National Committee for Fluid Mechanics Films

FILM NOTES
for
CHANNEL FLOW OF A COMPRESSIBLE FLUID*

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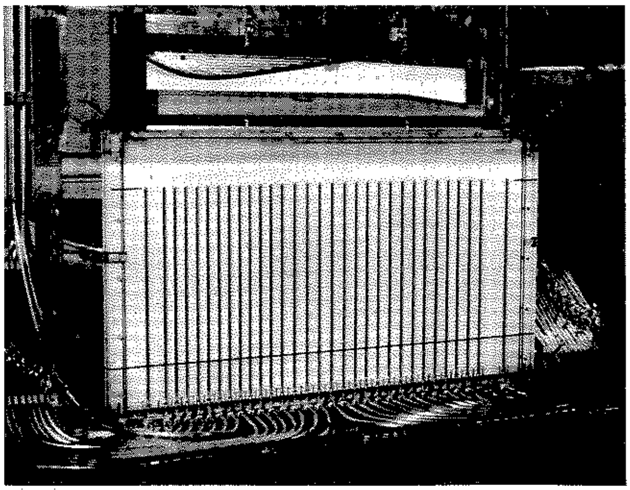
Introduction

The purpose of this film is to demonstrate several effects of compressibility which are important in any attempt to produce or control a supersonic internal flow. The heart of the subject is the behavior of a compressible gas flowing at high speed through a converging-diverging nozzle. The film therefore begins with the phenomenon of *choking,* or sonic flow at a throat. Some examples of shock waves and expansion waves are shown in passing. Finally, the film takes up the problem of bringing a supersonic stream efficiently to rest.



1. **Converging-diverging channel and manometer board at Aerophysics Laboratory, M.I.T.**

So that the whole flow can be viewed at one time, the demonstrations are performed in a channel small enough to fit within the 30-inch diameter beam of a conventional schlieren optical system. The apparatus is shown in Fig. 1. Room air enters the channel at the left and flows from left to right through the channel, through a control valve, and into a vacuum pump downstream. Each column on the manometer board shows the static pressure at the point directly above on the flat lower wall of the channel. The manometer is so connected that the mercury level rises when the pressure in the channel rises. The first column always shows the upstream stagnation pressure (the pressure in the room), and the last column always shows the downstream stagnation pressure, just ahead of the valve.

The manometer display is readily interpreted with the aid of the one-dimensional momentum equation for steady flow. If viscous forces are neglected, the fluid

CHANNEL FLOW OF A COMPRESSIBLE FLUID, a 16-mm B&W sound film, 29 minutes in length, was produced by Education Development Center, Inc. (formerly Educational Services Incorporated) under the direction of the National Committee for Fluid Mechanics Films, with a grant from the National Science Foundation. Information on purchase and rental may be obtained from the distributor:

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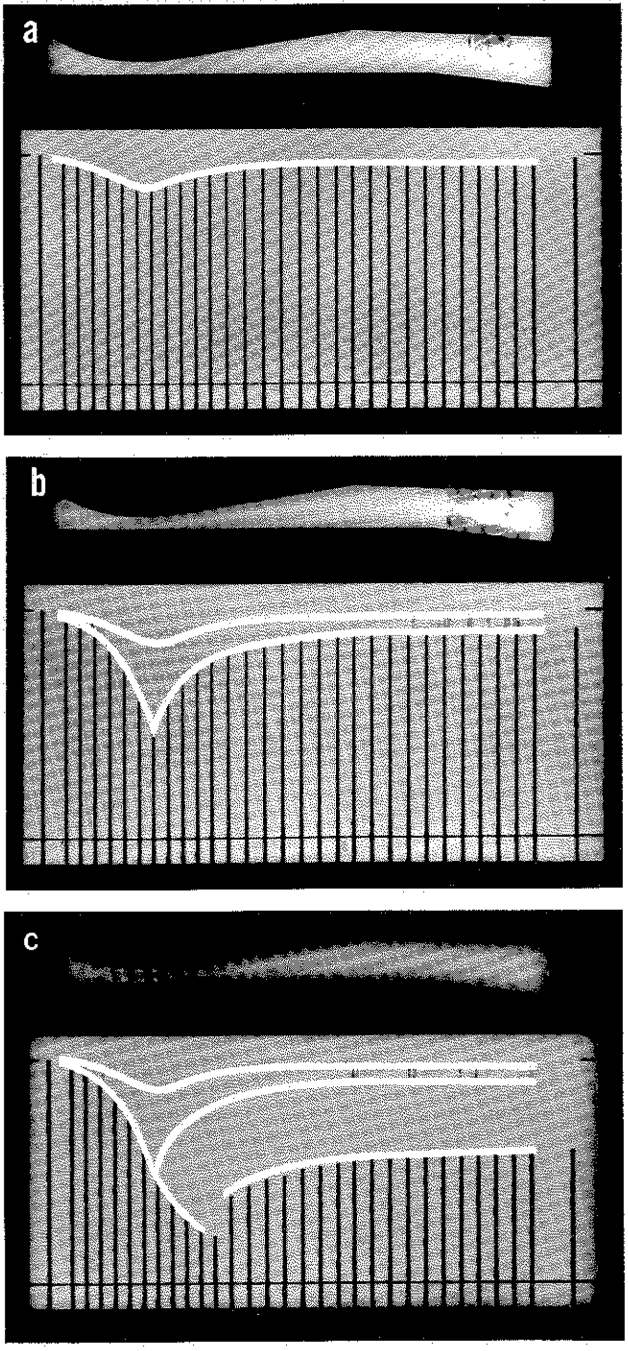
acceleration depends on pressure forces alone:*

