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National Committee for Fluid Mechanics Films

FILM NOTES for TURBULENCE*

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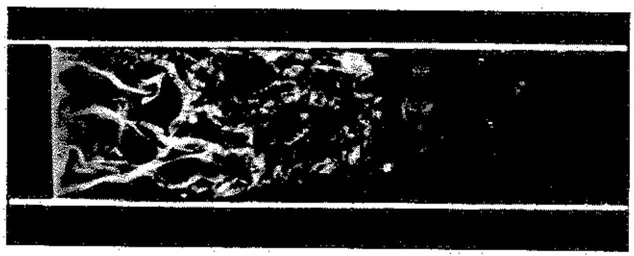
Introduction

"Turbulence" is not easy to define, but it is nearly ubiquitous. Tobacco smoke, industrial smoke, milk mixed into tea, all reveal turbulent motion.

Turbulent flows have common characteristics, one of the clearest of which is *disorder*. Fig. 1 shows a sheet of tiny bubbles advected by a channel flow. The disorder is so central that no matter how carefully one reproduces the boundary conditions, the flow is never reproduced in detail. On the other hand, *averages*, such as the mean speed of flow or correlation functions, are very well defined and "stable."



1. Channel flow visualized by hydrogen bubbles released uniformly from a wire stretched across the flow.

There are disordered fluid motions — for example some fields of water waves or of acoustic waves — which we prefer to exclude from the definition of turbulence, since they do very little *mixing* and mixing is an essential feature of turbulence. Thus disorder is necessary but not sufficient for description. A further characteristic of turbulence is the presence of vorticity, distributed continuously but irregularly in all three dimensions.

We can borrow a word from pathology and give a defining *syndrome*, or set of symptoms, for turbulence. It has disorder, irreproducible in detail, performs efficient mixing and transport, and has vorticity irregularly distributed in three dimensions. This distinguishes turbulence from various kinds of wave motion and excludes two-dimensional flows. Something like turbulent motion *can* occur in two dimensions; large-scale weather systems have some of this character. However, in strictly two-dimensional flows vorticity behaves as a scalar, and there is no vorticity production by vortex line stretching.** Thus the characteristics of two-

**The constant density vorticity equation is

$$\frac{D\omega}{Dt} = \omega \cdot \nabla \mathbf{V} + \nu \nabla^2 \omega.$$

In two dimensions the first term on the right must vanish, since ω is everywhere perpendicular to the plane of the flow. The equation then becomes exactly analogous to that for a conservative diffusible scalar like heat.

**TURBULENCE, a 16-mm color sound film, 29 minutes in length, was produced by Education Development Center under the direction of the National Committee for Fluid Mechanics Films, with the support of the National Science Foundation and the Office of Naval Research. Additional copies of the film notes and information on purchase and rental of the film may be obtained from the distributor:*

