JANUARY 2001
DEPARTMENT OF AERONAUTICS AND ASTRONAUTICS

WRITTEN QUALIFYING EXAMINATION
FOR
DOCTORAL CANDIDATES

Wednesday, January 24, 2001 37-212 9:00 a.m. – 1:00 p.m.

CLOSED BOOK AND NOTES

Answer a total of five (5) questions (no more or less).

You must answer at least two (2) questions from Column A.
Please answer each question in a separate blue book and indicate on the cover of the blue book which question is being answered.

Be sure that your name appears **on the cover of each of your blue books that you turn in to be graded.**

Oral examinations will be held on Tuesday, January 30, 2001. Please pick up your schedule on Monday, January 29, 2001 after 3:00 p.m. from Marya Klugerman in 33-208.

Results will be available on Wednesday, January 31, 2001 after 3:00 p.m.
Please contact your advisor.

Column A	Column B
Mathematics	Avionics
Physics	Fluids
Dynamics	Humans and Automation
	Instrumentation, Control and Estimation
	Propulsion
	Structures
	Systems
	Thermodynamics

Mathematics

Answer either **Math I** or **Math II**.

Math I

Written Qualifying Exam - Math

Given the matrix

$$A = \begin{pmatrix} 1 & -2 & -1 & 3 \\ 1 & 1 & 4 & 8 \\ -1 & 5 & 6 & 2 \end{pmatrix} = \begin{pmatrix} \cdots & r_1 & \cdots \\ \cdots & r_2 & \cdots \\ \cdots & r_3 & \cdots \end{pmatrix}.$$

1. Find the values of α_1 and α_2 such that $r_3 = \alpha_1 r_1 + \alpha_2 r_2$ where r_i is row i of A.

2. Find a basis for the row space $\mathcal{R}(A^T)$

3. Find a basis for the column space $\mathcal{R}(A)$

4. Find a basis for the nullspace $\mathcal{N}(A)$

5. Given the vector $x = (3, 3, 3, 3)$, split it into a component x_r from the row space and a component x_n from the nullspace such that $x = x_r + x_n$ where $x_r \in \mathcal{R}(A^T)$ and $x_n \in \mathcal{N}(A)$.

6. For a general matrix B, prove that $y^T z = 0$ for any $y \in \mathcal{N}(B)$ and any $z \in \mathcal{R}(B^T)$.

NOTE: The following vector space definitions might be helpful.

nullspace: The nullspace of a matrix A is denoted $\mathcal{N}(A)$ and is the set of all vectors x such that $Ax = 0$.

column space: The column space of a matrix A is denoted $\mathcal{R}(A)$ and is the set of all vectors which are a linear combination of the columns of A

row space: The row space of a matrix A is denoted $\mathcal{R}(A^T)$ and is the set of all vectors which are a linear combination of the rows of A

basis: A basis of a vector space is a linearly independent set of vectors which can be linearly combined to form any vector in that space.

Math II

Probability Problem

(submitted by Amedeo Odoni, arodoni@mit.edu, x 3-7439)

An elevator has a capacity of 2000 lbs. of weight. The weight (in lbs) of people who use this elevator is uniformly distributed between 120 and 220 lbs. Assume also that the weights of different individuals are statistically independent of one another.

(a) Given that 6 persons are in the elevator, what is the probability that exactly one of them weighs more than 200 lbs.?

(b) Given that 2 people are in the elevator and that their total weight is less than 400 lbs., what is the probability that one of them weighs more than 200 lbs.?

(c) Suppose that 10 people are waiting for the elevator at the instant when it arrives *empty* at the first floor. What is the probability that the weight capacity of the elevator will be exceeded? Please write an expression for an approximate answer to this question and explain briefly. A numerical answer is NOT required.

(d) Suppose that prospective riders arrive at the elevator's first-floor door in a Poisson manner at the rate of λ per unit of time. Suppose also that the time interval between successive stops of the elevator at the first floor is described by a random variable t whose pdf is negative exponential with parameter μ, i.e.,

$$f_t(t) = \mu e^{-\mu t}, \qquad t \geq 0$$

Given, that the elevator has just left the first floor taking all waiting elevator riders, write an expression for the probability that exactly 6 prospective elevator riders will be waiting at its first-floor door when the elevator next returns there. Do NOT evaluate this expression.

