National Committee for Fluid Mechanics Films

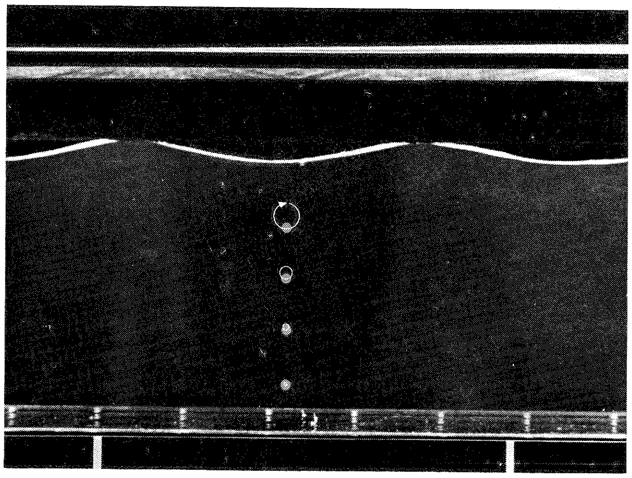
FILM NOTES
for
WAVES IN FLUIDS*

By
ARTHUR E. BRYSON
Harvard University

Introduction

Waves occur all around us. Most common, perhaps, are gravity waves on water and compressibility waves in air. Because waves in water are easy to observe, we will begin with them. Later we will see that many of the concepts we have developed with these gravity waves apply equally well to compressibility waves in gases.

A surface gravity wave is a region of increased or decreased depth that moves relative to the fluid. A large-amplitude wave is one that produces a change in depth large compared to mean depth as it goes by. When the change in depth is small compared to both mean depth and wave length, we call it a small-amplitude wave. Even when we cannot see the surface, we can detect the passage of a wave by the change in pressure at a fixed point underneath the surface. We can also detect it by the motion of the fluid particles. One way to see the motion of fluid particles is to introduce neutrally buoyant solid particles into water as shown in Fig. 1. The mean location of these solid particles is stabilized by using slightly buoyant particles (specific gravity approximately .999) and attaching a thread to them which extends to the bottom where a short length of miniature chain is attached; the particles lift one or two links of the fine chain and become neutrally buoyant.



1. Circular particle paths typical of periodic waves where wave length is comparable to or less than depth ("deep water waves").

Deep Water Waves

Superposed on Fig. 1 are the paths of the particles at various depths in a horizontal water channel when

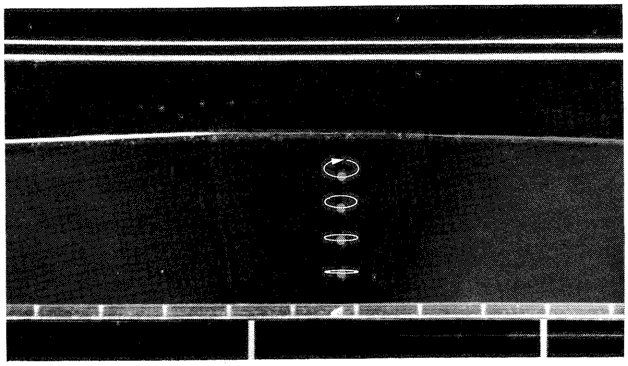
* *WAVES IN FLUIDS, a 16-mm B&W sound film 33 minutes in length, was produced by Educational Services Incorporated under the direction of the National Committee for Fluid Mechanics Films, with a grant from the National Science Foundation. Information on purchase and rental may be obtained from the distributor:*

Encyclopaedia Britannica Educational Corporation
425 North Michigan Avenue, Chicago, Illinois 60611
151 Bloor Street West, Toronto 5, Ontario

periodic waves of small amplitude go by. Here the wave length is comparable to the depth. In these waves the particles move in approximately circular paths. The diameter of the circles decreases with depth. The particles near the bottom hardly move at all. This is the pattern of fluid movement characteristic of waves on the surface of deep water, that is, where the wave length is comparable to or less than the depth.

Shallow Water Waves

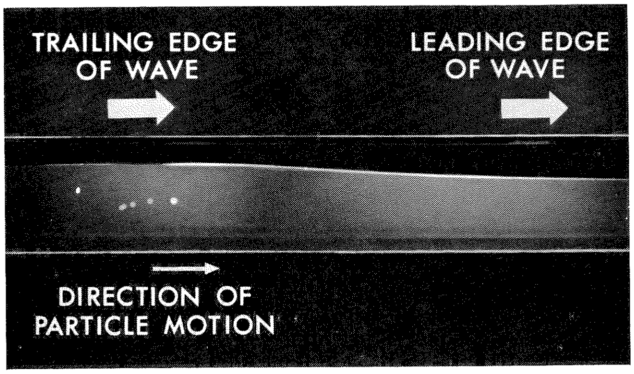
Shallow water waves are waves with wave length long compared to the depth. Fig. 2 shows the characteristic pattern of particle motion for shallow water waves. Notice that the horizontal amplitude of the particle motions is nearly the same at all depths. We say that such waves are essentially one-dimensional, since the horizontal fluid motion depends only on time and horizontal distance from a reference point. Because they are simpler and yet display most of the important features of wave motion, we will deal with one-dimensional waves hereafter.



