4:45 - 5:45

JANUARY 2002
DEPARTMENT OF AERONAUTICS AND ASTRONAUTICS

WRITTEN QUALIFYING EXAMINATION
FOR
DOCTORAL CANDIDATES

Wednesday, January 23, 2002 **Room 37-212** **9:00 a.m. – 1:00 p.m.**

CLOSED BOOK AND NOTES

Answer a total of five (5) questions (no more or less).

You must answer at least two (2) questions from Column A (one Math and one Physics), and three questions (3) from Column B (Professional Area Subjects). Please answer each question in a separate blue book and indicate on the cover of the blue book which question is being answered.

Be sure that your name appears **on the cover of each of your blue books that you turn in to be graded.**

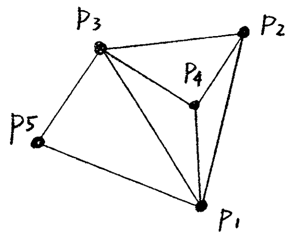
Oral examinations will be held on Tuesday, January 29, 2002. Please pick up your schedule on Monday, January 28, 2002, after 3:00 p.m. from the Aero Astro Student Services Office, 33-208.

Results will be available on Wednesday, January 30, 2002 after 3:00 p.m.
Please contact your advisor.

Column A
Mathematics (Discrete OR Continuous)
Physics (Dynamics OR Fields)

Column B
Autonomy ✓
Communication and Networks ✓
Control
Fluid Mechanics ✓
Human Factors Engineering ✓
Propulsion & Thermodynamics 1/2
Software Engineering
Structures and Materials
Vehicle Design and Performance ✓

Discrete Mathematics

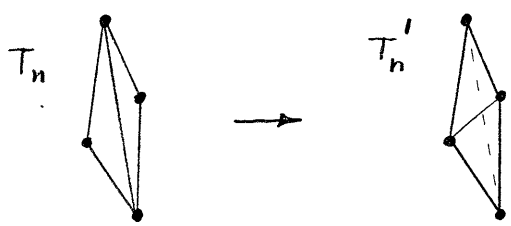


A given set of points $P_n = \{p_1\, p_2\, \ldots\, p_n\}$ is used to define a triangulation T_n which is a set of all possible edges connecting pairs of points such that
a) only triangles are formed by the edges (no squares, pentagons, etc), and
b) no pair of edges intersects.

1a) [10%] For a convex set P_n with n points, determine the number of its possible T_n configurations. Determine the unique number of edges $|T_n|$ common to all these configurations.

1b) [40%] If P_n is non-convex, the number of edges is not a unique function of n, but it can be bounded. Determine the upper and lower bounds on $|T_n|$. Prove your result by induction or otherwise.

The energy of a triangulation $e(T_n) \in I\!R$ is the sum of the lengths of all its edges. Any given P_n clearly has a minimum value of $e_{\text{min}}(P_n)$ among all its possible T_n. An elementary "annealing" state transition $T_n \rightarrow T_n'$ is one where one edge on the long diagonal of a quadrilateral is replaced by an edge on the short diagonal, thus reducing $e(T_n)$, or at least leaving it unchanged (if diagonals are equal).



2a) [20%] For a general non-convex P_n, determine the order of the worst-case operation count required to find e_{min} by direct search.

2b) [30%] A starting T_n^0 for a given P_n is to undergo a number of elementary state transitions in order to reach e_{min}. Can this always be achieved without increasing e at any one transition?

Continuous Mathematics

Consider the following initial-value problem for $x(t)$:

$$
\begin{aligned}
\ddot{x} + \dot{x}^3 + x &= 0 \\
x(0) &= 0 \\
\dot{x}(0) &= 100
\end{aligned}
$$

1. [15%] Qualitatively sketch

   A. Initial behavior of $x(t)$ for $t \ll 1$, and
   B. Long-term behavior of $x(t)$ for $t \gg 1$.

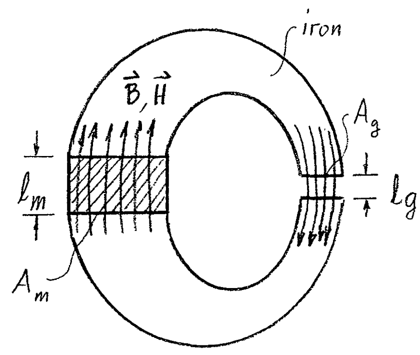
2. [40%] Quantitatively estimate the initial behavior of $\dot{x}(t)$ and $x(t)$ in A. Give criteria for the validity of your estimate.

3. [15%] Quantitatively estimate the long-term behavior of $\dot{x}(t)$ and $x(t)$ in B, neglecting any damping effects. Give criteria for the validity of your estimate.

4. [30%] For a given amplitude of $x(t)$ in B, estimate the average magnitude of the damping term, and thus estimate the decay rate of the amplitude.

