JANUARY 2004
DEPARTMENT OF AERONAUTICS AND ASTRONAUTICS

WRITTEN QUALIFYING EXAMINATION FOR DOCTORAL CANDIDATES

Wed., January 21, 2004 Room 37-212 9:00 AM – 1:00 PM

CLOSED BOOK AND NOTES

Answer a total of five (5) questions (no more or less)

You must answer at least two (2) questions from Column A, (one (1) Math and one (1) Physics), and three (3) questions from Column B (Professional Area Subjects). Please answer each question in a separate blue book and indicate on the cover of the blue book which question is being answered.

Be sure that your NUMBER (last four digits of your MIT ID number) appears on the cover of each of your blue books that you turn in to be graded.

Oral examinations will be held on Tuesday, January 27, 2004. Please pick up your schedule on Monday, January 26, 2004 after 3:00 PM from the Aero Astro Student Services Office (33-208).

Results will be available from your advisor on Wednesday, January 28 after 3:00 PM.

Column A

Mathematics (Discrete OR Continuous)
Physics (Dynamics OR Fields)

Column B

Autonomy
Communication and Networks
Control
Fluid Mechanics
Human Factors Engineering
Propulsion and Thermodynamics
Software Engineering
Structures and Materials
Vehicle Design and Performance

Continuous Math

1) Find the point in the plane defined by

$$Ax + By + Cz = D$$

which is nearest to the origin.

2) Second-order non-linear differential equations, not directly containing the independent variable, can be solved using the substitutions

$$\frac{dy}{dx} = p \qquad\qquad \frac{d^2y}{dx^2} = p\,\frac{dp}{dy}$$

a) Show why the second relation follows from the first.

b) Use these substitutions to obtain the general solution of the equation

$$y\,\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 0$$

3) Bernoulli numbers B_k are the coefficients in the power series expansion

$$\frac{t}{e^t - 1} = \sum_{k=0}^{\infty} \frac{B_k}{k!}\,t^k$$

a) Determine B_0 and B_1.

b) Prove that B_3, B_5, $\ldots B_{2k-1}$ are all zero.

HINT: Examine the function $\dfrac{t}{e^t - 1} - B_1 t$

Discrete Math

1) Consider a grammar $\{V, T, S, P\}$, where $V = \{S, A, B, a, b\}$ and $T = \{a, b\}$. Using set notation, describe the language generated by this grammar when P is: $S \longrightarrow AB$, $A \longrightarrow aAb$, $B \longrightarrow bBa$, $A \longrightarrow \lambda$, $B \longrightarrow \lambda$.

2) Write a context-free grammar over the alphabet $\{0, 1\}$ that generates the set of all palindromes, where a palindrome is a string that reads the same forwards as backwards, i.e. $w = w^R$, with w^R denoting the reversal of string w.

3) Create a Mealy machine (finite state automaton) that models the behavior of a robot with two hands. The robot receives a sequence of dishes and cups on a conveyor belt, and operates as follows on each received item:

1. If both hands are free, the robot picks up the item with its left hand, be it a dish or cup.
2. If the robot's left hand holds an item, the new item it picks up with its right hand must be of a different type.
3. The robot puts the cup over the dish and outputs them.
4. When the input stream terminates, both hands must be free.

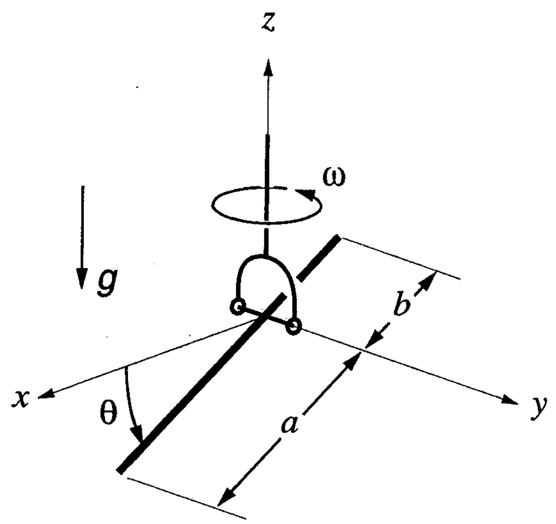
4) Let G_1 and G_2 be context-free grammars, generating the languages $L(G_1)$ and $L(G_2)$. Show that there is a context-free grammar generating each of the following sets:

1. $L(G_1) \cup L(G_2)$
2. $L(G_1)L(G_2)$
3. $L(G_1)^*$

Hint: To show that there is a context-free grammar for these sets, you just need to show how you would create such a grammar.

