JANUARY, 2006
DEPARTMENT OF AERONAUTICS AND ASTRONAUTICS

WRITTEN QUALIFYING EXAMINATION
FOR
DOCTORAL CANDIDATES

Wed., January 25, 2006 Room 37-212 9:00 AM – 1:00 PM

CLOSED BOOK AND NOTES

Answer a total of five (5) questions (no more or less)

You must answer at least two (2) questions from Column A, (one (1) Math and one (1) Physics), and three (3) questions from Column B (Professional Area Subjects). Please answer each question in a separate blue book and indicate on the cover of the blue book which question is being answered.

Do not put your name on any of the blue books.

You will use a NUMBER (last four digits of your MIT ID) to identify yourself; make sure it appears on the cover of each of the blue books that you turn in to be graded.

Oral examinations will be held on Tuesday, January 31st. Please pick up your schedule on Monday, January 30th after 3:00 PM from the Aero Astro Student Services Office (33-208).

Results will be available from your advisor on Wednesday, February 1st after 3:00 PM.

Column A

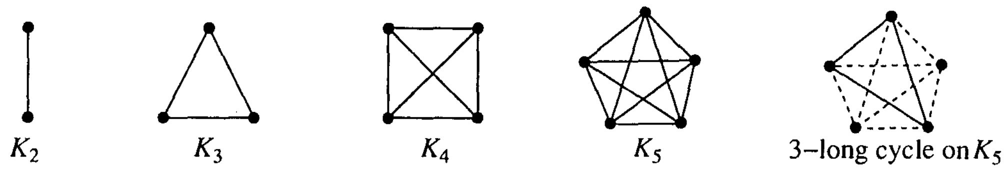
Mathematics (Discrete OR Continuous)
Physics (Dynamics OR Fields)

Column B

Autonomy
Communication and Networks
Control
Fluid Mechanics
Human Factors Engineering
Propulsion and Thermodynamics
Software Engineering
Structures and Materials
Vehicle Design and Performance

Discrete Math/Probability

K_n denotes the complete graph over n vertices. A *cycle* is a closed, connected sequence of edges with no repeated vertices or edges. Some examples:



1a) Determine the number of edges in K_n.

1b) Determine the number of 3-long cycles which can be defined on K_n.

1c) Determine the number of k-long cycles which can be defined on K_n, where k can take on any value $k = 3, 4, \ldots n$.

A tetrahedral die can show 1,2,3, or 4 with equal probablility. Two such dice are repeatedly tossed, and their outcomes i and j are noted.

2a) What is the most likely outcome of the sum $i + j$? What is the most likely outcome of the absolute difference $|i - j|$?

2b) The player bets B dollars that a declared sum $i + j$ will show. If the value doesn't show, the bet is lost. If the value does show, the player receives $B \times |i - j|$ dollars (the bet itself is not returned). What is the best sum $i + j$ value to declare?

2c) In the long run, can a player making the most rational declarations lose or make money?

Calculus

Answer the following questions:

a) Determine a vector normal to the plane defined by

$$ax + by + cz = 0$$

where a, b, c are given constants.

b) For some given point $\mathbf{v} = (v_x, v_y, v_z)$, find the point $\mathbf{w}$ on the plane given in part a) which is closest to $\mathbf{v}$. Specifically, find the matrix $\mathbf{P}$ which gives your result as

$$\mathbf{w} = \mathbf{P}\mathbf{v}$$

c) Evaluate $\mathbf{P}^n$.

d) For some given point $\mathbf{v}$, find the point $\mathbf{w}$ on the line defined by

$$\frac{x}{x_o} = \frac{y}{y_o} = \frac{z}{z_o}$$

which is closest to $\mathbf{v}$. Find $\mathbf{P}$ for this case.

e) Evaluate the boundary integral

$$I = \int_S \mathbf{u} \cdot \hat{\mathbf{n}} \, ds$$

where S is the boundary of the unit circle, $\hat{\mathbf{n}}$ is the outward unit normal, and the vector field is,

$$\mathbf{u} = (\sin x \, \cos y)\,\hat{\imath} \; - \; (\cos x \, \sin y)\,\hat{\jmath}$$

Dynamics

The thin solid disk of radius r and mass m is mounted on an effectively massless and frictionless shaft assembly shown below, which can freely pivot about the horizontal x axis. The disk is set spinning at rate p about its z axis. Gravitational acceleration g is downwards.