$$\rho V dV + dp = 0 \qquad \text{(Eq. 1)}$$

Consequently, changes in pressure and velocity always have *opposite* sign, no matter what happens to the density. A falling pressure always means a rising velocity, and vice versa.

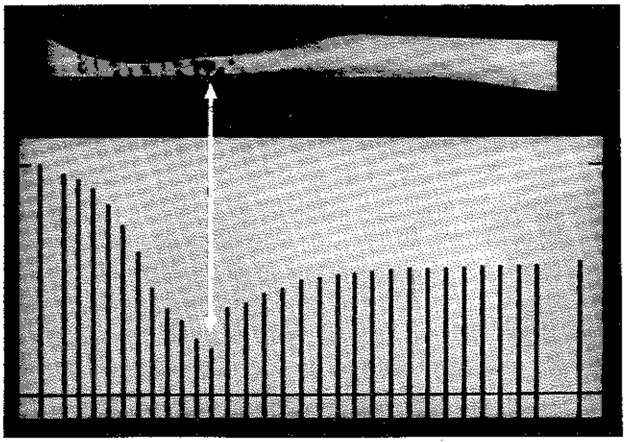


2. (a) Subsonic flow in entire channel; note symmetry of pressure distribution about the throat. (b) Choking at throat; note corner in pressure distribution. (c) Supersonic flow downstream of throat; pressure distribution no longer symmetrical. Farther downstream, the pressure rises abruptly. Note large loss of stagnation pressure.

Flow Near a Throat

At low rates of flow, the highest velocity in the channel is at the point of minimum area, called the throat (Fig. 2a). As this throat velocity is increased by opening the valve, a critical flow rate is reached, and the pressure distribution develops a sharp corner at the throat (Fig. 2b). As the valve is opened further, the symmetry disappears (Fig. 2c), and the pressure and velocity upstream of the throat no longer change. Just downstream of the throat, the falling pressure means that the velocity is increasing in the diverging part of the channel. Then the pressure rises almost discontinuously, but does not return to its original upstream value.

To show what is happening in the channel, the schlieren optical system* can be used to make density variations visible. As the valve is opened, a *shock wave* is seen to appear at the throat and to move into the diverging part of the channel. The position of the shock wave matches the discontinuity in pressure (Fig. 3).



3. Schlieren visualization shows that position of shock wave matches discontinuity in pressure distribution. Subsonic flow exists downstream of shock wave and supersonic flow exists between throat and shock wave.

The appearance of a shock wave in the channel means that the flow is locally *supersonic*. The difference between the symmetrical flow in Fig. 2a and the unsymmetrical flow in Fig. 2c is that the velocity in the latter case is *sonic* at the throat. This transition

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*Strictly speaking, Eq. (1) holds (except at discontinuities) along each streamline in steady inviscid flow. Provided that the streamline radius of curvature is large compared to the lateral extent of the flow region of interest, lateral changes in p, ρ, V, etc. can be neglected compared to changes in the direction of the flow. Some local effects of streamline curvature for the case of an incompressible fluid are demonstrated in the film "Pressure Fields and Fluid Acceleration" (Ref. 1).

*Schlieren and other optical methods for flow visualization are described in many places; e.g., Ref. 2 & 3. For all the illustrations in these notes, the knife edge is parallel to the general flow direction. The boundary layers usually show up clearly, but the shock patterns in some cases do not. Other knife-edge positions could not be used because the thickness of the glass side walls, which were also in the schlieren beam, was highly non-uniform in the flow direction.

to sonic flow at the throat, with all of its consequences, is called *choking.*

Choking

To see why choking plays an important part in compressible channel flow, consider the one-dimensional momentum equation, Eq. (1), for steady flow without viscosity. For isentropic flow, this relation becomes

$$\frac{d\rho}{\rho} + M^2 \frac{dV}{V} = 0 \qquad \text{(Eq. 2)}$$

where M is the Mach number (the ratio of the velocity of the fluid to the local speed of sound).

