JANUARY 2003
DEPARTMENT OF AERONAUTICS AND ASTRONAUTICS

WRITTEN QUALIFYING EXAMINATION
FOR
DOCTORAL CANDIDATES

Wed., January 22, 2003 Room 37-212

CLOSED BOOK AND NOTES

Answer a total of five (5) questions (no more or less)

You must answer at least two (2) questions from Column A, (one (1) Math and one (1) Physics), and three (3) questions from Column B (Professional Area Subjects). Please answer each question in a separate blue book and indicate on the cover of the blue book which question is being answered.

Be sure that your name appears on the cover of each of your blue books that you turn in to be graded.

Oral examinations will be held on Tuesday, January 28, 2003. Please pick up your schedule on Monday, January 27, 2003 after 3:00 PM from the Aero Astro Student Services Office (33-208).

Results will be available from your advisor on Wednesday, January 29 after 3:00 PM.

Column A

Mathematics (Discrete OR Continuous)
Physics (Dynamics OR Fields)

Column B

Autonomy
Communication and Networks
Control
Fluid Mechanics
Human Factors Engineering
Propulsion and Thermodynamics
Software Engineering
Structures and Materials
Vehicle Design and Performance

Discrete Mathematics

Well-formed formulae in prefix notation over a set of symbols and a set of binary operators are defined recursively by the following rules:

- if x is a symbol, then x is a well-formed formula in prefix notation.
- if X and Y are well-formed formulae and $*$ is an operator, then $*XY$ is a well-formed formula.

1) Which of the following are well-formed formulae over the symbols $\{x, y, z\}$ and the set of binary operators denoted by $\{\&, +, \%\}$?

a) $\& + + x\, y\, z$

b) $\& + \%\, x\, x\, \%\, x\, x\, x$

c) $\& \%\, x\, z\, \& \&\, x\, y$

2) Show that any well-formed formula in prefix notation over a set of symbols and a set of binary operators contains exactly one more symbol than the set of operators. [you need not do a formal proof, just an informal proof sketch of the reasoning underlying such a proof.]

Continuous Mathematics

Consider the nonlinear equation

$$x^2 + \ln(x/2) - 4p^2 = 0$$

where p is a free parameter. The general solution has the form $x = x(p)$. For the particular choice $p = 1$, the solution is $x = 2$.

1a) For the perturbed choice $p' = 1 + \epsilon$, determine the perturbed solution x' to first order in ϵ.

1b) Determine the solution's parameter-sensitivity dx/dp for any arbitrary chosen p. Give the result in terms of p and the corresponding x.

The above equation has the form $f(x;p) = 0$. Consider now two given simultaneous nonlinear equations in two unknowns x, y, both involving a free parameter p.

$$\begin{aligned} f(x,y;p) &= 0 \\ g(x,y;p) &= 0 \end{aligned}$$

The solution, however obtained, will have the form $x = x(p)$, $y = y(p)$.

2a) Determine expressions for the solution's derivatives dx/dp and dy/dp. Clearly state any conditions on the validity of your result.

With p held fixed at some specified value, an approximate numerical solution to the two nonlinear equations above is known to be $\tilde{x}, \tilde{y}$, with the equations violated to some computed small errors:

$$\begin{aligned} f(\tilde{x},\tilde{y}) &= \epsilon_f \\ g(\tilde{x},\tilde{y}) &= \epsilon_g \end{aligned}$$

2b) Determine expressions for an improved approximate numerical solution $\tilde{x}', \tilde{y}'$.

