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National Committee for Fluid Mechanics Films

FILM NOTES

for

FUNDAMENTALS OF BOUNDARY LAYERS*

By

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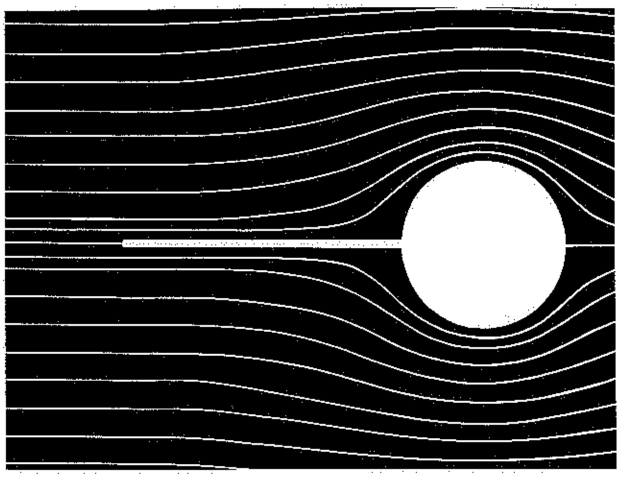
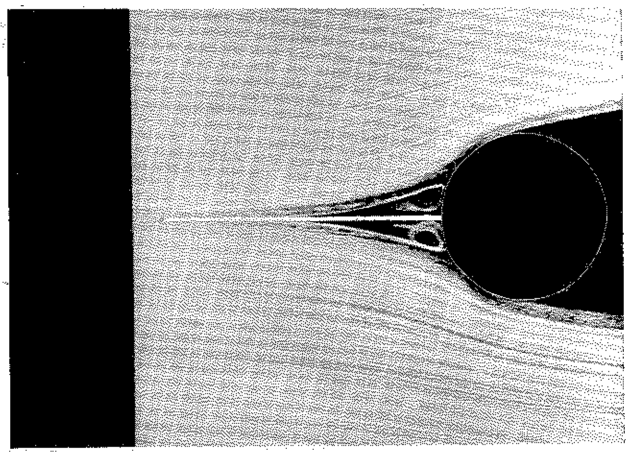
Introduction

In potential flows, which assume an ideal fluid without viscosity, only pressure and inertia forces determine the flow dynamics (Fig. 1). Real fluids do have viscosity, and the flow field can be very different (Fig. 2). *Boundary layers,* thin layers of fluid in which viscosity effects are significant, are formed along solid boundaries. In some cases these boundary layers, under the influence of pressure gradients, significantly affect the entire flow field.



1. Potential flow streamlines about a thin plate attached to a cylinder.



2. Hydrogen-bubble visualization of water flowing past the object in Fig. 1.

The streamline pattern of a real fluid, air, flowing past an airfoil at a small angle of attack (Fig. 3) is very nearly what one would predict from inviscid flow

***FUNDAMENTALS OF BOUNDARY LAYERS.** *A 16-mm B&W sound film, 25 minutes in length, was produced by Education Development Center under the direction of the National Committee for Fluid Mechanics Films, with the support of the National Science Foundation. Additional copies of the notes and information on purchase and rental of the film may be obtained from the distributor:*

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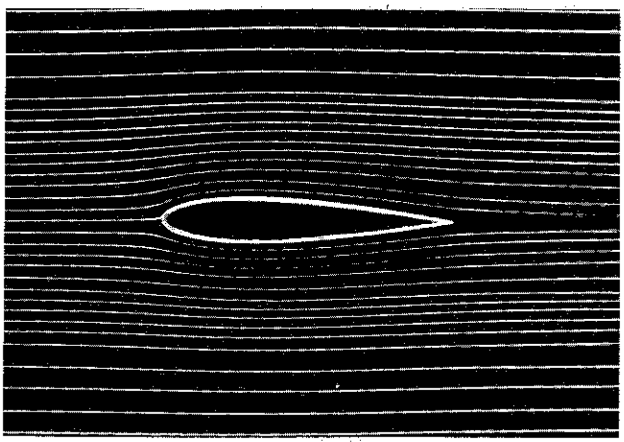
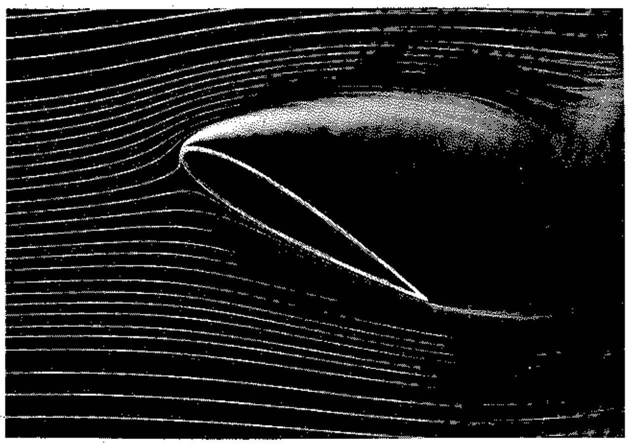