For nearly parallel flow, changes in density and velocity can be related to changes in channel area by the one-dimensional continuity equation $\rho AV =$ constant, or, in differential form,

$$\frac{d\rho}{\rho} + \frac{dV}{V} + \frac{dA}{A} = 0 \qquad \text{(Eq. 3)}$$

At the throat, where the area is locally constant, the last equation becomes

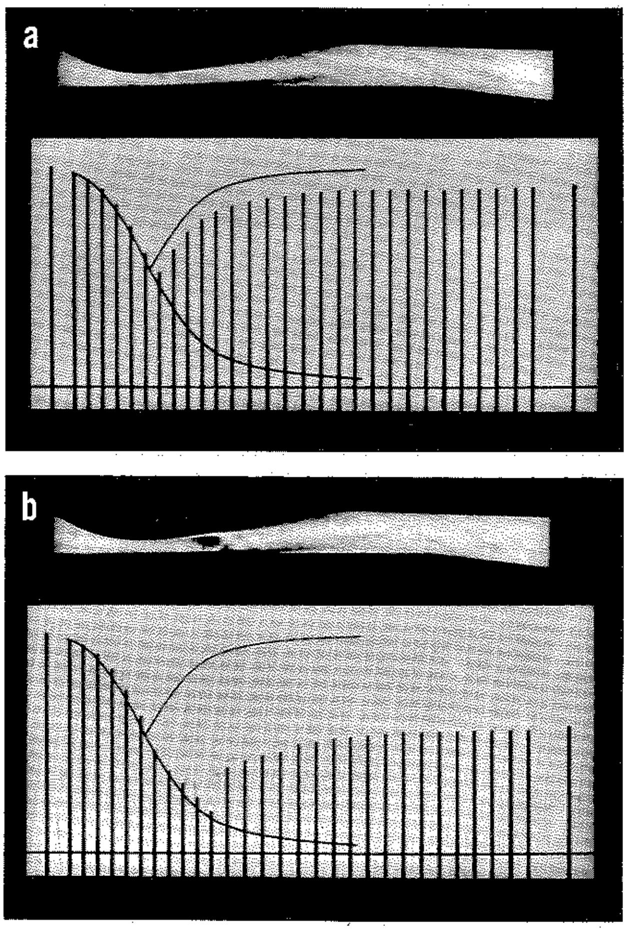
$$\frac{d\rho}{\rho} + \frac{dV}{V} = 0 \quad \text{(at throat)} \quad \text{(Eq. 4)}$$

Eqs. (2) and (4) are a pair of homogeneous algebraic equations for $d\rho$ and dV at a throat. If the Mach number at the throat is *not* equal to one, the two equations can only be satisfied if $d\rho$ and dV are zero. That is, the velocity, density, and pressure are locally constant at the throat, and must be going through a maximum or a minimum there. The flow has a kind of local symmetry about the throat (Fig. 2a).

On the other hand, if the Mach number at the throat is *equal* to one, then Eqs. (2) and (4) become identical. *This is the special case called choking.* Since both $d\rho$ and dV cannot be determined from a single equation, they need no longer be zero at the throat. The velocity, density, and pressure can increase or decrease continuously through a sonic throat, and the flow need not be symmetrical. In particular, the highest velocity need not occur at the throat (Fig. 2c).

Subsonic and Supersonic Flow

To illustrate these ideas, and to test the accuracy of the one-dimensional approximation, a theoretical pressure distribution can be computed* for the choked channel (Figs. 4a, 4b). The intersection of the curves marks the conditions for which air, accelerating from rest, should reach sonic speed ($p/p_o = 0.528$).



4. (a) Comparison of theory and experiment for subsonic flow just at choking. (b) Comparison of theory and experiment for supersonic flow after choking.

Downstream of the throat, there are two theoretical curves. The upper branch is *subsonic,* with the Mach number decreasing from one toward zero as the channel area increases. Such a subsonic flow behaves qualitatively as if the density were constant. The continuity equation, Eq. (3), is then satisfied (to a first approximation) by a balance between *area* and *velocity.* In a subsonic flow, the velocity and area change in opposite directions.

The small channel can also be used to demonstrate some properties of supersonic flow. As an aid in flow visualization, the flat wall of the channel is roughened with strips of plastic tape. There is no visible effect until the shock wave appears. Then characteristic disturbances, called Mach waves, appear in the region between the throat and the shock wave (Figs. 5a,

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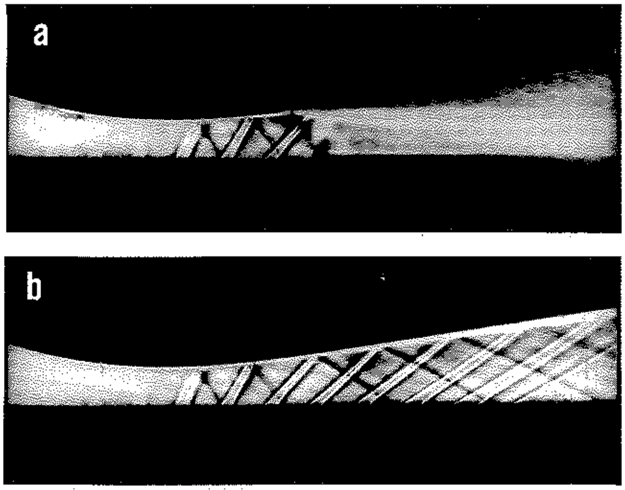
*The details of this calculation (for one dimensional isentropic flow in a channel of specified area) are described in any standard text on compressible fluid flow; e.g., Ref. 3 or Ref. 4. The nozzle used to illustrate choking in the early part of the film, incidentally, has one flat wall and is best described as a half nozzle. This geometry allows pressure changes along the nozzle to be spread out sufficiently so that details of the flow behavior near the throat can be easily seen.



