DEPARTMENT OF AERONAUTICS AND ASTRONAUTICS

WRITTEN QUALIFYING EXAMINATION
FOR
DOCTORAL CANDIDATES

Wednesday, January 20, 1999 37-212 9:00 a.m.-1:00 p.m.

CLOSED BOOK AND NOTES

Answer a total of five (5) questions (no more or less).

You must answer at least two (2) questions from Column A.
Please answer each question on a separate sheet.

Be sure that your name appears on **Every** sheet of paper you turn in.

Oral examinations will be held on Tuesday, January 26, 1999. Pick up your schedule on Monday, January 25, 1999 after 1:00 p.m. from Hillary in 33-208.

Results will be available on Wednesday, January 27, 1999 after 2:00 p.m.
Please contact your advisor.

Column A	Column B
Mathematics	Avionics
Physics	Fluids
Dynamics	Humans and Automation
	Instrumentation, Control and Estimation
	Propulsion
	Structures
	Systems
	Thermodynamics

Qualifier Written Question (Mathematics)

1a) Find the real symmetric matrix $\mathbf{A}$ that has the eigenvectors

$$\vec{u}_1 = \begin{Bmatrix} 1 \\ 1 \end{Bmatrix} \quad , \quad \vec{u}_2 = \begin{Bmatrix} 1 \\ -1 \end{Bmatrix}$$

and corresponding eigenvalues $\lambda_1 = a$, $\lambda_2 = b$.

1b) Determine the inverse matrix $\mathbf{A}^{-1}$.

1c) Consider the scalar quantity

$$s = \left\{ \vec{v}^T \right\} \begin{bmatrix} \mathbf{A} \end{bmatrix} \left\{ \vec{v} \right\}$$

for some arbitrary real symmetric $\mathbf{A}$. What are the conditions on $\vec{u}_1$, $\vec{u}_2$, λ_1, λ_2 of $\mathbf{A}$ to make s positive for *any* real vector $\vec{v}$? (if you state a summary answer, you must justify it)

2a) Find the general solution $y(x)$ governed by:

$$\frac{1}{2}y'' - y' + y = 4e^{-x} \sin x$$

2b) Find $y(\pi)$ for the specific case where

$$y(0) = 1 \quad , \quad y'(0) = 0$$

Physics Written

a) Gauss' Law in electrostatics can be stated as:

$$\oint_S \underline{D}.\underline{ds} = Q$$

Define the terms in this expression, and explain the physical significance in words.

b) The coaxial cable conductor shown below comprises a central hollow tube outer radius r_1, metal thickness, t_1, and a concentric tube outer radius r_2 made of metal thickness t_2. Calculate the capacitance per unit length if the space between the conductors is filled with an insulator of relative permittivity ε_1 to radius r_ε and an insulator of permittivity ε_2 otherwise.

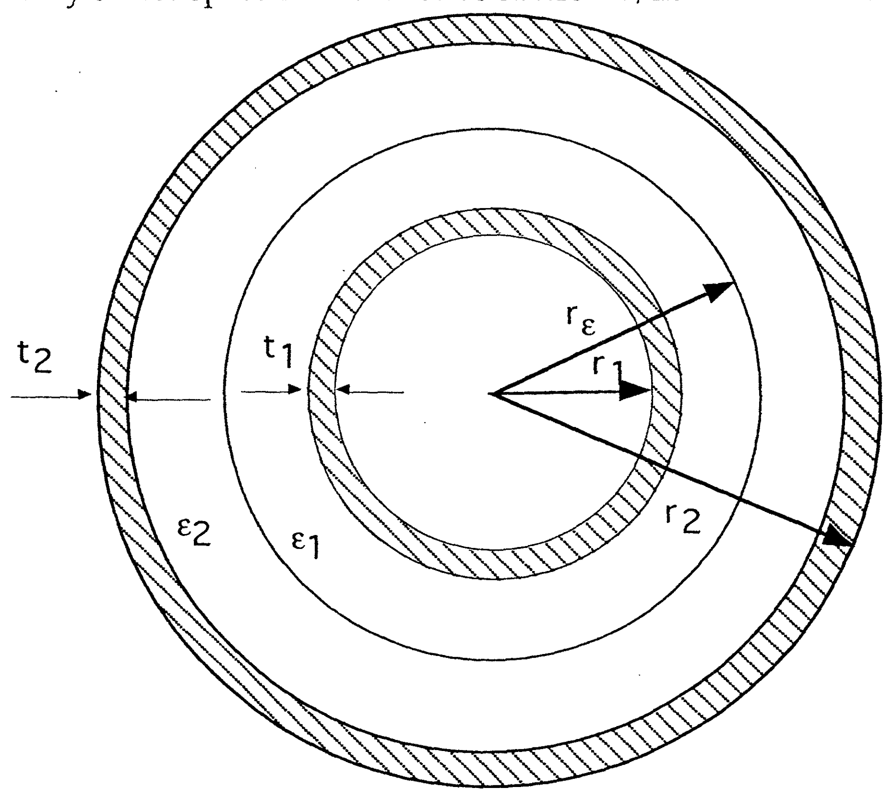
You may find it helpful to recall that the parameter D is related to the electric field E via:

$$D = \varepsilon_0 \varepsilon_r E$$

c) Calculate the capacitance per unit length for the case r_1=2 mm, r_2= 5 mm, t_1=t_2=0.5 mm, r_ε =3 mm, ε_1 =10 and ε_2 = 5

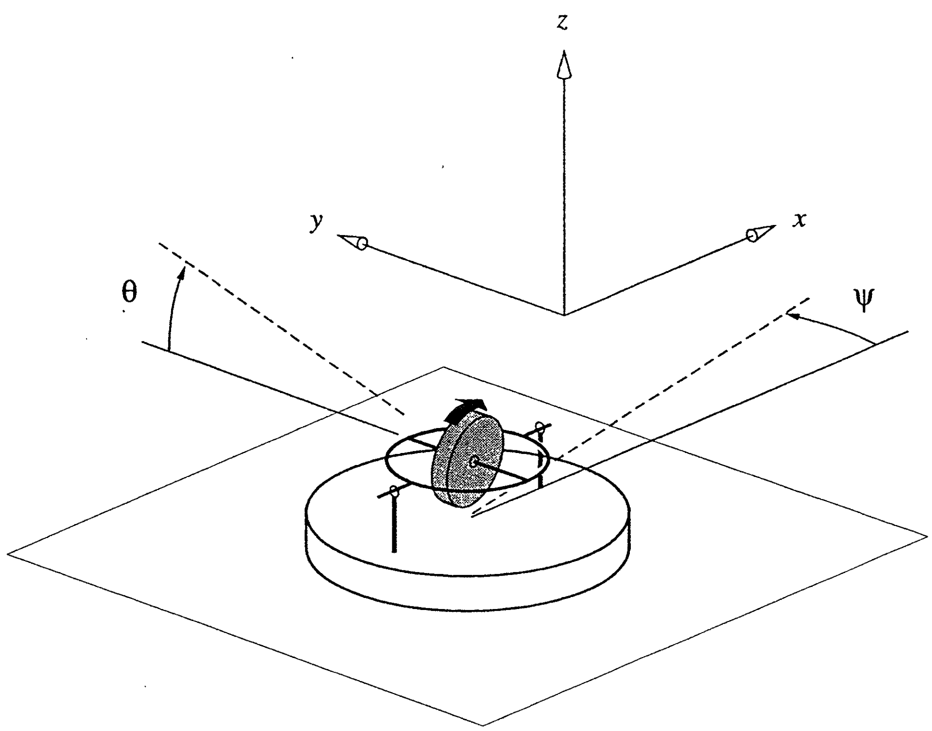
If the potential difference applied between the conductors is 100 V, make a sketch of the electric field versus radial distance from r=0 to r=6mm. What is the maximum electric field strength?

The permittivity of free space can be taken as 8.9×10^{-12} F/m



Qualifier Written Question (Dynamics)

A puck is free to slide and rotate over an effectively-frictionless air table. A small gyroscope with angular momentum magnitude h is mounted in a gimbal on the puck. The elevation angle θ of the gyro's axis can be controlled by rotating the gimbal about its horizontal axis as shown. The azimuthal angle of the puck relative to the inertial x-axis is ψ. The moment if inertia of the whole puck about the vertical z-axis is I. The moment of inertia of the gyro/gimbal unit about the gimbal axis is I_g.



1) [20%] Determine the angular momentum vector $\vec{H}$ of the puck/gyro system for a given $\theta, \psi, \dot{\theta}, \dot{\psi}$.