Written Doctoral Qualifying Exam

Physics

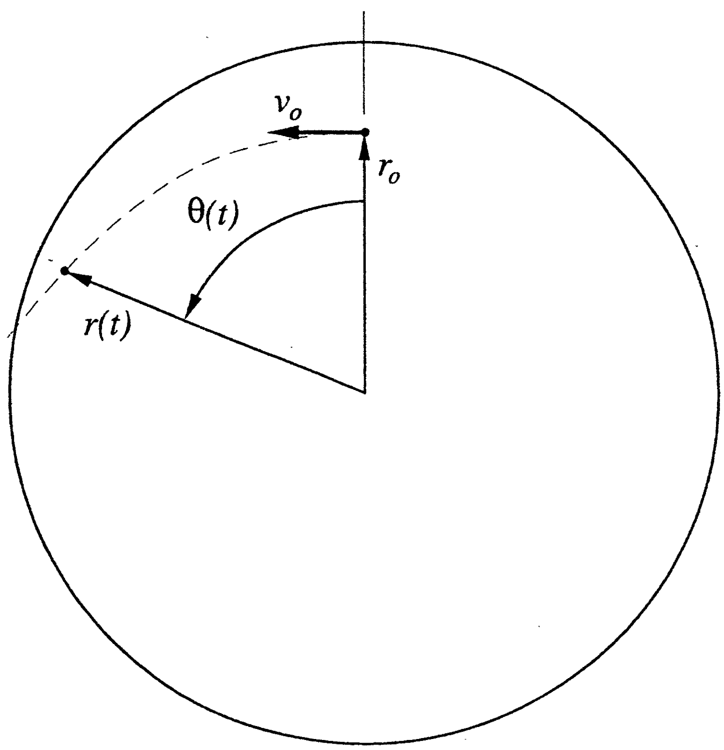
1) Assuming the earth is a perfect sphere of radius R and uniform density, determine the gravitational acceleration $a(r)$ for all radii $r = 0 \ldots \infty$ (assuming a known acceleration g at the surface $r = R$). Determine the potential energy/mass $\phi(r)$ associated with this acceleration for all radii.

2) A hypothetical particle which doesn't interact with matter except via gravity is released within the earth at radius r_o with tangential velocity v_o. Determine the energy conservation relation between the subsequent radial and tangential velocities $\dot{r}$ and $\dot{\theta} r$. Eliminate all θ dependencies in your relation using conservation of angular momentum.

3a) Determine the critical release velocity v_o which will allow the particle to just barely reach the earth's surface.

3b) Determine the critical release velocity v_o which will allow the particle to just barely escape to infinity.

4) Determine (but do not attempt to solve) the differential equation governing the trajectory shape $r(\theta)$.



Qualifier Written Question (Dynamics)

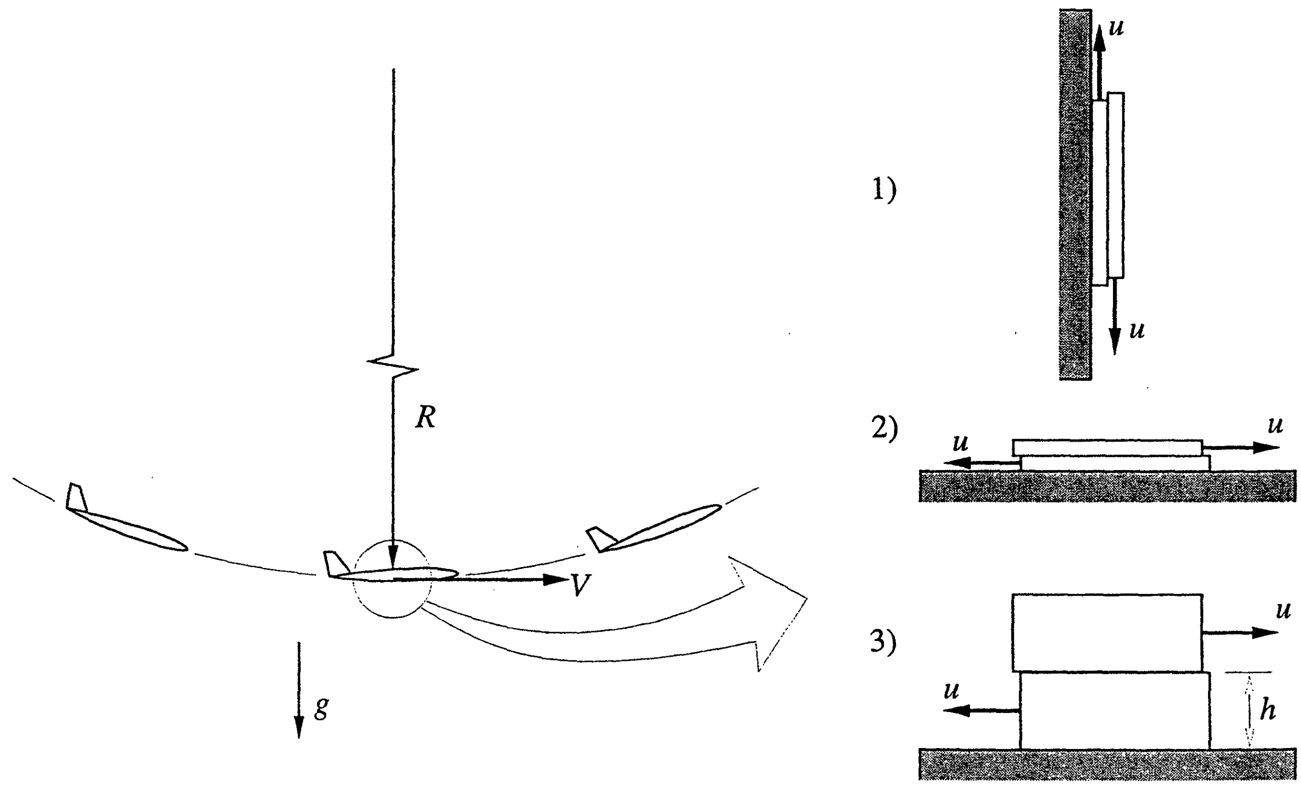
A sliding-mass experiment is being carried out in an aircraft flying in a circular path in a vertical plane at constant velocity V. The aircraft's rotation is only due to the circular-path motion. A gravitational acceleration g is present. Experiments are performed when the aircraft is at the bottom of the loop as shown:

1) Two thin material slabs each of mass m are pulled by vertical wires, and are made to slide on the face of a vertical table at speed $\pm u$ as shown. Determine the contact force at each of the two interfaces.