5. (a) Wave patterns confirming existence of supersonic flow downstream of throat. (b) The same flow as in (a) but with shock wave farther downstream.

5b).* These Mach waves, which originate at the edges of the plastic tape, are evidence of supersonic flow in the diverging part of the channel. So is the pressure distribution (Fig. 4b), because at first the pressure in the diverging part of the channel is decreasing, and hence the velocity is increasing [cf. Eq. (1)]. Downstream of the shock wave, the pressure is increasing and the velocity is decreasing in the same diverging channel, showing subsonic flow. The supersonic region ends at the shock wave.

The Mach waves and the pressure distribution thus confirm that the lower theoretical branch downstream of the throat (Fig. 4b) is *supersonic,* with the Mach number increasing from one as the channel area increases. To help in understanding this experimental fact of acceleration in a diverging channel, think of the extreme case where the Mach number is so large that the velocity is close to its maximum value for the energy available. But if the velocity is almost constant, the continuity equation, Eq. (3), is satisfied (to a first approximation) by a balance between *area* and *density*. The density goes down as the area goes up; the gas *expands* to fill the channel.

These results can be summarized in a single equation, obtained by eliminating $d\rho$ between Eqs. (2) and (3),

$$\frac{dV}{V} = \frac{1}{M^2 - 1}\,\frac{dA}{A} \qquad \text{(Eq. 5)}$$

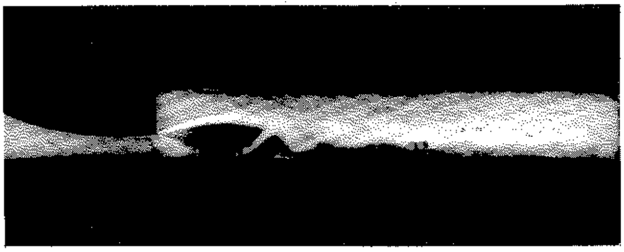
Eq. 5 is equivalent to the statement that *for steady acceleration from subsonic to supersonic velocity the area of the channel — or of any stream tube — must first decrease and then increase. There must be a throat. Since the fluid is accelerating at this throat, the Mach number there must be equal to one, and the channel is necessarily choked.*

Upstream Influence

The absence of upstream influence in a supersonic region is illustrated by the fact that the Mach waves in Fig. 5a are steady, although the shock wave itself is not steady.* When the downstream pressure in the choked channel is changed by the control valve, the shock wave moves, but there is no other influence on the flow upstream of the shock wave.** Because the area, velocity, and density are all fixed at the throat, so is the mass flow. A choked channel is therefore a convenient device for maintaining a specified mass flow from a reservoir which is held at constant pressure and temperature.

Flow at a Nozzle Exit

Even if the wall is cut away at some point downstream of the throat (Fig. 6), there is no change in the mass flow in a choked channel. As long as the back pressure, or chamber pressure, is lower than the nozzle exit pressure, downstream conditions do not affect the flow in the closed part of the channel. In Fig. 6, the Mach number is about 1.2 at the nozzle exit, and the flow is adjusting itself to low back pressure through an expansion wave. A nozzle operating under these conditions is said to be *underexpanded.*



6. Flow in underexpanded nozzle, with wall cut away in supersonic region.

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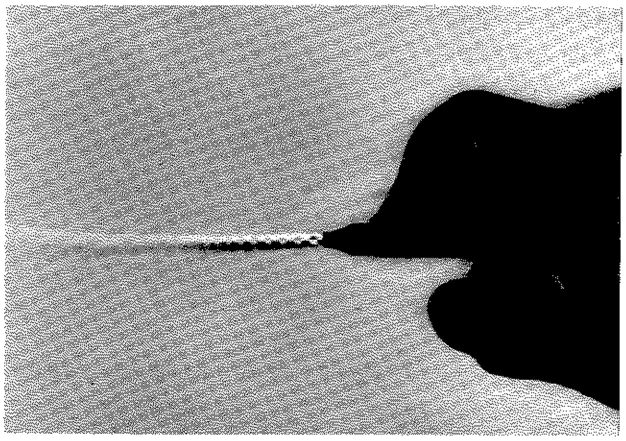
*Strictly speaking, Mach waves are infinitesimal disturbances, and therefore cannot be seen. The waves in Figs. 5a and 5b obviously have finite strength; in fact, they remain visible after reflection from the upper wall. Note also that the waves appear to become weaker in the downstream part of the channel. This is partly because the density (and hence the schlieren sensitivity) is decreasing rapidly in the downstream direction in the supersonic region; but it is also partly because the initial disturbances (produced at the base of the boundary layer on the lower wall) become weaker as the boundary layer increases in thickness.