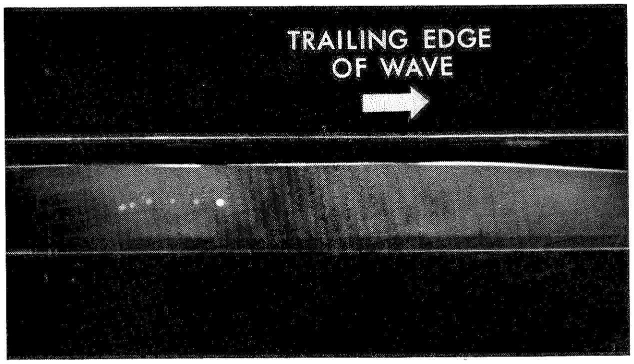
2. Elliptical particle paths typical of periodic waves where wave length is greater than depth ("shallow water waves").

Step Waves

Not all waves are periodic. The upper photograph in Fig. 3 shows a single progressive, depth-increasing wave moving to the right in the horizontal water channel. The wave was generated by moving a "piston" at nearly constant velocity at the left end of the channel. The two arrows show the leading and trailing edges of the wave. Ahead of the wave (to the right), the fluid is at rest. Superposed on the photograph are the positions of a particle at equal time increments after passage of the step wave. The particle moves through a much shorter distance than the wave. Since the particle positions were recorded at equal time intervals, it is clear from their spacing that the wave is a region of acceleration. As the step wave continues to the right, particles behind it stop accelerating and move at a constant speed as shown in the lower photograph. Fluid acceleration occurs only during the time the depth is changing; i.e., *waves are accelerators.*

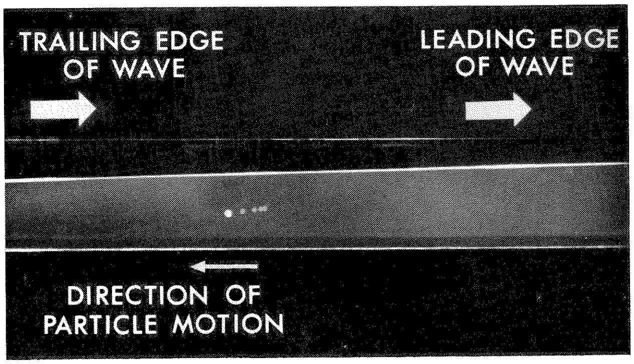


3. Particle position at equal time increments during (above) and after (below) passage of a step wave that increases depth ("compression wave"). Note forward acceleration of particle during wave passage and constant forward velocity of particle after wave passage.



The depth in a given length of the channel can only increase if more fluid is moving into it than is moving out (a simple statement of the conservation of matter). This is apparent in the upper photograph of Fig. 3, where the fluid is stationary at the leading edge of the wave and moving to the right at the trailing edge.

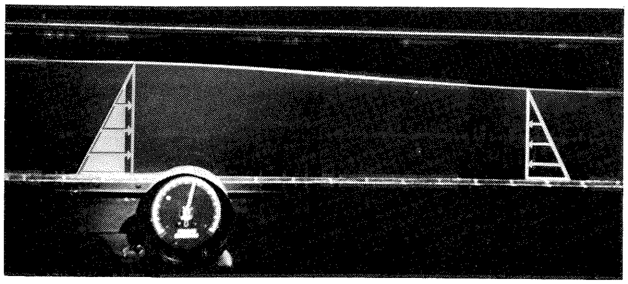
Fig. 4 shows a depth-decreasing wave moving to the right. It was generated by "pulling a gate," allowing fluid to flow toward a shallower region at the left. More fluid is flowing out of the left side of a given length of the channel than is flowing in on the right side, causing the depth to decrease. The decrease



4. Particle position at equal time increments during passage of a step wave that decreases depth ("expansion wave"). Note backward acceleration of particles during passage.

in depth is accompanied by an acceleration of the fluid toward the left, i.e., in the *opposite* direction of the wave motion, as the multiple exposure in Fig. 4 shows. As the step wave moves on to the right, the fluid particles behind it continue to move to the left with constant speed.

The force that causes the fluid to accelerate is a force arising from differences in pressure. The pressure at the bottom of the channel increases as the wave of Fig. 3 goes by. For shallow water waves the vertical acceleration of the fluid is small compared with the gravitational acceleration. Thus, the vertical pressure variation is nearly hydrostatic, that is, the pressure is atmospheric at the surface and increases almost linearly with depth. The pressure difference in a horizontal direction is nearly the same at all depths, as shown in Fig. 5, where the pressure distributions some distance apart are superposed. This means that the horizontal acceleration of fluid particles is nearly the same at all depths. As the wave of Fig. 3 passes, the pressure on a fluid particle increases. For this reason, we may call this type of depth-increasing wave a *compression wave.* Note that in a compression wave the fluid acceleration is in the direction of the wave motion.

