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FILM NOTES for SECONDARY FLOW*

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Introduction

A number of different types of flow have been called "secondary flow." We shall deal here only with the most common type, that which occurs when fluid is made to follow a curved path.

A classic example is the teacup experiment. If one stirs one's tea so as to give it a generally circular motion, the tea leaves invariably gather at the center of the bottom of the cup (Fig. 1).

Clearly, tea leaves are denser than the liquid, since they are found at the bottom of the cup. There will thus be a tendency for the leaves to be centrifugally separated from the liquid, and one might expect to find them at the perimeter rather than at the center of the bottom. Some opposing phenomenon must cause them to seek the center.

1. Tea leaves at the bottom of a cup gather at the center.

A Model of the Teacup Experiment

In order to improve understanding of the phenomenon, a model, larger than the teacup, was made (Fig. 2). So that the fluid inside the circular cylinder could be viewed tangentially with little optical distortion, a transparent square box was placed around the cylinder and the space between was filled with water. The cylindrical container was rotated about its axis at constant velocity for a long time in order to insure that initially, at least, the fluid motion would be simple rotation about the axis. In order to simulate the motion of the fluid in the teacup, we then stopped the rotation of the container. Except near the side wall and the bottom, the fluid continued to rotate as before. At these surfaces the velocity must be zero and near them the fluid is slowed by friction.

In developing a description of the actual flow, we

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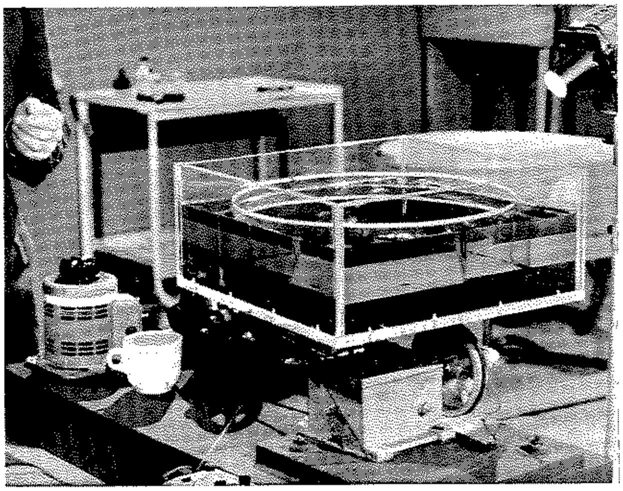
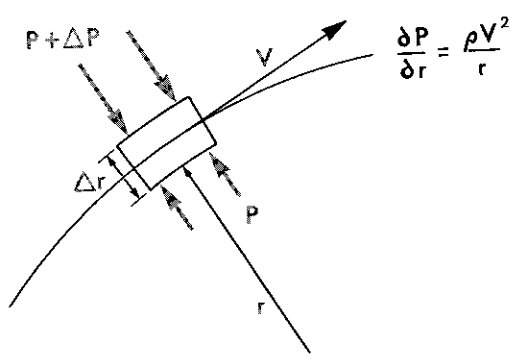


2. Apparatus modeling the teacup experiment.

first postulate a flow which we call "primary," assuming that each fluid element moves in a circle around the center of the container but that the elements near the bottom move more slowly because of friction. Clearly this flow satisfies the boundary conditions and satisfies continuity, but must be tested to see if it satisfies the momentum equation, which involves shear forces, gravity forces, pressure forces, and momentum.

Gravity forces produce a vertical pressure gradient, which affects the motion only insofar as it alters the shape of the free surface at the top of the liquid. In our experiment the rate of rotation is slow enough so that this surface remains nearly flat. We shall postulate that shear forces are negligibly small compared to pressure and inertia forces, and thus we may use Euler's equation to examine the momentum of the fluid.

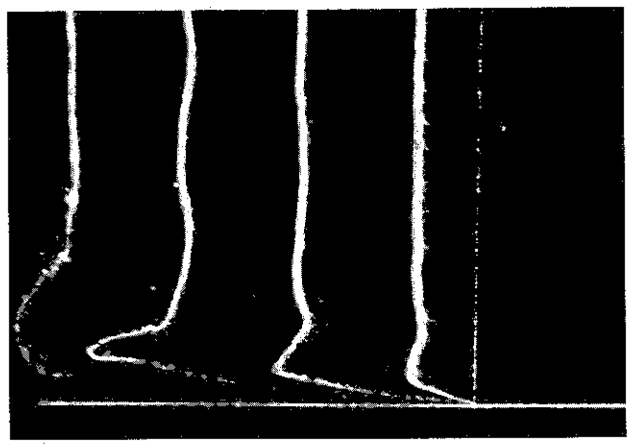
For motion of a fluid element in a circle Euler's equation in the radial direction becomes



For a given pressure gradient, $\frac{\partial P}{\partial r}$, the smaller the velocity, the smaller the radius the particle must follow. In our postulated primary flow there can be no vertical pressure gradients (other than that due to gravity, which has already been taken into account). Therefore, the pressure gradient toward the center, which keeps the fluid elements traveling in circles, must be the same at all depths. If we assume that the boundary layer occupies only a small part of the total depth, then the main flow outside the boundary layer fixes the pressure gradient. Since the circumferential velocity of fluid in the boundary layer is low, its centrifugal acceleration is insufficient to balance this gradient. This flow is therefore forced inward toward the center, carrying the tea leaves with it.

