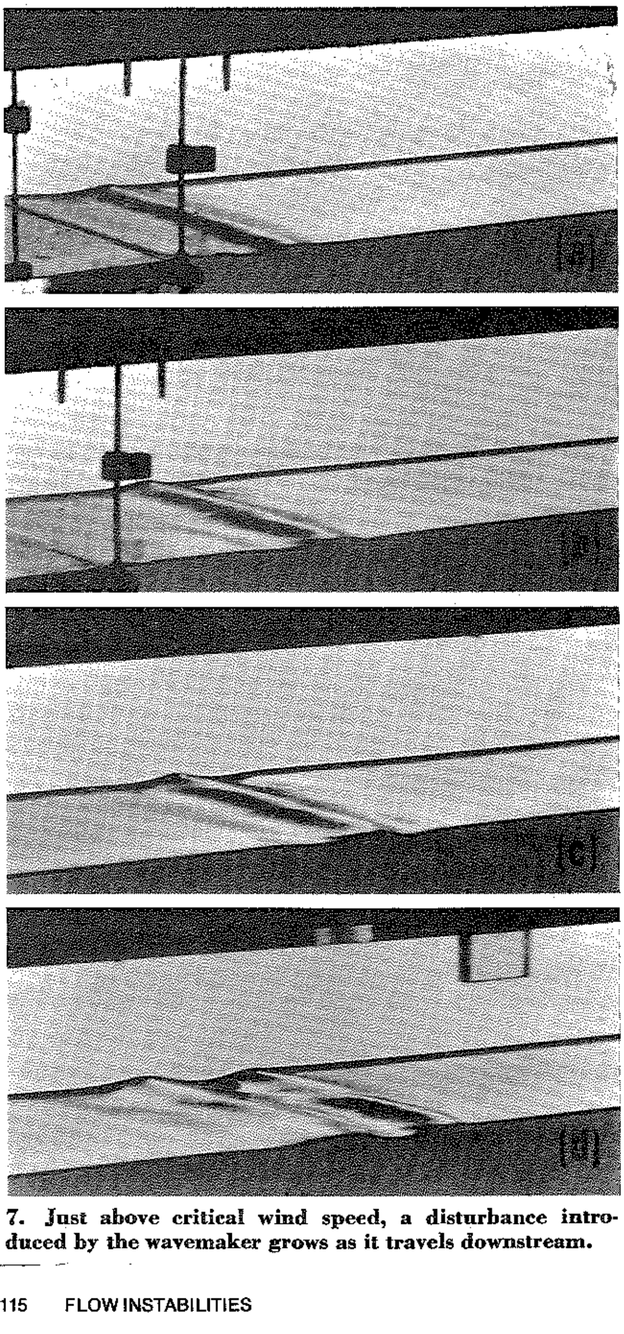
The flow of one fluid on top of another, heavier fluid can produce a shear instability. The closed rectangular tank of Fig. 5 contains two fluids, differing in density by 2 percent. The interface between the two fluids remains virtually horizontal when the tank is tilted very slowly. A sudden change in tank tilt causes internal waves. When the lower fluid flows to the right the upper fluid must flow to the left to fill up the space. (Fig. 5a) This produces a velocity shear at the interface between the two fluids. When the parameter $\rho\,\frac{(\Delta U)^2}{gl\Delta\rho}$ exceeds a critical value, the flow is unstable and small waves appear (Fig. 5b). Here l is a characteristic length, ΔU is the velocity difference between the upper and lower fluids, $\Delta\rho$ is the density difference, ρ is the average density, and g is gravity. This shear instability is often called Kelvin-Helmholtz instability. At still larger velocity shears, the waves break, resulting in turbulence. The particular fluids used in this experiment were: "Pine-X," an aliphatic hydrocarbon paint thinner, and "Doversol," a stove-fuel alcohol. The alcohol will weaken a plexiglass container in the course of less than a year.