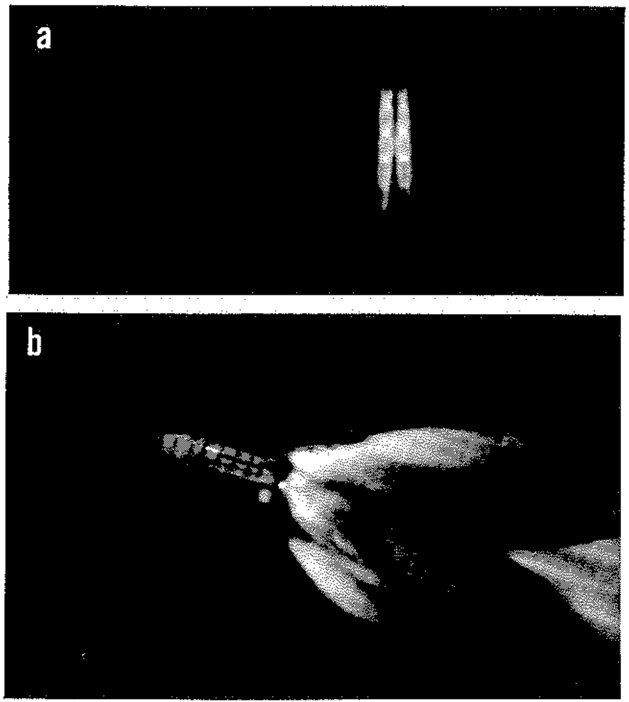
*The shock wave is not a single *normal shock,* but a complicated system of non-steady oblique shocks. This occurs because the boundary layers in the channel are relatively thick at the low Reynolds numbers which are typical of the small channel, and are separating from the walls. For more information about boundary layers and separation, see the films "Fundamentals of Boundary Layers," "Flow Visualization," and "Boundary-Layer Control" (Refs. 5-7).

**The sound track of the film includes, as background, the aerodynamic noise picked up by a microphone placed at the channel entrance. Noise from the valve and piping can only propagate upstream inside the channel if the entire flow is subsonic. The presence of supersonic flow is therefore accompanied by a decreased sound level.



7. **Wave pattern in jet from shop air hose and nozzle.**

Fig. 7 shows an ordinary air hose and nozzle from a machine shop. When the jet is viewed with a schlieren optical system, a typical pattern of expansion and compression waves appears. These waves are the mechanism of adjustment to the ambient pressure.



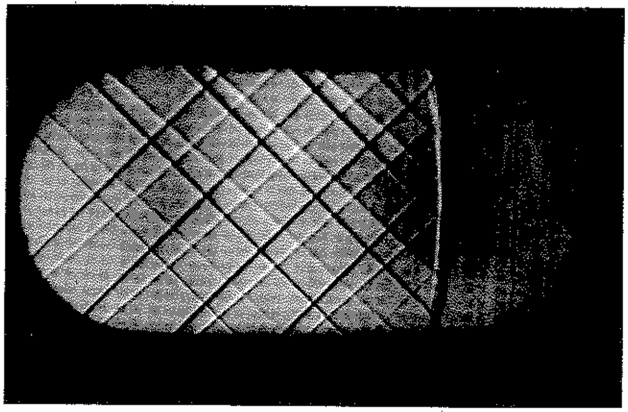
8. **(a) Rocket exhaust just after liftoff. (b) Rocket at 30 miles altitude. (Courtesy of NASA, Marshal S.F.C.)**

The same pattern can be seen in a rocket exhaust (Fig. 8a) when the outside pressure is not equal to the exit pressure. At very high altitudes, a poor match in pressure is unavoidable (Fig. 8b), and the expansion wave at the exit of an underexpanded nozzle may turn the flow through a spectacular angle.

Shock Waves

Converging-diverging nozzles are a means of *generating* supersonic flows. They can also be used to *slow down* supersonic flows. For example, large quantities of air may have to be brought almost to rest in the engine inlet passages of a supersonic air-breathing vehicle. Especially at high Mach numbers, the engine power is likely to depend strongly on the efficiency of this deceleration and compression process.

Without careful design, supersonic flow in a channel tends to slow down abruptly, through a strong shock wave, with a large loss in stagnation pressure. The deceleration in a normal shock wave (Fig. 9) is typically thousands of millions of gravities, and the word "shock" is certainly appropriate. There are very large gradients of velocity and temperature inside such a shock wave, and large losses in stagnation pressure due to viscous dissipation and heat conduction.



9. **Normal shock wave at a Mach number of 1.5.* (Courtesy of von Karman Gas Dynamics Facility, ARO, Inc., Tullahoma, Tennessee.)**

Supersonic Diffusion

Consider the problem of *controlling* a supersonic flow, in the sense of bringing it to rest as efficiently as possible. Just as a supersonic stream can be speeded up by increasing the channel area, it can be slowed down — at least in principle — by decreasing the area. A converging channel used for this purpose is called a *supersonic diffuser.* A suitable geometry for demonstrating this process in the small channel is shown in Fig. 10. Downstream of the inlet there is a short symmetrical nozzle, designed to give uniform flow at a Mach number of about 2.3 in the *test section* which follows. Then there is a supersonic diffuser, consisting of an adjustable *second throat* which can be opened or closed to vary the amount of supersonic compression. As a result, there are now *two* means of control over the flow in the channel; these are (a) the geometry of the second throat, and (b) the original control valve.