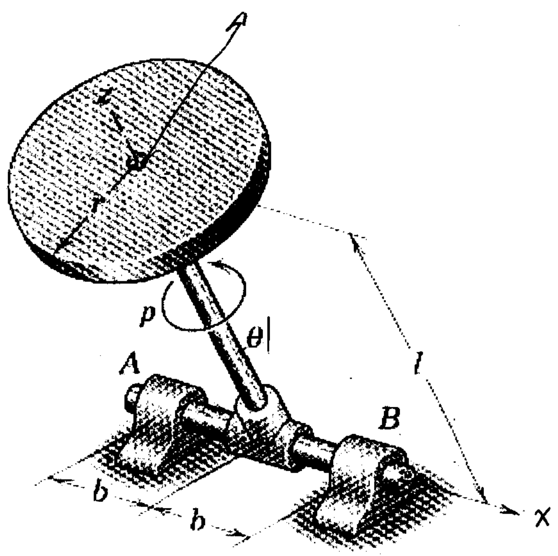
a) The assembly is released in the vertical position at $\theta = 0$ and $\dot{\theta} = 0$, and then tips over. Determine the energy conservation relation between θ and $\dot{\theta}$ for any instantaneous $\theta > 0$ position. Compute $\dot{\theta}$ when the disk's shaft passes the horizontal position ($\theta = \pi/2$).

b) Determine the total force vector $\vec{F} = (F_x, F_y, F_z)$ which is being applied to the disk as it passes the horizontal position. Note that in this position, y is vertical.

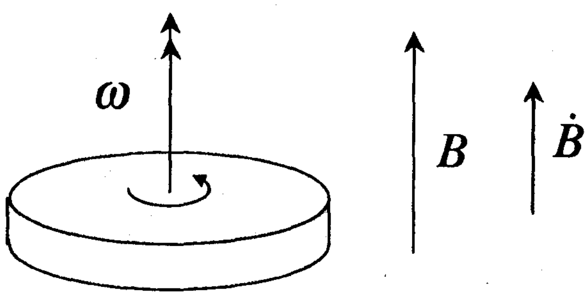
c) Determine the reaction force vectors $\vec{F}_A$ and $\vec{F}_B$ being applied by the two bearings.

Note: The moment of inertias of the disk are

$$
\begin{aligned}
I &= mr^2/2 && \text{about the } z \text{ axis} \\
I &= mr^2/4 && \text{about any axis orthogonal to } z
\end{aligned}
$$



Fields



Consider a metallic reaction wheel of radius R and thickness T, rotating with an angular velocity ω in a uniform magnetic field B. The field is aligned in a positive sense with the angular rate vector as shown.

a) Assume first that the magnetic field is constant in time ($\dot{B}=0$). What is the distribution of the electric field generated within the wheel?

b) Assume now that the magnetic field is increasing at a rate $\dot{B}$. What is the current density distribution within the wheel, assuming an electrical resistivity of σ.

c) What is the total power dissipated in the wheel?

d) Qualitatively, would you expect the wheel to accelerate or decelerate as a result of the changing magnetic field? Is this related to the power calculated in part c)?