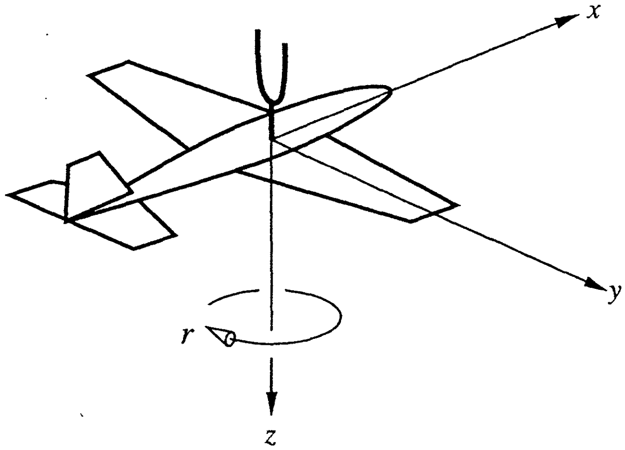
2) [20%] Determine a conservation law for the vertical component of $\vec{H}(t)$ in terms of $\theta(t)$ and $\psi(t)$. The puck is initially at rest when $\theta = 0$ and $\psi = 0$. When the gimbal is then rotated at a constant rate $\dot{\theta}$, what is the motion of the puck?

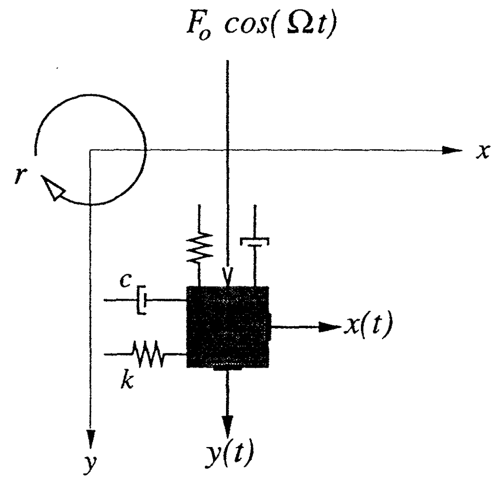
3) [25%] Beginning from rest at $\psi = 0$, the puck is to rotate as fast as possible to $\psi = \pi$, and then stop. How must $\theta(t)$ be varied to execute this maneuver?

4) [35%] The gyro axis is first brought to small angle $\theta = \theta_0$, with $\theta_0 \ll 1$. The gimbal actuator is then disconnected, so that the gimbal is free to pivot about its bearings. What is the subsequent free motion $\theta(t)$, $\psi(t)$?

Qualifier Written Question (Avionics)

A tuning fork is positioned vertically on an aircraft as shown. One fork arm is modeled as a mass with two equal springs of stiffness k and two equal dampers with a very small damping constant c. A small exciting force $F(t) = F_o \cos(\Omega t)$ is applied to the arm along the y axis.





1) [25%] Derive the equations of motion for $x(t)$ and $y(t)$ of the arm in the presence of a constant aircraft z-rotation rate r.

2) [25%] At some constant r, what are the long term limit-cycle $x(t)$ and $y(t)$?

3) [15%] The $x(t)$, $y(t)$ positions of the arm are sensed. Which of the two signals is suitable for use as a yaw-rate indicator?

4) [35%] Estimate the frequency response of this sensor.

Written Qualifying Exam - Fluids

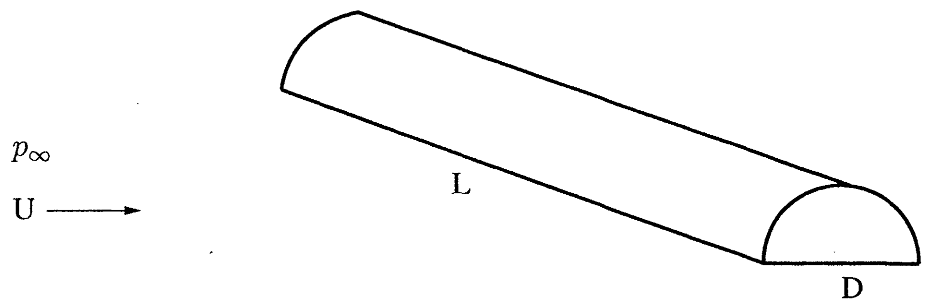


Figure 1: Semi-cylinder aircraft hangar with length L and diameter D.