2) Now one slab is pulled on a horizontal table towards the front of the aircraft, and the other slab on top is pulled the other way. Determine the contact forces at the two interfaces.

3) The case in 2) is repeated, but this time with significantly thick slabs of thickness h. Determine the contact forces.

4) Assume the slabs are very flexible. At the moment the slabs are aligned, is the contact force/area in 1,2,3 uniform over each interface?



Avionics Qualifying Exam Question

The reliability of avionics systems is usually evaluated in terms of the probabilities that elements of the system are working after some specified period of time. Typically this probability is modeled in terms of a differential equation as follows:

$$\frac{dp}{dt} = -\lambda p$$

where

p =probability that the element is working at the time t
λ = instantaneous failure rate of the element

Using this model we wish to analyze the reliability of a dual redundant satellite avionics system. In particular we want to evaluate the integrity of the avionics system at the point when the satellite is inserted into its orbit, at the end of a boost phase lasting 360 seconds. The avionics system has two identical channels, either of which is capable of carrying out the mission of the satellite. Ground tests indicate that during the boost phase the failure rate of a channel will be a function of the level of vibration it experiences. The relationship is

$$\lambda = \lambda_0 + \lambda_1 a_{rms} \qquad \text{where} \quad \begin{array}{l} \lambda_0 = 10^{-4}\,hr^{-1} \\ \lambda_1 = 10^{-1}\,hr^{-1}\,g^{-1} \end{array}$$

and a_{rms} is the root mean square level of acceleration due to vibration experienced by the channel. This level of vibration increases in proportion to the decrease in mass of the boost vehicle as fuel is burned off during the launch. Thus a_{rms} is given as

$$a_{rms} = a_0 + a_1 t \qquad \text{where} \quad \begin{array}{l} a_0 = .01g \\ a_1 = .01g\ \sec^{-1} \end{array}$$

Assuming both channels are working at the beginning of the boost phase (t=0), both channels experience the same levels of vibration, and that the probabilities of failure of the two channels are independent-

1. What is the probability that at least one channel will have <u>failed</u> by the end of the 360 second boost phase?

2. Now assume that the outputs of the two channels are constantly compared in order to detect a failure. When a failure is detected a built-in-test (BIT) system is used to identify which channel has failed. The probability of success of the BIT system is .99. Once identified the failed channel is disabled and the other channel performs the avionics functions. Using this system what is the probability that the avionics function will <u>not</u> be performed correctly at the end of the boost phase?

3. Is the BIT system a good idea?

Written Doctoral Qualifying Exam – **Fluids**

A 2-D airfoil with chord c is operating at a small angle of attack α and speed U in inviscid incompressible flow. The airfoil has negligible thickness and camber. The instantaneous velocity field $\vec{u}$ as seen by an observer fixed in the fluid far away is described by a perturbation potential: $\vec{u} = \nabla\varphi(x, y)$.

1) The airfoil location approximated by placing it on the x-axis. To solve the governing equation $\nabla^2\varphi = 0$ for the true geometry, what boundary conditions must be applied on the relocated airfoil?

2) After solution, the tangential perturbation velocity on each surface turns out to be uniform, so that $\partial\varphi/\partial x = \pm u$ on the top and bottom surfaces (i.e. relative to the airfoil, the velocity is $U \pm u$). What must be the potential difference

$$\Delta\varphi(x) \;=\; \varphi(x, 0^+) - \varphi(x, 0^-)$$

between the top and bottom of the airfoil? What is the airfoil's lift?

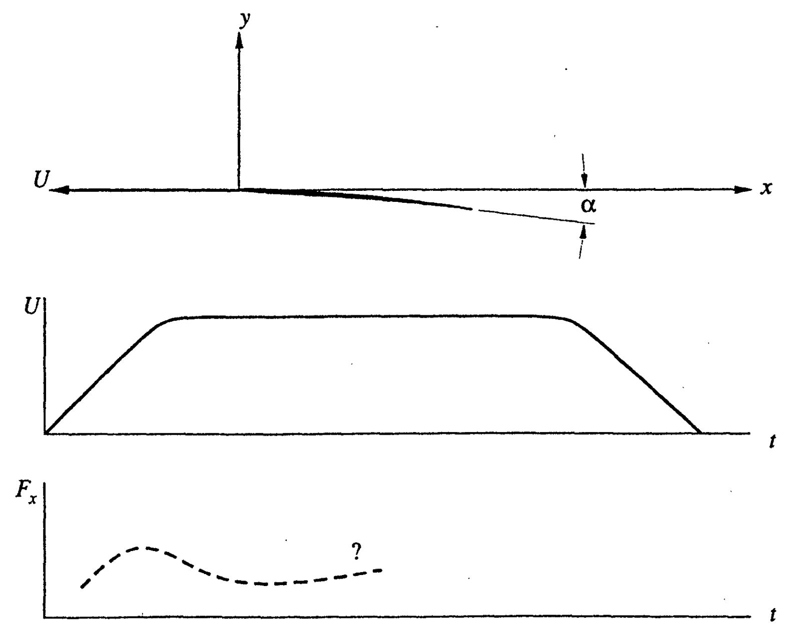
3) Determine the kinetic energy E of the fluid motion in the entire flowfield.