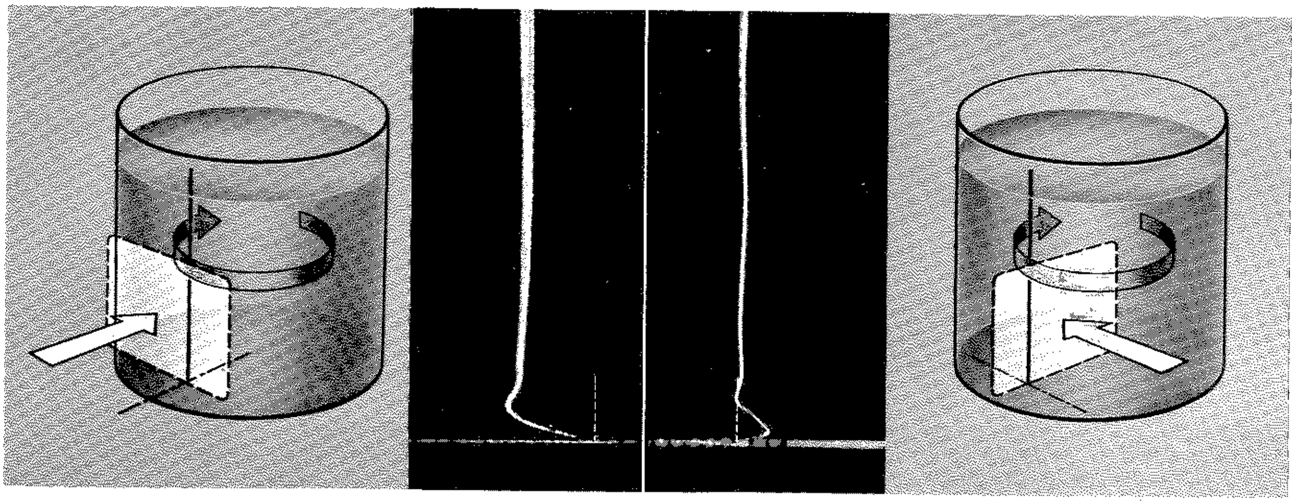
In other words, the postulated primary flow does not satisfy the momentum equation in the radial direction everywhere, and a correction to this flow is necessary to give a reasonable approximation to the actual flow. It is this correction which we call *secondary flow*.

We can observe the secondary flow by watching a line of hydrogen bubbles produced by electrolysis from a vertical wire by a short pulse of current. The wire is positioned about one third of the radius in from the circumference and is shown in radial view in Fig. 3. The bubbles are carried by the fluid; therefore, the wire marks the initial position of a set of fluid particles, and a line of bubbles a later position. If the



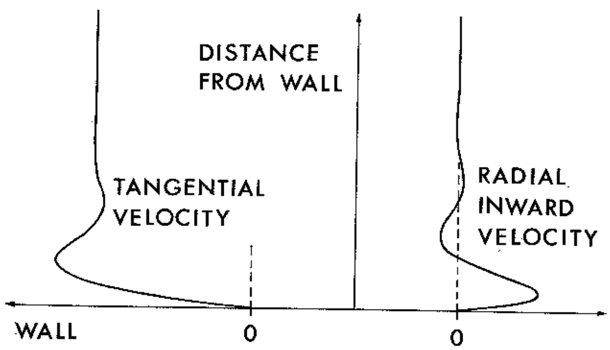
3. Radial view of vertical wire and bubble lines in teacup model.

vertical velocity of the bubbles is small, the line of bubbles represents a velocity profile with the wire as origin. By looking inward along a radius, we observe the tangential component of the velocity at any depth. Similarly, by looking in the tangential direction we can see the radial component of velocity at any depth. These two velocity profiles are shown in the composite photograph (Fig. 4). The tangential velocity shown at the left is seen to be nearly the same at all depths, except in the boundary layer where it goes to zero at the floor and shows an interesting overshoot near the outer edge of the boundary layer. This curious overshoot is a result of fluid in the radially-inward secondary flow being carried inward by its own radial momentum past the position of equilibrium where its tangential velocity matches that of the primary flow above. The right-hand bubble line (tangential view) shows the radial velocity components with the center of rotation toward the right. The radial velocity is essentially zero except in the boundary layer where there is considerable inward flow with again a few minor wiggles near the outer edge of the layer.



4. Tangential (left) and radial (right) velocity profiles.

Bödewadt (1, 2) gave a mathematical solution of a steady flow that is quite similar to the flow in our rotating tank. He postulated a semi-infinite body of fluid rotating uniformly about an axis perpendicular to a stationary bounding plane. Thus his solution corresponds approximately to what happens near the center of a very large and very deep tank wtih a stationary floor but with an outer wall which rotates with the fluid. Bödewadt's solution is shown graphically in Fig. 5.



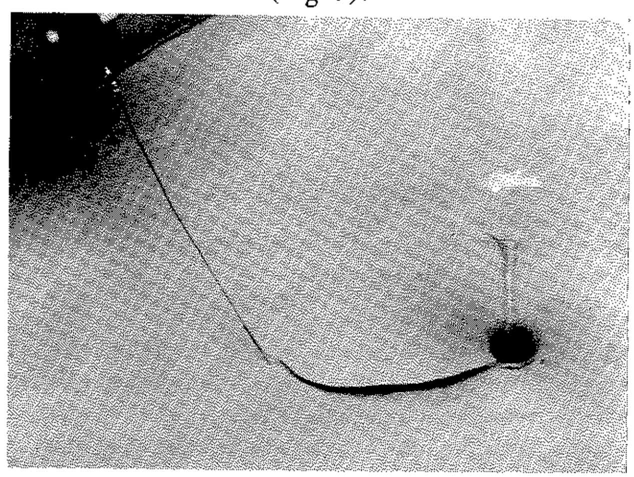
5. Computed tangential and radial velocity profiles (after Bödewadt).