Wind Blowing Over a Liquid Surface

The apparatus shown in Fig. 6 is used to study the generation of surface waves by wind. Air enters at the left, flows through the test section, and is sucked out by vacuum cleaners. The air speed may be varied and disturbances of different frequency may be introduced into the upstream end of the liquid surface by vertical oscillation of a horizontal rod placed across the tank. The liquid in the tank is a mixture of glycerine and



6. Wind tunnel for studying wave generation by wind.



7. Just above critical wind speed, a disturbance introduced by the wavemaker grows as it travels downstream.

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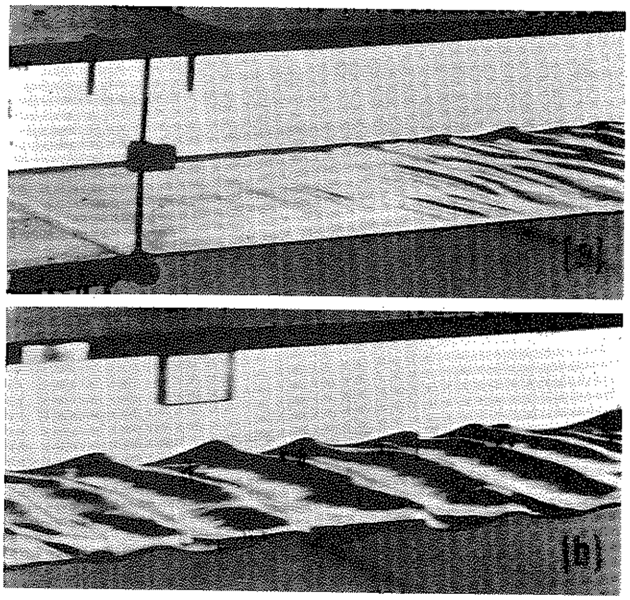
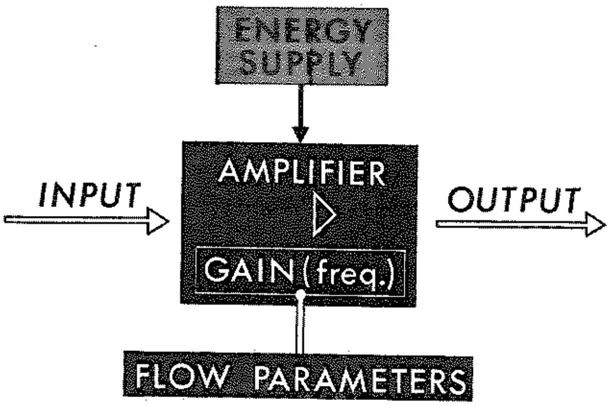
* (2) Ch. 2. A more extensive account can be found in Ref. 1, Ch. VII.

water, which causes capillary waves to be strongly damped, leaving just gravity waves unstable. At zero or low wind speeds a disturbance is damped as it travels away from the wavemaker. At wind speeds above the critical speed the same disturbances grow (Fig. 7). At air speeds well above critical speed, even with the wavemaker off, small accidental disturbances in the air stream are sufficient to make waves which grow (Fig. 8a). Downstream these waves appear quasi-regular (Fig. 8b) because the wave length having the highest growth rate ultimately dominates.

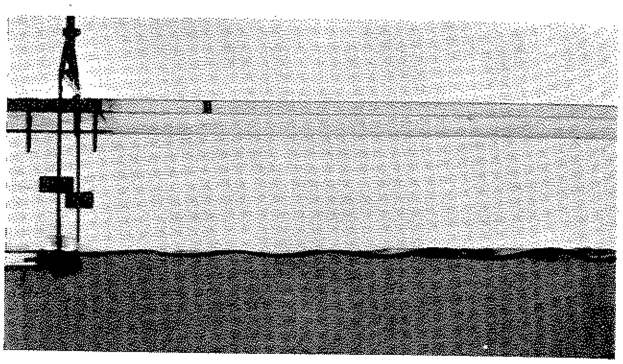


8. Well above critical wind speed, waves appear spontaneously and grow (a). A narrow range of wave lengths is predominant (b).