For reference:

$$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0} \qquad \nabla \times \vec{E} = -\frac{d\vec{B}}{dt}$$

$$\nabla \cdot \vec{B} = 0 \qquad \nabla \times \vec{B} = \mu_0 (J + \varepsilon_0 \frac{\partial \vec{E}}{\partial t})$$

Autonomy

Answer the following problems:

1. (50%) Linear Programming

2A. (25%) A* Search

2B. (25%) DFS-BB Search

Problem 1 Linear Programming

Use the simplex method (by hand), to solve the following optimization problem

$$
\begin{aligned}
\text{Maximize:} \quad & x_1 + 2x_2 + 3x_3 \\
\text{Subject to:} \quad & 2x_1 + 3x_2 + 2x_3 \leq 12 \\
& x_3 \leq 3 \\
& 2x_1 - x_2 \leq 1 \\
& x_1,\ x_2,\ x_3 \geq 0
\end{aligned}
$$

Optimal cost ______, Optimal assignment x_1 __, x_2 ___, x_3 ____

Simplex Derivation (continue on next page if needed):

Repeat of problem statement:

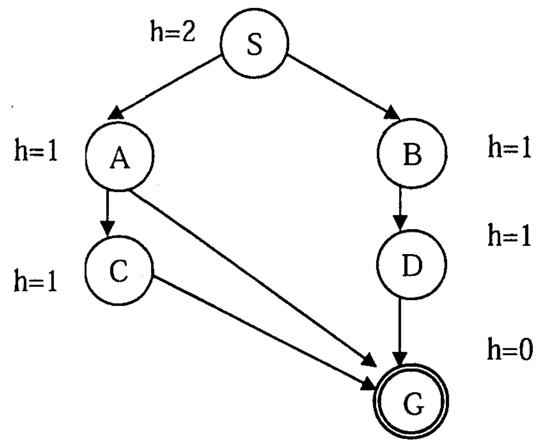
$$\begin{aligned}
\text{Maximize:} \quad & x_1 + 2x_2 + 3x_3 \\
\text{Subject to:} \quad & 2x_1 + 3x_2 + 2x_3 \leq 12 \\
& x_3 \leq 3 \\
& 2x_1 - x_2 \leq 1 \\
& x_1,\ x_2,\ x_3 \geq 0
\end{aligned}$$

Simplex Derivation Continued:

2/4

Problem 2 Informed Search

Your task is to find the shortest path from the state **S** to state **G** in the following directed graph using two search methods (A* and Branch and Bound):



Assume the following:

- All edges are length 1.
- h denotes heuristic cost for each state.
- Given two or more paths with equal estimated cost, explore them in **alphabetical (lexicographic) order**.
- Search stops as soon as the algorithm **guarantees it has found a shortest path**.

For each search method, give the sequence of **partial paths expanded** by that method, and their **associated cost**. A path is **expanded** when the method takes it off the queue, and attempts to create its children. A **path** and its associated cost is specified as a **list, starting with a cost, and followed by a list of states in order**. For example, "2 (S A C)" denotes a path of estimated cost 2, starting at S, going to A, and ending at C.

Part A: A* Search

Show the A* sequence of expanded paths (we have started it for you). **To the right of each cost/path** in your expansion, give the current state of the search queue.

Path expanded with Cost	Queue
	(2 S)
(2 S)	(2 A S) (2 B S)

3/4

Indicate the shortest path found by A*.

(______________ S)

Is the heuristic H admissible? Yes, or No [ ]

Part B Depth-first Branch and Bound (DFS-BB)

Show the sequence generated by a Branch and Bound search in which the partial paths are expanded in depth first order (we have started the search for you). Similar to A* search, bound the cost of a partial path using the function f = g + h. Recall that g is the cost of the partial path, while h is an estimate of the cost of completing the path; h is labeled on the graph, next to each state in the graph. **Specify the successive expanded paths (in order of expansion)** on the numbered lines below. To the right of each expanded path P, specify three pieces of information: First, specify the **bound on path P that is computed by DFS-BB**. Second, specify the path that is the **current incumbent**, when P is expanded. Third, specify the **cost of that incumbent**.

	Path Expanded	Bound on Path	Current Incumbent Path	and Cost
1.	(S)	3	none	infinity
2.	(A S)	3	none	infinity
3.				
4.				
5.				
6.				
7.				
8.				
9.				
10.				
11.				
12.				
13.				
14.				
15.				
16.				