While the experimental profile is slightly unsteady, it often shows the overshoot in tangential velocity predicted by Bödewadt's computation. The shape of the radial velocity profile also agrees qualitatively with his computation. We should not necessarily expect quantitative agreement, since Bödewadt's model is one of steady flow with a semi-infinite reservoir of rotating fluid to maintain the motion against friction, whereas our experiment involves a flow in a finite tank nonsteady in the sense that the entire flow is slowing down.

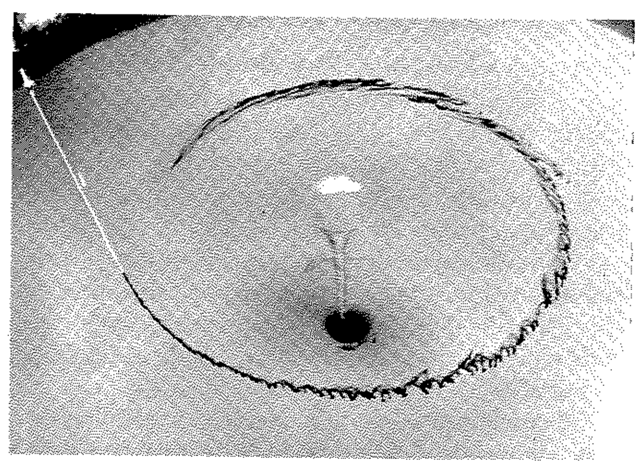
Steady Sink-vortex Flow

An example of a similar secondary flow occurs in a sink vortex (Figs. 6 and 7). Flow enters a circular tank tangentially and flows out a hole in the center of the bottom.

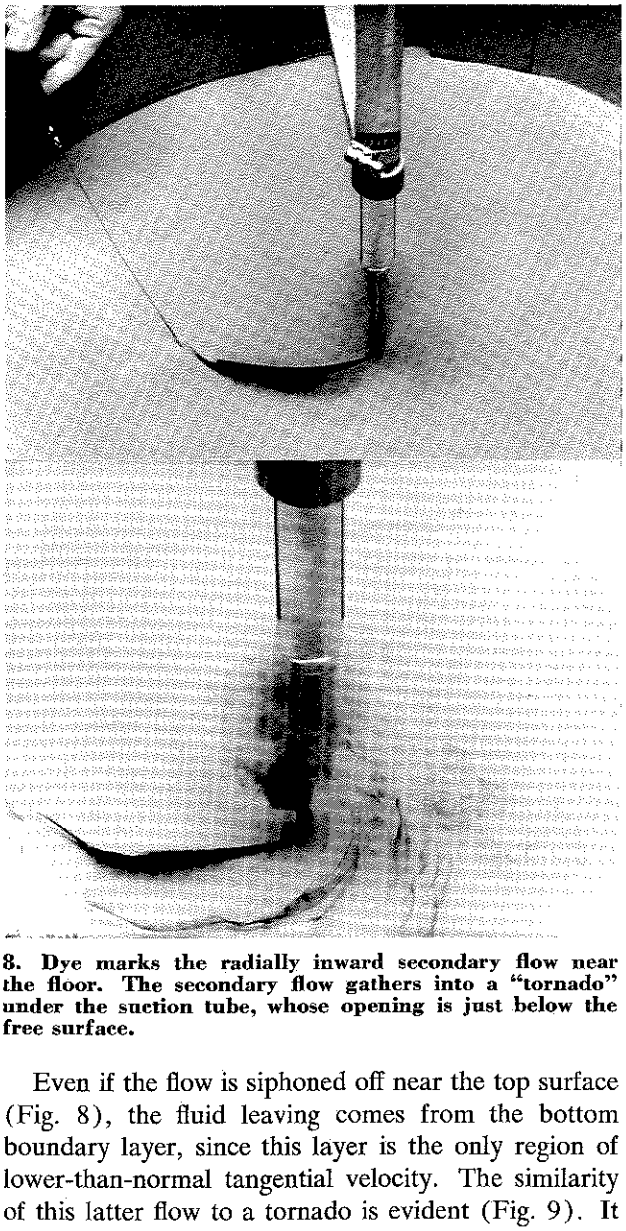
As might be expected, the slower-moving fluid in the boundary layer near the bottom is again forced inward by the radial pressure gradient established by the more rapidly moving fluid above. In fact, it appears that all of the flow out the drain hole comes from the boundary layer (Fig. 6) and the fluid above does indeed travel in circles (Fig. 7).