The instability mechanism can be pictured as an amplifier (Fig. 9). The airstream is its energy supply, and its gain and frequency characteristics depend upon wind speed and other flow parameters. The input is disturbances in the liquid or the air which may be intentional or accidental. Surface waves are the output, and they may be damped or amplified. When the amplification ratio, the gain, exceeds unity we have instability. Fig. 10 shows the wavemaker producing sinusoidal disturbances with the wind speed above critical value. The waves grow with distance from the wave generator.

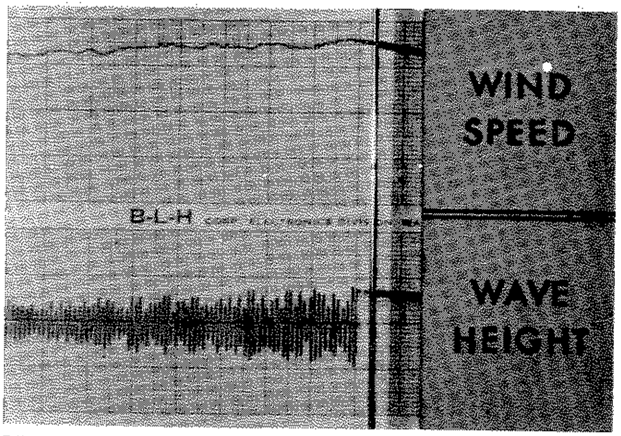


9. The mechanism of instability can be thought of as a frequency-selective amplifier.

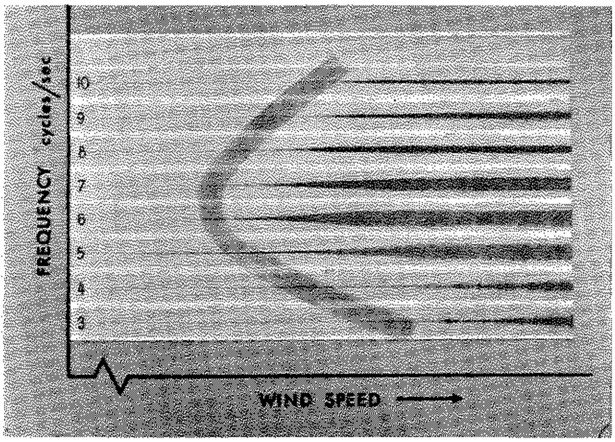


10. Wavemaker at constant frequency, wind above critical value, waves grow with distance.

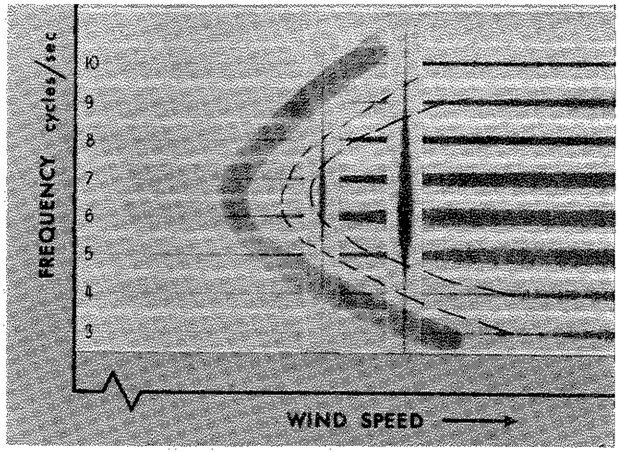
The essential features of the amplifier analogy are demonstrated in the generation of waves by wind (Ref. 3). We measure the amplifier gain as a function of the frequency of input disturbances and wind speed. In the experiment, waves of one frequency are being generated and the wind speed is increased continuously from a low value. The amplitude of the waves is measured downstream with a variable-capacity transducer. Fig. 11 shows a portion of a typical recording of downstream wave height vs. wind speed. The experiment is repeated at the same wave-generator amplitude but at different frequencies. When the recorder tracings are mounted on a chart of frequency vs. wind speed, a pattern becomes apparent (Fig. 12). The curve passing through the points where amplification first occurs is the neutral curve. To the left of this curve the waves are damped; the flow is stable. To the right of the curve, the waves are amplified and instability is present. To emphasize the fact that the amplifier has a limited band width, the experiment is performed the other way. Keeping the air speed constant, the frequency is slowly increased, starting at zero. The resulting record also fits the neutral curve