3. Smoke visualization of air flow past an airfoil at a small angle of attack.

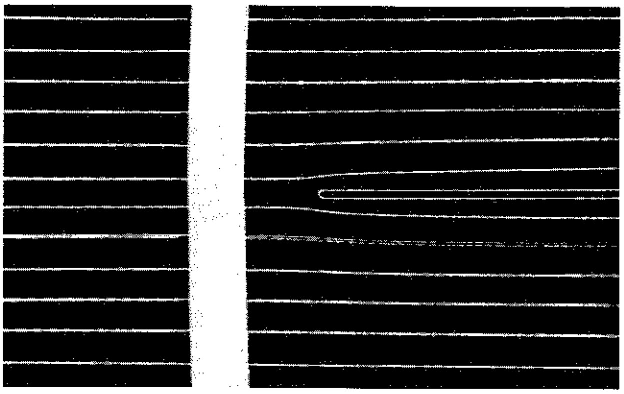
theory. Because the Reynolds number is large, the influences of viscosity are confined to a narrow region close to the surface of the wing. The primary effect of viscosity is to create a drag force on the wing through the integrated effect of surface shear stresses. When the angle of attack of the airfoil is increased, viscous effects become very pronounced and change the flow field in a qualitative manner. Pressure gradients imposed on the boundary layers become so large that separation of the boundary layer occurs on the upper surface (Fig. 4). A region of recirculating flow is formed over most of the upper surface of the wing, which is then said to be stalled. An understanding of how viscous forces can influence the entire flow field, as shown in Figs. 2 and 4, is intimately related to an understanding of the behavior of boundary layers. The film shows the causes of boundary layers, how they grow, how they respond to pressure gradients, and the differences in the behavior of laminar and turbulent boundary layers.



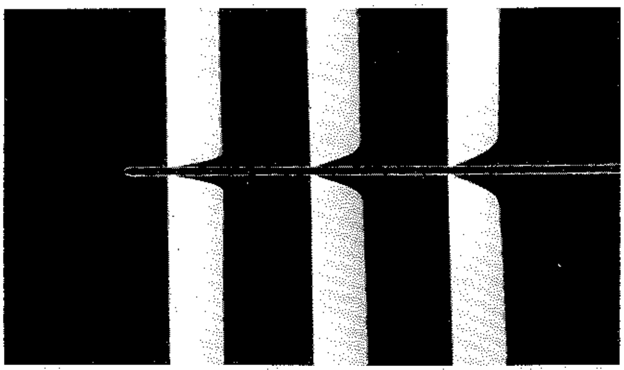
4. Same airfoil, at a large angle of attack.

Flow Along a Flat Plate

We first examine a boundary-layer flow where pressure gradients are negligible — two-dimensional uniform flow over a long flat plate (Figs. 5 and 6). The flow is visualized by hydrogen bubbles generated by electrolysis along wires oriented perpendicular to the plate. Upstream of the plate the front and back edges of hydrogen-bubble patches remain perpendicular to the streamlines, showing that the flow is uniform and free of vorticity (Fig. 5). Downstream of the leading edge of the plate the flow is still uniform and free of vorticity except in a narrow region adjacent to the plate (Fig. 6). This narrow region containing vorticity is the viscous boundary layer. In this layer, both viscous forces and inertial forces are important. Outside of this boundary layer viscous forces can be neglected.



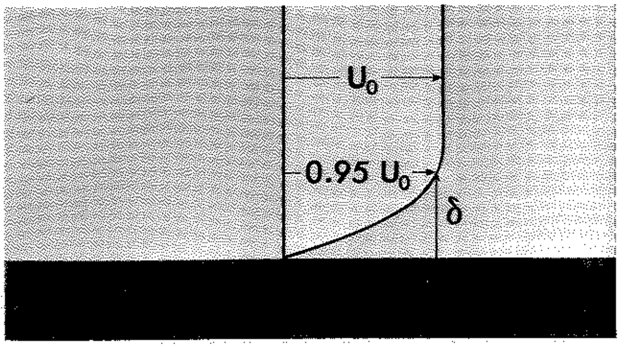
5. Flow approaching a flat plate in a water channel.