$$E \;=\; \frac{1}{2}\rho \iint |\vec{u}|^2 \, dx\, dy$$

where the integration is over the entire flowfield. Use of the following vector identity and Gauss's Theorem will be useful:

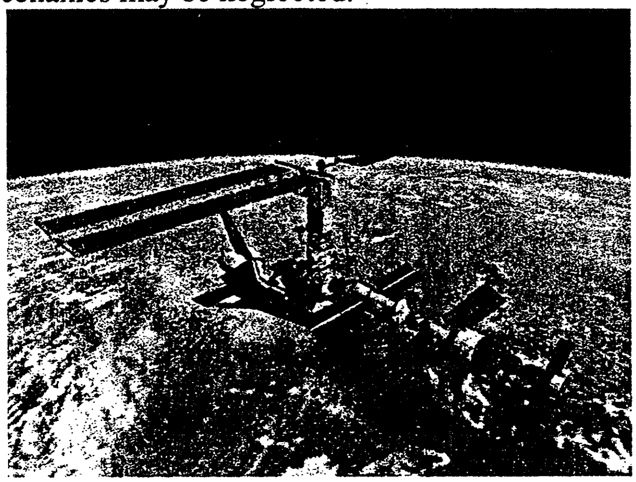
$$\nabla \cdot (f\,\nabla g) \;=\; \nabla f \cdot \nabla g \,+\, f\,\nabla^2 g \qquad\qquad \iint \nabla \cdot \vec{F}\, dx\, dy \;=\; \oint \vec{F} \cdot \hat{n}\, ds$$

4) The airfoil is smoothly accelerated from zero up to U, and then later decelerated back down to zero as shown in the graph below. Sketch a plot of the x-force F_x which you expect to see acting on the airfoil during this whole time, Explain your reasoning.

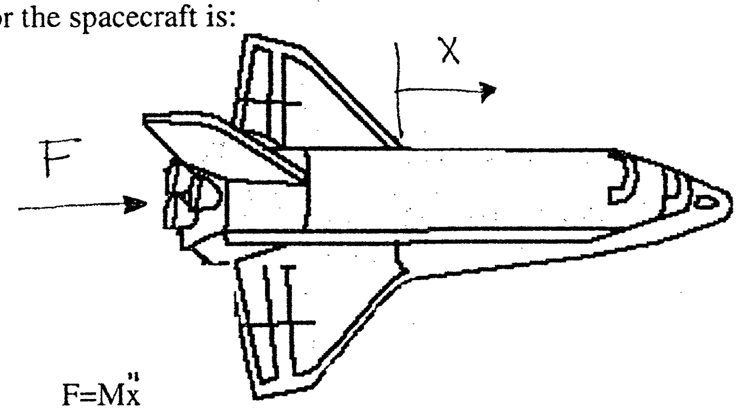


H&A Qualifier Question 2001: International Space Station (ISS) Assembly

Consider the position control of a simple representation of a spacecraft (e.g., the Space Shuttle) during a recent assembly flight to the International Space Station (ISS). Assuming small displacements, orbital mechanics may be neglected.



The equation of motion for the spacecraft is:



$$F = M\ddot{x}$$

F is the force generated by the reaction jets, M is the spacecraft mass, and x is the displacement.

A) Assume a human operator is manually controlling the spacecraft by observing its displacement, δ, from the desired location, x_r, and directly controlling F. Draw a block diagram of the closed-loop feedback system. Label each element.

B) A simple model of the human operator is one where his/her control output is proportional to the spacecraft displacement. Is this sufficient to stabilize the closed-loop system? State why or why not.

C) If the human operator had available a display of position (x) and velocity ($\dot{x}$), show how feedback can model operator control behavior to achieve second order dynamics.

D) What is important to the operator in this human-machine control task?

E) Explain the effect of the human operator reaction time on the performance of the closed-loop control system.

Control Qualifying Question **January 2001**

Consider the simple system of an inverted pendulum for small angles θ. Assume that the mass m is at the end of a massless rod of length L, that there is a torque actuator T that acts at the base of the rod, and that we can measure θ.

(a) [10%] Show that the linearized equations of motion can be written in the form

$$\frac{\theta(s)}{T(s)} = \frac{1/mL^2}{s^2 - g/L}$$

Assuming that $mL^2 = 1/2$ and $g/L = 1$, draw a root locus for this system for positive feedback gains.