Physics (Fields)

It is desired to produce a nearly uniform magnetic field strength $B_g = 0.5\text{Tesla}$ in a gap of $\ell_g = 0.5\text{cm}$ and area $A_g = 4\text{cm}^2$, using a Samarium-Cobalt permanent magnet mounted in a yoke of silicon-iron. Figure 1 shows the configuration, the governing equations for the magnetic field $\vec{B}$ and the forcing field $\vec{H}$, and gives the permeabilities μ_o and μ.



$$\nabla \cdot \vec{B} = 0$$
$$\nabla \times \vec{H} = 0$$

$\mu_o = 1.256 \times 10^{-6}$ Tesla-m/A (air)

$\mu = 10^4 \mu_o$ (iron)

Figure 1: Geometry

The fields $\vec{B}$ and $\vec{H}$ can be assumed to be parallel everywhere, and the relationships between their magnitudes B and H in the various components are idealized as shown in Figure 2.

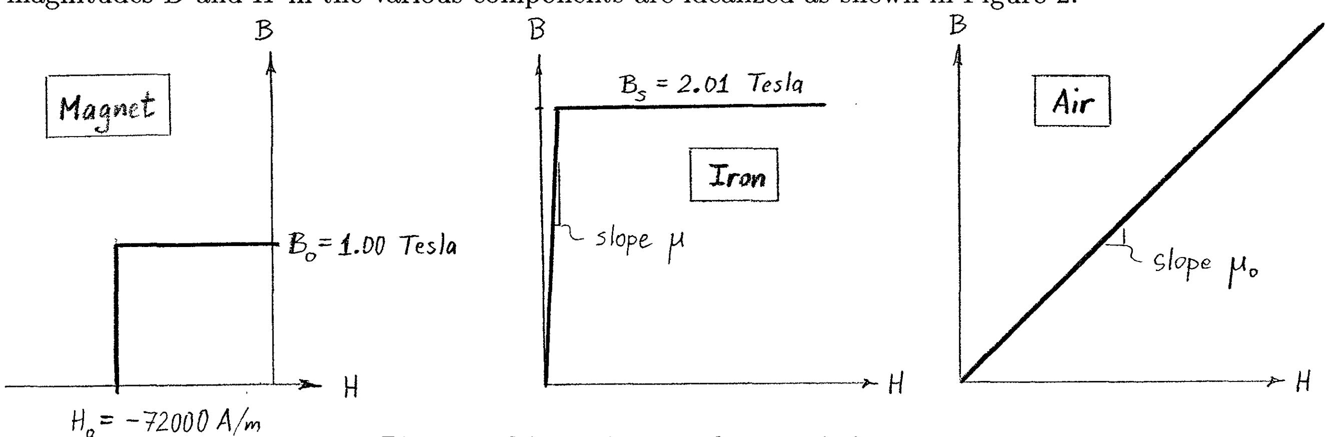


Figure 2: Magnetization characteristics

The design should use the smallest possible volume of the expensive magnetic material, and should avoid saturation of the iron (i.e. $B < B_s$ in the iron) to prevent flux leakage outside of the circuit.

1) [10%] Relate B to the local cross-sectional area A anywhere in the magnetic circuit.

2) [10%] Determine the line integral $J = \oint \vec{H} \cdot d\vec{\ell}$ taken along a closed contour around the magnetic circuit. In what parts of this circuit can H assumed to be nearly negligible?

3) [30%] Determine B_g in terms of the geometry and magnetic properties. Consider two cases — i) magnet unsaturated, and ii) magnet saturated.

4) [35%] Consider the "design space" ℓ_m/ℓ_g vs A_m/A_g. Identify regions where the magnet is saturated and unsaturated, and the line along which the design requirement $B_g = 0.5\text{Tesla}$ holds.

5) [15%] Compute the minimum magnet volume along the design line.

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Dynamics

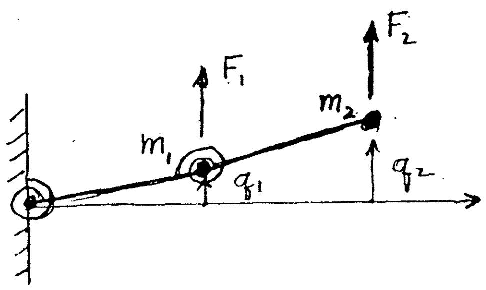
A cantilever beam is simply represented by the 2 rigid element, spring-mass system as shown below. The corresponding dynamic equations for this representation are given below.

(a) Determine the natural frequencies ω_1 and ω_2 of this system.

(b) Determine the corresponding natural modes of vibration.

(c) A step function force $F_2 = F_0$ is given to mass 2 of the system. Determine the response q_2 at mass 2 after the force is applied.