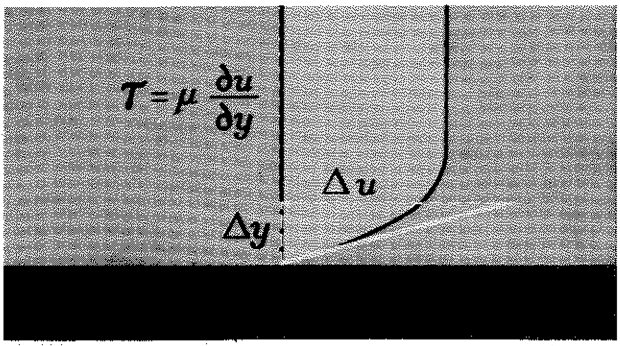
6. Timelines produced at wires perpendicular to the plate correspond closely to velocity profiles.

The experimental fact that there is no slip between the plate and the layer of fluid immediately adjacent to it is shown in Fig. 6. The velocity of the liquid at the surface of the plate is zero. This is called the *no-slip boundary condition* of viscous flow.

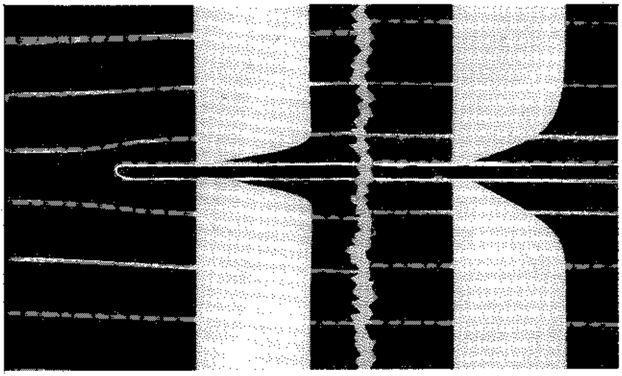
The thickness of the boundary layer increases along the length of the plate. Physically, fluid deceleration is transferred successively from one fluid layer to the next by viscous shear stresses acting in the layers. The boundary-layer thickness is sometimes defined as the distance, δ, from the surface to where the velocity, U, reaches some fixed percentage (say 95%) of the free stream value (Fig. 7a). The local shear stress, τ, is related to the velocity gradient normal to the surface



7a. Definition of boundary-layer thickness δ.



7b. Relationship of shear stress τ to velocity gradient at wall.

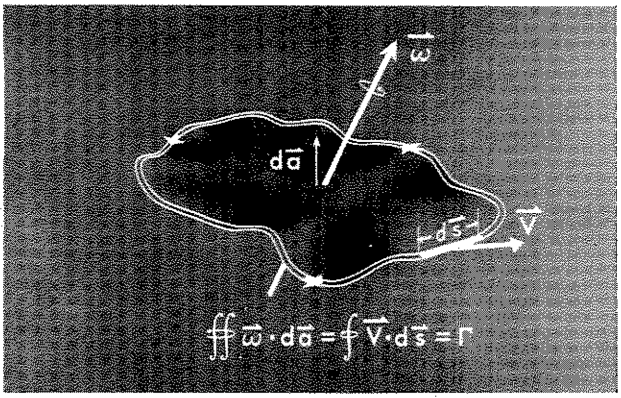


8. Boundary-layer velocity profiles near the leading edge (left) and far downstream (right).

by $\tau = \mu \frac{\partial u}{\partial y}$, where μ is the fluid viscosity (Fig. 7b). The composite photograph of Fig. 8 compares the velocity profiles at upstream and downstream stations along the plate. The velocity gradient at the wall is less downstream than upstream, indicating that the wall shear stress decreases along the plate.

One way to understand the mechanism of boundary-layer growth is to consider the time history of the vorticity within the boundary layer. Stokes' theorem states that the area integral of the vorticity vector, ω, bounded by a closed contour, is equal to the line integral of the velocity vector around the bounding contour, which is called the circulation, Γ. (See Fig. 9.)