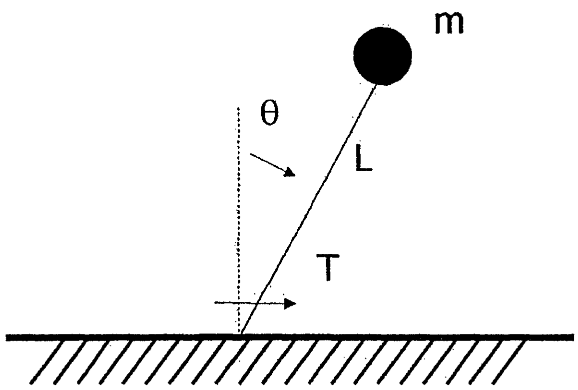


Figure 1: Inverted Pendulum

(b) [25%] Use the root locus in part (a) to design a simple (first-order) controller to give the dominant closed-loop pole a settling time of less than 1 second.

(c) [25%] Now assume that there is a pure delay of T_d=0.25 sec in the actuator. Sketch a Bode plot of this delayed system, and find the phase margin (PM) of your design in part (b).

(d) [40%] With a Pade approximation of this delay, use a root locus to re-analyze your design in part (b). Is the closed-loop system stable with this delay? Is you conclusion consistent with part (c)? Should it be?

$$\text{Pade Approximation} \quad e^{-sT} \approx \frac{1 - sT/2}{1 + sT/2}$$

Written Doctoral Qualifying Exam – **Propulsion**

A low-speed actuator disk propulsor is ingesting the viscous wake behind an upstream body. Assuming no viscous losses are incurred by the propulsor's action, the kinetic energy balance applied to the control volume enclosing the propulsor is

$$P = \oint p\,\vec{q}\cdot\hat{n}\,dA + \oint \tfrac{1}{2}q^2\,d\dot{m}$$

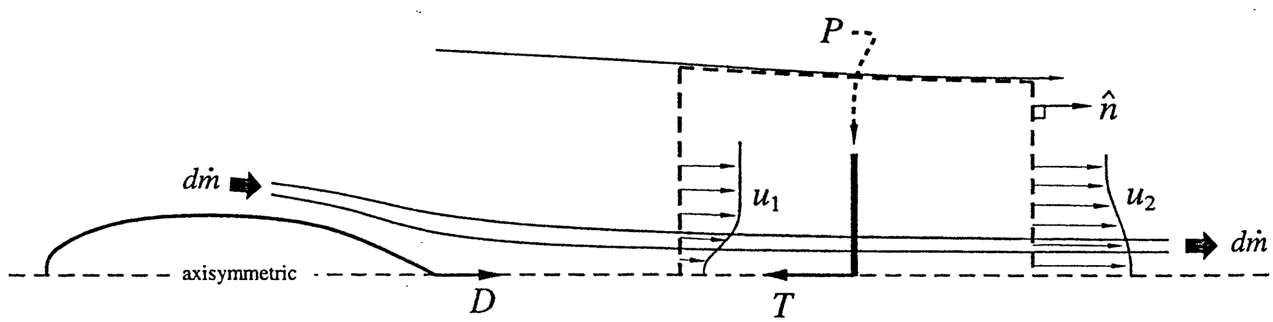
where P is the shaft power, $q = |\vec{q}|$ is the speed, p is the static pressure, and $d\dot{m} = \rho\vec{q}\cdot\hat{n}\,dA$ is the mass flow increment.

1) Relate P to the total pressure $p_o(r)$ and axial velocity $u(r)$ distributions upstream and downstream of the propulsor.

2) Relate the body drag D and propulsor thrust T to the static pressure $p(r)$ and axial velocity $u(r)$ distributions upstream and downstream of the propulsor. The upper streamline boundary is assumed to be far away.

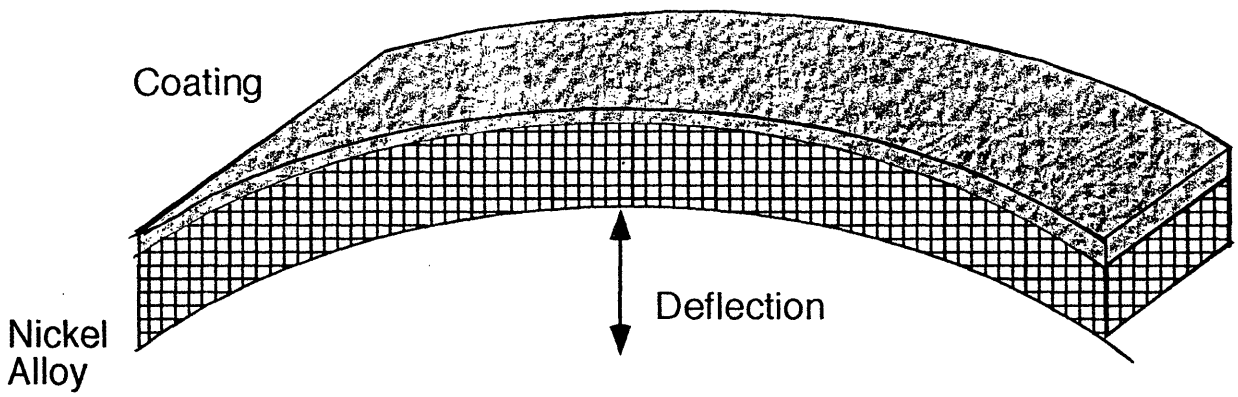
3) The shaft power is supplied by a shaft rotating at rate Ω, so that the torque applied to the propulsor disk is $Q = P/\Omega$. Relate Q to the swirl velocity distribution $v(r)$ downstream of the propulsor (not shown in the figure). No swirl is present in the body's wake.