(d) A step displacement of magnitude $q_2 = u_0$ is given to mass 2 of the system. Determine the response q_1.



$$m_1 \ddot{q}_1 + K_{11} q_1 + K_{12} q_2 = F_1$$
$$m_2 \ddot{q}_2 + K_{21} q_1 + K_{22} q_2 = F_2$$

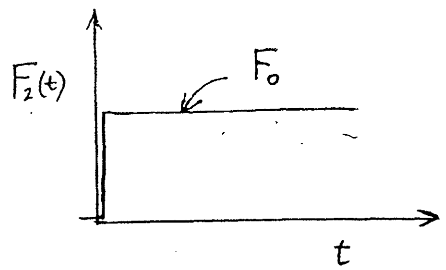
$m_1 = .100$ Kg

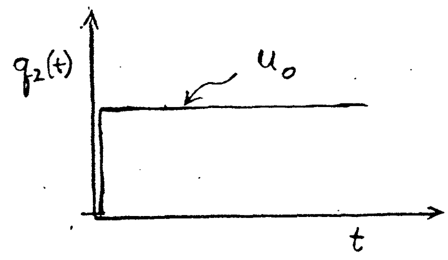
$m_2 = .050$ Kg

$K_{11} = 6000$ N/m

$K_{22} = 1000$ N/m

$K_{12} = K_{21} = -2000$ N/m





Autonomy

1) Use the simplex method (by hand), to solve the following optimization problem

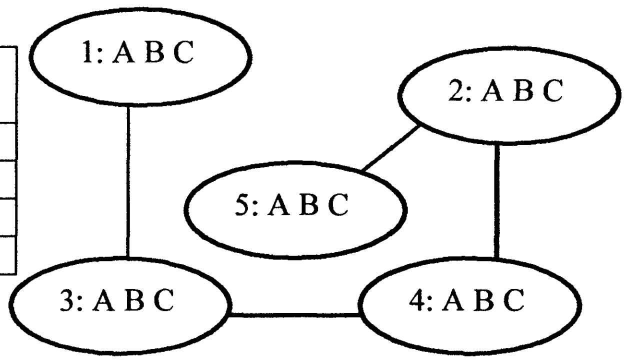
Maximize: $x_1 + 2x_2 + 3x_3$

Subject to:

$$9x_1 + 3x_2 + 9x_3 \leq 12$$
$$x_3 \leq 3$$
$$2x_1 - x_2 \leq 1$$
$$x_1, x_2, x_3 \geq 0$$

2) Consider the following constraint graph. Each variable has the same domain {A, B, C}. The only valid assignments to pairs of constrained variables are given in the following table.

Constraint ($V_i - V_j$)	**Valid Assignments (V_i, V_j)**
1-3	(A, A) or (B, B)
2-4	(A, A) or (B, B)
3-4	(A, B) or (B, A)
2-5	(B, A) or (B, C)



I. Repeatedly perform constraint propagation on the above constraint graph until you achieve arc consistency. Cross out the eliminated values on each node of the graph.

II. What is the maximum number of possible solutions, based only on knowledge of the remaining values?

In general, does constraint propagation guarantee that all infeasible solutions are pruned?

Communications and Networks

A source generates letters from the alphabet {a1, a2,a3, a4 } with corresponding probabilities {1/2, 1/4, 3/16, 1/16}. Letters are generated independently at a rate of one letter per second.

A) What is the binary entropy of the source (you may use: $\log_2(3) = 1.585$)?

B) What is the minimum required average codeword length to represent the source for error free reconstruction? What data rate would be needed to transmit this source?

C) Design a Huffman code for the source. What is the average codeword length of your code?

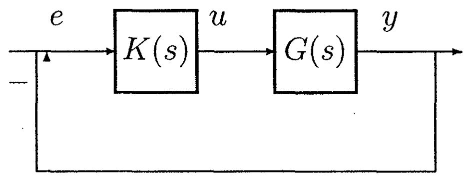
D) How can you improve your code to achieve a smaller average codeword length?

Control Qualifying Question **January 2002**

Consider the simple system with dynamics given by

$$G(s) = \frac{s+2}{s^2 - 3\sqrt{2}s + 9}$$

in a negative feedback loop.

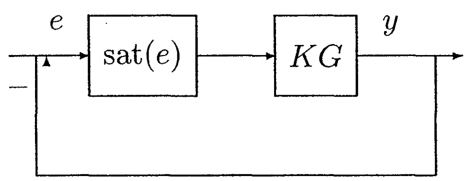


(a) [15%] Sketch the root locus for this system for positive values of the control gain $K(s) = k$.

(b) [30%] Sketch the Nyquist plot for this system assuming that the control gain $K(s) = 5$. Do your Nyquist and root locus sketches yield consistent conclusions about the stability of the closed-loop system for this control gain?

(c) [35%] Design a controller $K(s)$ that gives a closed-loop system with a rise time (10% to 90%) of approximately 0.5 seconds and a settling time (1%) of approximately 2.3 seconds. What type of controller is this?