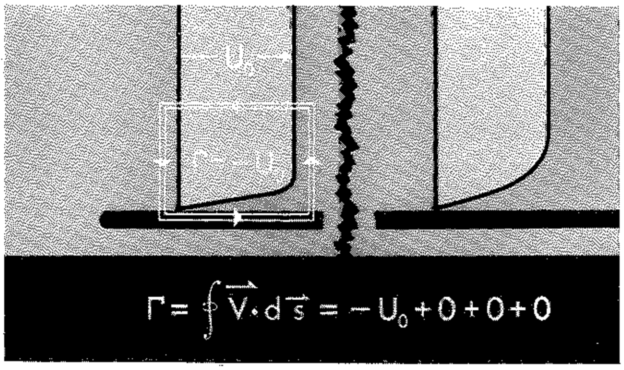
$$\oiint \omega \cdot d\mathbf{a} = \oint \mathbf{V} \cdot d\mathbf{s} = \Gamma$$



9. The area integral of vorticity ω equals the line integral of the velocity around the bounding contour.

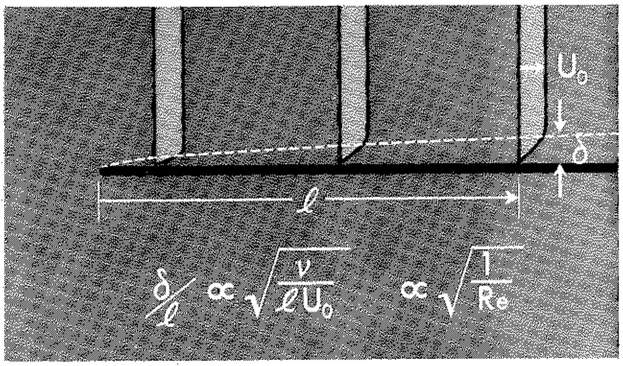
In other words, the circulation around a closed contour is the sum of the vorticity enclosed within it. The contour shown in Fig. 10 is at the upstream station; it is of unit length along the plate, and more than a boundary-layer thickness high. The free-stream velocity is parallel to the top of the contour but is directed in the opposite sense. This contributes $-U_o$ times a unit length to the value of the circulation. The components of vertical velocity along the right and left parts of the contour are virtually zero and, because there is no slip, the velocity contribution to the circulation at the surface is exactly zero. Therefore, the total circulation is $-U_o$ times a unit length. At any downstream station the circulation is also equal to $-U_o$ times a unit length. Therefore the total amount of vorticity within each contour is the same. Because there is no vorticity upstream of the plate and because the circulation per unit length along it is constant, we conclude that all of the vorticity in the boundary layer is introduced at the leading edge as a consequence of the no-slip boundary condition.

Even though the total amount of vorticity contained in the boundary layer per unit length of the plate is the same, the distribution of vorticity normal to the plate does change along its length. Viscosity acts, through the mechanism of molecular diffusion, to spread the vorticity transversely as it is convected downstream. The local boundary-layer thickness can be thought of as a measure of the distance vorticity has



10. Evaluation of the circulation per unit length about a contour at the upstream station.

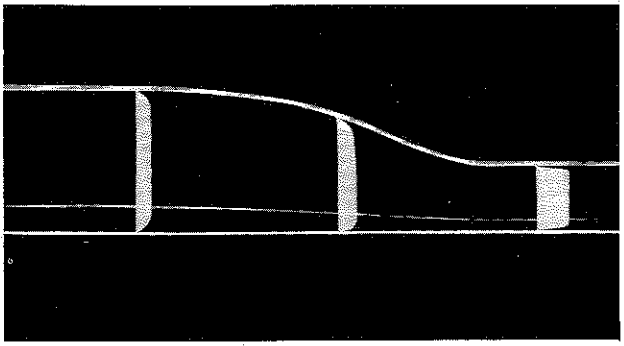
diffused away from the plate. We can relate the factors controlling this growth process in the following approximate way by considering that the transverse diffusion length, δ, is of the order of $\sqrt{\nu t}$, where ν is the kinematic viscosity and t is the time of diffusion. At a distance l from the leading edge, the time during which vorticity has diffused is approximately $t \simeq l/U_0$ (Fig. 11). Thus $\delta/l \propto \sqrt{\frac{\nu}{lU_0}} = \sqrt{\frac{1}{\text{Re}}}$. This relationship is valid only at high Reynolds numbers, where $\delta/l << 1$. As an example of the Reynolds-number dependence, note that increasing the flow velocity decreases the boundary-layer thickness at a given station along the plate. With a higher main-stream velocity, at any position along the plate the boundary layer thickness is less because it has had less time to grow.