6. Dye marks the radially inward secondary flow near the floor of the sink-vortex apparatus.



7. Dye marks the circular streamlines of the primary flow high above the floor.



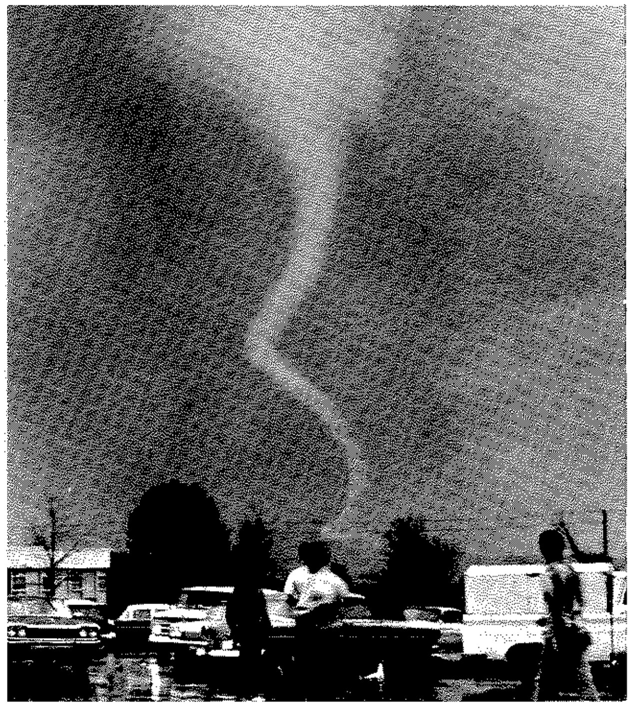
8. **Dye marks the radially inward secondary flow near the floor. The secondary flow gathers into a "tornado" under the suction tube, whose opening is just below the free surface.**

Even if the flow is siphoned off near the top surface (Fig. 8), the fluid leaving comes from the bottom boundary layer, since this layer is the only region of lower-than-normal tangential velocity. The similarity of this latter flow to a tornado is evident (Fig. 9). It is also of interest to note the "burst" or sudden swelling of the vortex core in both Figs. 8 and 9. This phenomenon has also been observed in vortices arising from delta-wing airplanes.

Flow in a Channel with a Bend

Another model which exhibits secondary flow is a channel with a bend (Fig. 10). If the velocity of a vertical line of fluid elements entering the bend is the same at all depths (Fig. 11a) there will be no tendency for secondary flow in the bend (Fig. 11b). However, if we introduce a flow in which velocity varies with depth (Fig. 12a), in the bend, we can expect the slower-moving fluid to be swept inward, while the more rapidly moving fluid will be forced outward by the radial pressure gradient (Fig. 12b).

The concept of vorticity is useful in explaining and analyzing this flow. If we select a right-handed set of coordinates x, y, z, the corresponding components of



9. **Tornado. (Wide World Photos.)**

vorticity ω_x, ω_y, ω_z can be written in terms of the components of velocity u, v, w thus:

$$\omega_x = \frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}$$

$$\omega_y = \frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}$$

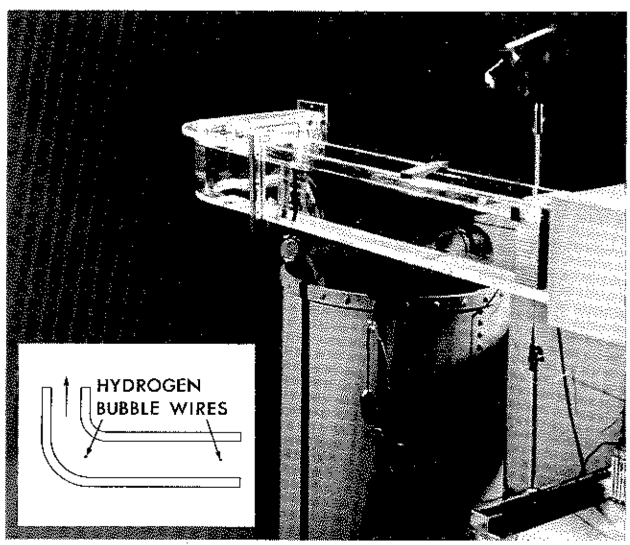
$$\omega_z = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$$

If we take x in the direction of flow, z vertically upward, and y in the transverse direction, we see that the initial vorticity of the flow shown in Fig. 12a will be