Dynamics

A uniform slender bar of mass/length ρ is freely pivoted about the y-axis at the clevis, which rotates about the vertical z-axis with a constant angular velocity ω. Gravitational acceleration g is in the $-z$ direction.



1) Determine the equilibrium angle θ_0 of the bar.

2) When the bar is at θ_0, what is the moment M_z which needs to be applied to the clevis to maintain the constant angular velocity ω?

3) The bar is perturbed by a small amount $\delta\theta$ from the equilibrum position θ_0 and then released. Determine the bar's period of oscillation about θ_0.

HELP: The bar's tensor of inertia with respect to the axes xyz rotating with the clevis is

$$\bar{\bar{\mathbf{I}}} = \frac{\rho}{3}\left(a^3 + b^3\right)\begin{bmatrix} \sin^2\theta & 0 & \sin\theta\cos\theta \\ 0 & 1 & 0 \\ \sin\theta\cos\theta & 0 & \cos^2\theta \end{bmatrix}$$

Fields

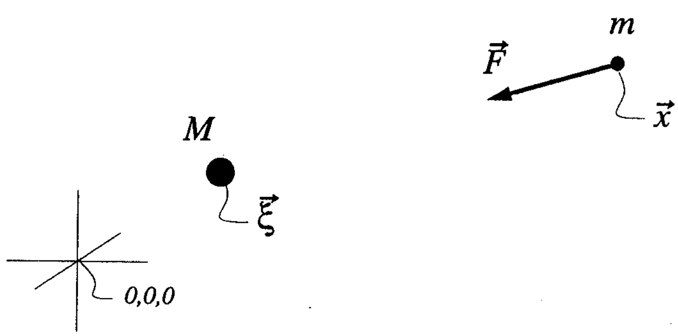
The gravitational force on a mass m at location $\vec{x}$ by a mass M at location $\vec{\xi}$ is

$$\vec{F} = -GMm\frac{(\vec{x}-\vec{\xi})}{|\vec{x}-\vec{\xi}|^3}$$

This can be expressed as $\vec{F} = -m\,\nabla\phi$, where

$$\phi = -\frac{GM}{|\vec{x}-\vec{\xi}|}$$

is the Gravitational Potential Field of mass M, and $\nabla()$ is the gradient with respect to $\vec{x}$.



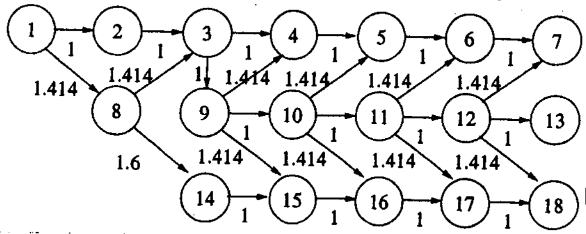
1) If M is distributed in space with mass density $\rho(\vec{\xi})$, express $\vec{F}(\vec{x})$ and $\phi(\vec{x})$ as volume integrals over $\vec{\xi}$.

2) Show that your $\phi(\vec{x})$ satisfies Poisson's equation, $\nabla^2\phi = f$, and determine f for this case.
HINT: Apply your result from 1) to a small sphere around point $\vec{\xi}$.

3) Determine the gravitational acceleration $\vec{g}(r) = \vec{F}/m$ as a function of the radius r, inside a planet of uniform density.

4) Determine $\vec{g}(r)$ inside a hollow sphere with a uniform mass/area density μ.