Indicate the shortest path found by Depth-first Branch and Bound (DFS-BB):

(____________ S)

Communications and Networks

Consider a binary communication system that conveys a bit of information by transmitting $x(t) = bs(t)$, where b takes on only the values +1 and -1 with probabilities $\mathbb{P}(b = -1) = P_0$ and $\mathbb{P}(b = +1) = P_1$ respectively. The transmitted signal waveform $s(t)$ has time support $[0, T]$ and energy E. Now, the signal $x(t)$ is sent over a channel and the received signal is given by

$$y(t) = \alpha x(t) + n(t)$$

where α is a random channel gain and $n(t)$ is white Gaussian noise with spectral density $N_0/2$.

(a) Determine the optimum receiver when α is known to the receiver.

(b) Derive the probability of error when α is known to the receiver.

(c) Assume that $\mathbb{P}(b = -1) = \mathbb{P}(b = +1) = 1/2$. Derive the average probability of error given that α is Rayleigh distributed. You may use the alternate expression of the Q function given as follows

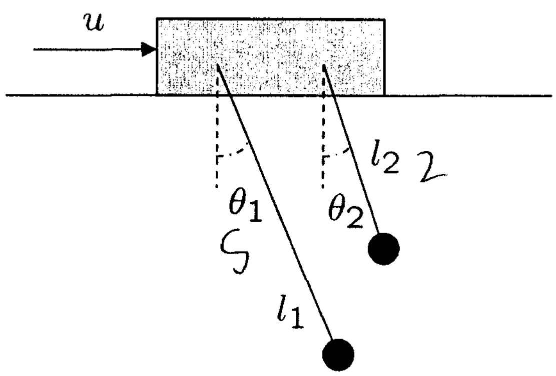
$$Q(x) = \frac{1}{\pi} \int_0^{\frac{\pi}{2}} \exp\left(\frac{-x^2}{2\sin^2\theta}\right) d\theta$$

Also note that the moment generating function for a unit mean exponential random variable Y is $\mathbb{E}\{e^{sY}\} = 1/(1 - s)$ for $s < 1$.

Controls

Consider the system consisting of a massless cart, which is free to slide horizontally and carries two pendulums of length l_1 and l_2, respectively.

- The actuator input to the system is the acceleration of the cart.
- The sensor output is the sum of the two angles of the pendulums, $\theta_1 + \theta_2$.



With state $x = [\ \theta_1\ \ \dot\theta_1\ \ \theta_2\ \ \dot\theta_2\ ]^T$ and $\omega_i = \sqrt{g/l_i}$, a linearized statespace model for the system can be written as

A B

$$
\begin{aligned}
\dot{x} &= \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\omega_1^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -\omega_2^2 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ -1/l_1 \\ 0 \\ -1/l_2 \end{bmatrix} u \\
y &= \begin{bmatrix} 1 & 0 & 1 & 0 \end{bmatrix} x
\end{aligned}
$$

C D

1. Is this system controllable for all values of l_1 and l_2? Provide a mathematical condition and provide a physical interpretation of your answer.

2. Sketch the frequency response for the system when $\omega_1 = 1$ rad/sec and $\omega_2 = 1.2$ rad/sec.

3. With $\omega_1 = 1$ rad/sec and $\omega_2 = 1.2$ rad/sec, the goal is to design a controller that damps the motion of the pendulums. Using classical techniques (i.e., root locus and/or Bode), design a controller that adds as much damping as possible to each of the two pendulums. To start, you might find instructive to consider the effect of proportional and/or rate feedback for this system.