Physics

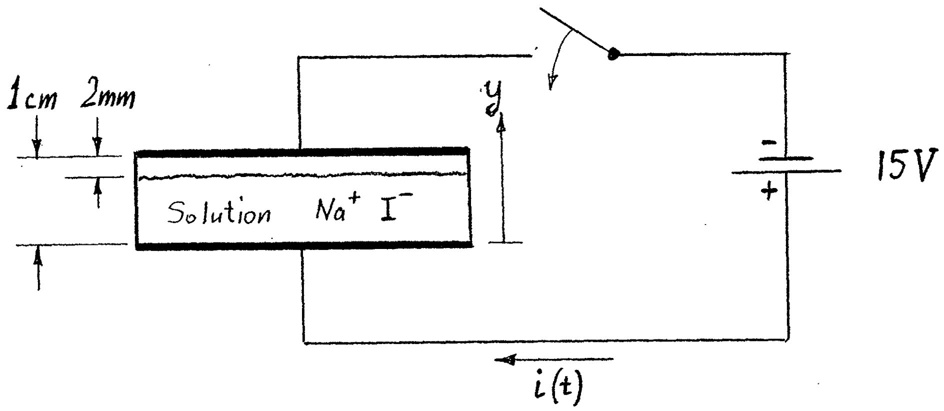
A solution of sodium iodide (Na I) in water, with a molecular concentration of $6.0 \times 10^{23}/\text{m}^3$, is placed in a shallow container of 10cm^2 area. The container has electrode plates on top and bottom, spaced 1cm apart. A 2mm air gap exists between the top electrode and the surface of the solution.

The Na^+ ions have a charge of 1.6×10^{-19}Coul, and move in response to the local electric field E with a speed $v_i = \mu_i E$, where $\mu_i = 4 \times 10^{-8} \frac{\text{m/s}}{\text{V/m}}$. By comparison, the I^- ions are effectively stationary.

The permittivity of vacuum is $\varepsilon_o = 8.89 \times 10^{-12}$Farad/m. The relative dilectric constants are $\varepsilon \simeq 1$ for air, and $\varepsilon = 64$ for water. The divergence equation below relates these quantities to the electric field vector $\vec{E}$ and the volume charge density ρ.

Edge effects are to be neglected, so quantities vary only along y.

$$\nabla \cdot \left(\varepsilon \varepsilon_o \vec{E}\right) = \rho$$



Using a very fast switch, a potential difference of 15V is applied between the plates.

1. Explain what happens after the switch is closed. Sketch the electric potential distribution $\varphi(y)$ between the plates for $t = 0$ and $t \to \infty$.

2. Derive an expression for the ion charge density σ (per unit area) on the surface of the solution, in terms of the fields E_{air} and E_{solution}.

3. Relate E_{solution} to the current density $j(\text{A/m}^2)$ due to the moving ions, and also to $d\sigma/dt$.

4. Formulate a first-order ODE governing j in time.

Dynamics

An aircraft is landing aboard an aircraft carrier. The carrier weighs 10^8kg and is traveling North at 10m/s. The aircraft weighs 10 000kg and is proceeding North at 50m/s.

1) About how fast is the carrier moving at the instant the aircraft comes to rest on the carrier deck?

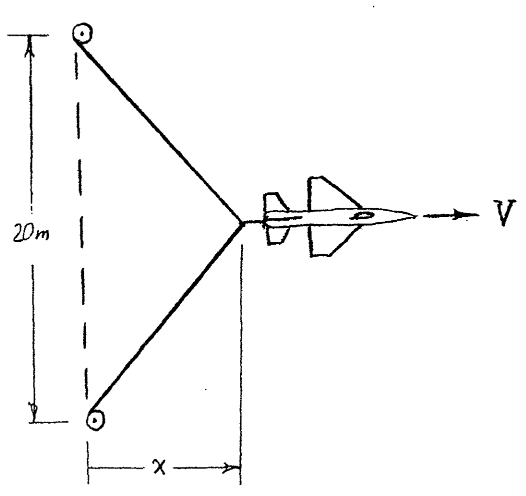
2) The greatest acceleration magnitude the aircraft should experience is 30m/s^2. What is the minimum distance over which an ideal arresting device can apply the deceleration force?

The actual arresting device is a cable which is hooked by the aircraft. The cable is initially stretched out perpendicular to the landing path, between two drums 20m apart which pay out the cable so as to maintain a constant cable tension T. The cable mass is negligible compared to the aircraft's mass.

3) Determine the speed of the aircraft $V(x)$ as a function of the distance x it has traveled after hooking the cable.

4) Determine the cable tension required to stop the aircraft in the minimum distance determined in 2), and the maximum acceleration magnitude seen by the aircraft for this tension.

5) Propose a change in the arresting-cable system geometry to reduce the maximum acceleration magnitude towards the ideal theoretical minimum determined in 2).