11. Recorder output.



12. **The neutral curve.**

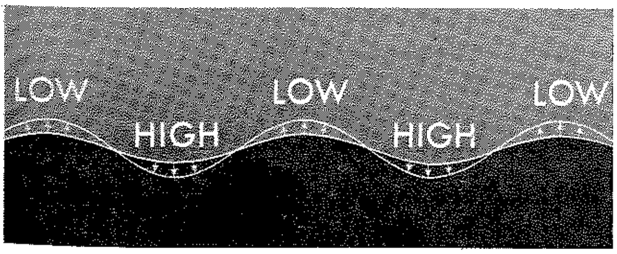


13. **Varying the frequency at constant air speed shows the amplified frequency band. Contours of constant amplification have been added.**

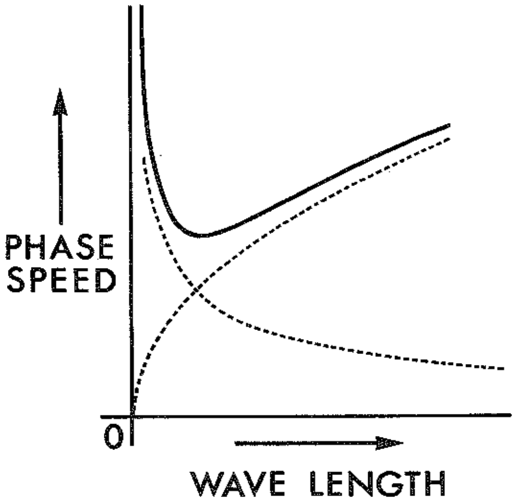
(Fig. 13). Crude contours of constant amplification rate are shown in Fig. 13. Without the wave generator, but with the wind speed above critical, accidental disturbances present will trigger the instability. The amplified waves occur first in the frequency range and at the wind speed corresponding to the nose of the neutral stability curve. A classical example of this experimental approach is given in Ref. 4.

We can develop a simplified physical explanation of this instability. The pressure pattern on the liquid surface depends on the speed of the wind relative to the waves. If the air were inviscid, pressure minima would occur at the wave crests and pressure maxima in the troughs as shown in Fig. 14. The wave speed depends on wave length, liquid depth, gravity, and surface tension. Since waves of different wave lengths have different speeds of propagation (Fig. 15), they feel different relative winds. For all except the very shortest waves, which are dominated by surface tension, the longer the wave the higher its phase speed and the less the relative wind for a given air speed. Thus, very long waves tend to be stable. Viscous damping in the liquid dominates the very shortest waves, making them stable. Therefore, only waves with an intermediate range of wave length may be unstable. The boundary layer in the air must be taken into account to obtain the final clue as to how the pressure forces do work on the water. The air velocity profile relative to moving waves is shown in Fig. 16. Across the *critical layer,* where the relative air velocity is zero, there is an exchange of vorticity as the air moves across the waves, resulting in a shift of phase of the pressure distribution (Fig. 17) so that high pressure occurs on the upwind face of the waves. This is how pressure does work on the liquid, putting energy into the waves.