Fluid Dynamics

The Concorde is flying at its cruising altitude of 60,000 ft where the atmospheric temperature and pressure are 217 K and 0.05 bar respectively. Suddenly one of the cockpit windows cracks and fails, leaving a hole of 0.05 m diameter. The initial pressure and temperature in the cockpit are 1 bar and 290K. The cockpit has a volume of 10 m^3 and can be considered to be well insulated and sealed from the rest of the aircraft. Assume that air is a perfect gas with $\gamma = 1.4$.

a) Sketch the evolution of the Mach number through the opening as a function of time during the cockpit "blow down". Why did you draw it that way? (A one to several answer justifying the form of the sketch is required)

b) Similarly sketch the evolution of the mass flow as a function of time and comment on the physical reasons that support your sketch

c) At what cabin pressure might the behavior of the Mach number change qualitatively?

d) Find an expression for the mass flow in terms of the cockpit conditions and the Mach number through the opening.

e) Find a relation between cockpit pressure and temperature that holds for any instant in time during the event. State all your assumptions and describe the attributes of the thermodynamic process.

f) If the pilots become unconscious at a pressure of 0.1 bar, how much time do they have to put on their oxygen masks?

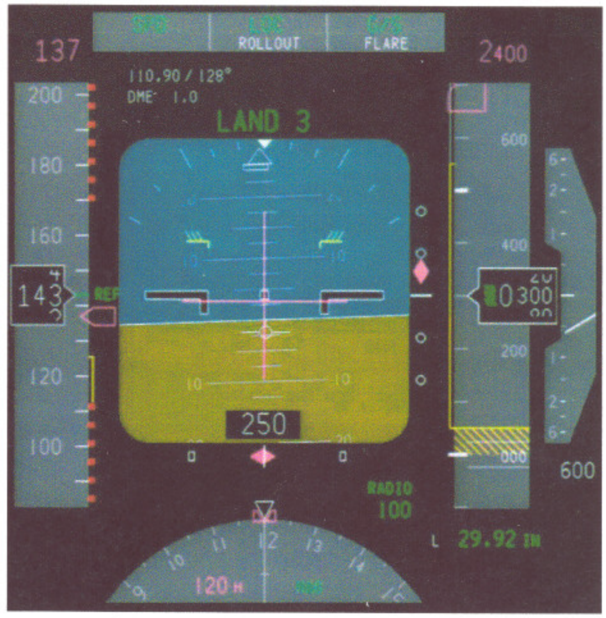
Concorde cockpit and view at 60,000 ft altitude.

Humans and Automation

1. Discuss how Display A does or does not meet the following design principles.
    - Proximity-compatibility principle
    - Principle of pictorial realism
    - Principle of the moving part
    - Inside-out vs. outside-in

2. Which primary flight display* below do you think is better and why?

A

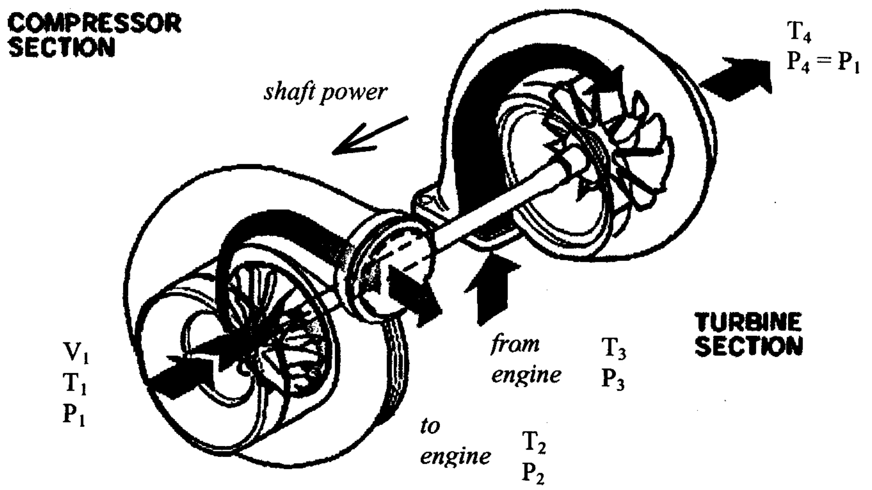


B

3. How would you test your claim in #2? Specifically address independent and dependent variables, including subjective and objective metrics.