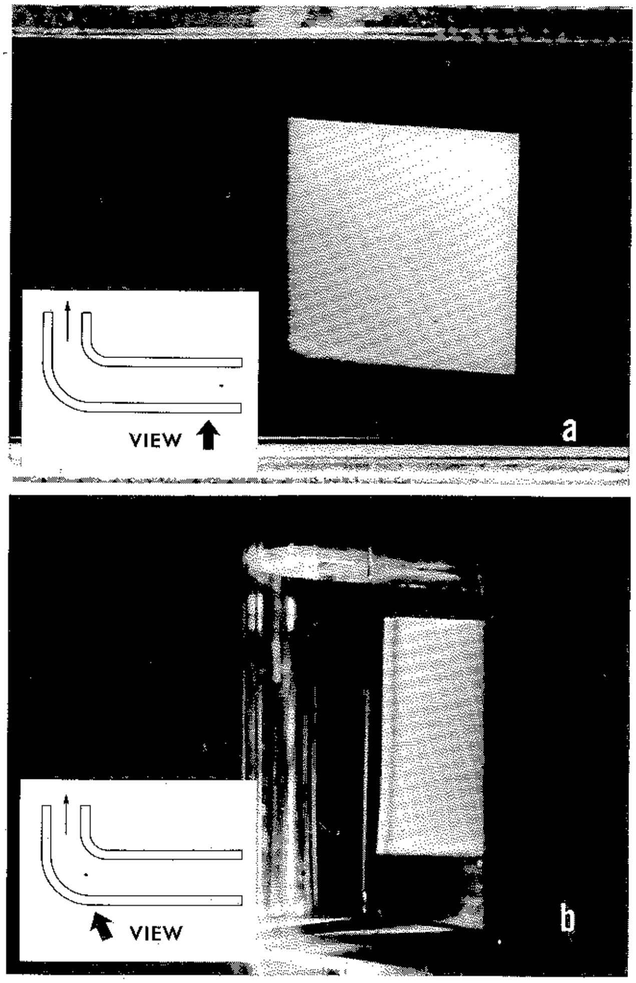
$$\omega_x = 0$$

$$\omega_y = \frac{\partial u}{\partial z} \approx \text{constant}$$

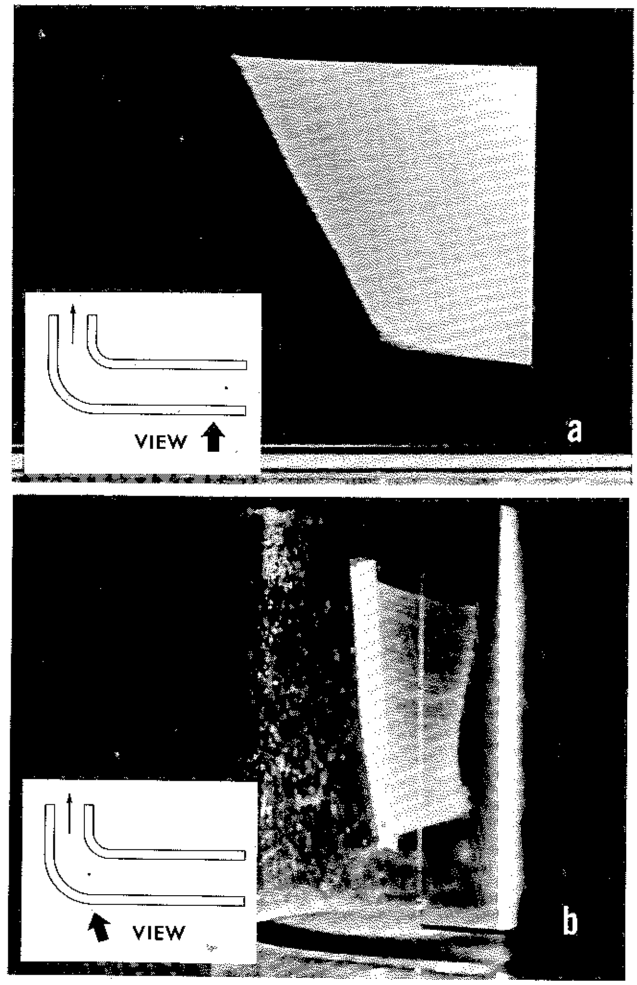
$$\omega_z = 0$$



10. **Water channel with a bend.**



11. Uniform flow. (a) Straight section, (b) Tangential view in bend.

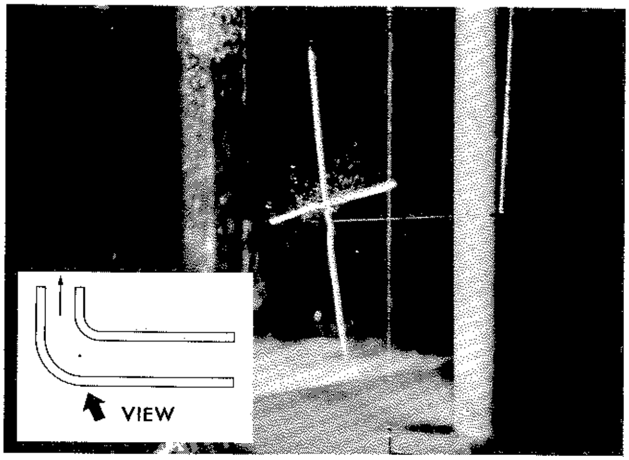


12. Shear flow. (a) Straight section. (b) Tangential view in bend shows that transverse velocities are induced by the radial pressure gradient.

Vortex lines (lines everywhere parallel to the vorticity vector) are, in this case, horizontal, and perpendicular to the flow direction.

The Helmholtz theorem states that in an inviscid flow, vortex lines are transported by the fluid. We shall assume that the fluid acts in an essentially frictionless manner (an assumption which will be validated later). Fig. 13 views the water channel from above and shows the displacement of a bubble cross produced upstream of the bend. One leg of the cross is a part of a streamline and the other a part of a vortex line. As the cross enters the bend the streamline leg must turn. The rate of turn is $\frac{\partial v}{\partial x}$. The rate of

13. A cross composed of a streamline and vortex line is formed upstream (a) and enters the bend (b).



14. Tangential view of bend. Inclination of initially erect fluid cross shows streamwise vorticity.

turn of the vortex line is $\frac{\partial u}{\partial y}$. If the vertical vorticity, ω_z, is to remain zero

$$\frac{\partial v}{\partial x} = \frac{\partial u}{\partial y}$$

that is, the initial rate of turn of the vortex lines is equal and opposite to the initial rate of rotation of the streamline. The two lines, therefore, cease to be at right angles, and close together like scissors.

There is horizontal vorticity entering the bend in Fig. 13, and it is marked by a bubble line. The fact that this vortex line ceases to be perpendicular to the streamline indicates that a component of vorticity is appearing in the streamwise direction. An initially upright bubble cross viewed in the streamwise direction (Fig. 14) shows by its tilt additional evidence of this streamwise component of vorticity. It is evident from this picture that vertical velocities have been produced.