(d) [20%] Now consider the possibility that the system actually has a saturation nonlinearity in the feedback loop, as shown in the following figure.



where

$$\text{sat}(e) = \begin{cases} e, & |e| \leq 1 \\ 1, & e > 1 \\ -1, & e < -1 \end{cases}$$

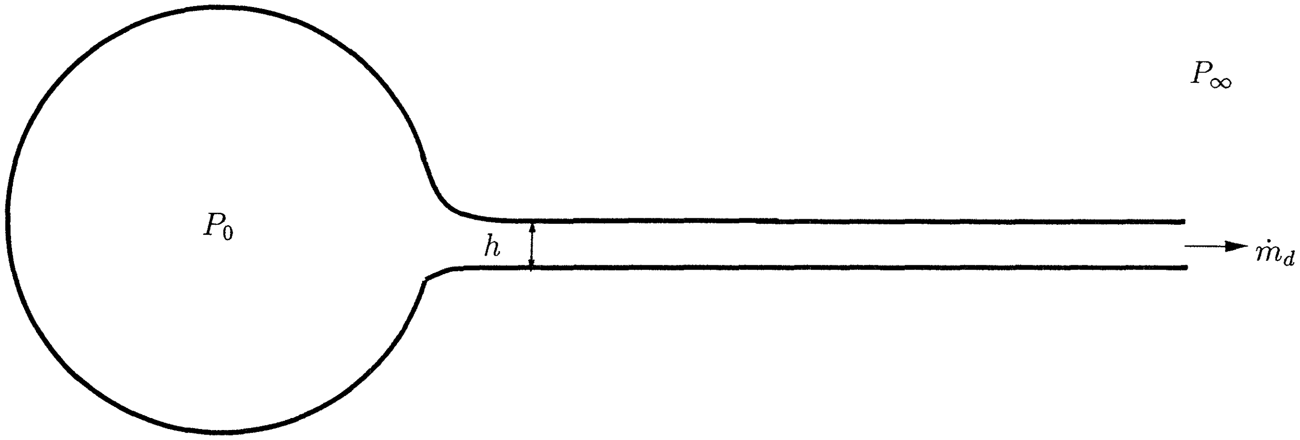
Using the controller you designed in part (c), qualitatively describe how you would expect the system to respond to both:

- A unit step input ($r = 1$)
- A step input of $r = 4$

10/1.

Fluids

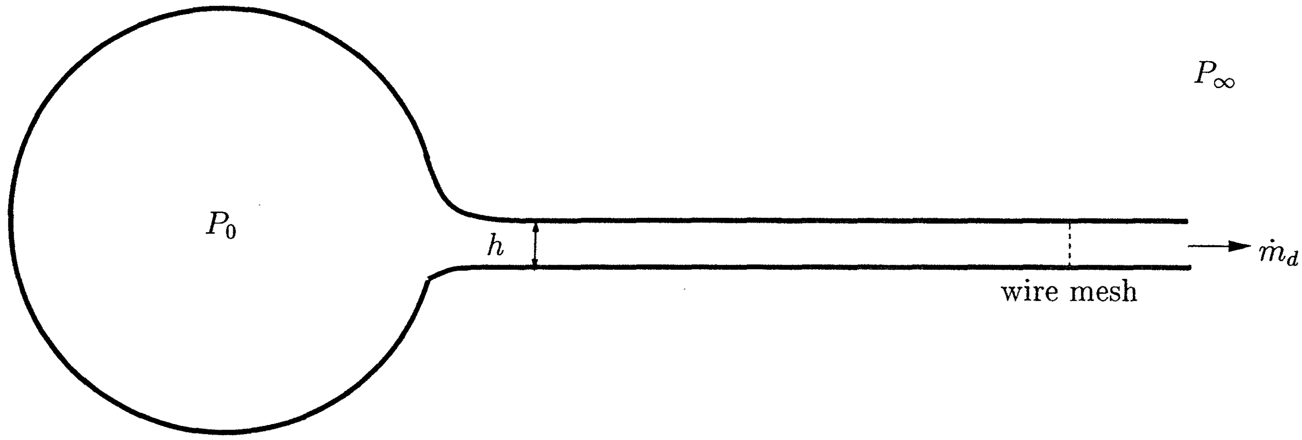
Part 1 [10%]: A two-dimensional duct is connected to a large reservoir with pressure P_0 and dumps into the atmosphere with pressure P_∞. Assume that the wall shear stresses are negligible and the flow is incompressible with density ρ. Calculate the mass flow leaving the duct $\dot{m}_d$.



Part 2 [10%]: Next, a wire mesh is introduced into the duct. For flow to pass through the wire mesh, a pressure drop is required and given by,

$$p_u - p_d = \frac{1}{2}\rho u_{mesh}^2 K$$

where p_u and p_d are the upstream and downstream static pressure, u_{mesh} is the velocity through the mesh, and K is a positive constant. Calculate $\dot{m}_d$. How does $\dot{m}_d$ compare to that determined in Part 1?