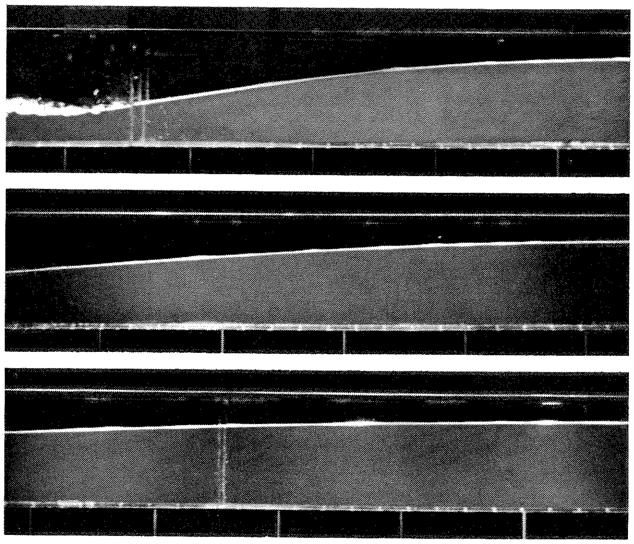


5. Pressure distribution at leading and trailing edges of a compression wave (superposed at left). Note that difference in pressures is constant at all depths.

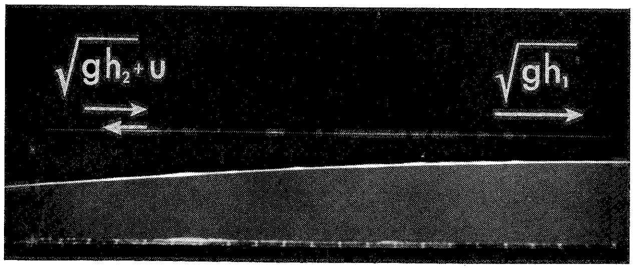
In the wave of Fig. 4 the fluid acceleration is in a direction opposite to the wave motion. This is so because the pressure is higher at the leading edge of the wave than at the trailing edge. As this depth-decreasing wave passes, the pressure on the fluid particle decreases; hence, we may call this type of wave an *expansion wave,* in contrast to the compression wave. In both cases, the horizontal pressure gradient arises because of the gravitational force acting on the fluid. Thus it is really gravity that causes these waves. Whenever the fluid surface is not horizontal, a horizontal pressure gradient is produced by gravity that accelerates the fluid horizontally toward the shallower depth.

Expansion Waves Flatten Out

As it advances, the expansion wave flattens out, the trailing edge falling behind the leading edge as shown in Fig. 6. Differences in local fluid velocity and depth within the wave itself alter its shape as it advances. There are two reasons for this. Where the depth is greater, that portion of the wave travels faster, since the wave speed relative to the fluid is given by $\sqrt{gh}$ where h is the depth. The velocity of the leading edge of the expansion wave relative to the fluid is $\sqrt{gh_1}$, where h_1 is the depth at the leading edge. Similarly, the velocity of the trailing edge relative to the fluid is $\sqrt{gh_2}$, where h_2 is the depth at the trailing edge. Clearly the local wave speed at the shallower trailing edge is less than the local wave speed at the deeper leading edge. This is one reason why an expansion wave tends to spread out as it moves.

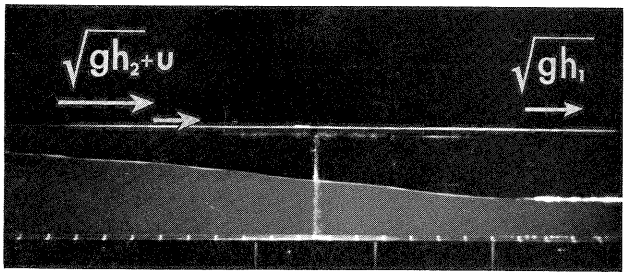


6. Camera moving with an expansion wave shows how it "flattens out" as it moves.



7. Speeds of leading and trailing edges of an expansion wave. Rearward drift speed at trailing edge further decreases trailing edge speed relative to leading edge.

But there is another factor at work here. The local wave velocity given above is relative to the local fluid velocity. Now we have already seen that the fluid at the trailing edge of an expansion wave is moving away from the fluid at the leading edge with a velocity u (Fig. 4). So, relative to the leading edge, the slower speed of the trailing edge is rendered even slower by virtue of a local fluid velocity there in the opposite direction. This is shown in Fig. 7.