Autonomy

Consider the following linear program

$$\textbf{Maximize} \quad 3x + y$$

$$\textbf{Subject to} \quad \begin{cases} 2x + 2y & \leq & 16 \\ y & \leq & 5 \\ x & \leq & 5 \\ 3y + x & \leq & 18 \\ x & \geq & 0 \\ y & \geq & 0 \end{cases}$$

1. What is the optimal value for this optimization problem? Which design parameters optimize this problem?

2. Assume **in this question only** the first constraint becomes

$$2x + 2y \leq 16 + \delta$$

   Write a formula showing how the optimal cost of the problem changes with δ. What is the cost value for $\delta = 1$?

3. Assume **in this question only** that the cost function is now $x+3y$. What is the optimal value of this problem. Which design parameters optimize this problem?

4. Assume **in this question only** that the cost function is $x + 3y$ and that the fourth constraint becomes

$$3y + x \leq 18 + \mu.$$

   Write a formula showing how the optimal cost of the problem changes with μ. As μ evolves from -1 to $+1$, describe the set of optimal design parameters.

Communications and Networks

White noise of (single-sided) intensity N_0 Watts/Hz disturbs a communication channel in which one of two equal energy and equally probable signals are transmitted.

1. Illustrate the optimal receiver.
2. Determine the error probability of the optimum receiver for
    (a) Binary antopodal signals (Binary Phase Shift Keying (BPSK) signals), i.e., with the following signal constellation.

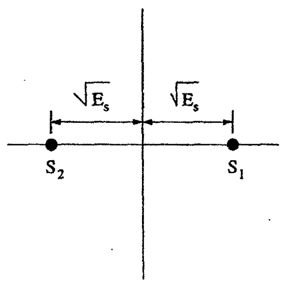


Figure 1: Signal constellation for binary antipodal signals.

    (b) Binary orthogonal signals i.e., with the following signal constellation.

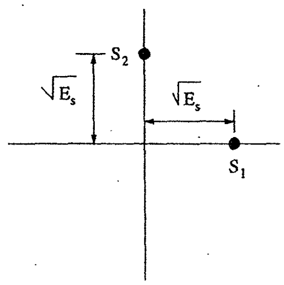


Figure 2: Signal constellation for binary orthogonal signals.

Control

Consider an aircraft that is initially flying horizontally with velocity V and then starts a coordinated turn. In this case the linearized dynamics are given by:

$$\begin{aligned}\dot{p} &= \frac{-1}{\tau}p + L_\delta \delta \\ \dot{\phi} &= p \\ \dot{\psi} &= \frac{g}{V}\phi\end{aligned}$$

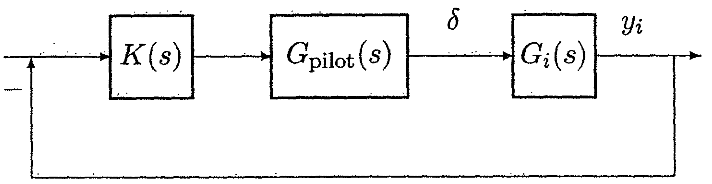
where p is the roll rate, ϕ is the bank angle, ψ is the heading angle, δ is the aileron input, the speed is $V = 50\text{m/s}$, $L_\delta = 1$, and $\tau = 0.25$ sec. The aircraft will be controlled by a pilot, the effective time delay of which we model as a simple first order system

$$G_{\text{pilot}}(s) = \frac{1}{s/12+1}.$$

There are two sensor options to display to the pilot:

- The "turn indicator", which measures $y_1 = r \approx \dot{\psi}$
- The "turn coordinator", which measures the combination $y_2 = r + \gamma p$, with $\gamma \approx 0.2$

The closed-loop system is shown in the figure, where the plant we use, G_i, depends which output we select y_1 or y_2.



1. Plot an accurate root locus for a positive gain K using (a) sensor y_1 and (b) sensor y_2.
2. Plot an accurate Bode diagram using (a) sensor y_1 and (b) sensor y_2.
3. Design a controller $K(s)$ for both cases that will approximately achieve the following specifications:
   - Bandwith of 7 rad/sec
   - A phase margin of 70 degs
   - Be as simple as possible

   Use whatever design method you prefer (*e.g.*, Bode or root locus), but you should design two controllers using the different sensors that meet the specifications.
4. In urgent situations, pilots tend to resort to gain feedback. Given that, which of these two sensors would you recommend be installed?
5. Discuss quantitantively what complications could occur if we incorrectly installed the second sensor into the aircraft so that it measured $y_3 = r - \gamma p$.