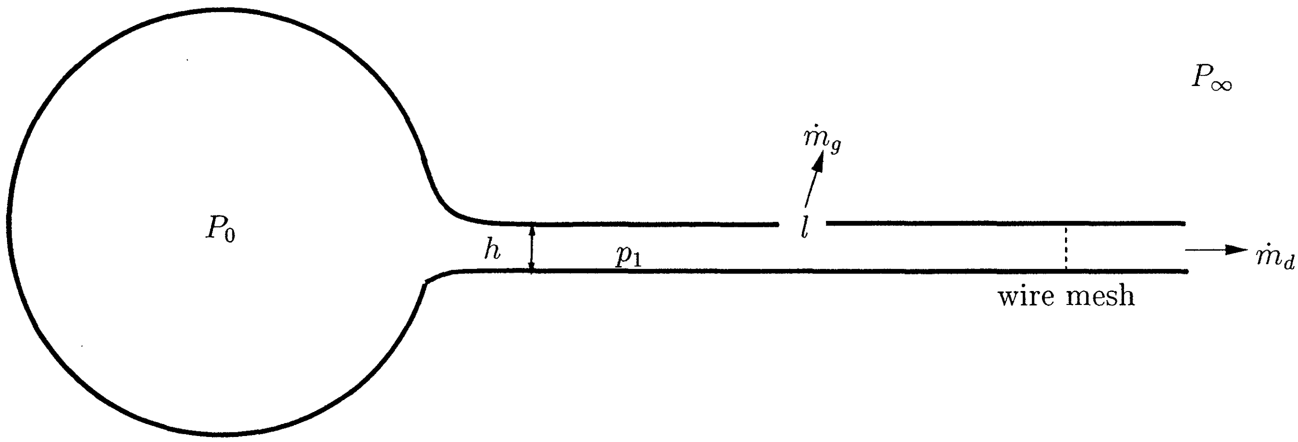
PARTS 3 and 4 ARE ON FOLLOWING PAGE

10/2

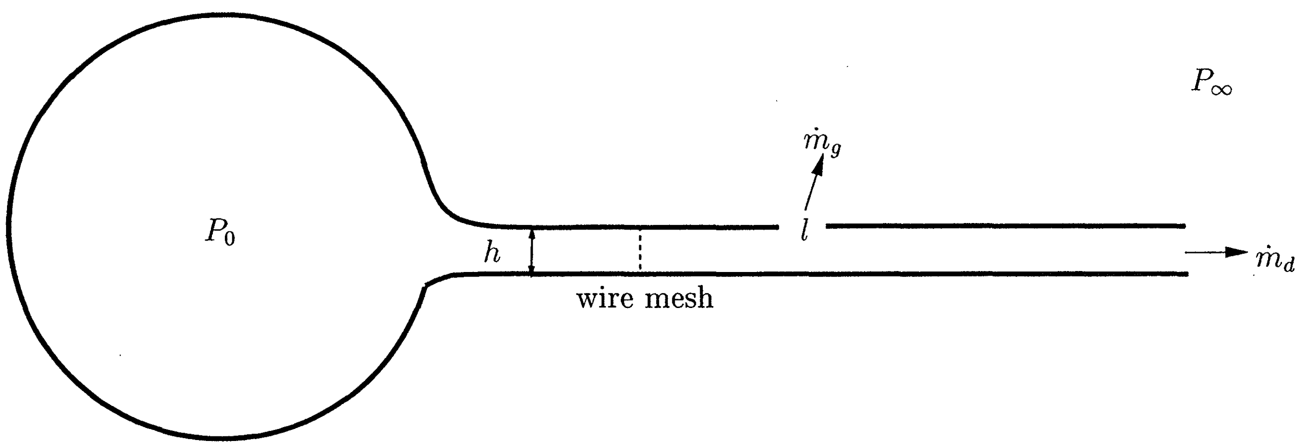
Part 3 [60%]: A small gap of length l (assume that l is small compared to h) is now introduced upstream of the wire mesh. Assuming that $P_\infty < p_1$, the air flows through the gap with mass flow given by

$$\dot{m}_g = l\, C_d \sqrt{2\rho\,(p_1 - P_\infty)}$$

where p_1 is the static pressure in the duct upstream of the gap and C_d is a positive constant. Calculate $\dot{m}_d$ and $\dot{m}_g$. How does $\dot{m}_d$ compare to that determined in Part 2?



Part 4 [20%]: For the final part, the wire mesh is now moved upstream of the gap. Calculate $\dot{m}_d$ and $\dot{m}_g$. How do $\dot{m}_d$ and $\dot{m}_g$ compare to those determined in Part 3?