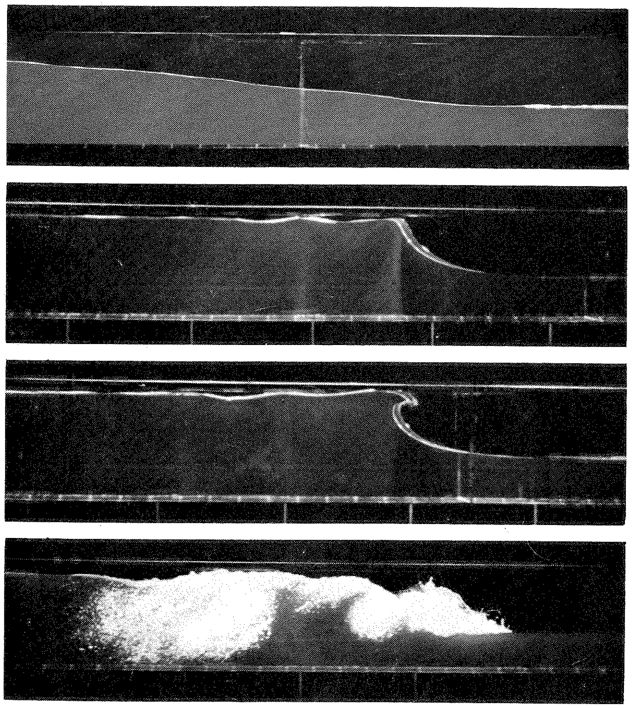
8. Speeds of leading and trailing edges of a compression wave. Forward drift speed at trailing edge further increases trailing edge speed relative to leading edge.

Compression Waves Steepen

The fact that the local wave velocity increases with depth also explains why the compression wave tends to steepen. The local wave velocity is higher at the deeper trailing edge of the wave than at the shallower leading edge. Furthermore, in compression waves the fluid at the trailing edge is moving toward the leading edge. This local fluid speed, *u,* adds to the increased local wave speed, $\sqrt{gh_2}$, at the trailing edge, causing the trailing edge to overtake the leading edge even more quickly. This is shown in Fig 8.

The Breaking of a Compression Wave

What happens when a compression wave becomes so steep that its slope is vertical? Anyone who has been to the beach knows the answer: the wave topples over as shown in Fig. 9. The leading edge of the toppled wave travels faster than it did before it toppled. Furthermore, the toppled wave continues in its turbulent condition, does not change its shape, and travels at constant speed.



9. Camera moving with compression wave shows how it steepens and topples over to form a positive surge wave.

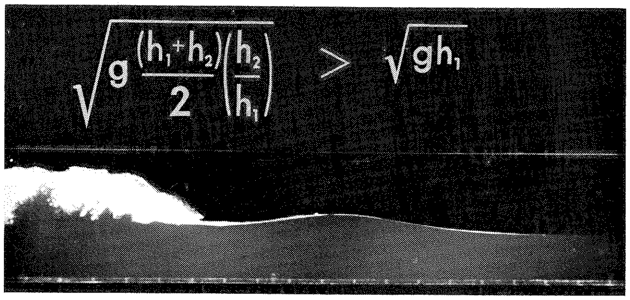
The Positive Surge Wave

The speed of such a toppled wave, which is called a *positive surge wave,* depends on both the depth ahead (h_1) and the depth behind the wave (h_2).

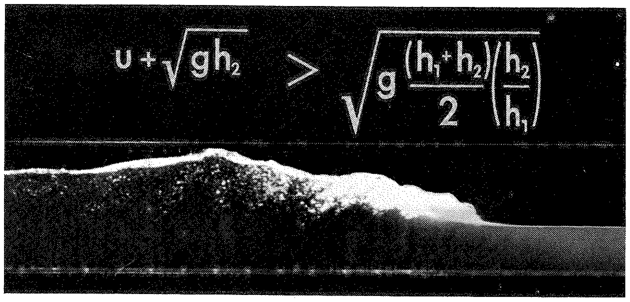
The larger the depth behind the wave, the faster the wave travels. In fact, it can be shown that the speed of a positive surge wave, V_s, is given by:

$$V_s = \sqrt{g\frac{(h_1+h_2)}{2}\left(\frac{h_2}{h_1}\right)}$$

Small-amplitude waves ahead of the surge wave move at the speed, $\sqrt{gh_1}$, relative to the fluid. If you compare these latter two expressions, you will see that this local wave speed is always less than the speed of the surge wave. Thus the surge wave catches up to small-amplitude waves ahead of it as shown in Fig. 10. Small-amplitude waves behind the surge wave move relative to the fluid at a speed, $\sqrt{gh_2}$. However, behind the surge wave the fluid is moving; there is a drift speed of the fluid $u = V_s\left(1-\frac{h_1}{h_2}\right)$. The drift speed plus the local wave speed, $\sqrt{gh_2}$, is always greater than the speed of the surge wave (Fig. 11). Thus, small-amplitude waves behind the surge wave, and moving in the same direction as the surge wave, always catch up to it.