Propulsion and Thermo

The compressor of a turbocharger shown below has an inlet flow rate of 7.5 m^3/minute of air as measured at the inlet condition. This condition is P_1 = 1.013 bar and T_1 = 288 K. The compressor has a pressure ratio of 1.5 and an adiabatic efficiency of 80%. The compressor is driven directly by an exhaust turbine with an adiabatic efficiency of 70%. The turbocharger and engine are arranged such that both turbine and compressor operate under conditions of steady flow. The turbine pressure ratio is the same as that for the compressor and the mass of engine fuel may be neglected. Assume air and exhaust gas are perfect gas with γ = 1.4 and R = 287 J/kg-K.



a) Sketch the compression in the compressor and the expansion in the turbine in an *h-s* diagram. Indicate the shaft work and how the inlet and exit pressures of the turbine relate to the ones of the compressor.

b) Calculate the inlet temperature to the turbine.

c) Calculate the turbine exit temperature.

d) Calculate the shaft power produced by the turbine.

Software

Answer one of the following (but not both):

A) Software Engineering, or

B) Real Time Software Systems

A) Software Engineering

The NASA Check Out and Launch Control System (CLCS) that was being developed to control the launch of the Space Shuttle at Kennedy Space Center (until the project was canceled because it was so late and so overbudget) had a mandate to use COTS (Commercial Off-The-Shelf) software whenever possible. Of the estimated 1.8 million lines of code, 1.4 million was to be COTS.

a) What factors might have been involved in the decision to mandate the use of COTS?

b) What are some of the technical risks involved?

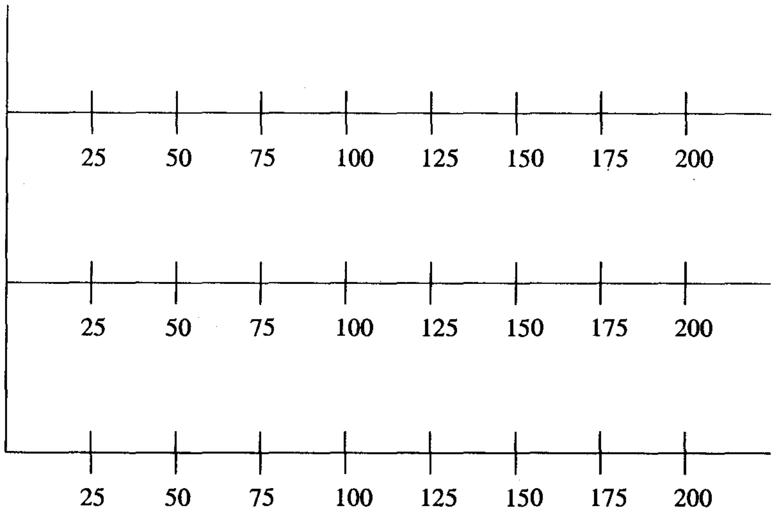
c) What are some management risks?

d) If you were the CLCS program manager, what might you have done to mitigate each of these risks?

B) Real-time Software Systems

- **Scheduling**

  Suppose we have three tasks, with parameters T1=(0, 50, 25, 25), T2=(0, 35, 10, 20), T3=(125, 125, 25, 50). The 4 parameters are (phase, period, execution time, relative deadline). Please schedule this job using Deadline-Monotonic scheduling. Circle any missed deadlines.



- **Usability**

  Assume that we schedule the following tasks using rate-monotonic scheduling.

Task	Period	Execution Time	Phase	Deadline
1	.1	.01	0.1	.05
2	.2	.1	2	.1
3	.1	.005	0.5	.05
4	.2	.04	0	.1

  Please give an O(1) expression for determining if the scheduler is guaranteed to find a feasible schedule for this set of tasks. Is this schedule feasible?