HUMANS AND AUTOMATION

A new cockpit display is needed to aid pilots in landing a helicopter on the deck of a small ship in very rough seas at night in the fog. The helicopter is subject to vertical turbulence that could cause sudden vertical velocity changes of up to 3 ft/s. Landing with a relative velocity greater than 10 ft/s is considered a crash. Assume that fore-aft and lateral displacement errors are taken care of automatically, as are attitude errors, so you are only concerned with a display for height control. Assume further that the helicopter allows direct control of the lift vector, with a maximum lift 1.25 times the vehicle weight.

The approach starts at 200 ft above mean sea level. The maximum wave height is ± 30 ft relative to mean sea level with a period of 20 seconds. Barometric pressure and a radar altimeter measurement (showing the altitude of the helicoter relative to the ship) are available in the helicopter.

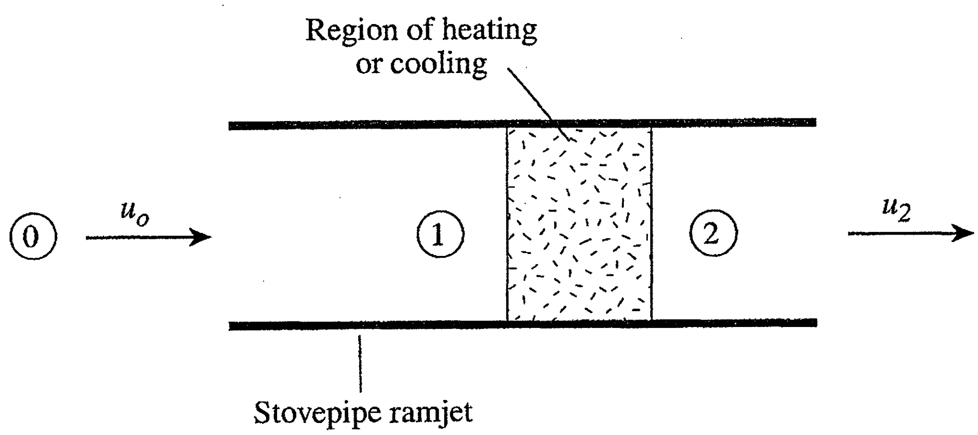
1. Clearly define all your variables and show them in a sketch.

2. Sketch a design for each of the displays below, and describe as completely as possible how each display would operate, referencing your variables as necessary. Also discuss the benefits or limitations of each display type for this task.

    (a) Compensatory display
    (b) Pursuit display
    (c) Predictor display
    (d) Quickened compensatory display
    (e) Command display using a flight director

Propulsion & Thermodynamics

An idealized model of a basic propulsion system, the ramjet, is a tube flying through the air, with a region inside the tube in which the air is heated. The situation is shown below. There is an isentropic increase in pressure from location 0 to location 1 and heat addition at constant area from location 1 to location 2.



Tube with heating or cooling (idealized ramjet)

If heat is added the density decreases from location 1 to location 2. From continuity, the velocity increases. As a result the exit velocity, u_2, will be different from the inlet velocity and thrust will be produced.

a) [30%] Sketch the velocity on a line through the ramjet center from station 0 to 1 to 2. In this take the flight Mach number as low enough so that the only density change that needs to be considered is that between station 1 and station 2.

b) [20%] Sketch the static pressure, with respect to atmospheric pressure, on a line through the ramjet center from station 0 to station 1 to station 2.

c) [20%] Sketch the streamlines into the engine.

d) [20%] Thrust can be viewed as the sum of the fluid forces on all wetted area in the engine. However, in this idealized device, the engine surfaces are only parallel to the direction of flight so that (seemingly) pressures could only produce a net force on the engine which is normal to the direction of flight. For this idealized ramjet, where do the pressure forces act to produce a net thrust?

e) [10%] Suppose instead of heating addition, the air is cooled from location 1 to location 2. Would such a device produce thrust? Would it go backwards? What happens?

Software Engineering

1) Why, in general, are effective metrics more difficult to devise for software than for hardware?

2) For each of the following categories of metrics, briefly (not more than a quarter of a page) describe a specific metric that has been proposed for it and critique it. What are the assumptions that underlie each of the metrics you chose and how realistic do you think those assumptions are?

a. A static code metric

b. A dynamic code metric

c. Algorithmic project cost modeling

Structures

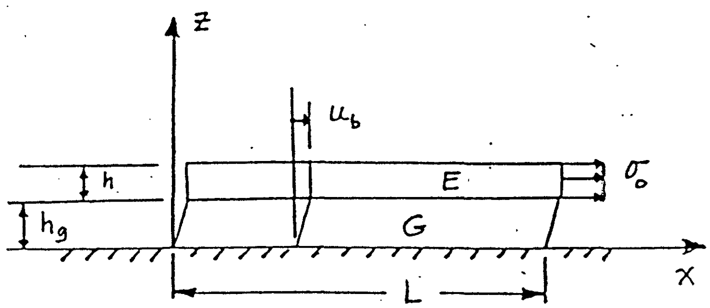
Given an elastic beam of length L, width b, and thickness h, which is glued to an essentially rigid base by a thin layer of glue of thickness h_g. The beam has an elastic modulus of elasticity E and can stretch, while the glue is assumed to have only a shear modulus G and can only shear. If the beam is pulled by a uniform edge stress σ_o (N/m^2) as shown below,

(a) Determine the shear stress distribution τ_{xz} in the glue. What is the maximum shear stress? Sketch roughly the distribution with x.