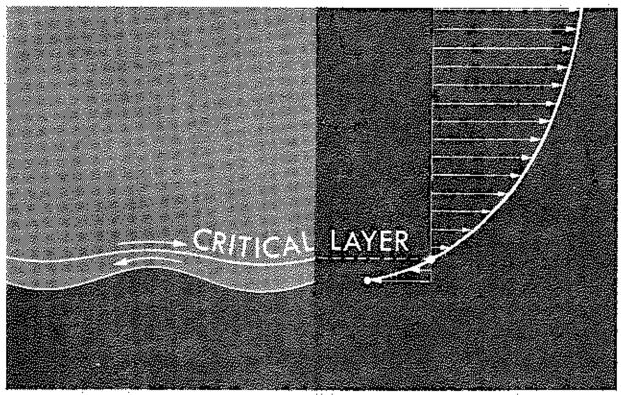
In summary, the mechanism of flow instability be-



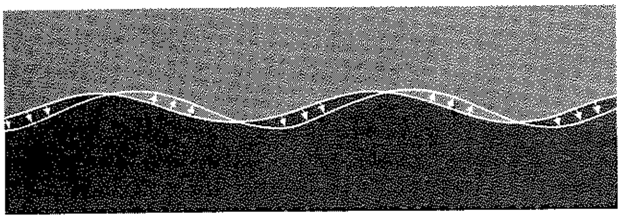
14. **Potential flow would create high air pressure at the troughs and low pressure above the crests.**



15. **The wave phase speed depends upon wave length.**



16. **Air velocity profile and the critical layer in a coordinate system moving with a wave.**

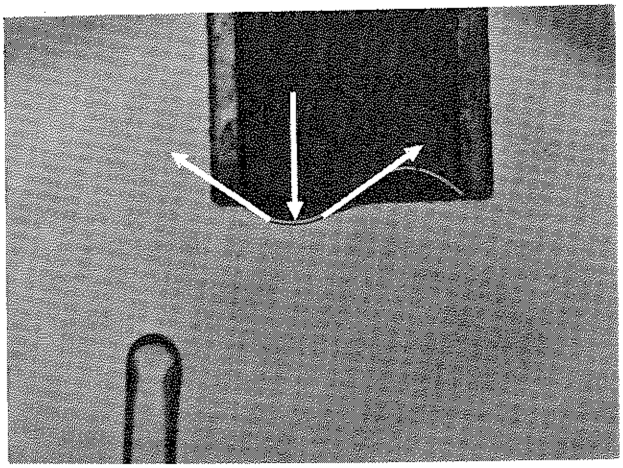


17. Approximate actual pressure distribution.

haves like a frequency-selective amplifier whose characteristics depend on flow parameters. The instability can be investigated experimentally by means of controlled disturbances, measuring the amplification as a function of flow parameters. The experiment may provide a valuable guide to an explanation of the physical mechanism underlying the instability.

A Density-Difference Surface-Tension Instability

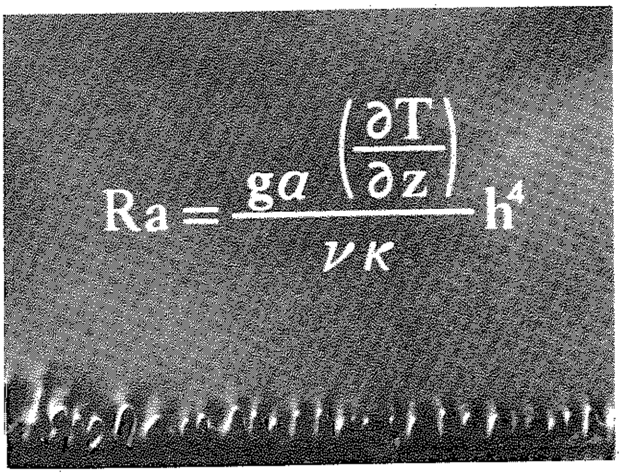
A layer of heavy fluid above a lighter fluid will tend to fall down, exchanging places with the lighter fluid, unless there are restraining effects. Surface tension may provide this restraint in a container of finite size. Fig. 18 shows a container with one fluid placed above a lighter fluid where the bottom of the container is open. If the interface is disturbed, it returns to its equilibrium position. However, if a surface-active agent is added near the interface to lower the surface tension, instability results; the heavier fluid pours down, the lighter fluid pours up. The neutral-stability case lends itself to a simple mathematical analysis. If $\eta(x)$ is the vertical position of the interface (x being horizontal distance), the pressure perturbation at the surface is $g\Delta\rho \cdot \eta(x)$ where $\Delta\rho$ = density difference, g = gravity. This pressure difference must be held in check by surface tension: $\sigma \frac{d^2\eta}{dx^2} = g\Delta\rho\eta$. Solving for $\eta(x)$ with boundary condition $\eta = 0$ at $x = 0$ and $x = l$, one finds neutral stability when $\frac{gl^2\Delta\rho}{\sigma} = (2\pi)^2$, after having imposed the constraint that the average deflection is zero $\left(\int_0^l \eta\, dx = 0\right)$.