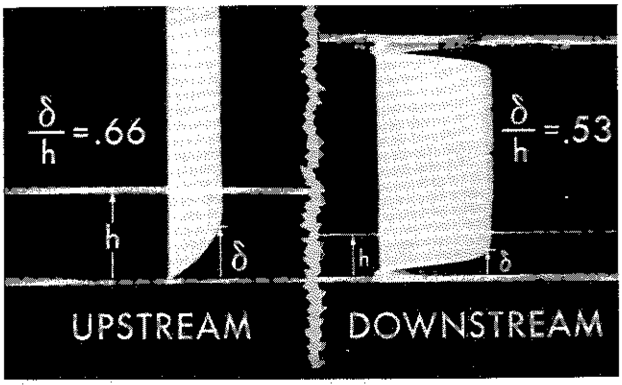
11. Boundary-layer growth along a flat plate.

Favorable Pressure Gradients

The pressure gradients in the flow direction along the flat plate in Figs. 5 and 6 were negligibly small. In most other flow situations there are regions of decreasing pressure and regions of increasing pressure in the flow direction. By using the two-to-one contracting flow channel of Fig. 12 and observing the behavior of the boundary layer along the flat side, we can examine the effects of a pressure distribution which decreases in the flow direction (a *favorable* pressure gradient).



12. Flow in a converging channel (two-to-one contraction ratio).



13. Composite blowup of upstream and downstream boundary-layer profiles from Fig. 12.

The boundary layer upstream of the contracting portion of the flow channel is much thicker than the boundary layer emerging from it (Figs. 12 and 13). Most of this decrease in boundary-layer thickness through the flow contraction is attributable to the two-to-one decrease in flow area. However, using the local distance h from the lower wall to the nearby streamline as a reference dimension (Fig. 13), we see that the boundary-layer thickness relative to this dimension has also decreased. This decrease in relative thickness of the boundary layer can be explained using the vorticity arguments just developed.

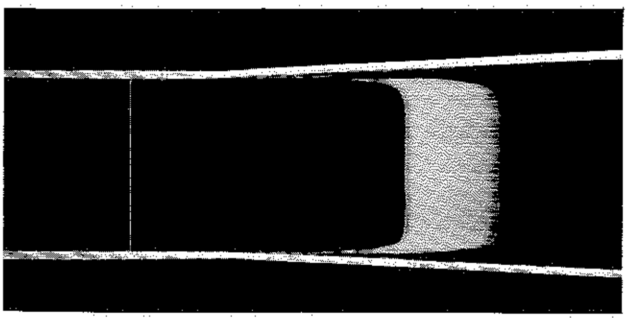
The amount of vorticity contained in a contour of a unit length along the plate and of height h is twice as large downstream as it is upstream because the free-stream velocity has doubled through the contraction. This new vorticity is of course added to the boundary-layer fluid at the wall. It is as though a new boundary layer were being created within the older one at each increment along the way. The combined profile at the exit is relatively thinner because there has been little time for lateral diffusion of the new vorticity in the boundary layer. Downstream, therefore, a larger percentage of the total vorticity is near the wall than upstream. This results in a relatively thinner boundary layer.

Instead of discussing vorticity concentration and diffusion, the same conclusions can be reached using force arguments. At each differential increment in distance along the contracting portion of the channel the pressure gradient causes a corresponding incremental increase to the main flow velocity, according to Bernoulli's equation. In the outer portions of the boundary layer, where changes in shear stress are small, the velocity increases by almost the same increment. It is only very near the wall that the incremental increase in velocity is substantially different from the free-stream value. The fluid velocity at the wall remains zero because of the no-slip condition. It is as though a new and therefore thin boundary layer were being added to the existing one at each step along the contraction.

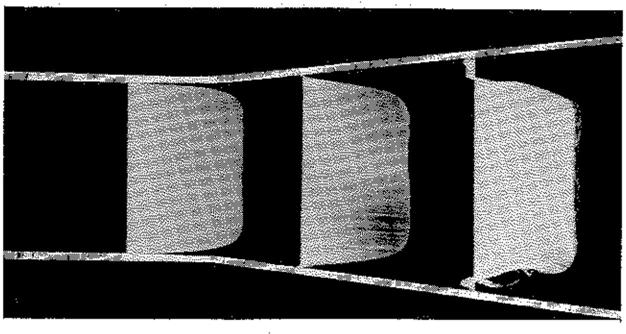
The integrated effect is to enhance the already high shear stress near the wall and decrease the lateral distance required for the velocity to attain 95 per cent of the free-stream value.