Autonomy



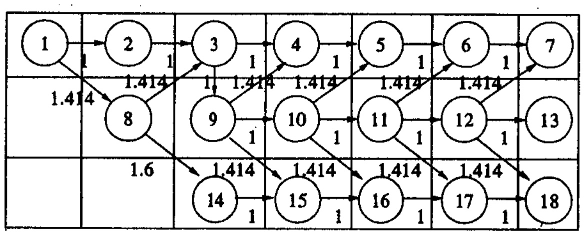
Use Dijkstras Algorithm to find the shortest path from Node 1 to Node 18 in the figure above. Assume that when two nodes have the same value, the node with the lowest index is expanded first.

1a) What is the total length of the path from 1 to 18?

1b) By what criterion is a node selected for expansion?

1c) In what order are the nodes expanded?

1d) Show the results of relaxation after the third node is expanded. List each node and its corresponding relaxed value.



In the above figure, the graph is now embedded in a grid world; each node has an x and y position. This means that we can use Euclidean distance $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$ to measure the distance between two nodes. Use A^* with Euclidean distance as a heuristic to find the shortest path from Node 1 to Node 18. Assume that when two nodes have the same value, the node with the lowest index is expanded first.

1e) By what criterion is a node selected for expansion?

1f) In what order are the nodes expanded? (Not all nodes are expanded).

1g) List each node that remains in the queue after the goal is expanded, along with its cost (path length + heuristic)?

Your goal is to optimize NASA's launch schedule. Commercial payloads generate \$20 million dollars per unit. Scientific payloads generate \$5 million dollars per unit. Launches with military payloads generate no revenue. However, you are not allowed to launch more commercial missions than military missions, and you are not allowed to launch more military missions than scientific missions. Of course, you can only launch 100% of total capacity.

2a) What is the objective function for this linear program?

2b) What are the constraints for this linear program?

2c) Perform the first step of the simplex method in order to solve your linear program, and write down the resulting set of equations (or tableau).

2d) What is the worst-case computational complexity of the number of iterations required by the simplex method, given n variables, each bounded by $[0, 1]$, in Big-O notation?

2e) Why is simplex used so often, given this complexity?

Communications and Networking

Consider a binary coherent phase –shift keying (BPSK) system which signals equally likely messages (m_0 or m_1) using the following known signals

$$s_0(t) = \sqrt{2S_0}\cos(2\pi f_0 t + \theta), \qquad 0 \le t \le T$$

$$s_1(t) = -\sqrt{\frac{S_1}{S_0}}\, s_0(t)$$

in an additive white Gaussian noise channel with single-sided power spectral density N_0 Watts/Hz.

(a) Show the vector representation of the transmitted signals.

(b) Derive the optimal receiver and sketch the receiver block diagram.

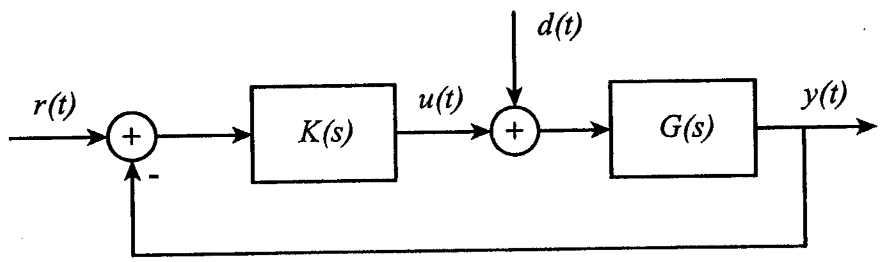
(c) Evaluate the error probability of the optimum receiver.

(d) If the messages are not equally likely, discuss how (b) and (c) changes.

Your task is to design a position controller for a large dish antenna. The azimuthal position of the antenna (i.e., the direction it points relative to north) is controlled by an applied motor torque, $u(t)$. The transfer function from motor torque to azimuth angle is given by

$$G(s) = \frac{1}{s(10s+1)}$$

The antenna is also subject to the wind, which produces a disturbance torque, $d(t)$. The goal is to have the azimuthal angle, $y(t)$, track a desired reference input, $r(t)$, by the use of a control loop of the form



Because the wind can be steady or gusting, the disturbance signal $d(t)$ is time-varying. Preliminary analysis shows that the magnitude of the disturbance torque is bounded, and the performance of the antenna control system will be satisfactory if the transfer function from disturbance to output is bounded by

$$|T_{yd}(j\omega)| = \left|\frac{Y(j\omega)}{D(j\omega)}\right| \leq 2$$

In addition to the requirement on disturbance rejection, system should have at least 45 deg of phase margin, to ensure adequate stability and good time response to a step change in the reference input. Finally, the control system should have a bandwidth (as measured by the crossover frequency) of no more than 1 rad/s.