18. Heavy fluid above lighter fluid. A surface deflection downward increases the hydrostatic pressure, but surface tension pulls the bulge back up.

A Convective Thermal Instability

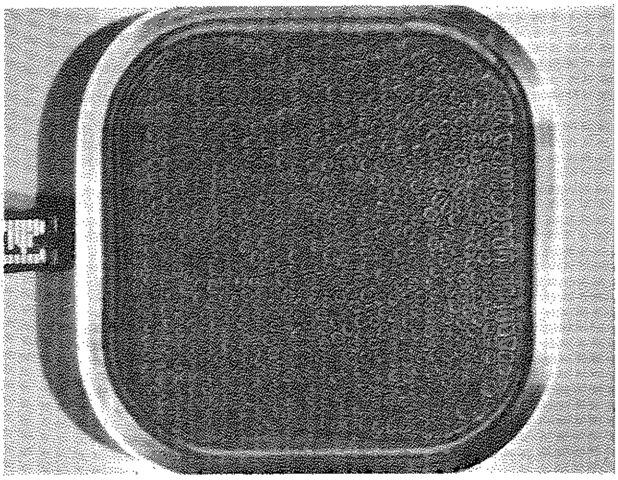
The density of most fluids decreases with increasing temperature. Heating a fluid from below will therefore create an unstable density stratification, and the lighter fluid will tend to rise, producing thermal convection. The speed with which a fluid parcel rises depends upon its size, its excess buoyancy, and the fluid viscosity. As it rises, it encounters colder fluid and loses heat, and therefore buoyancy, to its surroundings. If it rises fast enough to get to colder (and therefore denser) surroundings faster than its own rate of increase in density, it will keep on going and we have instability (Fig. 19). The critical parameter is the Rayleigh number $Ra = \frac{g\alpha\partial T/\partial z}{\kappa\nu}h^4$ where $\alpha = \frac{1}{\rho}\frac{\partial\rho}{\partial T}$, and z is distance, increasing downward. The stratification is given by $\frac{g}{\rho}\frac{\partial\rho}{\partial z} = g\alpha\partial T/\partial z$, and the larger this term is and the smaller the viscosity and the thermometric conductivity κ, the less stable the system will be. The smaller the depth, h, the more efficient are the restraining influences of viscosity and thermal conductivity. Instability occurs above a critical value of Rayleigh number. For an analytical treatment, see Ref. 1, Ch. 2.



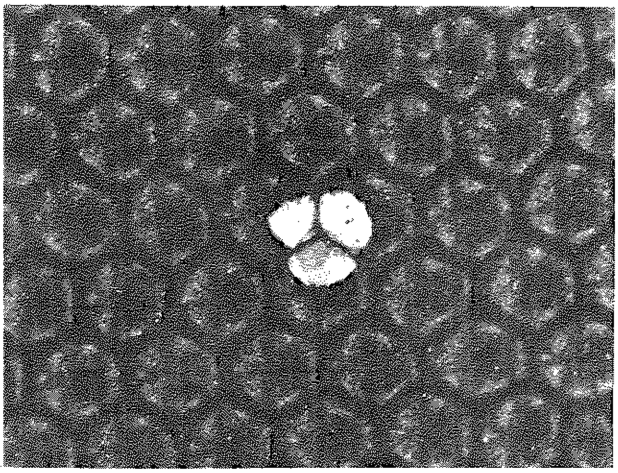
19. A thin layer of fluid confined between parallel vertical plates with a heating element near the bottom. Above a critical value of the Rayleigh number it goes unstable.