  You might find these values useful: $(2^1 = 2, 2^{.5} = 1.414, 2^{.33} = 1.2599, 2^{.25} = 1.1892, 2^{.2} = 1.1487, 2^{.1667} = 1.1225)$

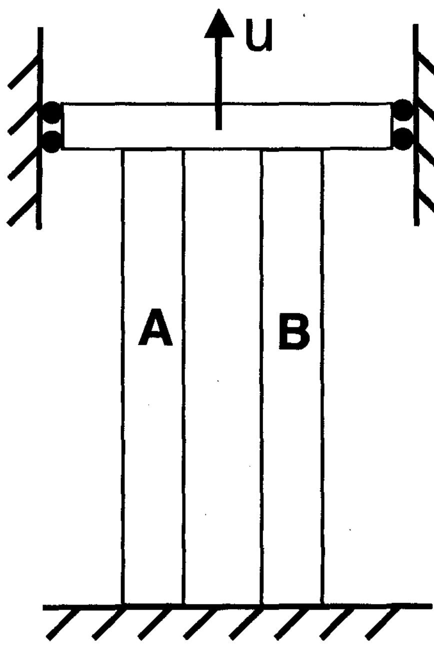
- **Concurrency**

  In many concurrent models of computation, a concurrent process can signal to other waiting processes that some resource has been released. When a process starts to wait for a signal, it must be inside a critical region (a region of mutual exclusion). Why?

  When a process sends a signal that a resource has been relased, it must be leave the critical region immediately. Why?

Materials and Structures

Engineers are designing a structural arrangement to measure temperature. The configuration is as shown in the accompanying diagram. Two bars, A and B, are rigidly attached at one end to a rigid frame. At the other end, each bar is rigidly attached to a rigid, horizontal cross-piece that makes contact with two side walls via rollers. This arrangement is such that no vertical load is transferred to the side walls, and the bars are constrained so that their total lengthwise deformation, u, must be the same. Each bar is of length L, has a square cross-section, w to a side, and is made of an isotropic material. Material A is 50% stiffer than Material B. The coefficient of thermal expansion of Material B is twice that of Material A.



The design engineers are looking to determine the displacement of the rigid cross-piece, u, as a function of temperature change, ΔT. The reference temperature is room temperature and that is the temperature at which the structural configuration will be assembled.

Help the engineers by determining a plot of displacement versus temperature. Describe the behavior manifested in the structural configuration using equations as appropriate. Indicate issues to be considered in the determination of this behavior for various ranges of temperatures that the engineers may consider and show the effects on the displacement-temperature response. Throughout your answer, be as quantitative as possible.

Systems (Vehicle Performance)

In both aircraft and spacecraft, a fundamental issue is the effective impulse that can be achieved with the onboard fuel or propellant. A good model for a rocket or turbojet is to assume the thrust T is proportional to the fuel flow rate $\dot{m}_{\mathrm{f}}$,

$$T = I_{\mathrm{sp}} g \, \dot{m}_{\mathrm{f}}$$

where I_{sp} is the specific impulse, and g is the earth-surface gravitational acceleration. The velocity $V(t)$ along the flight path is then governed by

$$m\dot{V} = T - F_r$$

where F_r is a retarding force opposite to the flight direction.

a) Consider the limiting case $F_r = 0$. The vehicle has an initial mass m_0. Determine the change in velocity ΔV resulting from burning a substantial mass of fuel Δm_{f}.

b) Consider the initial part of a vertical rocket launch, where the aerodynamic drag is still neglible. The rocket has liftoff mass m_0. Determine the change in velocity ΔV resulting from burning a substantial mass of fuel Δm_{f}.

c) Consider the case of level horizontal flight at constant speed V. The aircraft starts flying at an initial mass m_0. Determine the range R, or distance flown, resulting from burning a substantial mass of fuel Δm_{f}. Assume a fixed lift/drag ratio L/D.

d) Consider the general case where the thrust is a prescribed function of time $T(t)$, and the retarding force is a known function of speed $F_r(V)$. Write down (but do not try to solve!) the mathematical relations required to compute the velocity history $V(t)$ of the vehicle. Specify the inputs which will be required to execute this calculation.