A. Suppose that the controller is simply a gain, so that $K(s) = 1$. It turns out that this control law will not meet all of the requirements above. For each requirement (disturbance rejection, phase margin, and bandwidth), determine whether the system meets the requirement, and explain.

B. Design a dynamic controller, $K(s)$, that does meet all of the requirements.

PROBLEM CONTINUED ON NEXT PAGE!

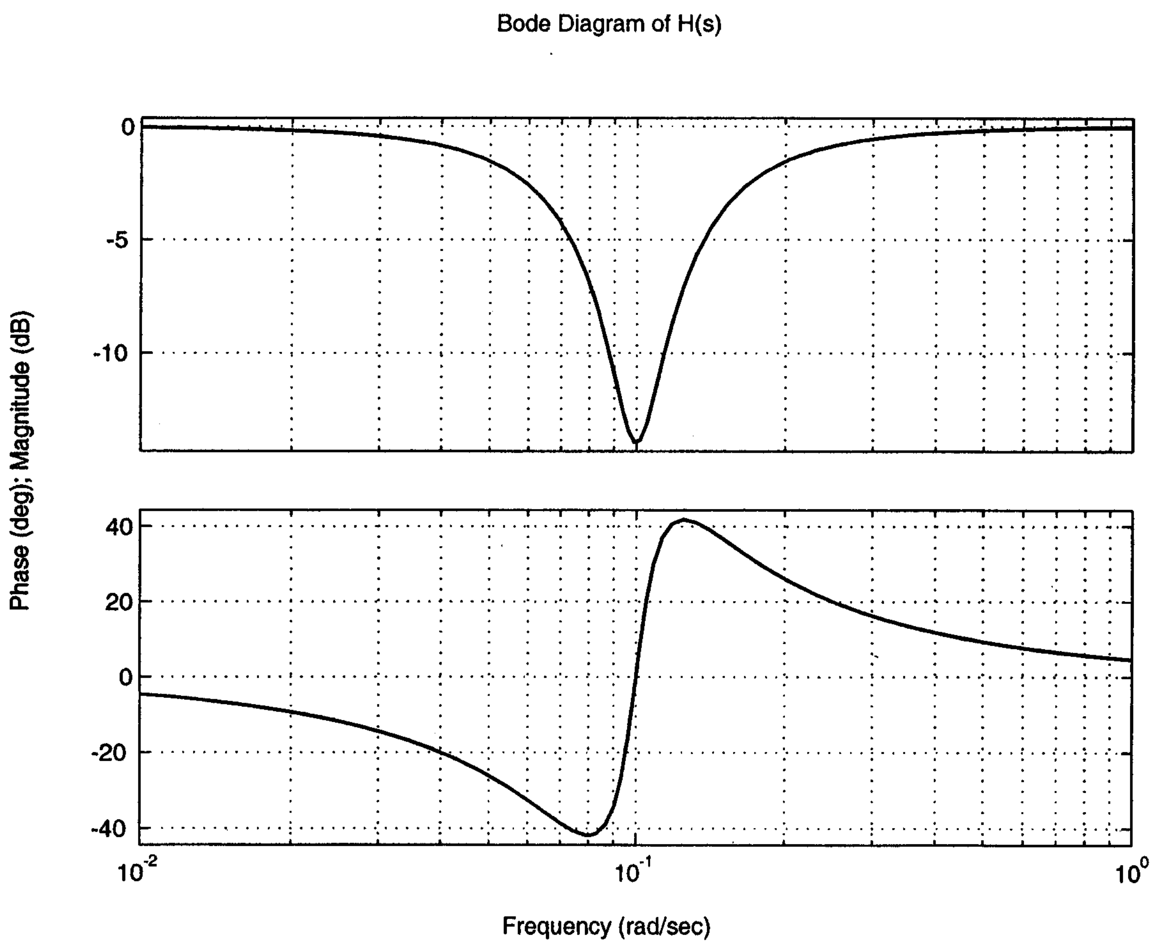
C. A more careful analysis of the wind loads show that antenna may be subject to near-periodic disturbances at frequencies in a narrow range around of 0.1 rad/s, due to periodic shedding of vortices from the antenna. The disturbance torques due to periodic shedding may be substantially larger than the torques due to steady winds or wind gusts. As a result, the requirement on disturbance rejection has been tightened, to be