Notched Flow Around a Bend

A flow with a wake or defect in velocity at the center was created upstream of the bend. We will call this a "notched" flow (Fig. 15a). A tangential view in the bend (Fig. 15b) shows the familiar inward flow of the low-velocity region. The limited extent of the vorticity, and the proximity of two layers of vorticity with opposite sign tend to minimize vertical velocities.

If we assume that vertical velocities are zero everywhere, and that vertical vorticity remains zero throughout the flow, the components of vorticity in the bend become:

$$\omega_x = -\frac{\partial v}{\partial z}$$

$$\omega_y = \frac{\partial u}{\partial z}$$

$$\omega_z = 0$$

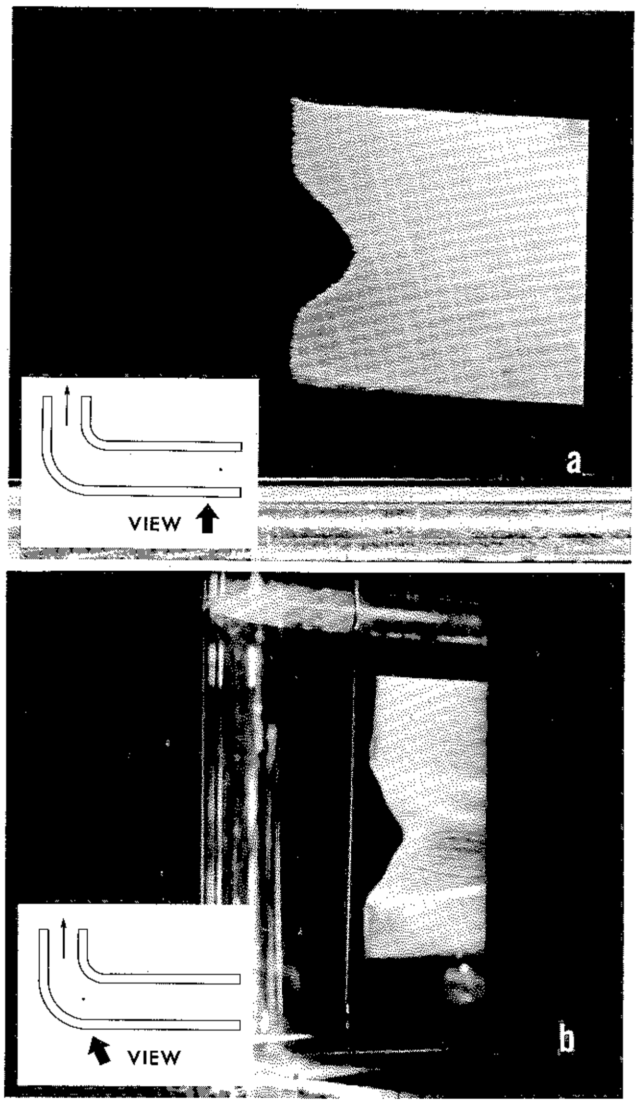
$$\vec{\omega} = \mathbf{i}\,\omega_x + \mathbf{j}\,\omega_y = -\mathbf{i}\,\frac{\partial v}{\partial z} + \mathbf{j}\,\frac{\partial u}{\partial z}$$

Fig. 16a is a view from above of an initially vertical bubble line in the notched flow in the bend, taken a short time after the bubble line was formed. We note that it lies in a vertical plane; that is, it appears from above as a nearly straight line. It can be verified that the direction of this line in the horizontal plane is

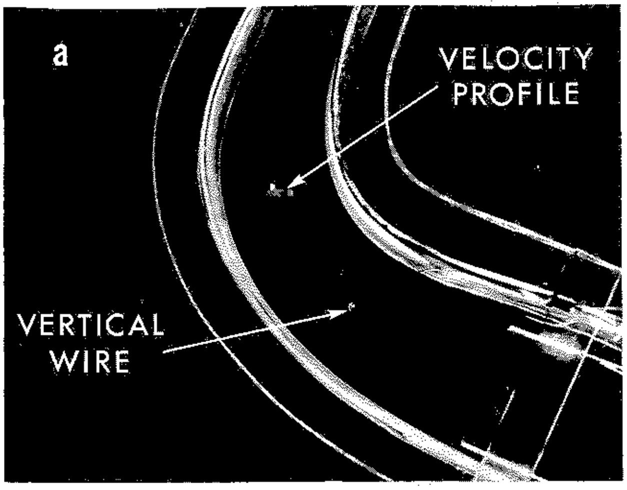
$$\mathbf{i}\,\frac{\partial u}{\partial z} + \mathbf{j}\,\frac{\partial v}{\partial z}$$

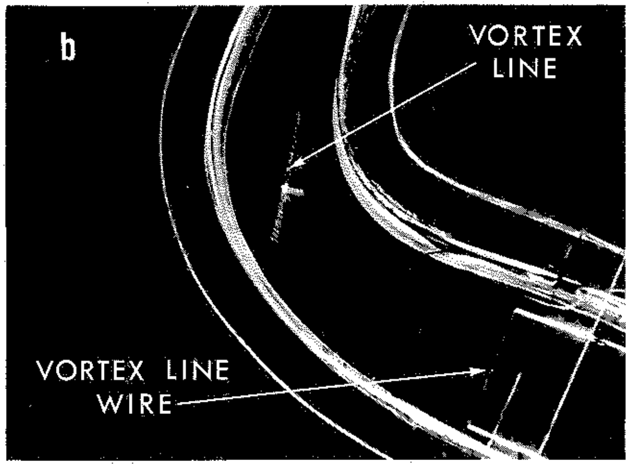
and that it is perpendicular to $\vec{\omega}$. Thus, if vertical components of velocity can be neglected, the plane determined by the velocity profile is perpendicular to the local vorticity.