4) It is proposed that the same propulsor be operated in "clean" flow away from the body's wake. The same net propulsive thrust $T - D$ must still be obtained. Will this alternative arrangement require a smaller or larger shaft power P?



Materials and Structures Written Exam Question

During the development of a thermal barrier coating for a gas turbine blade an experiment is performed in which a relatively thin layer of the ceramic (zirconium-oxide-based) coating is applied to one of the large area faces of a rectangular cross-section strip of the nickel alloy of which the blades are made. The strip is observed to bend.



a) Explain this observation.

b) Develop a model for the deflection of the strip with the coating applied that relates the deflection at the center of the strip relative to its ends to the relevant material, geometrical and processing temperature. Clearly state all assumptions that you make and define all symbols that you use. Make sure to adhere to the maxim attributed to Einstein that "a model should be as simple as possible but no simpler" .

c) If the nickel alloy strip is 300 mm long, 3 mm thick and 10 mm wide and the coating is 0.1 mm thick and the central deflection of the strip, relative to its ends, is found to be 1mm, estimate the temperature at which the coating is applied. Comment on the likely accuracy of your estimate.

Some potentially useful material properties

Young's modulus of the nickel alloy: 200 GPa
Poisson's ratio of the nickel alloy: 0.3
Thermal expansion coefficient of the nickel alloy (at room temperature): 13 $\times 10^{-6}$/K
Density of the nickel alloy: 8000 kg/m^3
Yield Strength of the nickel alloy: 520 MPa

Young's modulus of the ceramic coating material: 100 GPa
Poisson's ratio of the ceramic coating material: 0.22
Thermal expansion coefficient of the ceramic used for the coating: 10$\times 10^{-6}$/K
Density of the coating ceramic: 5750 kg/m^3
Fracture strength of the coating ceramic: 635 MPa

Written Qualifying Exam
Systems

Systems activities involve dealing with the system as a whole and mitigating the impact of complexity. This can include working with the system's internal and external interfaces in an effort to ensure that the system's elements or components function synergistically together for some intended purpose or primary behavior. Additional effort is required to ensure that the designed interacting components do not exhibit unwanted or harmful secondary behaviors. Two systems have been provided for consideration. Please see the attached graphics. Select one of these systems and answer the following questions.

1) Identify (list) the external interfaces that this system must be compatible with. Note that interfaces may include elements that are related to humans, operating environments, and other engineering systems.

2) Illustrate the dynamic character of these interfaces by showing how the external interfaces change with time during a typical operational sequence, period, or mission. Do this by generating a sequence of schematics. In each schematic, the interfaces included should be appropriate for a particular point in time.

3) List the system's primary subsystems. Characterize each subsystem. For example, an *Electric Power Generation/Distribution Subsystem* might include generators, batteries, wires or power buses, converters, rectifiers, control units, etc.

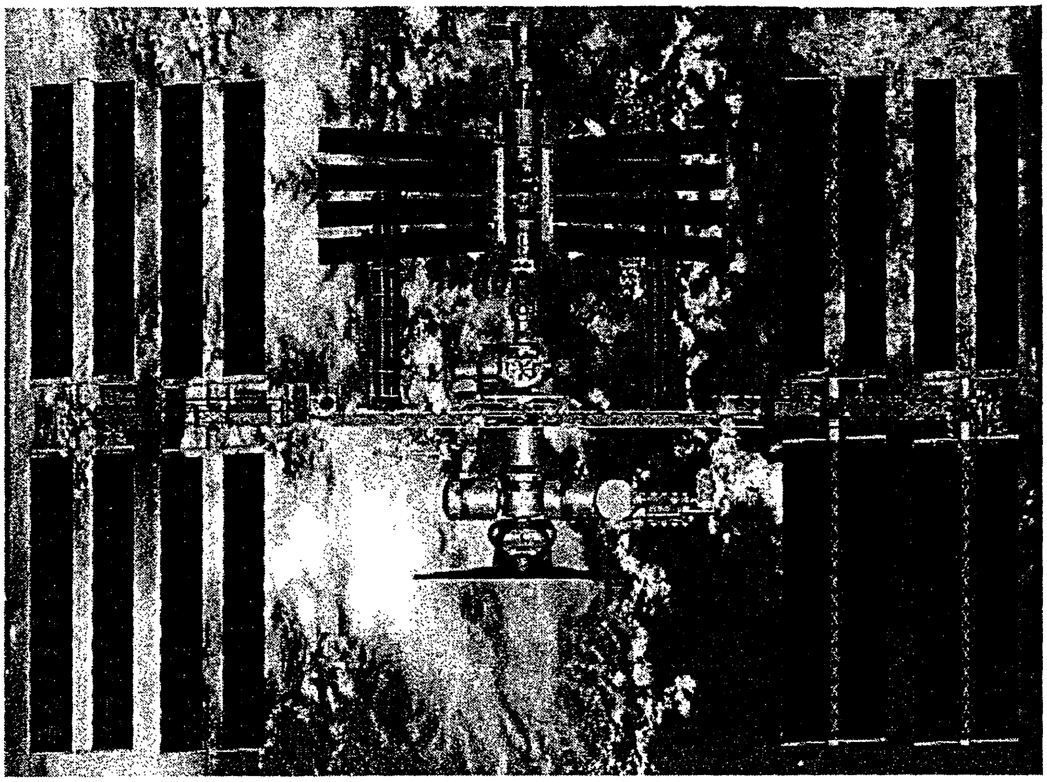
4) Use a schematic block diagram to show how the subsystems in Question #3 are connected. Label these internal interfaces (mechanical, fluid, electrical, signal, pneumatic, etc.). Note: Try to use the aforementioned types instead of physical implementations such as databus, hydraulic line, wire, fuel line, power cable, etc.