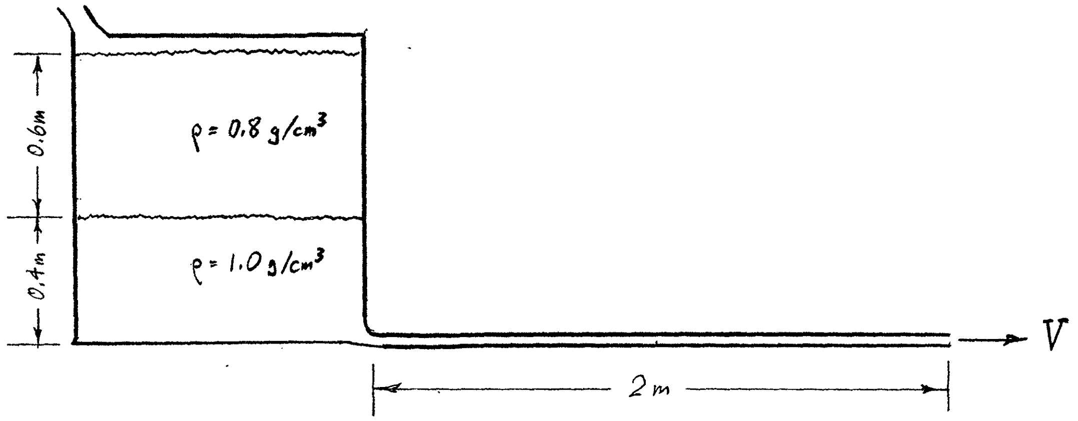
Fluids

A fuel tank in the shape of a 1m cube has 0.4m of collected water on the bottom, with 0.6m of fuel on top. The density of water is $\rho = 1.0\text{g/cm}^3$, and the density of the fuel is $\rho_f = 0.8\text{g/cm}^3$. The top of the tank is vented to the atmosphere.

The water is to be drained from one of the bottom edges through a hose with the small cross-sectional area $A = 4\text{cm}^2$ and length $L = 2\text{m}$. With steady flow through it, the hose has a frictional pressure drop equal to

$$\Delta p_{\text{friction}} = K \tfrac{1}{2} \rho V^2 \quad , \quad K = 0.4$$

where V is the average volume flow rate velocity. The junction between the hose and tank is smoothly faired. The hose end is at the same level as the tank bottom.



1) When the hose end is opened, a nearly-steady outflow of water is quickly established. Determine the pressure at the bottom of the tank as a function of the water height $h(t)$ in this quasi-steady condition.

2) Determine $V(t)$ as a function of $h(t)$. How long does it take to drain out all the water?

Consider now the initial unsteady situation just after the hose end is opened.

3) Assuming that the frictional forces in the hose are nearly independent of Reynolds number (i.e. K is constant), determine an equation for the acceleration $\dot{V}$ of the fluid in the hose, in terms of $V(t)$.

4) Numerically determine the initial acceleration $\dot{V}(t = 0^+)$ immediately after the hose is opened. Estimate the characteristic time to reach steady flow.

Human Factors

Consider the task of screening passenger carry-on bags for weapons at an airport security checkpoint. A human inspector monitors a new type of scanning machine that gives a numerical readout corresponding to the likelihood that a weapon is present as well as a color X-ray view of the bag's contents. If no weapon is present in a bag, the number indicated by the machine (x) is a random variable with a probability density function $f_0(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$. If a weapon is present in a bag, x is described by the probability density function $f_1(x) = \frac{1}{\sqrt{2\pi}} e^{-(x-2)^2/2}$. The machine is designed such that an audio alarm is triggered whenever $x > x_c$ where x_c is a threshold value defined by the system designer. Assume that $x_c = 0$ and that the *a priori* probability of a weapon being in a bag is estimated to be 0.001. The cost of a false alarm is 1 unit, and the cost of a missed detection is 1000 units.

a) Sketch a plot that shows both of the probability density functions and x_c. Clearly show the conditional probabilities of false alarm, missed detection, correct detection, and correct rejection from the machine.