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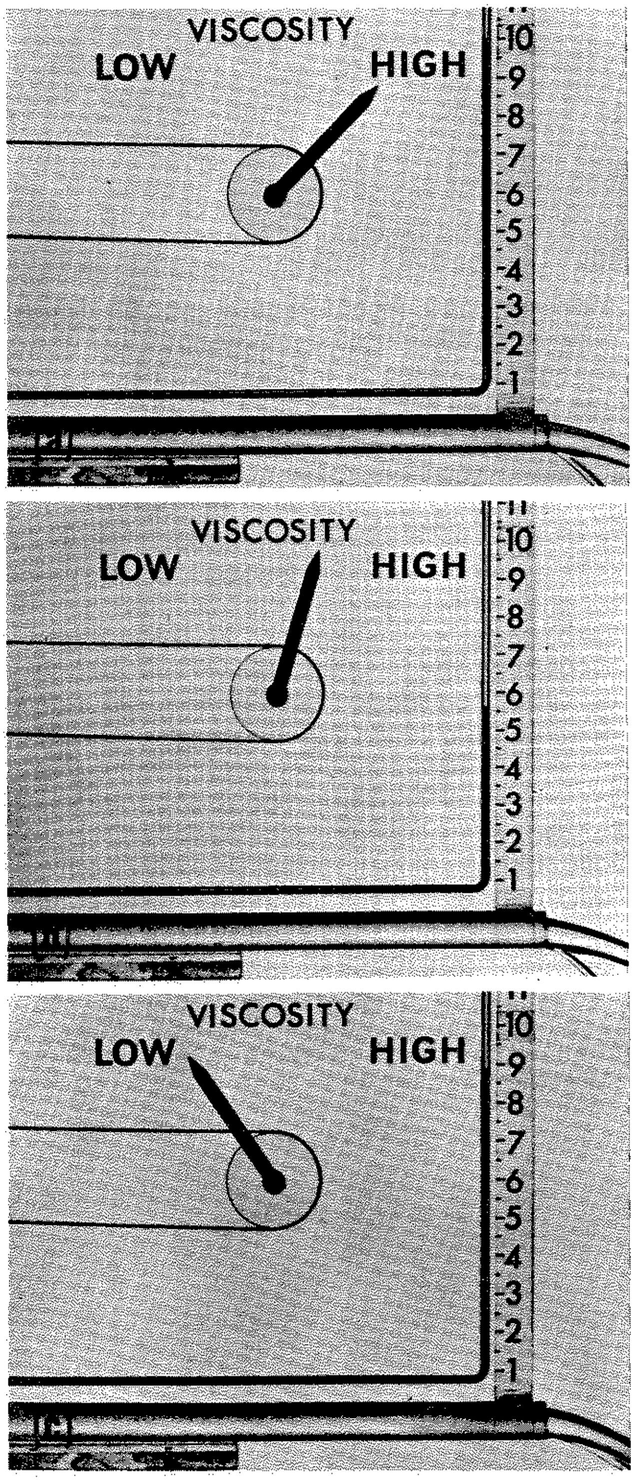
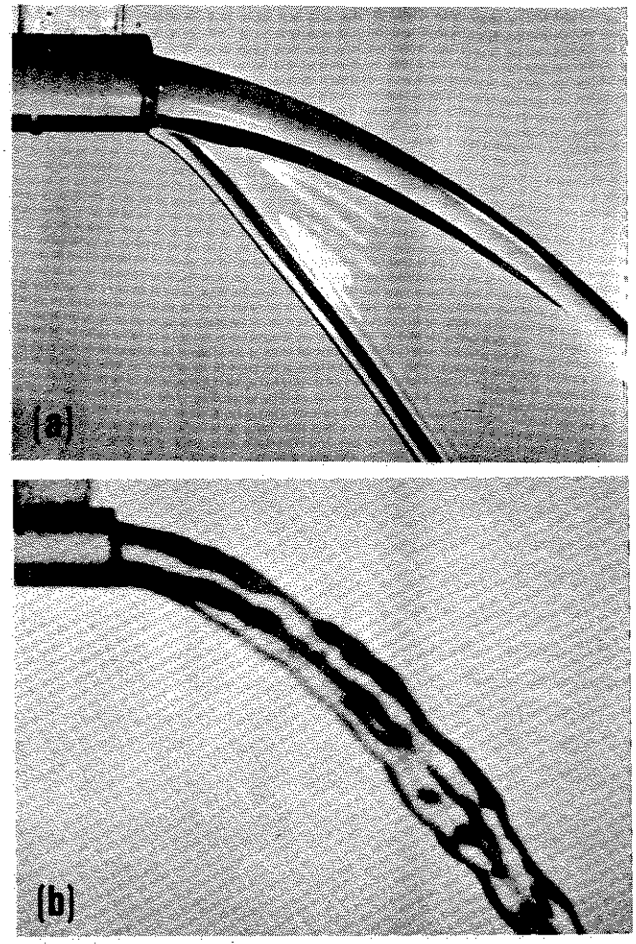
dimensional flows are quite different from those of three-dimensional turbulent flows.

Reynolds Number

Some flows are clearly turbulent. Others, with similar boundary conditions, are equally clearly not. What determines whether a flow is turbulent?



2. Pressure drops across a tube at constant flow rate and variable viscosity. In (a) and (b) the flow is laminar. In (c) it is turbulent.



3. Flow issuing from the end of the pipe shown in Fig. 2. In (a) the flow corresponds to Fig. 2(a) and (b). In (b) the flow corresponds to Fig. 2(c).