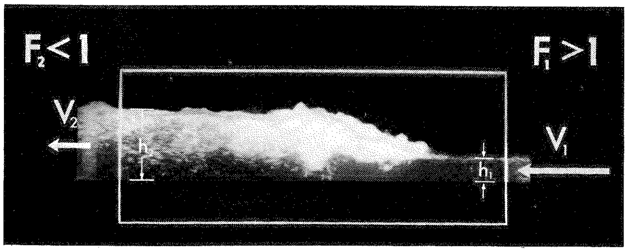
10. Positive surge wave overtaking a small-amplitude wave.



11. Small-amplitude wave overtaking a positive surge wave.

The Hydraulic Jump

As the positive surge wave moves down the channel, there is zero velocity ahead of it and a drift velocity behind it. In a reference frame moving with the surge, the fluid ahead appears to be moving into the wave front. The wave appears stationary and the fluid behind has a net velocity in the opposite direction to the drift speed. The flow is steady in this reference frame.



12. Control surface for analyzing conditions upstream and downstream of a hydraulic jump. Froude number ahead, F_1, is greater than one; Froude number behind, F_2, is less than one.

Exactly this situation occurs in a flume, as shown in Fig. 12. This is a stationary wave called a *hydraulic jump*. The structure of this turbulent wave is very complicated and difficult to analyze. However, if we consider the fluxes of mass and momentum into and out of a control surface, like the one in Fig. 12, the velocity V_2 and depth h_2 behind the hydraulic jump can be determined in terms of the velocity V_1 and depth h_1 ahead of it:

$$V_2 h_2 = V_1 h_1 \qquad \text{(conservation of mass)}$$

$$\frac{1}{2}\rho g h_2^2 + \rho V_2^2 h_2 = \frac{1}{2}\rho g h_1^2 + \rho V_1^2 h_1 \quad \text{(conservation of momentum)}$$

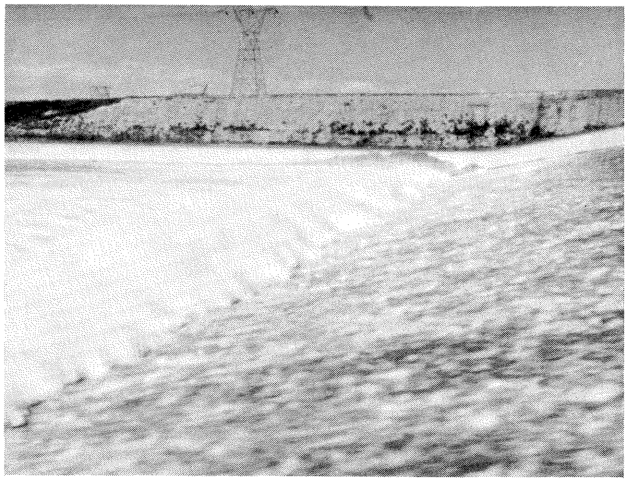
Solving these two relations for V_2 and h_2 gives:

$$\left.\begin{aligned} \frac{h_2}{h_1} &= \frac{\sqrt{1+8F_1^2}-1}{2} \\ \frac{V_2}{V_1} &= \frac{2}{\sqrt{1+8F_1^2}-1} \end{aligned}\right\} \quad \text{where } F_1 = \frac{V_1}{\sqrt{gh_1}}$$

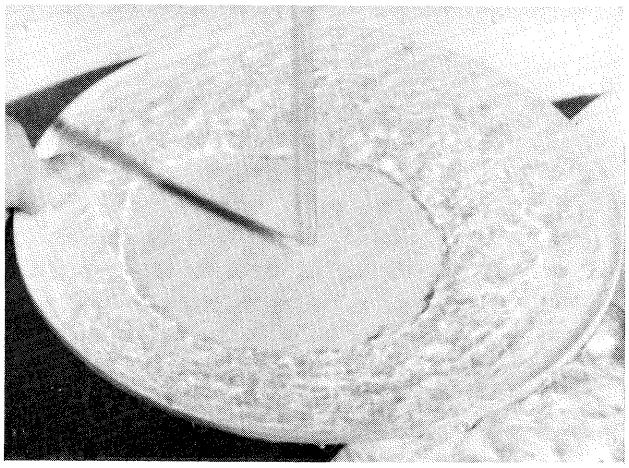
The ratio of entering fluid speed V_1 to the local wave speed $\sqrt{gh_1}$ is called the Froude number ahead of the jump, F_1. It is always greater than unity, that is, the fluid speed entering the jump is greater than the local wave speed. We say the flow ahead is supercritical (or "shooting"). The Froude number behind the jump, F_2, is always less than unity — that is, the fluid speed behind is subcritical (or "tranquil"). This is easily demonstrated in a flume. A disturbance behind the jump moves upstream into the jump. A disturbance ahead is washed downstream into the jump. This is just another way of describing the effects shown in Figs. 10 and 11 in a different reference frame. Small-amplitude waves behind a positive surge wave catch up to it. Small-amplitude waves ahead are overtaken.