$$|T_{yd}(j\omega)| \leq 2\,|H(j\omega)|$$

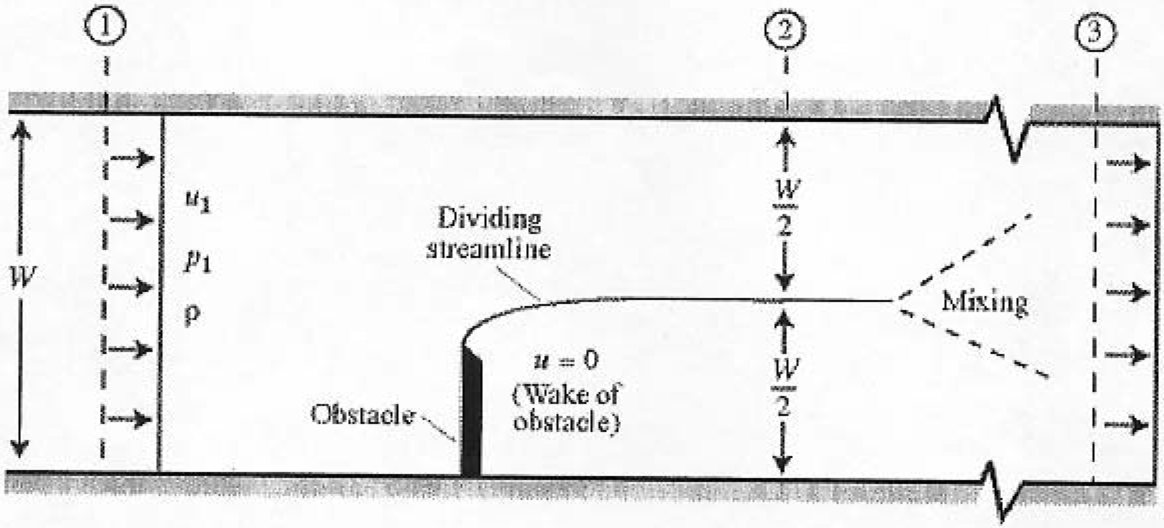
where

$$H(s) = \frac{100s^2 + 2s + 1}{100s^2 + 10 + 1}$$

For convenience, the transfer function of $H(j\omega)$ is shown plotted below. The requirements are otherwise unchanged. Design a controller, $K(s)$, that best meets these new requirements, or comes as close as possible to meeting them.



FLUIDS



The figure shows the two-dimensional incompressible flow past a solid obstacle in a duct. Behind the obstacle there is a wake with zero velocity. At station 2 the streamlines are parallel, with the dividing streamline marking the boundary between the wake and the "free stream". At this station the wake occupies one-half the duct height (see the sketch)

1) What is the velocity distribution at station 2 where the streamlines are parallel? (sketch and describe)

2) If p_1 is the (uniform) static pressure at station 1, what is the static pressure distribution at station 2? Express the answer in terms of $\frac{(p_1 - p_2)}{\rho u_1^2/2}$

3) What is the force on the obstacle?

4) Some distance downstream of station 2 the flow "mixes out" to a uniform state at station 3 which is far downstream. If p_1 is the (uniform) static pressure at station 1, what is the static pressure at station 3? Express the answer in terms of $\frac{(p_1 - p_3)}{\rho u_1^2/2}$

Humans Factors Engineering

a) Sketch the status of an aircraft using conventional (western) "inside-out" displays, showing the airplane pitched 10 degrees down, banked 30 degrees left.

b) Now show the status using the "outside-in" convention, and comment on the pros and cons of the difference from a)

c) How would you design an experiment to test these two displays to determine which one is more effective? Be sure to address the experimental conditions and performance metrics.

d) How would you go about testing workload and situation awareness in the above experiment?