In the film this question is discussed with the aid of an apparatus like that used by Hagen in the middle of the nineteenth century for a study of flow through pipes. A mixture of glycerin and water is pumped at a constant rate through a tube some 4 m long and 3 cm in diameter. At the downstream end the liquid issues into the free atmosphere. The pressure drop in the tube is shown by a manometer (Fig. 2a) which is tapped at an upstream position, and carried down to the open end. By varying the ratio of glycerin and water, the viscosity of the fluid can be controlled. According to the Hagen-Poiseuille law for laminar flow of a Newtonian fluid flowing through a circular pipe of length L and radius r, ($L >> r$), the pressure drop is given by $\Delta P = 8\mu LQ/\pi r^4$, where μ is the viscosity of the fluid and Q the volume flow rate.

As is seen in Fig. 2 (a) and (b), when the viscosity is reduced somewhat the pressure drop decreases, consistent with this formula. However, when the viscosity is reduced still further, as shown in Fig. 2 (c), we find that the pressure drop *increases*.

At the higher viscosities the flow issuing from the end of the pipe (Fig. 3a) is smooth and steady. At the lowest viscosity (Fig. 3b) high-speed photography reveals a time-dependent irregularity of the edges of the stream. Thus when we pass to the lowest viscosity we find that the Poiseuille law is not obeyed; instead of decreasing, the pressure drop increases. The flow in the pipe has become turbulent, revealed both by the irregular motion of the outcoming stream and by the greatly increased pressure drop down the tube.

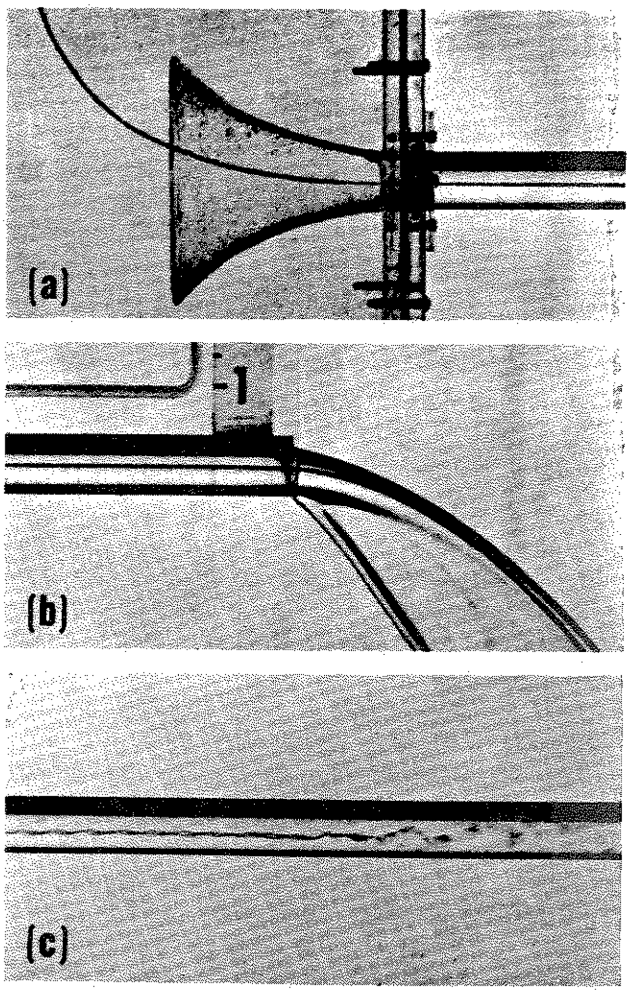
In the early 1880's Osborne Reynolds did a series of experiments on flow through tubes and came to the conclusion that the criterion for the onset of turbulence depended upon a dimensionless function of the flow parameters which has since been called Reynolds number.

There is usually some arbitrariness in the choice of parameters for the definition of Reynolds number. For pipe flow we may take it as: $\mathrm{Re} = \frac{VD}{\nu}$, where D is the tube diameter, V is the average speed of the flow and ν is the kinematic viscosity. Although the question is still under investigation, it seems that if the Reynolds number so defined is appreciably less than 2000, the flow is not turbulent and perturbations are damped out by viscosity. At higher Reynolds numbers the flow may or may not be turbulent. Poiseuille's relation corresponds to a solution of the dynamical equations which is valid at all Reynolds numbers. At sufficiently large Reynolds number, however, this flow is unstable to certain perturbations. Whether or not a particular pipe flow is turbulent depends upon the length of the pipe and upon the nature and amplitude of perturbations, as well as upon the Reynolds number. If great care is taken to reduce such perturbations it is possible to push the Reynolds up to the neighborhood of 100,000 without turbulence.