We now have a means of checking whether friction is important: if a vortex line marked upstream coincides with the vorticity as indicated locally by the perpendicular to the projection of the velocity profile on the horizontal plane, then Helmholtz's theorem holds, an observation which is consistent with the absence of friction. Fig. 16b includes a vortex line sent down



15. "Notched" flow. (a) Straight section. (b) Tangential view in bend.



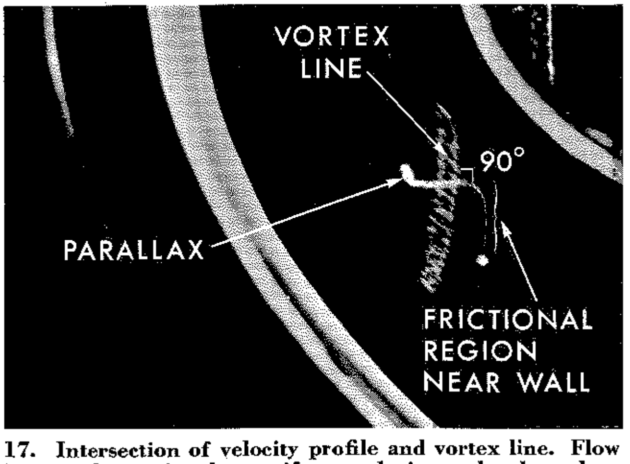


16. Notched flow. (a) Initally vertical bubble line remains in a vertical plane. (b) The plane of the velocity profile is perpendicular to a vortex line convected from upstream.

from upstream which is indeed nearly perpendicular to the velocity profile.

Boundary-layer Flow Around a Bend

Secondary flow due to the boundary layer on the bottom surface shows behavior similar to that of the notched flow. Here vertical velocities are suppressed by the floor of the channel.



17. Intersection of velocity profile and vortex line. Flow in straight section has uniform velocity and a boundary layer on the floor.

In this case the bubble line is no longer a plane curve (Fig. 17). Looked at from above it appears as two approximately straight lines joined by a short curved section.* One of the straight portions is indeed perpendicular to the vortex line from upstream. This portion is in the outer part of the boundary layer, where local friction is unimportant; hence this part of the boundary layer behaves like the notched flow. The other straight portion is in a part of the boundary layer very near the wall, where friction is important. This part is not perpendicular to the vortex line, as might be expected. The curved portion is a transition region.

Transport Phenomena Associated with Secondary Flows

Secondary flow in a curved channel acts to replace the slow-moving fluid near the walls with faster-moving fluid, thereby greatly increasing viscous friction at the wall. Frictional losses are increased not only in the bend itself, but also downstream, as a result of the persistence of the secondary flow there.

Materials as well as momentum may be transported by secondary flow. The causes of the meandering of rivers are complex, but the phenomenon is undoubtedly associated with the transport of silt by secondary flows.

Heat transfer and momentum transfer follow similar laws, so that changes in the flow distribution which promote heat transfer generally cause increased friction, and vice versa. Heat exchangers are sometimes made with wavy passages, to promote secondary flow. The resultant heat transfer (and friction) can be more than double that of a comparable straight passage (3).

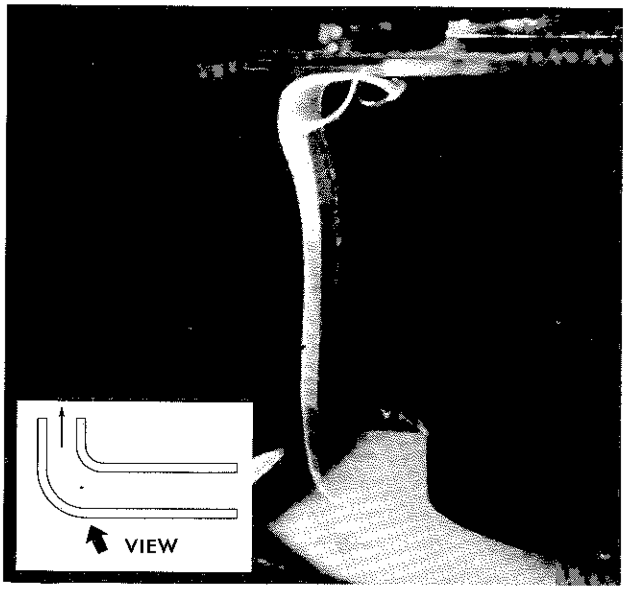
Turbines and compressors have blades which change the direction of the working fluid. There are boundary layers not only on the blades, but also on the hub and casing, and secondary flows are often important. In general, the fluid in the hub and casing boundary layers is turned more than the fluid outside the boundary layers, and impinges on the next set of blades at an increased angle of incidence.

Other Secondary Flows

The causes of some secondary flows are difficult to determine. For example, Fig. 18 shows a secondary flow arising from an oily film on the water surface. Such films can behave very much like solid walls.