(b) Determine the displacement distribution u_b in the elastic beam.

(c) How does this shear stress compare with the shear stress assumed uniformly distributed over the beam?

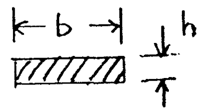
(d) How are the stresses affected by the thickness of the glue layer h_g?



elastic beam can stretch due to stress σ_x

glue can shear due to stress τ_{xz}

cross section



$E = 72 \times 10^9$ (N/m^2)

$G = 6 \times 10^9$ (N/m^2)

h = 2 mm   L = 20 mm $h_g/h \ll 1$

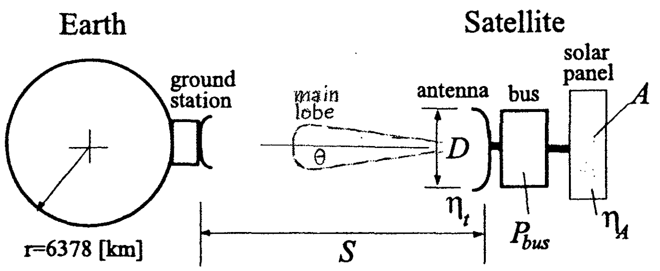
h_g = 0.2 mm   b = 10 mm

Vehicle Design and Performance

You may solve **either** question (a), which deals with the design of a communications satellite **or** question (b), which deals with the design of an unmanned aerial vehicle (UAV). Both questions are equivalent in terms of degree of difficulty and methodology.

(a) Spacecraft Design Problem

You should assist in the design of a communications satellite at geosynchronous altitude S, see Figure 1. **The performance requirement is a transmission data rate of** $R_{req} = 100$ [Mbps]±20% at a radio frequency of $f = 2$ [GHz][1]. The satellite is made up of a solar panel of area A [m^2] with efficiency η_A, a standard bus that consumes P_{bus} power, and a parabolic antenna of diameter D [m] with transmission efficiency η_t.



S=35786	[km]	GEO altitude
c=3 $\cdot 10^8$	[m/s]	speed of light
$W_o = 1358$	[W/m^2]	solar constant
$\eta_A = 0.12$	[-]	solar panel efficiency
P_{bus} =300	[W]	bus power
$\eta_t = 0.8$	[-]	transmission efficiency
$\alpha = 2.4 \cdot 10^{21}$	[J^{-1}]	link constant

Figure 1: Geosynchronous communications satellite with important constants

It is your job to negotiate between the power and payload teams to find a solution for the design vector $x = [D\ A]^T$ that will satisfy the performance requirement R_{req}, while not violating any constraints. Some equations that might be useful are given below, where λ is the electromagnetic wavelength [m] and P_A is the power generated by the solar panel:

$$\begin{array}{ll} \textbf{data rate [bps]} & R = \alpha \cdot P_t \cdot G_t \cdot L_s \\ \textbf{transmission power [W]} & P_t = P_A - P_{bus} \\ \textbf{parabolic antenna gain [-]} & G_t = \dfrac{\pi^2 D^2 \eta_t}{\lambda^2} \\ \textbf{path loss [-]} & L_s = \left(\dfrac{\lambda}{4\pi S}\right)^2 \end{array} \quad (1)$$

(a1) Assuming 45 degree incidence of sunlight onto the solar panel, find $P_t = f(A)$. Compute a lower bound A_{min} for $P_t = 0$.

(a2) The two subsystem teams suggest an initial design $x_o = [D\ A]^T = [1\ 5]^T$. Compute the performance $R(x_o)$. Does this design meet the requirement? Explain.

(a3) If design x_o meets the requirement, set $x_1 = x_o$ and continue directly with **(a4)**. Otherwise, use x_o as a starting point for finding an acceptable design x_1. Plot your path from x_o to x_1 in the (D,A)-design space. The space is bounded by $0.5 \leq D \leq 3$ [m] and $A_{min} \leq A \leq 12$ [m^2]. **Hint:** Plot the (D,A)-space on a separate sheet.

(a4) Management likes your design x_1, but points out that it is not unique. Find a second acceptable design, x_2, at some distance from x_1. Sketch the line that corresponds to the requirement $R_{req} = 100$ [Mbps] in (D,A)-space (isoperformance contour).

(a5) The accounting department gives you the cost estimation relationships for the antenna $C_D = 2500 \cdot D^2$ [$] and solar panel $C_A = 12000 \cdot (A+1)$ [$], respectively. Find a design x_3 that will minimize the total cost $C_A + C_D$ **and** meet the requirement R_{req}.