Propulsion & Thermodynamics

A turbojet engine on an aircraft takes in air at *static temperature* T_0 from the atmosphere and exhausts air at a *static temperature* of T_1 at its exit. The aircraft is in level steady flight at velocity u_0. The exhaust velocity of the engine, relative to the aircraft is u_1. Assume air and the combustion gas to be perfect gases with the same specific heat c_p.

a) Sketch the thermodynamic cycle in an h-s diagram and indicate work and heat transfer in each segment of the cycle.

b) What is the heat rejected to the atmosphere per unit of mass flow?

c) What is the *net heat input* to the engine per unit of mass flow?

d) What is the thermal efficiency $\eta_{thermal}$ of the engine?

e) What is the overall efficiency ($\eta_{overall} = \eta_{thermal} \times \eta_{propulsive}$) of the engine?

Express your answers in terms of the given quantities.

Software

1) In Bollinger's article, "The Interplay of Art and Science in Software," he argues: "The creation of genuinely new software has far more in common with developing a new theory of physics than it does with producing cars or watches on an assembly line." Discuss his argument for this statement.

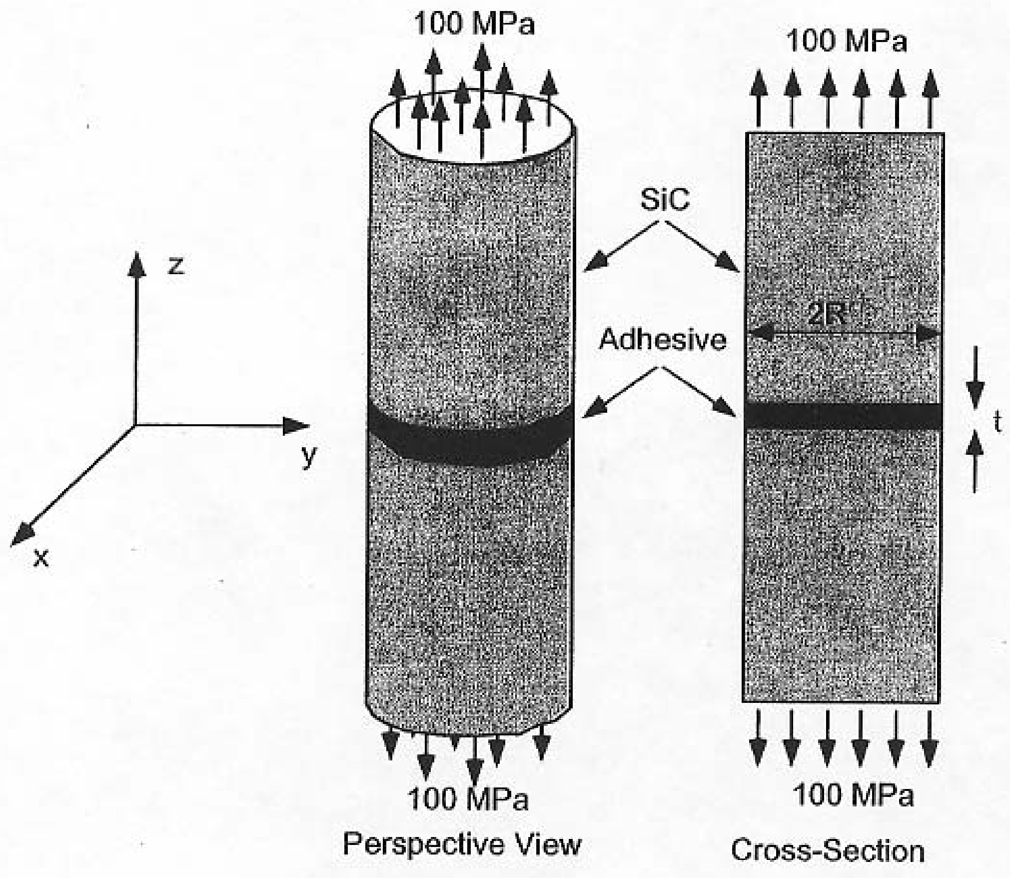
2) How would a supporter of CMM counter this argument?