Mixing

If a thin streamer of dye is introduced into the flow, as in Fig. 4 (a), mixing can be examined. At low Reynolds number the dye filament maintains its identity with very little change right to the end of the tube, as shown in Fig. 4 (b). The only mixing is molecular, so the process is very slow. If the Reynolds number is increased, perturbations can be seen in the dye flow, and at the onset of turbulence it seems to explode; the dye is rapidly mixed across the tube, as in Fig. 4 (c).

We can regard the increase in pressure drop with the onset of turbulence, shown in Fig. 2, as a manifestation of mixing too — mixing of momentum. When the flow is laminar, slow-moving fluid from close to the wall produces the steeply dropping portion of the stream shown in Fig. 3 (a). Flow in the center of the tube is much more rapid and produces the flat trajectory which forms the upper right boundary of the stream. When the flow becomes turbulent, the mixing of momentum causes the flow speed in the pipe to be much more uniform. The fastest fluid is not quite so fast, and there is so little slow fluid that it can be dragged along with the rest, producing the trajectory shown in Fig. 3 (b). The fluid motion vanishes at the wall, so we can regard it as the sink for momentum. The turbulence increases the rate at which momentum is transferred toward the wall. Thus, with turbulence we need a larger pressure gradient to replace the momentum lost to the wall.



4. Dye filament introduced (a) at the entrance to a tube retains its identity (b) in laminar flow but "explodes" and mixes rapidly across the flow (c) when the flow becomes turbulent.

Turbulent Transport and Reynolds Stress

Although the principal motion of the fluid in the channel of Fig. 5 is downstream, because of the turbulence there is appreciable cross-stream motion. Fluid moving across the stream tends to carry its properties with it. Thus the darker dye which marks fluid originally in the center of the stream has moved, in some

5. Dye injected near the center and near the wall of a turbulent channel flow. The walls of the channel have been deliberately roughened to increase the ratio of the turbulent to the mean flow speeds.

places, quite close to the wall. The lighter-colored dye, marking fluid originally close to the wall, has moved toward the center of the channel. This ability of turbulence to carry fluid properties is referred to as *turbulent transport,* and occurs whenever there is some gradient of a mean property, be it momentum, dye concentration or whatever, within the turbulent fluid. For example, in the flow shown in Fig. 5 the region near the wall continuously gains momentum at the expense of the region near the center of the flow.

Analytically, in tensor form, we may write the Navier-Stokes equation for the velocity component V_i as

$$\frac{\delta \rho V_i}{\delta t} + \frac{\delta}{\delta x_j}\rho V_i V_j + \frac{\delta P}{\delta x_i} - \frac{\delta}{\delta x_j}\,\mu\left(\frac{\delta V_i}{\delta x_j} + \frac{\delta V_j}{\delta x_i}\right)$$
$$= \text{body force}$$

Now if we define some suitable average* velocity U_i (space, time, or ensemble, depending upon the situation) we can put

$$V_i = U_i + u_i,$$
$$<V_i> = U_i \qquad <u_i> = 0$$

and write the equation as

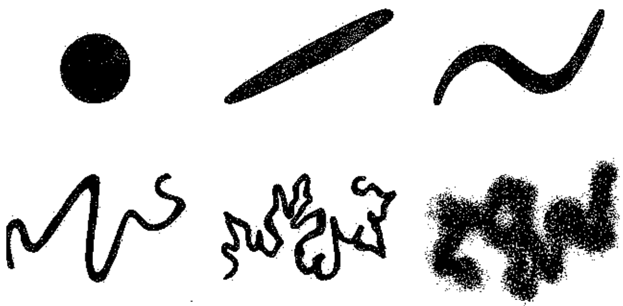
$$\frac{\delta \rho U_i}{\delta t} + \frac{\delta \rho U_i U_j}{\delta x_j} + \frac{\delta}{\delta x_j}<\rho u_i u_j> + \frac{\delta <P>}{\delta x_i}$$
$$-\frac{\delta}{\delta x_j}\,\mu\left(\frac{\delta U_i}{\delta x_j} + \frac{\delta U_j}{\delta x_i}\right) = <\text{body force}>$$

the expression $<-\rho u_i u_j>$ appears in the equation in the same way as does the viscous stress

$\mu\left(\frac{\delta U_i}{\delta x_j} + \frac{\delta U_j}{\delta x_i}\right)$. Thus it acts like a stress. It is called the *Reynolds stress.*

Consider now the mixing of a scalar. If two miscible liquids are carefully placed in a vessel, one floating on top of the other, after a week or two molecular diffusion does a fair job of mixing. However, much more thorough mixing can be accomplished in less than a minute if we make the fluid turbulent. In this case too, the end result is intimate mingling on a molecular scale — although the turbulent motions themselves are not much smaller than a millimeter. The role of the turbulence is to make inhomogeneities more vulnerable to the effects of molecular diffusion. This is illustrated schematically in Fig. 6.