Dissipation of Energy in a Hydraulic Jump

Energy is dissipated in a hydraulic jump. Traversing a pitot tube downstream through the jump shows that the total head decreases. Mechanical energy is converted into internal energy. The jump in Fig. 12 is dissipating about one-third of a horsepower. Below big dams like the one in Fig. 13, hydraulic jumps may dissipate thousands, even millions, of horsepower. The temperature rise is hardly noticeable, however, since a total head loss of 778 feet is required to heat water one degree Fahrenheit.



13. Large hydraulic jump below a dam (U.S. Army Corps of Engineers).



14. Circular hydraulic jump on a plate below a kitchen water tap.

It is straightforward to show that the loss in total head is given by

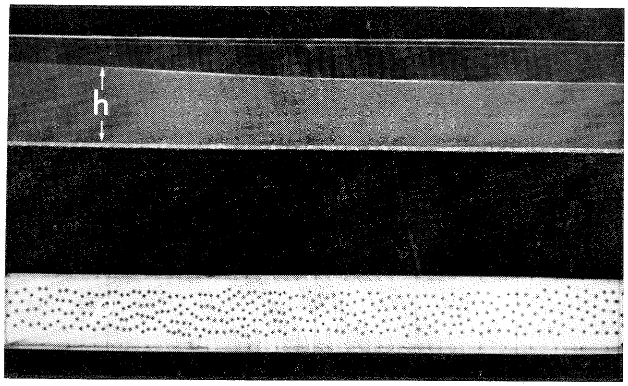
$$H_1 - H_2 = \frac{(h_2 - h_1)^3}{4h_1 h_2}$$

where H_1, H_2 represent total head ahead of and behind the jump, respectively.

You can see hydraulic jumps all around you. Fig. 14 shows one you can see in your kitchen sink. The water spreads out at a supercritical speed from under the tap, goes through a cylindrical hydraulic jump, and moves at subcritical speed to the edge of the dish.

Sound Waves and Shock Waves

Although gravity waves in liquids are easily seen, they are not the only common waves we encounter in fluids. Compressibility waves in gases are also very common; for instance, sound waves are small-amplitude compressibility waves. Large-amplitude compressibility waves are *shock waves.* Compressibility waves move at very high speeds; furthermore, they can usually be seen only with the aid of special optical devices (such as schlieren and shadowgraph systems). On rare occasions, if the light is just right, shock waves may be seen with the unaided eye. However, the passage of a compressibility wave is easily detected by the changes in pressure associated with it; our ears, for instance, are very sensitive detectors of pressure changes.



15. Analogy between depth *h*, in a surface gravity wave and particle density, ρ, in a compressibility wave.

As a model for a gas, the bottom half of Fig. 15 shows many particles floating on the surface of water. The number of particles per unit of surface area represents the density. As a compression wave comes along, the particles accelerate in the direction of the wave motion, and the density increases — that is, the particles move closer together (see Fig. 15).

Much of what we have discussed concerning one-dimensional surface gravity waves applies equally well to compressibility waves if we simply replace the depth of the fluid, *h,* by the density of the fluid, ρ, as shown in Fig. 15. In fact, a compressibility wave may be described as a region of increased or decreased density that moves relative to the fluid. Just as depth is proportional to the amount of fluid per unit length in an open channel, so density is proportional to the amount of fluid per unit length in a closed channel. The net flow into a length of open channel increases the depth. The net flow into a length of closed channel increases the density. In a compressibility wave, the pressure depends on the density, just as, in a gravity wave, it depends on the depth.

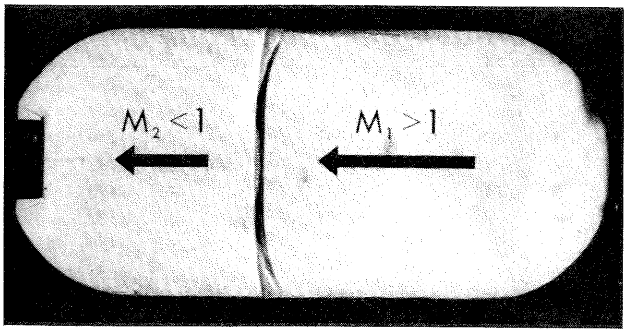
Although surface gravity waves and compressibility waves behave quite similarly, they are propagated by entirely different physical mechanisms. The fluid acceleration in waves in gases is due to elastic forces associated with the compressibility of the fluid rather than to gravitational forces.