5) Identify three primary functions that the system perfoms and illustrate/describe how the interfaced subsystems work together to satisfactorily execute each function.

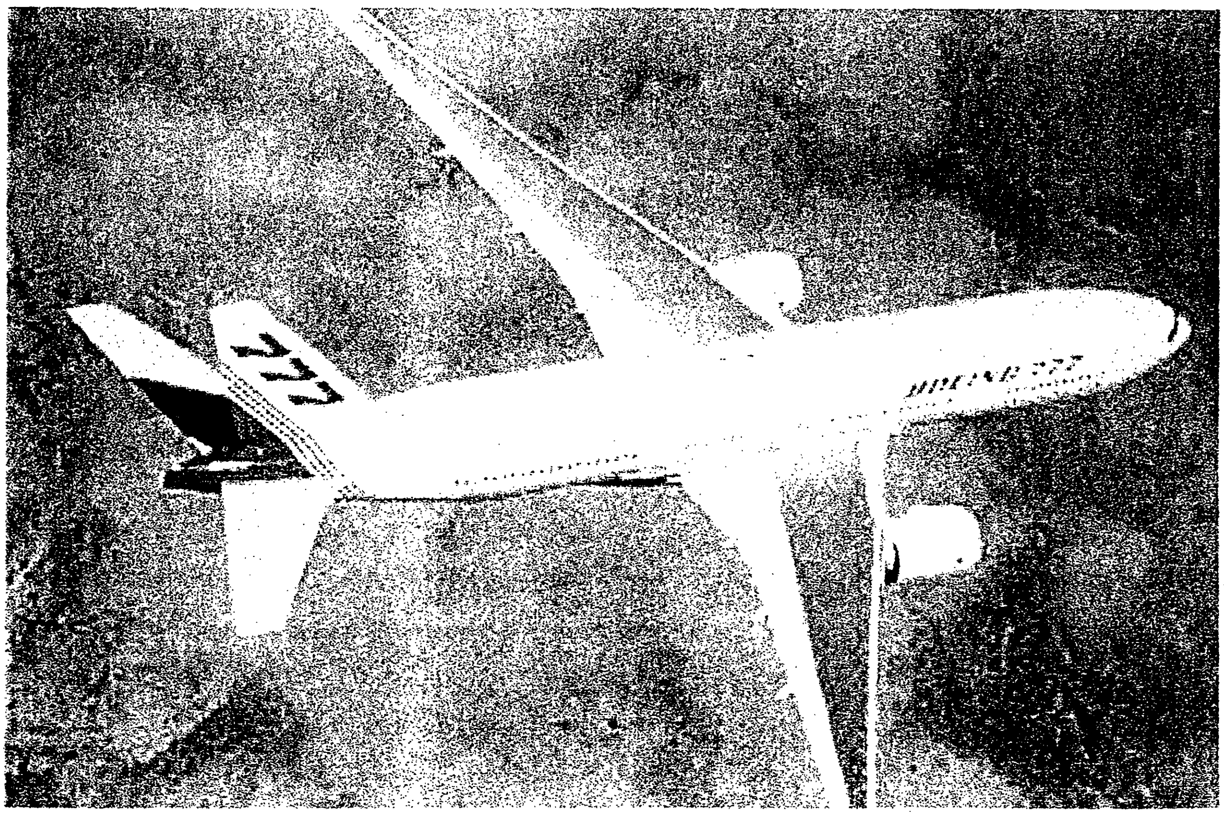
6) Identify three examples of secondary or unintended behaviors that might result from the system being architected as characterized in Question #4 above.

7) Select one of the subsystems in Question #3 and explain/illustrate how you would propose to improve this subsystem's reliability if it was determined during a design review that aerospace reliability standards had not been met. Here, *reliability* means "the duration or probability of failure-free performance under stated conditions."