Unfavorable Pressure Gradients

In the slightly divergent channel (a *diffuser*) of Fig. 14, the free-stream static pressure *increases* in the flow direction, thereby subjecting the wall boundary layers to a positive (or *unfavorable*) pressure gradient. If the unfavorable gradient is small enough (as it is in the flow of Fig. 14), then the increasing pressure in the free stream causes a corresponding decrease in the free-stream velocity, increases the boundary-layer thickness, and decreases the wall shear stress, *without causing flow separation.* These effects can be deduced from either of the two arguments used in the previous section. Using pressure and velocity arguments it follows that the positive pressure gradient decreases the free-stream velocity and decreases the boundary-layer velocity by almost the same increment except very near the wall. The size of the increment decreases rapidly near the wall and must be zero at the wall. A major consequence of this incremental decrease in velocity is to decrease the velocity gradient or shear stress at the wall. This change in boundary-layer profile can be seen by comparing the profiles at the first two stations of Fig. 15.

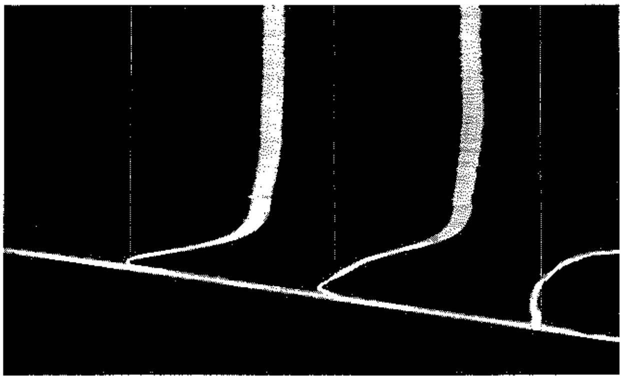


14. Flow in a small-angle diffuser.



15. A diverging channel with a larger diffuser angle.

The deceleration of the flow imposed by a positive pressure gradient cannot be very large or sustained too long by the boundary-layer fluid without the wall shear stress going to zero, followed downstream by local flow reversal. For the small-angle diffuser of Fig. 14, the positive pressure gradient is very small, and no flow separation occurs. The large-angle diffuser of Fig. 15 imposes a larger positive pressure gradient which the boundary layer cannot sustain without separating from the wall between the second and third stations in Fig. 15. At the second station the flow near the wall is to the right, while at the third station the flow near the wall is to the left. The point on the wall where the fluid in the upstream boundary layer meets the fluid from the region of flow reversal is called the separation point. The wall shear stress is zero there. Downstream of this point the fluid which was in the upstream boundary layer is no longer in contact with the wall, and is separated from it by the region of reversed or recirculating flow. The boundary layer is said to have *separated.* The point of separation of the laminar boundary layer in Fig. 16 is just upstream of the first bubble wire. A comparison of the flow fields of Figs. 3 and 4 illustrates the enormous changes that boundary layer separation can cause.



16. Bubbles generated at three wires in the downstream section of the diffuser of Fig. 15 show reversed flow near the wall.

Laminar to Turbulent Transition

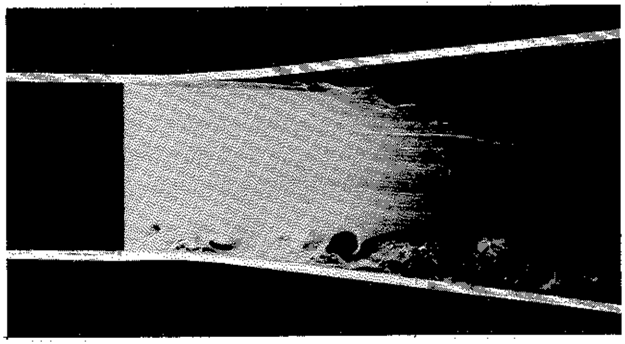
In most practical situations the Reynolds number is large and the boundary layers are turbulent rather than laminar. Stages in the transition from a laminar to a turbulent boundary layer are shown in Fig. 17. In Fig. 17 a slight adverse pressure gradient causes transition to occur within the field of view. The steps in the transition are complicated and interdependent. First, there is the growth of nearly two-dimensional waves, Tollmien-Schlichting waves, followed by the appearance and growth of three-dimensional disturbances, which contain streamwise vorticity. Further downstream turbulent spots can be seen. Finally, fully turbulent flow appears.

The transition process is influenced by many factors: free-stream disturbances, plate roughness, pressure gradients, vibration, sound, etc. Therefore, the position where the transition process starts varies with time in a random way.