Materials and Structures

State all assumptions (this is a key component of the problem)

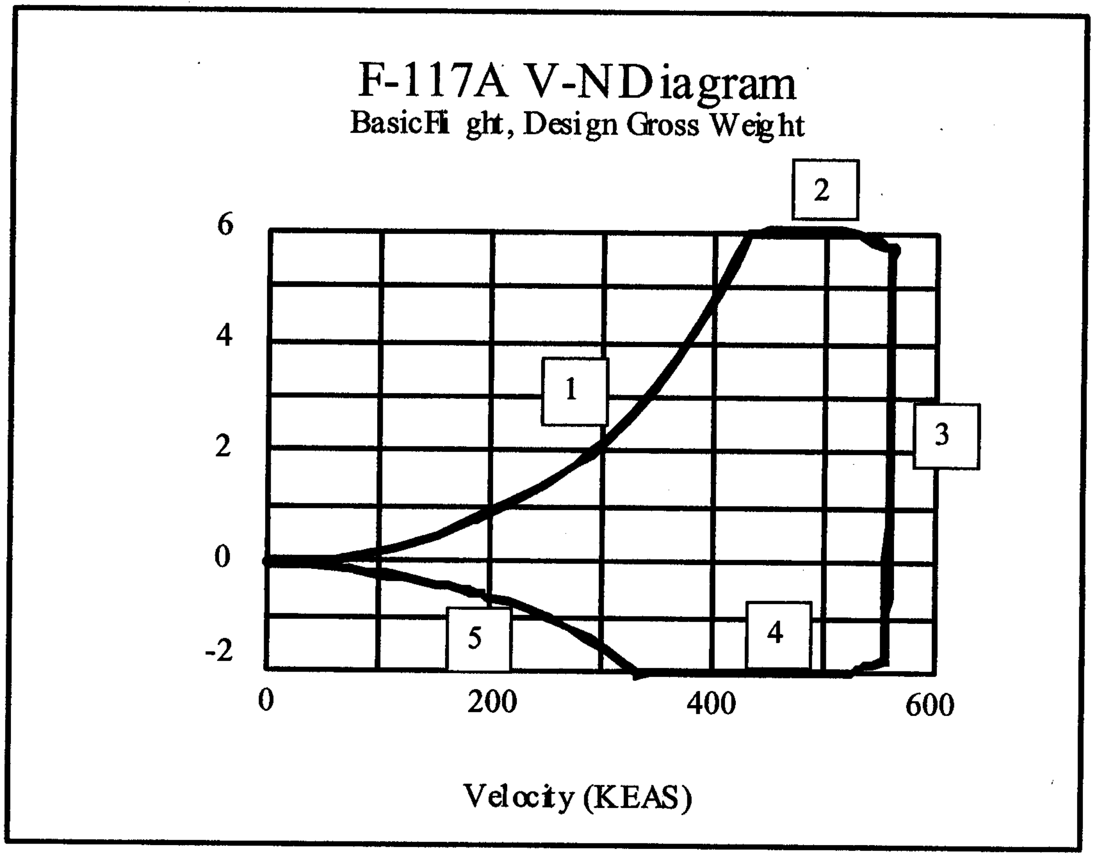
a) (i) State the definition of the engineering elastic constants required to describe the elastic response of an isotropic material: Young's modulus, E, and Poisson's ratio, ν. (ii) Given these definitions derive the constitutive law allowing the determination of the six components of strain given the six components of stress in a linear-elastic, isotropic material. (You may find it useful to present this in the form of a matrix)

b) A thin ($t<<R$) layer of an epoxy adhesive is used to join two silicon carbide bars of circular cross-section, as shown below. An axial (z direction) tensile stress of 100 MPa is applied to the joint as shown. Calculate the axial strain (ε_{zz}) in the adhesive. The Young's modulus of the epoxy adhesive is 3 GPa, the Poisson's ratio is 0.3. The Young's modulus of the SiC is 450 GPa and the Poisson's ratio is 0.2



c) The joint is to be designed to withstand the stress of 100 MPa. The manufacturer occasionally observes penny shaped (ie. flat, circular) defects in the plane of such adhesive joints. Outline what analysis you might perform to evaluate whether the joint will carry its design stress and what additional information you would need.