*Symbolized by the bracket $< - >$.



6. Schematic representation of the turbulent mixing of a scalar. The turbulent motions stretch and distort a blob of inhomogeneous fluid, until both the increase in surface area and the increase in property gradients enable molecular effects to occur rapidly.

Analytically, the transport of a scalar can be described as follows: the Eulerian equation for the concentration C of a conservative scalar property is

$$\frac{\delta C}{\delta t} + \nabla \cdot \mathbf{V}C = K\nabla^2 C$$

where K is the diffusivity appropriate to C.
If we again break the fluid velocity $\mathbf{V}$ into mean and turbulent parts $\mathbf{U}$ and $\mathbf{u}$, we get

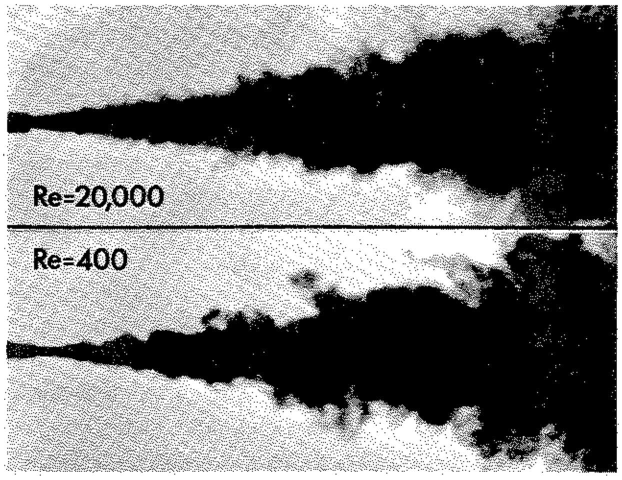
$$\frac{\delta <C>}{\delta t} + \nabla \cdot \mathbf{U}<C> + \nabla \cdot <\mathbf{u}C> = K\,\nabla^2 <C>.$$

The vector $<\mathbf{u}C>$ represents the turbulent transport of the property C.

Notice that in both this example and the previous one, which showed the origin of the Reynolds stress, the turbulent effects were analytically derived from the "advection" terms $\nabla \cdot \mathbf{V}C$ and $\frac{\delta}{\delta x_j}\rho V_i V_j$. In the Navier-Stokes equation this term is non-linear. It is this essential non-linearity that leads both to the complexity of turbulence and to the great difficulty of treating it analytically. Typically, in turbulent situations, the non-linear term is as important as any other in the equation and so cannot be treated adequately by the usual perturbation methods.

The Influence of Reynolds Number on Fully Developed Turbulent Flows

One of the curious properties of turbulence is the fact that, although the Reynolds number is very im-

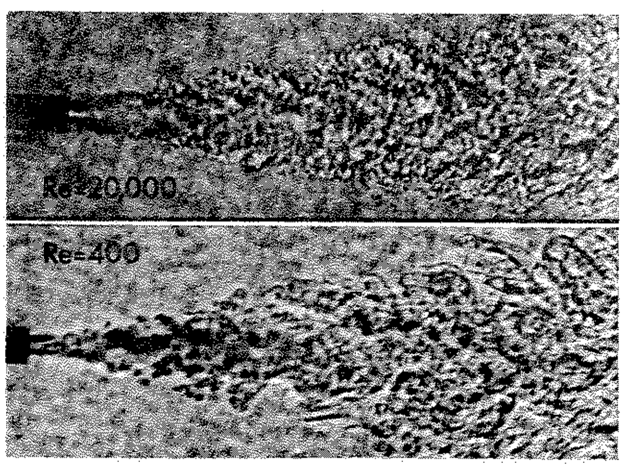


7. Turbulent jets showing that the Reynolds number does not much affect the appearance, so long as it is sufficiently large that the jet is indeed turbulent. The upper jet has a Reynolds number 50 times that of the lower.

portant in determining whetner or not a particular flow will be turbulent, once it has become turbulent the Reynolds number is of very little importance so far as the *large-scale motion* is concerned. This is illustrated in Fig. 7, which shows two jets, identical in every way except for the viscosity of the fluids (and therefore the Reynolds number), which differs by a factor of fifty. Evidently the large-scale features of the flow are comparatively insensitive to Reynolds number.