A Thermal Instability Resulting in Convection Cells

When a shallow layer of fluid is heated from below and cooled from above, the instability discussed in the previous section results in a strikingly uniform array of convection cells called Benard cells. In Fig. 20 these

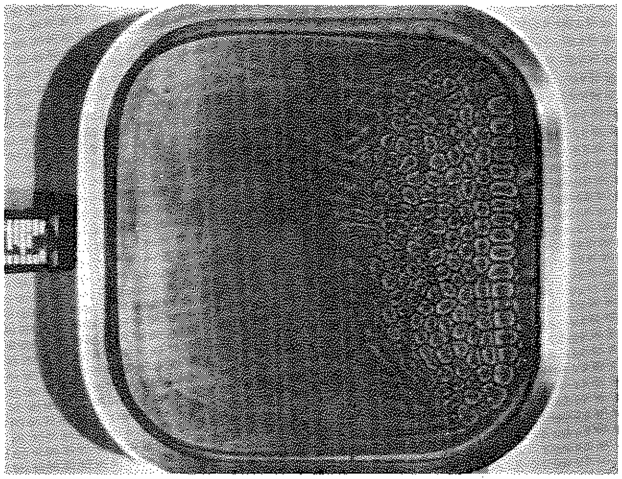


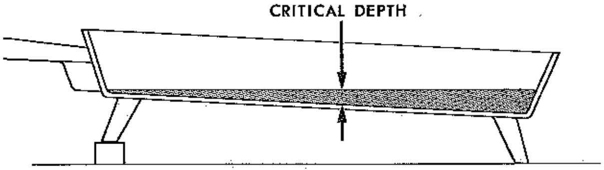
20. **Above the critical Rayleigh number, Benard cells appear in a layer of fluid.**



21. **A drop of shiny fluid put at the corner between three cells will spread along the bottom and rise to the surface, showing the location of a cell as seen from below.**

have been produced in a frying pan* with silicone oil. The flow is made visible with aluminum flakes and iron oxide. Each hexagonal cell is made up of six equilateral triangles (Fig. 21). The cell pattern as seen from below also consists of hexagons, but these are made up of two triangles from each of three adjacent hexagons as seen from above. Since the Rayleigh number, Ra $= \frac{g\alpha\Delta T h^3}{\kappa\nu}$, varies as the third power of the depth, h, the critical condition for instability is clearly demonstrated by tilting the frying pan (Fig. 22). No cells are visible until the depth reaches the critical value. At depths greater than critical, the cell size increases roughly proportional to depth. Cellular convection patterns may also be seen occasionally in the atmosphere where clouds may make them visible.

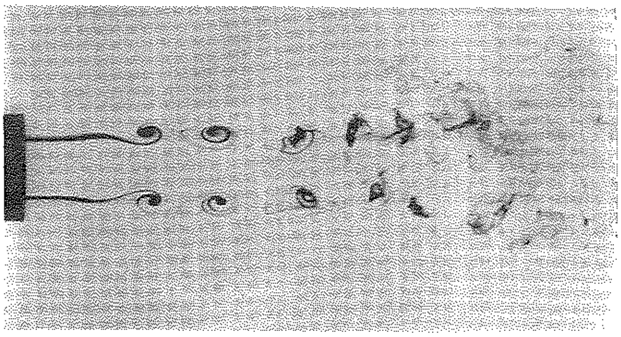




22. **Varying the Rayleigh number by tilting the pan. The Rayleigh number is below critical on the left and supercritical on the right.**

Shear Instability Without Stratification

Shear flows are especially unstable, without stabilizing density stratification. A jet of water into water (Fig. 23) creates shear layers which become unstable with a characteristic "most unstable" wave length.