If the second throat is smaller than the first one, the

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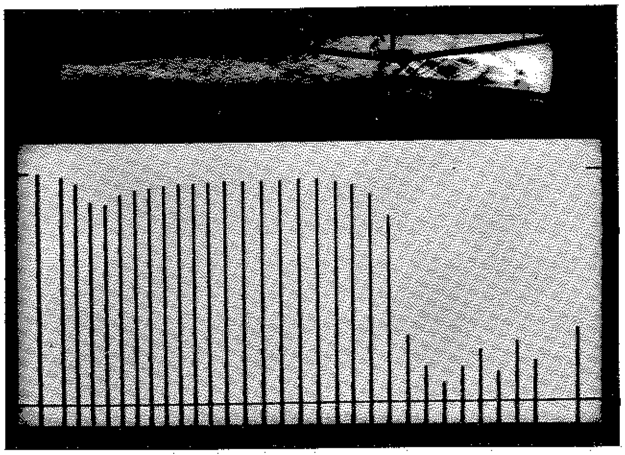
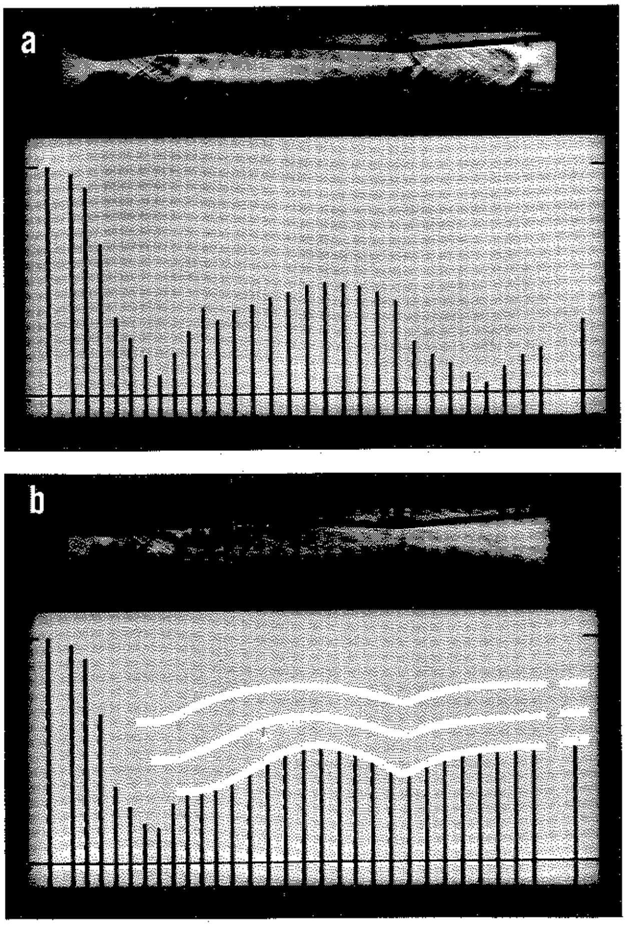
*This shadowgraph shows the central portion of a normal shock wave in a channel much larger than that of Fig. 1. The flow is therefore much less subject to boundary-layer separation and other adverse effects of viscosity. The Mach waves remove any uncertainty about the direction of flow.



10. Choking at a second throat.

channel chokes at the *second* throat as the valve is opened (Fig. 10). The upstream flow is subsonic and independent of downstream conditions. All that the control valve can do is move the shock wave in the region downstream of the second throat without affecting the flow upstream of the shock.

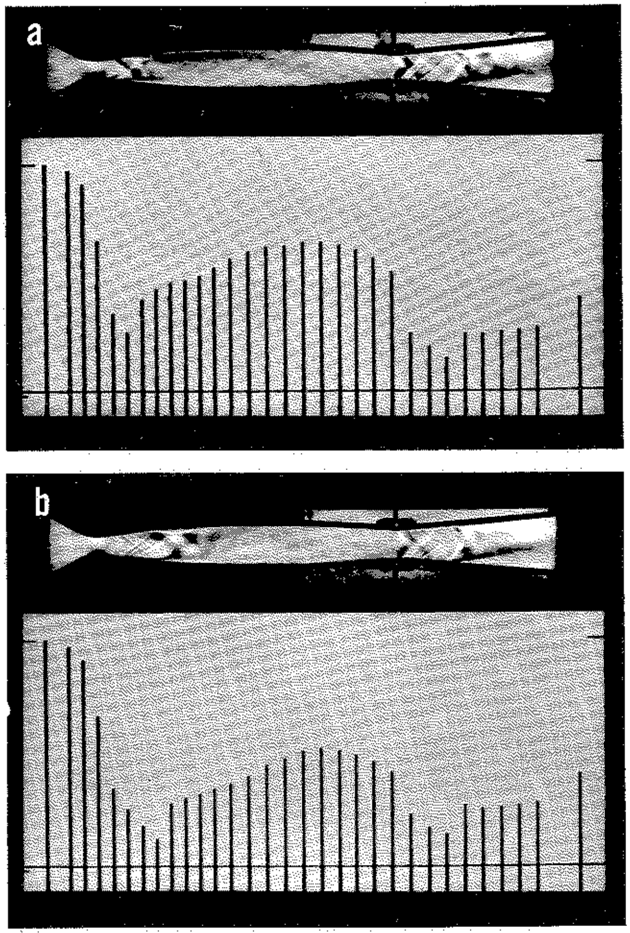
If the second throat is slightly larger than the first one, the channel chokes at the *first* throat as the flow rate increases, and a shock wave appears in the nozzle and moves downstream. Then the second throat chokes (Fig. 11a), and it is no longer possible to affect the first shock wave with the control valve. Under these conditions the channel is said to be *blocked,* with the first shock wave *trapped* in the supersonic nozzle.