Other secondary flows are very difficult to analyze. For example, when the flow in a bend has progressed around a considerable corner, a portion of it originating from the boundary layer rolls up into what is called a "passage vortex" (Fig. 19). Note the start of a passage vortex in Fig. 18.

* The apparent upward turn at the left-hand end of the line is due to parallax; the line of sight was along the wire and as the bubble line moved away from the wire, the camera looked at an angle to the line of bubbles.

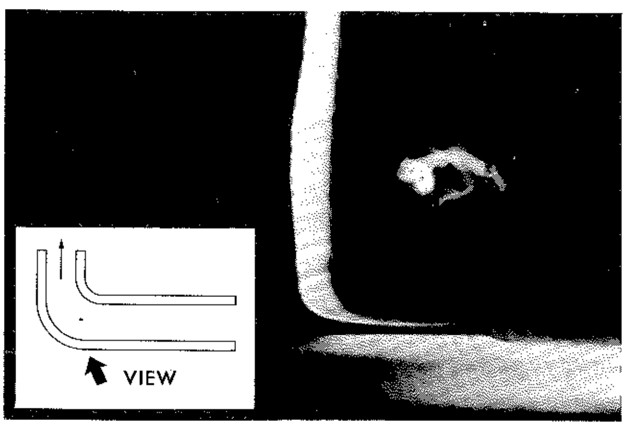


18. Secondary flow in a bend resulting from dirt film on free surface.

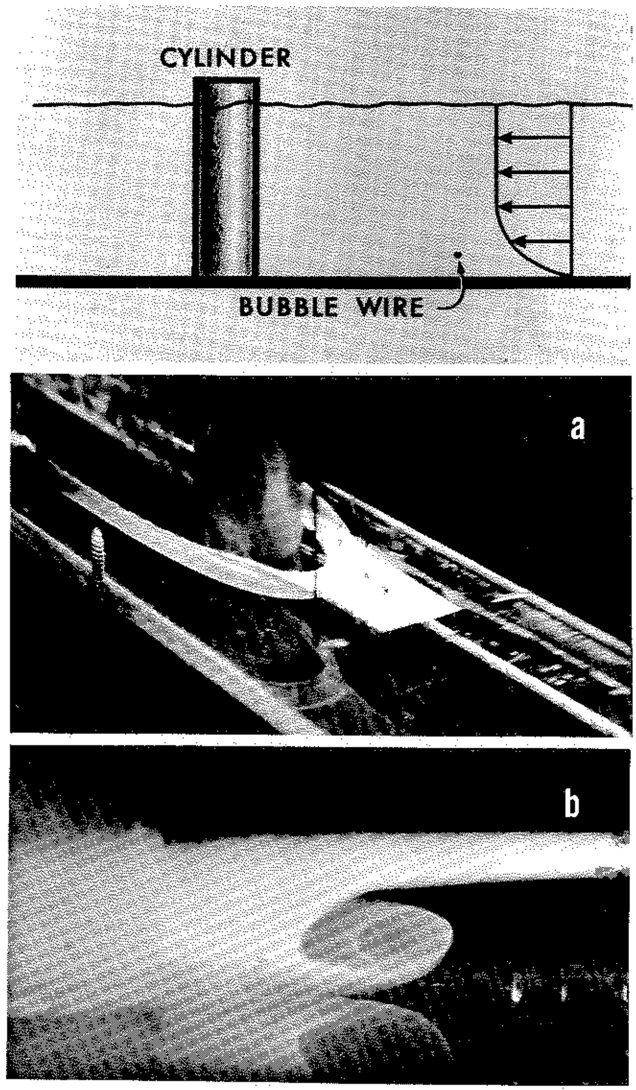
All of the pictures were taken with laminar flow to insure that dye or bubble lines would not be diffused by turbulence but would persist long enough to permit observation. Secondary flow phenomena, however, are not confined to laminar flows, but are present in the turbulent flows which often occur in pipes, channels, and ducts, as well as in rotating machinery.

Another more complex secondary flow is the horseshoe vortex formed when the boundary layer on the bottom of the channel meets an obstruction (Fig. 20). Fluid from the boundary layer cannot reach a stagnation point on the obstruction, since its total pressure is less than the static pressure at such a point. The static pressure in the center of a horseshoe vortex is low, and the low-energy fluid finds its way to this point. This particular flow pattern results in very high shear stresses on the bottom wall directly under the vortex and in front of the obstruction. (Note that the fluid near the bottom is flowing upstream at this point and the friction force on the wall is *upstream.*)

After a blizzard you may have noticed the way the snow is scooped out on the windward side of trees and telephone poles. This is evidence of horseshoe vortices resulting when the boundary layer on the ground encounters obstructions. These beautiful snow patterns have counterparts underwater near bridge piers in streams. The resulting undermining of the earth in front of and around such piers can seriously affect the structure.



19. A "passage vortex" in a bend.



20. Horseshoe vortex. (a) Oblique view. (b) Cross section at plane of symmetry.

Many other flows can be analyzed by postulating a primary flow and deducing a secondary flow. The book *Boundary Layer Theory* by H. Schlichting gives several examples, such as turbulent flow in pipes of non-circular cross section (pp. 415-416) and oscillating disks, spheres, and cylinders (pp. 196-197).

References

1. Bödewadt, U. T. See also Schlichting pp. 176-180
2. Schlichting, H. *Boundary Layer Theory,* McGraw-Hill, 1960
3. Kays, W. M. and London, A. L. *Compact Heat Exchangers,* McGraw-Hill, 1958