The International Space Station

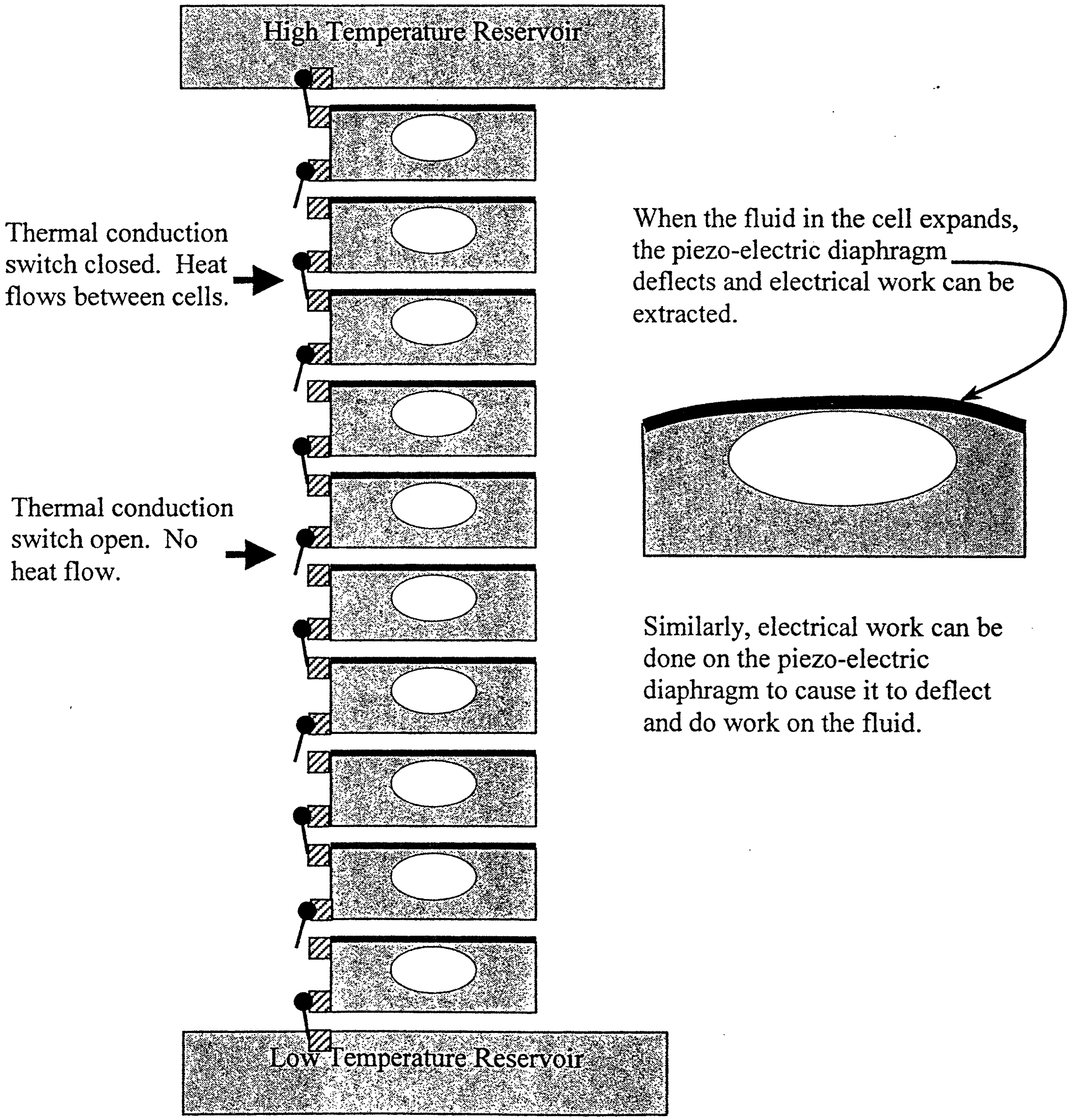


Boeing's first flight 777 flying over Washington State.



Thermodynamics

A new heat engine concept has been proposed that could be constructed using micro-electronic fabrication processes. The device is composed of 10 layers of cells. Each cell contains a mixture of liquid water and water vapor and is capped with a piezo-electric diaphragm (shown by the thick dark line on top of each cell). There are a series of thermal conduction switches between the cells that have two positions. In the open position, heat cannot flow through the switch. In the closed position, heat can flow between the cell and its nearest neighbor (either the cell above it or the cell below it). Assume that the thermodynamic state of each cell can be independently set as desired during fabrication.



Similarly, electrical work can be done on the piezo-electric diaphragm to cause it to deflect and do work on the fluid.

a) Using sketches and words, briefly describe how this device would operate as i) an engine and ii) a refrigerator. For example, what is the sequence of processes (switches opening and closing, heat being added and removed, electrical work being done or extracted, etc.).

b) Assuming that the maximum temperature difference that can be sustained across any one cell is 10K, what is the best efficiency that can be achieved with a 10-layer device when operating as a heat engine? Assume the low temperature reservoir is at 300 K.

c) Assuming that the heat flow rate is set by the thermal conductivity of water (0.68 W/mK) and that each layer is 100 microns thick, what is the maximum power a 1cm x 1 cm square, 10-layer device could produce?

d) Discuss other practical limits on the performance of this device. For example, given your knowledge of other heat engines, how close to the best efficiency do you anticipate this device could operate and why?