Steepening of Sound Waves into Shock Waves

Recall that the local wave speed of a surface gravity wave increases with depth; the speed of a small-amplitude isentropic compressibility wave increases with density. This increase of sound speed with density causes compression waves to steepen. When we say that a compression wave steepens, we mean that the density and velocity gradients in the wave increase as the wave propagates. What happens when these gradients become very high, that is, when the wave becomes very thin? The compressibility wave cannot topple over like the surface gravity wave; instead, the density and velocity gradients reach limiting high values in a thin region and then remain constant. In compressibility waves, these limiting high gradients are determined by the effects of viscosity and heat conduction which, up to this point, we have been able to ignore. The reason that viscous forces become significant here is that fluid particles going through a steep wave experience high rates of deformation and high temperature gradients; viscous forces and heat conduction thus become enormously large. The steepening tendency of inertia is therefore opposed by the diffusive actions of viscosity and heat conduction; the steep wave of stationary form that results is called a shock wave.

Standing Shock Waves

Standing shock waves can be produced in a supersonic wind tunnel (Fig. 16). The gas moves at supersonic speed ahead of the shock (Mach number M_1 greater than one) and at subsonic speed behind the shock (Mach number M_2 less than one). This is analogous to the stationary hydraulic jump in the flume (Fig. 12) with fluid moving at supercritical speed ahead of the jump and with subcritical speed behind it. Recall that the total pressure (total head) dropped in going through a hydraulic jump. In the same way, total pressure drops in going through a shock wave because some of the mechanical energy is changed into internal energy by the dissipative action



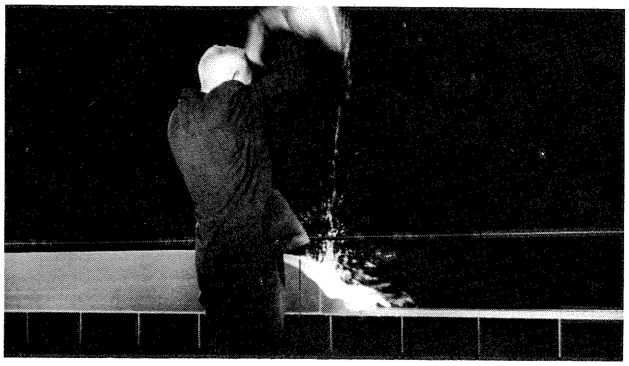
16. Stationary normal shock wave in a supersonic wind tunnel. Mach number ahead, M_1, is greater than one; Mach number behind, M_2, is less than one.

of viscosity and heat conduction. Unlike the hydraulic jump, a shock wave is extremely thin, on the order of a few mean free paths thick, about one-millionth of an inch at room temperature and pressure.

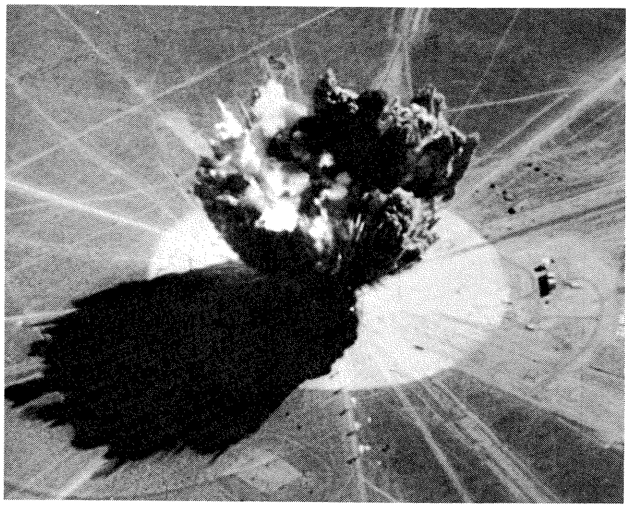
Moving Shock Waves

A moving shock wave is what you hear when there is an explosion; it is the thunder that accompanies lightning; it is the sonic boom you hear from a supersonic airplane. We can produce moving shock waves in the laboratory in a *shock tube,* like the one shown in Fig. 17. We put high-pressure gas on one side of a frangible diaphragm, and low-pressure gas on the other side. When the diaphragm is broken, a shock wave is created which moves at supersonic speed down the tube. By pulling a gate in the hydraulic channel, we produce a positive surge wave, as shown in Fig. 18; this is analogous to breaking the diaphragm to produce a shock wave in the tube. There is a wind behind a shock wave, just like the drift speed behind a positive surge wave. An explosion sends out a spherical shock wave followed by a strong wind, which is the air moving at the drift speed (Fig. 19).