17. Side view of a long cylinder* with its axis aligned with an air flow. The faired nose of the cylinder is out of view to the left (upstream). A sheath of smoke generated upstream develops patterns which show stages of boundary-layer transition. (Courtesy F. N. M. Brown, University of Notre Dame.)

***The cylinder appears to be tapered because the camera is looking at a slight angle upstream.**

Placing an obstruction in a boundary-layer flow stimulates the naturally occurring processes and hastens the onset of transition. In Fig. 18, the boundary layer on the lower wall of the diffuser has been made turbulent by inserting a trip rod upstream. The turbulent boundary layer is able to withstand the adverse pressure gradient in the diffuser and does not separate, while the laminar boundary along the top wall is separated, with reverse flow along the wall (Fig. 18a). In the turbulent boundary layer the flow is downstream (compare Figs. 18b and 16), and no flow reversal is evident.

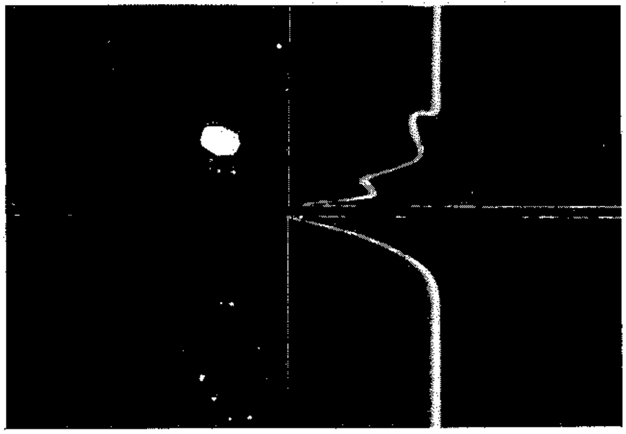


18a. Flow in channel of Fig. 15 with lower boundary layer made turbulent.

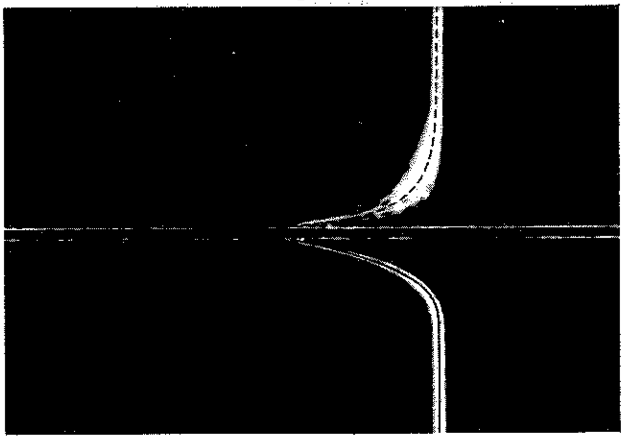
18b. Three bubble wires show unseparated, turbulent flow along the bottom wall.

A Turbulent Boundary Layer Along a Flat Plate

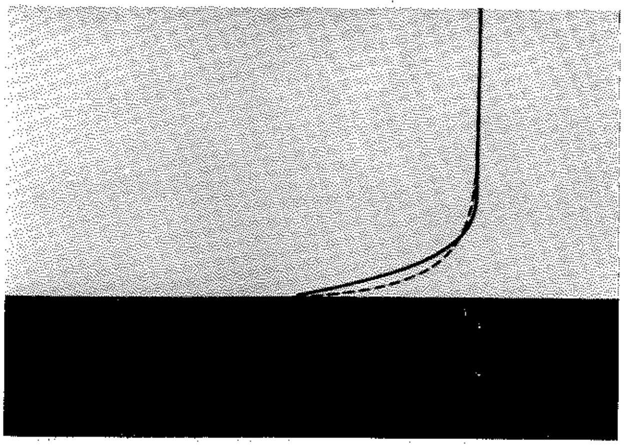
Laminar and turbulent boundary layers are different, and the differences explain why a turbulent boundary layer is able to withstand without separating a larger unfavorable pressure gradient than a laminar boundary layer. Consider again the flow along a long flat plate. In Fig. 19, the boundary layer on the lower side is laminar and two-dimensional; the boundary layer on the upper side has been tripped by a wire upstream and is turbulent. The motions in the turbulent boundary layer are unsteady and three-dimensional. Some motions are perpendicular to the plane of view. Because the displacement of a bubble line corresponds closely to an instantaneous velocity profile, superimposing a number of individual displacement lines provides a method of obtaining a mean velocity profile for the turbulent layer. The superposi-



19. Instantaneous displacement profiles for flow along a thin plate. The boundary layer on the upper surface has been made turbulent, while the flow along the lower surface is laminar.