However, the small-scale motion, as revealed in shadowgraphs such as those in Fig. 8, is markedly affected. The higher Reynolds number jet has a much finer scale structure than the other. This can be understood if we consider the energy dissipated. These two jets differ only in viscosity; all other conditions are the same, including the rate of energy input into the jet. Therefore they dissipate energy at the same rate.

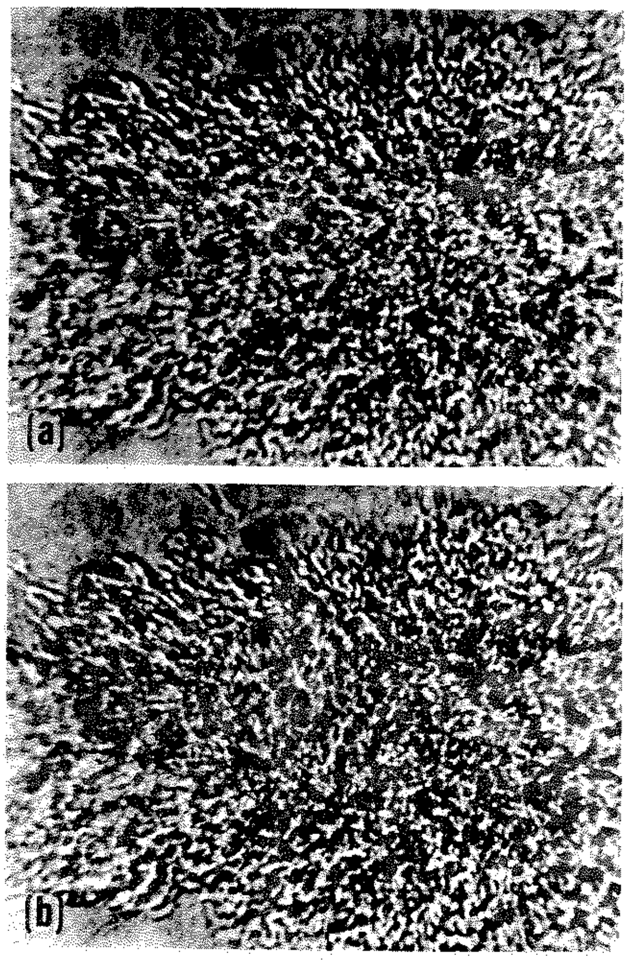
Dimensionally the dissipation rate must be given by $\nu V^2/\lambda^2$, where λ is a characteristic scale important to the dissipation process, and V a characteristic speed. Clearly, the larger ν is, the larger λ must be.



8. Shadowgraphs of the jets shown in Fig. 7. Note how much finer grained is the structure in the high-Reynolds-number jet than that in the low-Reynolds-number jet.

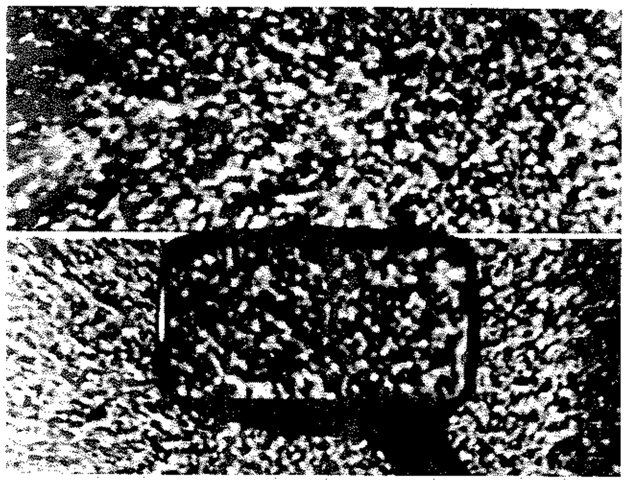
Turbulent Energy Cascade and Small-scale Similarity

This leads to one of the most important concepts in the study of turbulence: the idea of the *energy cascade.* As we have seen, under certain circumstances a large-scale motion can become turbulent. Some of the energy in the large-scale motion is converted into turbulent energy. The largest scales of the turbulence are usually smaller than, although comparable with, the scale of the basic flow, as can be seen in Fig. 1 and 7. However, usually these large-scale motions are themselves unstable and break into smaller-scale motions which take energy from them. Finally the energy passes down to scales like those revealed in the shadowgraphs of Fig. 8, which are so small that their Reynolds number is too low for instability. Their energy is then dissipated by the action of viscosity.