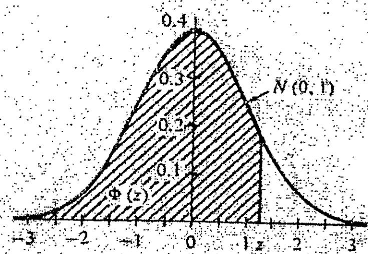
b) Briefly describe the tradeoffs involved in moving the decision threshold to the left or the right.

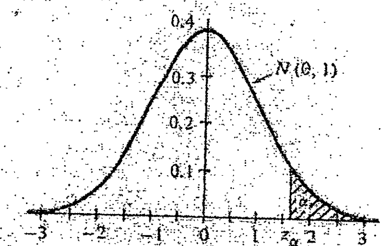
c) Find the expected cost of the machine in operation (you may refer to the attached table of values of the integrated Normal distribution). Assume that the human inspector simply follows the machine's advice and does not use the X-ray view.

d) Describe how an optimal decision threshold x_c could be determined for the machine.

e) From a signal detection theory point of view, describe how the human inspector would incorporate the machine's alarm information with the X-ray view to decide whether to actually open and inspect the bag.

Table IV The Normal Distribution





$$P(Z \le z) = \Phi(z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{-w^2/2}\, dw$$

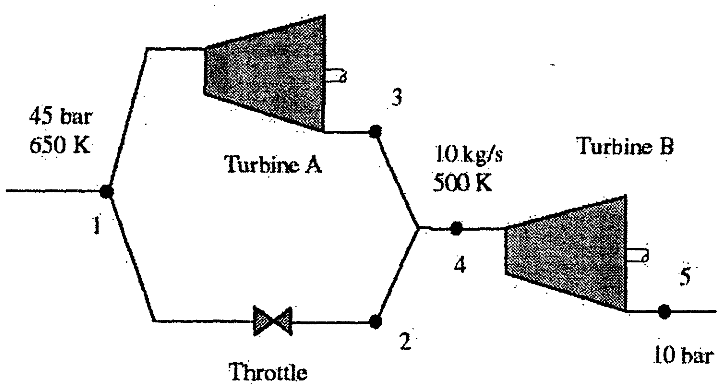
$$[\Phi(-z) = 1 - \Phi(z)]$$

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7703	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990

α	0.400	0.300	0.200	0.100	0.050	0.025	0.010	0.005	0.001
z_α	0.253	0.524	0.842	1.282	1.645	1.960	2.326	2.576	3.090
$z_{\alpha/2}$	0.842	1.036	1.282	1.645	1.960	2.240	2.576	2.807	3.291

<u>Propulsion & Thermodynamics</u>

The power system shown below consists of an adiabatic throttle and two ideal turbines. Both turbines operate at steady state. The pressure ratio p_4/p_5 of turbine B is 1.5. Kinetic and potential energy effects can be neglected and the working fluid can be assumed a perfect gas with c_p=1 kJ/kgK and γ=1.4.



a) Explain how the states change across the throttle and sketch the throttling process from state 1 to state 2 in a T-s diagram.

b) Sketch the expansion through the turbine on the same T-s diagram and indicate the curve of pressure p_3.

c) Sketch processes 3 to 4 and 2 to 4 in the same T-s diagram. Explain in a sentence or two what happens in these processes.

d) What is the mass flow rate through turbine A in kg/s? *(Hint: you may want to define an appropriate control volume that includes the flows at stations 2, 3 and 4.)*

e) What is the total power produced by the turbines (turbine A plus turbine B)?