11. (a) Blocking in a channel with two throats. (b) Evolution of pressure distribution during blocking as control valve is opened.

Blocking

To understand blocking, consider the original shock wave after it has formed just downstream of the first throat. This shock wave moves downstream as the control valve is gradually opened. The Mach number of the supersonic flow entering the shock wave therefore increases, and the dissipation of energy in the shock wave also increases. As a result, the downstream stagnation pressure gradually decreases. Several stages in the evolution of the pressure distribution are shown in Fig. 11b. Since the *mass flow* is fixed — because the first throat is choked — and since the *area* of the second throat is constant, the fact that the *density* is going down means that the *velocity* at the second throat must be going up. Finally the second throat goes sonic, or chokes, and the channel is blocked (Fig. 11a).

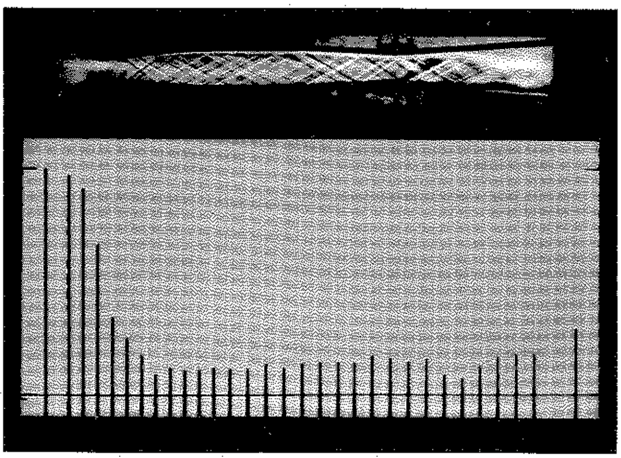


12. (a) Blocked channel with reduced second throat area; first shock has moved upstream (compare Fig. 11a). (b) Same flow with larger second throat area; first shock moves downstream.

The alternative means of control over the flow can be demonstrated by changing the area of the second throat while the channel is blocked (Figs. 12a, 12b). The shock wave in the nozzle moves, even though the motion of the walls is hardly visible. As long as the flow is sonic at the second throat, the density there has to change when the area does, because of the fixed mass flow. The shock wave in the nozzle must therefore move in such a way that the proper stagnation pressure and density through the second throat are maintained.*

Starting

To avoid blocking the channel, the second throat must be opened a little more, so that it will not choke even for the strongest possible shock wave (in this case at the nominal test-section Mach number of 2.3). As the control valve is opened, the first throat chokes, but the second throat does not. As the back pressure continues to decrease, the shock wave moves downstream in the channel . . . and suddenly jumps through the second throat to the subsonic diffuser (Fig. 13). The channel is now said to be *started,* or running; the original shock wave has been *swallowed* by the second throat. The wave pattern ahead of the second throat, and the pressure distribution, both show that the flow is now supersonic all the way from the first throat to the terminal shock wave.

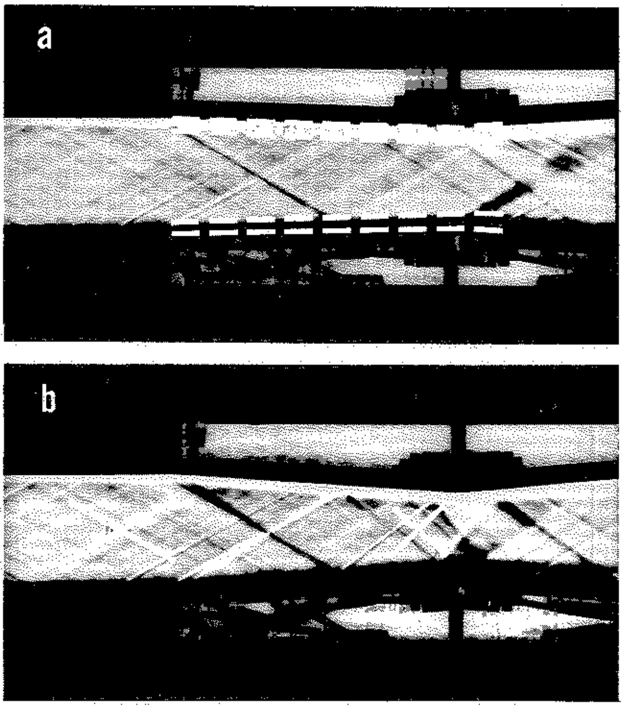


13. Started or running channel.

If the control valve is now slowly closed, the terminal shock pattern is moved upstream toward the second throat, to a position where the Mach number is lower than the test section value because of the supersonic compression. Too large an increase in back pressure will unstart the channel, or disgorge the shock wave, but the channel can readily be restarted by again opening the valve.