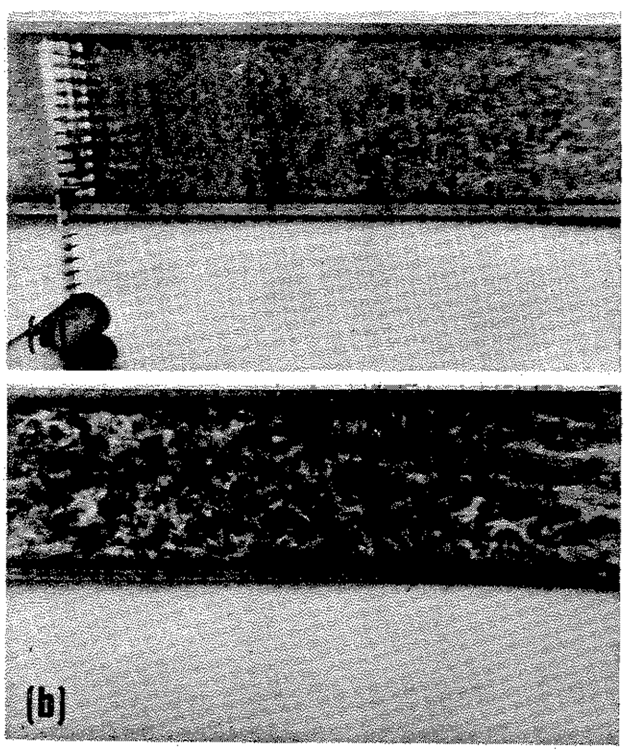


9. Shadowgraph of a high-Reynolds-number jet. The only difference between these two photographs is the fact that a circle from the center of Fig. 9(a) has been rotated through some 80 degrees to produce Fig. 9(b). The fact that this circle is hard to locate in Fig. 9(b) indicates that the small-scale turbulence is approximately both isotropic and homogeneous.

In this turbulent-energy cascade, at the smaller scales of motion it is only the rate of energy dissipation which is of any consequence. Other information associated with the large-scale motion is lost in the transfer process. Thus at high-enough Reynolds number the small-scale turbulence loses all directional orientation. It becomes locally isotropic, as is illustrated in Fig. 9.



10. Shadowgraphs show similarity of small-scale structure. The upper half of the frame is a shadowgraph from a jet, like that of Fig. 9. The lower half is from a channel flow, in part magnified so that the scale will be comparable to that of the jet.



11. Flow visualization of decaying turbulence behind a grid. Photograph (b) is taken several seconds later than photograph (a). Although the energy-transfer mechanism passes energy largely from large scale to small, the decay of the small-scale motion is comparatively rapid, so that it is the large-scale motions that are last to die.

Moreover, at high Reynolds number the small-scale turbulent structure ceases to depend upon the nature of the large-scale flow. Macroscopically the difference between a jet and a channel flow is marked. However on the very small scale revealed by shadowgraphs, the difference in structure disappears, as is shown in Fig. 10. Because of the size difference, the similarity between the small-scale structures may not be obvious unless suitable magnification is used, as in Fig. 10. This is a kind of "similarity": similar structure despite differences in scale. (The velocity scales may differ, as well as the length scales.) We have already seen that the large-scale motion does not depend much upon the Reynolds number. We now find that the structure of the small-scale motion is similar for all kinds of turbulence. What the Reynolds number does is to determine the ratio of the largest scales to the smallest scales.

In decaying turbulence, energy seems, paradoxically, to pass from small scales to large. In fact, however, the energy transfer is still mostly from large scale to small. The large-scale motions are the last to die, because the small scales dissipate more rapidly. Fig. 11 illustrates the effect.