(a6) You think you are all done. Unfortunately, now the attitude control system (ACS) team is complaining about your design x_3. What is their problem? **Hint:** Think about the effect of D on the beamwidth θ.

[1] 1 [Mbps] is one million bits per second

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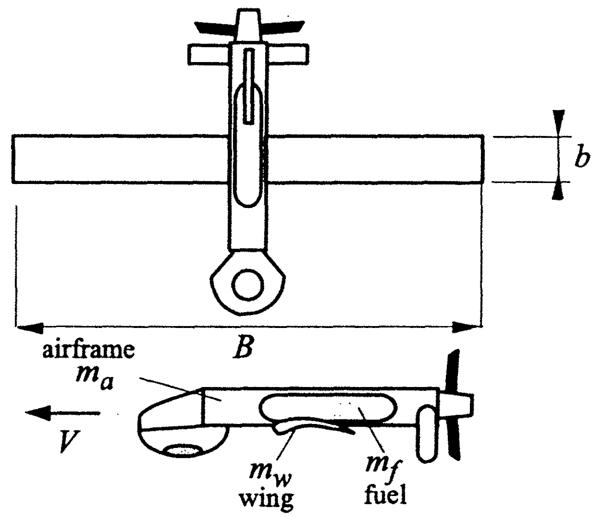
(b) Unmanned Aerial Vehicle (UAV) Design Problem

You should assist in the design of a UAV to be used for reconnaissance missions. **The performance requirement is an endurance E_{req} of 20 [hours]±10%.** Your job is to find a good design, represented by the vector $x = [f\ \mathit{AR}]$, where f is the fuel mass fraction and AR is the wing aspect ratio. Figure 2 shows a side and top view of the UAV along with important constants.



$\eta = 0.8$	[-]	propeller efficiency
$c = 10^{-6}$	[m^{-1}]	specific fuel burn
$C_L = 1.2$	[-]	lift coefficient
$C_{Do} = 0.05$	[-]	zero lift drag coefficient
m_a=220	[kg]	airframe mass (excluding wing)
$\rho = 0.6$	[kg/m^3]	air density at 6500m
V=40	[m/s]	cruise speed
g=10	[m/s^2]	gravitational acceleration

Figure 2: UAV design with important constants

The aspect ratio is $\mathit{AR} = B/b$, while the wing area S is equal to $S = Bb$ and remains fixed. The mass of the wing is dependent on the aspect ratio as $m_w(\mathit{AR}) = (m_a/18)\mathit{AR}$. Some other useful equations for solving this problem are:

$$
\begin{aligned}
&\textbf{Endurance [sec]} && E = a \cdot C_D^{-1} \cdot \left(\sqrt{1+f} - 1\right) \\
&\textbf{total drag coefficient [-]} && C_D = C_{Do} + \frac{C_L^2}{\pi \mathit{AR}} \\
&\textbf{empty mass [kg]} && m_1 = m_a + m_w = m_a\left(1 + \mathit{AR}/18\right) \\
&\textbf{gross takeoff mass [kg]} && m_o = m_1 + m_f = m_1 \cdot (1+f)
\end{aligned} \qquad (2)
$$

(b1) First compute the constant a used in the endurance equation: $a = 2\eta C_L/(cV)$. Then compute the wing area S [m^2], assuming that $f = 0.25$ and $\mathit{AR} = 6$. **Hint:** The lift is $L = (\rho/2)V^2 S C_L$ and $L = g \cdot m_o$ during straight and level flight.

(b2) Compute the performance $E(x_o)$ for the initial design vector $x_o = [f\ \mathit{AR}] = [0.25\ 6]$. Does this design meet the performance requirement E_{req}?

(b3) If design x_o meets the requirement, set $x_1 = x_o$ and continue directly with **(b4)**. Otherwise, use x_o as a starting point for finding an acceptable design x_1. Note that the design space is bounded by $0.1 \le f \le 0.5$ and $5 \le \mathit{AR} \le 16$, respectively. **Hint:** Try adjusting the fuel mass fraction, f.

(b4) Management likes your design x_1, but points out that it is not unique. Find a second acceptable design, x_2, at some distance from x_1. **Hint:** This time, try adjusting AR, starting from design x_o.

(b5) Plot your path from x_o to x_1 and x_2 in the (f, AR) design space. Sketch the line that corresponds to the requirement $E_{req} = 20$ [hours], i.e. the iso-endurance contour.

(b6) The UAV lifecycle cost is the sum of the production cost $C_p = 1000 \cdot m_1$ and the total fuel cost $C_f = 5000 \cdot \gamma \cdot f \cdot m_1$. Which design, x_1 or x_2, has the lower lifecycle cost $C_p + C_f$, given that $\gamma = 4$ [$/kg]? Briefly explain the tradeoff between both designs.

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