Changes of Second Throat Area

When the channel is running, the argument about blocking (in terms of the relative area of the second throat) does not apply, since there is no strong shock wave in the nozzle to lower the stagnation pressure of the flow. However, if the second throat is gradually closed, the flow becomes blocked almost immediately. A close-up view (Fig. 14a) shows part of the reason why the second throat cannot be closed to the same area as the first one. Room must be allowed for the wall boundary layers, which are very thick in the supersonic region and which are not carrying their share of the mass flow. With the second throat closed almost to the point of blocking, it can be deduced from the wave angles (Fig. 14b) that the Mach number in the test section is about 2.1, while the lowest supersonic Mach number obtainable at the second throat is about 1.4.



14. (a) Overlaid dashed lines show thickness of boundary layers in supersonic diffuser. Boundary-layer region is dark on the lower wall and light on the upper wall because the schlieren knife-edge is parallel to the walls. (b) Overlaid crossed lines call attention to change in wave angle and therefore to decrease in Mach number in supersonic diffuser.

In normal operation of a supersonic channel, the channel is first started, and then the second throat is closed almost to the point of blocking the channel (Fig. 15a). Advantage is then taken of the supersonic compression by closing the control valve a little, to move the terminal shock pattern upstream to a lower Mach number. The result is an optimum running condition, with a relatively small dissipation of energy (Fig. 15b). The advantage of using an adjustable second throat to

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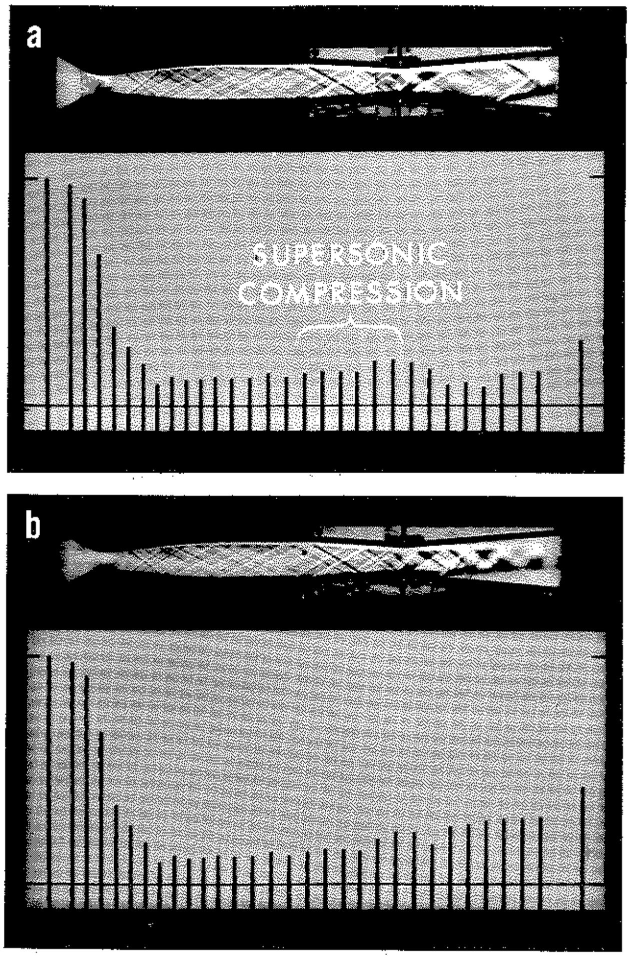
*Notice that the control valve, which is a sonic orifice of variable area, has been operating in this manner all along, acting as a second or third throat in the flow channel. The different function of such a control valve in a choked channel and in an unchoked one should be carefully noted.



15. (a) Started channel with second throat closed almost to point of choking. (b) Same flow but with optimum pressure ratio; note increase in downstream stagnation pressure compared to 15a.

control the air flow is that the stagnation pressure losses, and hence the power required to run the channel, can be reduced below the requirement for starting.

Summary

This film has illustrated three important phenomena which occur in adiabatic channel flow. *Choking* implies sonic flow at a throat, with an upstream flow which is independent of downstream conditions (e.g., Figs. 5a, 10). *Blocking,* or choking at two throats in succession, occurs if the second throat area is too small (e.g., Fig. 11a). *Starting* implies swallowing of a shock wave (e.g., Fig. 13), and is a necessary step in the achievement of supersonic compression or diffusion in channel flow of a compressible fluid.

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