23. **The shear layer surrounds a circular jet of water into water. It is unstable above a critical Reynolds number. The dye streamers are introduced at the top and bottom of the jet.**

Non-linear effects and secondary instabilities appear which finally result in turbulence. The critical parameter for the initial instability is the Reynolds number based on shear velocity difference and shear layer thickness.

The Karman Vortex Street

A familiar example of flow instability is the flow around a cylinder. Below a critical value of Reynolds

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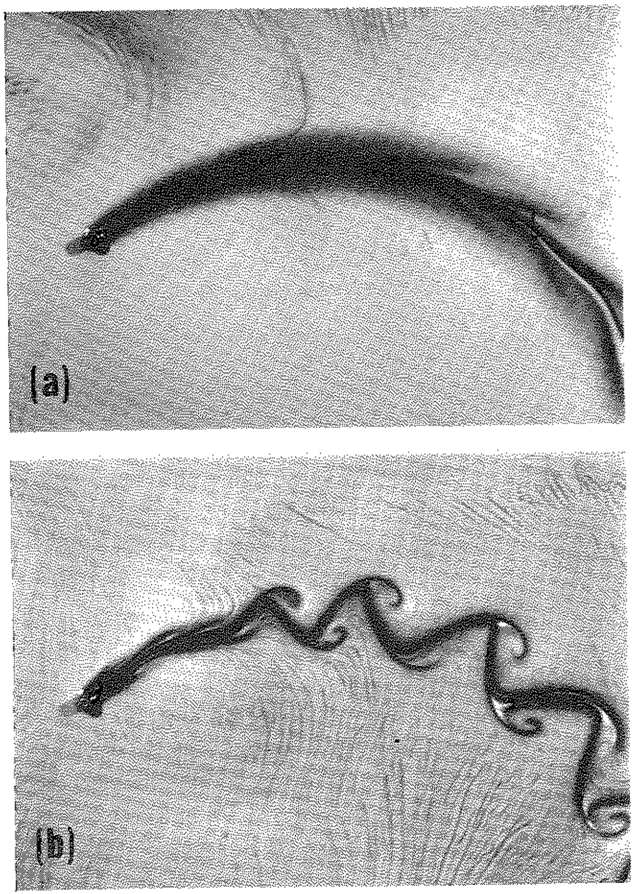
* Modified as follows: a layer of "cerrobend," a dense, low-melting-point (73°C) alloy was melted in the bottom of the pan. A flat aluminum plate was floated on the alloy and the alloy was allowed to solidify. This produced a level bottom surface at uniform temperature.



24. Flow around a cylinder. (a) Steady, stable flow with a slight wake instability. (b) Unstable flow.

number, $\mathrm{Re} = \frac{Ud}{\nu}$, the flow is steady (Fig. 24a), although there is a weak instability in the wake. Above the critical Reynolds number (Fig. 24b), the steady flow around the cylinder becomes unstable, resulting in a periodic Karman vortex street in the wake. Just at critical Reynolds number the flow stability is precarious, and a small disturbance will cause the flow to become unstable. This instability causes telephone wires to "sing." The sound comes from small pressure fluctuations associated with the periodic vortex shedding.

Summary

Steady flows may become unsteady for certain ranges of the relevant flow parameters. The instability takes its energy from the mean flow or externally supplied heat and the mechanism can be thought of as a frequency selective amplifier whose characteristics are governed by flow parameters.

One can observe flow instabilities in the atmosphere both on the large and on the small scale, in smoke from chimneys, in rivers, in flickering flames, and in fact almost everywhere, along with their ultimate consequences: turbulence or random waves.

References

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