Consider the V-n diagram[1] below for the F-117A Nighthawk stealth aircraft where n = L/W and KEAS = knots equivalent air speed.



Part I - V-n diagram explanation (6 points)

a.) In two to three sentences, describe what an V-n diagram is and what information it provides to engineers designing the air vehicle and its subsystems.

b.) For each segment [1] through [5] of the V-n diagram:

- Explain the physical significance of the boundary and what determines its location on the V-n diagram. Use equations to support your explanation, if appropriate.
- Explain what would happen if the aircraft attempts to operate outside of the boundary segment.

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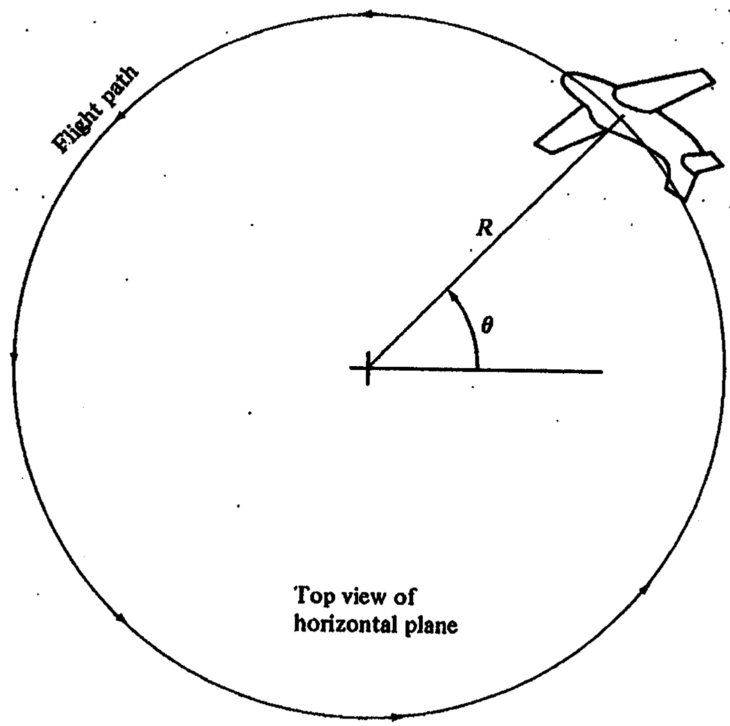
[1] George Zielsdorff, *Structural Developments of Recent Aircraft: F-117*, AIAA 95-1470 36th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics and Materials conference, New Orleans LA, April 10-12, 1995.

Part II - Performance (4 points)

Consider an airplane executing a level turn (constant altitude turning) as shown below. By analysis, show which point on the V-n diagram corresponds to the smallest turning radius R the F-117A can achieve.



Qualifier Written Question
Space Vehicle Performance

Rocket staging has clear advantages in increasing the amount of payload mass, as a fraction of initial mass, which can be accelerated to a given final velocity. Building upon the rocket equation below, derive an expression that compares a single stage system to a two stage system. Clearly show that the two stage rocket accelerates more payload mass than a single stage rocket assuming that each starts with the same initial mass. Assume that each stage in the two stage system delivers the same change in velocity. Remember, an empty propellant tank accelerated to the final velocity is not a part of useful payload mass. Assume that tank mass is proportional to the mass of the propellant it carries. This solution can be performed analytically and does not require numerical calculations.

$$\frac{m_f}{m_i} = e^{-\frac{\Delta v}{I_{sp} g}}$$

where

m_f is the final mass
m_i is the initial mass
Δv is the change in velocity
I_{sp} is the specific impulse
g is the acceleration due to Earth's gravity at sea level