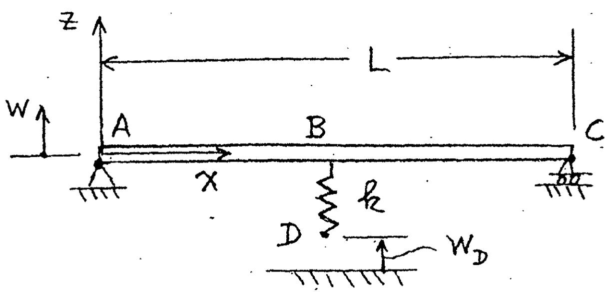
Software Engineering

Briefly describe the main features of three different software engineering process models. What are the important differences between these three models? Evaluate each with respect to the type of project for which they might be most appropriate, or in other words, what factors would you use to make a decision about the development process to be used for a particular project?

Structures

Given a uniform beam of length L, which is pushed up at its midpoint by a compressive spring k. The beam is pinned at both ends and the compressive spring moves up a distance w_D at it free end, as shown below.

1. Assuming the beam end at C is free to move horizontally and the spring has a value of $k = 10\ EI / L^3$, determine the beam deflection w_B for a given displacement w_D at the free end of the spring. Also, determine the reactions at A and C.
2. If the beam end at C is pinned but prevented from moving horizontally, would you expect greater or less deflection at the midpoint w_B for a given w_D? Describe the equations you would use to analyze this situation. How does this situation differ from that in 1. above?



Uniform beam
EI = constant

Vehicle Design and Performance

Answer either **Part A** or **Part B** (not both)

A

Vehicle Design and Performance

1. Assume the drag coefficient of an aircraft is given by

$$C_D = C_{D_0} + \frac{C_L^2}{\pi A\!\!R}$$

(i) Find the C_L which maximizes $\frac{L}{D}$.

(ii) How would the maximizing C_L be affected if C_{D_0} depends significantly on Reynolds number?

2. Rather than maximizing $\frac{L}{D}$, a better goal for jet aircraft design is to maximize $\frac{ML}{D}$, where M is the cruise Mach number. In order to do this, we must include the compressibility drag contribution:

$$C_D = C_{D_0} + \frac{C_L^2}{\pi A\!\!R} + K(M - M_c)^4$$

where

$$K = \begin{cases} 0 & M < M_c \\ 200 & M > M_c \end{cases} \tag{1}$$

and M_c is the critical Mach number.

(i) Find an expression for $\frac{\partial \frac{D}{ML}}{\partial M}$, assuming C_L, M_c, $A\!\!R$ and C_{D_0} are constant.

(ii) Evaluate this derivative for Mach numbers from 0.8 to 0.95 in steps of 0.05. Use $C_L = 0.5$, $C_{D_0} = 0.015$, $C_L^2/\pi A\!\!R = 0.01$, $M_c = 0.85$.

(iii) Give a rough estimate (± 0.01) of the Mach number that maximizes $\frac{ML}{D}$ for these parameters.

3. In aircraft configuration development, typically C_L is not fixed, but instead $C_{L_\perp} = \frac{C_L}{\cos^2 \Lambda}$ is fixed, where Λ is the wing sweep angle.

(i) How would you expect M and Λ to be related?

(ii) What is the aerodynamic reason for holding $C_{L_\perp}$ fixed?

4. Qualitatively, explain how the optimal M and C_L are influenced by the modification in 3.

B

Vehicle Design and Performance

A Mars surface mission rover (similar to Fig.1) that carries a research laboratory to a variety of sites and communicates scientific measurements back to Earth.

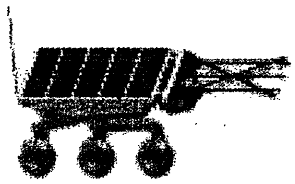


Fig 1. Mars surface rover

1) Perform a first-level functional decomposition of this system.
2) List subsystems that are important in the design, clearly rank their relative importance in determining performance and cost and briefly explain why.
3) Map the functions you determined in 1) to the subsystems from 2). Which subsystem carries out what functions?
4) Define a set of quantitative metrics that can be used to formulate requirements.
5) From among two or three sub-systems listed in 2) define a trade-off situation. What is the most important tradable parameter and how do cost, performance, and your metrics in 4) change as this parameter is varied?

Caution:

- There is not a single correct answer to this question.
- Be as quantitative as you can.
- Be brief and succinct. Don't spend more than 45 minutes. It is more important to answer all questions (1)-(5) somewhat, than to dwell extensively on only one or two.
- Demonstrate your ability to think like a vehicle designer or systems engineer.