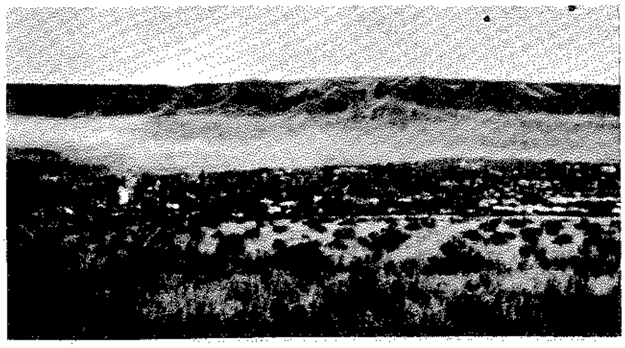
Effect of Buoyancy on Turbulence

The Reynolds number is not the only important parameter in determining the likelihood of turbulence. In some cases the Reynolds number may be enormous, many millions, and no turbulence will exist, because of the presence of some other influence like rotation, density stratification or, for conducting fluids, magnetic fields.

Of these, buoyancy effects are easiest to understand. If the fluid at the bottom is less dense than that at the top, convective activity sets in and can greatly increase the turbulence present — or even produce turbulence when none would otherwise exist. On the other hand, if the fluid on top is less dense, turbulence is inhibited, because the buoyancy effects operate in the other direction and take energy out of the turbulence.

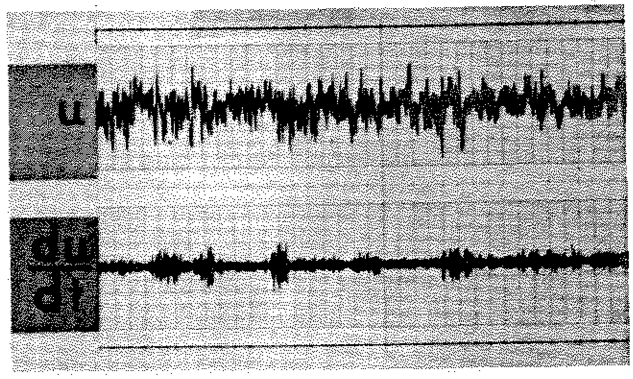
In the atmosphere both stable and unstable buoyancy effects occur frequently. In Fig. 12 we see a smoke layer in an atmosphere which is stable because the air close to the ground is colder and heavier than the air above it. This situation is called an *inversion* by meteorologists. Vertical turbulent motions are strongly inhibited and any motion which occurs tends to be almost horizontal. Smog can accumulate when an inversion at some height above the city prevents pollution from mixing upwards.

On the other hand, air close to the ground is often heated. This can produce vigorous convection, as shown in Fig. 13. Buoyant convection occurs nearly



12. **Smoke layer in a stable atmosphere. Because the air above is warmer and lighter than the air below, turbulence is greatly inhibited. All motion tends to be horizontal.**

13. **When the air is heated from below, convection is likely to result. In the case of cumulus clouds of this type, the convective activity is greatly enhanced by the release of latent heat when water vapor condenses into droplets to form a cloud.**



14. **Chart recordings of hot-wire anemometer measurements of the downwind turbulent velocity component in an atmospheric boundary layer. The upper trace shows the measured velocity. The distribution is very nearly Gaussian. The lower trace is the time derivative of the signal. (It is best to interpret this signal as a spatial derivative, since the time rate of change is mostly produced by the turbulent structure blowing past the probe, rather than by changes in the structure itself. Thus this signal is related to vorticity.) The distribution here is very intermittent, and clearly non-Gaussian. This effect increases with increasing Reynolds number.**

always when a fluid is heated from below, whether in a porridge pot or in the surface layers of the sun.

Small-scale Intermittency

In the defining syndrome of turbulence we did not employ the word "random," although it would seem to be apropos, because, to some at least, it carries with it the connotation of a Gaussian process. Turbulent distributions are more complicated than that. In Fig. 14 we see the output of a hot-wire anemometer operated in an atmospheric boundary layer. The large-scale motion, as shown by the horizontal velocity component *u*, is closely Gaussian. However, if we differentiate the signal, or examine any other property that is strongly dependent upon the small-scale motions, we find that activity seems to be distributed in concentrated bursts separated by regions which are comparatively quiescent. The effect is illustrated in Fig. 14.

The non-Gaussian, intermittent character of the small-scale structure becomes more marked as the Reynolds number increases. It seems to be fundamental to the nature of the turbulent cascade, but as with many other aspects of turbulence we do not have a fully satisfactory theoretical explanation. It is another manifestation of its baffling but fascinating complexity.

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