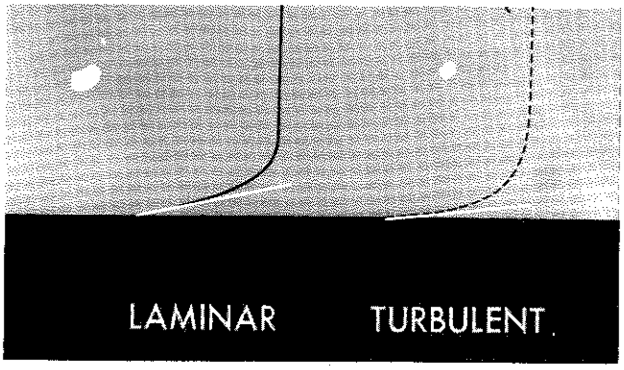
20a. The upper boundary layer is turbulent; the lower, laminar. Superposition of many instantaneous velocity profiles suggests mean velocity profiles.



20b. The mean laminar (solid) and turbulent (dashed) profiles are compared.

tion also gives an experimental notion as to where the turbulent fluctuations occur and how large they are in the plane of mean motion. Figure 20a was constructed by such a superposition — Fig. 20b compares the mean laminar and turbulent profiles.

The velocity gradient perpendicular to the plate is larger for the turbulent layer than for the laminar layer (Fig. 21), and therefore the turbulent layer has the larger wall shear stress or drag. The circulation is the same for both layers, since the free-stream velocity is the same. Both boundary layers therefore contain the same total amount of vorticity per unit length of the plate. However, the distributions of vorticity in the two layers are very different. In the turbulent layer more vorticity is concentrated near the plate, even though some vorticity has also spread farther from the plate (Fig. 20b).



21. Velocity gradients for the profiles are compared.

The distribution of momentum in the two boundary layers is also different. In the turbulent layer high-momentum fluid is transported toward the plate, and low-momentum fluid is transported away from the plate, by unsteady random rotary motions associated with vorticity aligned in the flow direction. There is more momentum near the wall in the turbulent boundary layer, even though the turbulent boundary layer is thicker. In the diffuser experiment (Figs. 18a and b), the extra momentum near the wall in the turbulent boundary layer along the bottom wall enabled it to withstand the unfavorable pressure gradient without separating.

Similarly, turbulent boundary-layer flow on the upper surface of an airfoil delays large-scale separation, or stall, until higher angles of attack are reached. Vortex generators, small blades set perpendicular to the surface of airplane wings, are often used to delay the onset of separation. They are so named because they introduce additional axial vorticity which enhances the naturally occurring rotary momentum interchange in already turbulent boundary layers, and thereby increase the momentum of the fluid near the surface.

Summary

At large Reynolds numbers, boundary layers, thin layers of fluid in which viscosity effects are significant, are formed along solid boundaries, because viscous fluids cannot slip at solid boundaries. In the absence of pressure gradients the boundary layer along a flat surface increases in thickness as $l\sqrt{\frac{1}{\mathrm{Re}}}$. Negative (or favorable) pressure gradients in the flow direction, which accelerate the flow, decrease the boundary-layer thickness and increase the velocity gradient at the wall. Positive or unfavorable pressure gradients tend to decelerate the flow, to increase boundary-layer thickness, and to decrease the velocity gradient at the wall. Unfavorable pressure gradients can cause boundary-layer separation, which often results in drastically altered flow patterns and losses in performance of such devices as airplane wings and diffusers.

At relatively low values of Reynolds number, boundary layers tend to be laminar. At higher Reynolds numbers, a boundary layer is unstable to small disturbances. The disturbances grow, resulting in transition to a turbulent boundary layer. Most practical flow situations involve high Reynolds numbers and turbulent boundary layers. Because of three-dimensional interchanges of momentum, a turbulent boundary layer is thicker and has a larger wall velocity gradient than a laminar layer at the same Reynolds number. The increased momentum near the wall allows a turbulent boundary layer to withstand a larger unfavorable pressure gradient than a laminar layer without separating, but results in higher wall shear stress and drag.

References

1. **Schlichting, H., *Boundary Layer Theory,* McGraw-Hill, 1960.**
2. **Hazen, D. C., "Boundary-Layer Control" (An NCFMF Film).**
3. **Goldstein, S., *Modern Developments in Fluid Mechanics,* Dover, 1965.**