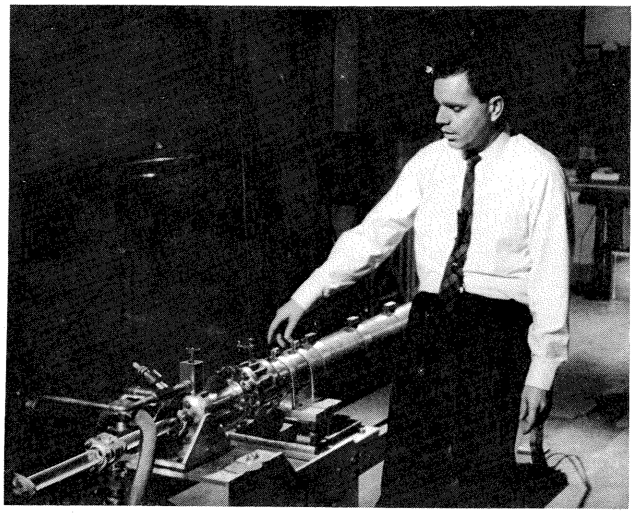
17. Shock tube for producing moving normal shock waves in the laboratory.



18. Surge wave produced by pulling a gate in a water channel.

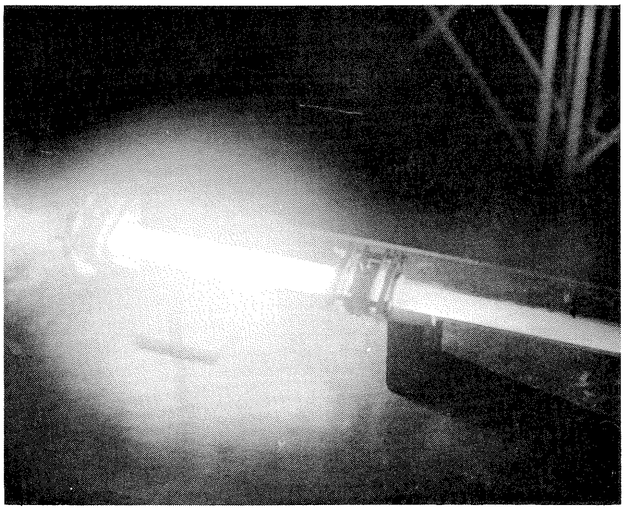


19. Circular trace on ground of a spherical shock wave produced by an explosion.



20. Shock tube for producing very strong normal shock waves.

We can produce very strong shock waves in the shock tube shown in Fig. 20. The high-pressure section is made of steel and the low-pressure section is made of pyrex tube. The diaphragm in the tube of Fig. 20 is

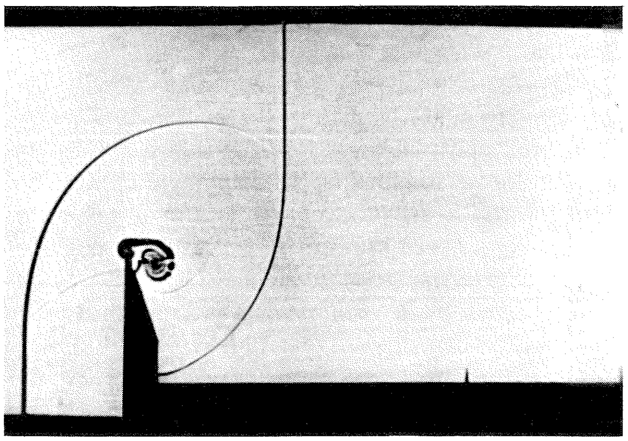


21. Ionization flash produced by passing strong shock wave through argon in a pyrex tube.

made of .050″ thick copper, because it must withstand very strong pressure forces. The temperature behind a very strong shock wave can become so high that the gas molecules are torn apart (dissociation), and even electrons stripped off the atoms (ionization). Ionized gases emit light, and hence this phenomenon is easily seen. Fig. 21 shows how the argon in the low-pressure section of the tube ionizes when a strong shock passes through it.

Diffraction of Shock Waves Over Obstacles

So far we have concentrated on one-dimensional waves; two and three-dimensional waves involve the same basic concepts but can become very much more complicated. Fig. 22 shows the shock pattern shortly after passage of a normal shock wave (moving toward right) over a sharp-edged vertical plate. Notice the vortices produced at the sharp edges, and the shock reflections and interactions.



22. Shock pattern shortly after passage of normal shock wave (moving toward right) over sharp-edge vertical plate.

Summary

There are two main requirements to produce wave motion in fluids. First, it must be possible to have a *local accumulation of mass,* which is due to depth changes in the gravity wave and to density changes in the compressibility wave. Second, there must be *restoring forces,* pressure forces, which are due to gravity and depth gradient in the gravity wave and to density gradient in the compressibility wave.

References

1. Prandtl, L. *Essentials of Fluid Dynamics,* Hafner, New York, 1952, pp. 86-98, 271-277
2. Rouse, H. *Fluid Mechanics for Hydraulic Engineers,* Dover, New York, 1961, Chs. XIII, XVI
3. Liepmann, H. W., and Roshko, A. *Elements of Gasdynamics,* Wiley, New York, 1957, pp. 57-84, 329-331
4. Shapiro, A. H. *The Dynamics & Thermodynamics of Compressible Fluid Flow,* Ronald Press, New York, 1953, pp. 112-158 of Vol. I