Consider the semi-cylinder aircraft hangar shown above. Assume:

- Far upstream of the hangar, the wind has uniform speed U and is perpendicular to the hangar axis. The upstream pressure is p_∞.
- the end effects are small.
- the flow is inviscid.

Answer the following questions.

A. Determine the wind velocity everywhere in the flowfield.

B. Determine the pressure distribution on the outer circular surface of the hangar.

C. Assume the interior pressure is some p_H. The circular roof can withstand a maximum net vertical force of F_{max}. What is the maximum U the hangar can withstand?

D. Small vents are to be placed on the roof at position θ_v as shown in Figure 2. What should θ_v be to make the net vertical force on the roof zero?

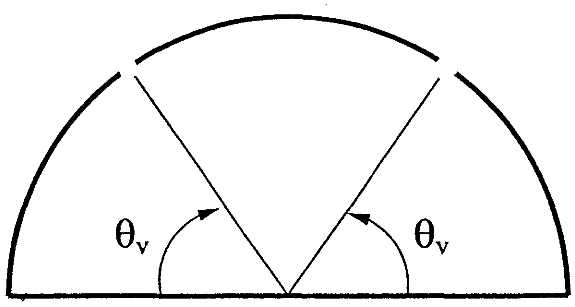


Figure 2: Hangar cross-section showing vents at angle θ_v.

Written Doctoral Qualifying Exam

Humans and Automation

Consider a remotely-controlled unmanned space resupply vehicle while it is in deep space attempting to dock with another spaceship. The space vehicle has thrusters which can provide six independent linear and angular accelerations about the vehicle's x, y, z axes. An operator on earth commands the six accelerations via an uplink. Sensors on the vehicle downlink to the operator the following parameters, all relative to the docking port:

- vehicle position
- linear velocity
- linear acceleration
- attitude (pitch, roll, yaw)
- angular velocity
- angular acceleration
- a video image from the resupply vehicle looking through its docking port

1) Describe a design for the operator controls, including input device type(s), location(s), and mapping from controller input to the uplinked data for the space vehicle.

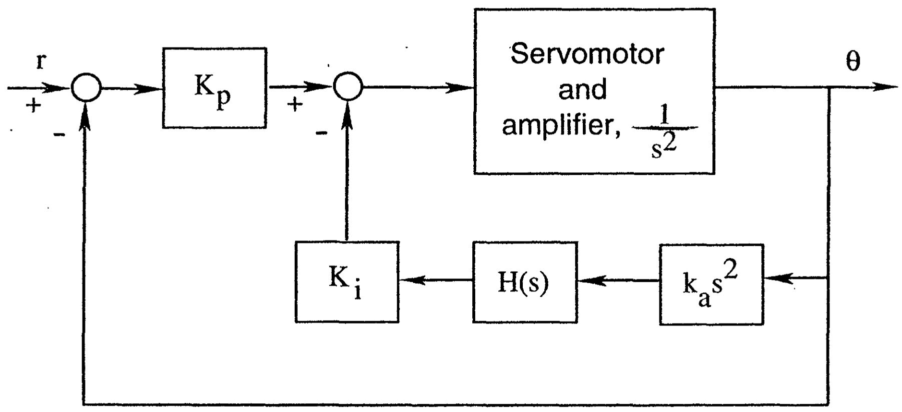
2) Describe a display design that allows the operator to monitor progress toward docking. Sketch the display, clearly showing the information that is depicted.

3) What issues arise as the vehicle gets farther from earth? Specifically, discuss how operation and/or display elements may need to change as the communication delays between earth and the vehicle become large.

In all parts, be sure to note any assumptions you make.

Controls Qualifier Question
January 1999

The figure below is a block diagram of a servomotor system for commanding motor position θ. Motor angular *acceleration* and position are measured and fed back as shown.



(a) If no compensation is used, i.e. H(s)=1, can this feedback scheme stabilize the system? Provide a brief explanation.

(b) Find K_p, K_i and H(s) (if necessary) that will lead to a critically damped system with a natural frequency of 10 rad/sec. Your answer will be in terms of the accelerometer calibration constant k_a.

(c) Show how you would arrange one resistor and one inductor to form an appropriate network H(s) to accomplish the same goals as those in part (b). Assume that connecting such a circuit to the rest of the system will not alter its properties, that the accelerometer provides a voltage output, and that the amplifier K_i requires a voltage input. Find numerical values for the resistance and inductance required, as well as K_p and K_i. You may assume that $k_a = 1$ for this part.

Propulsion/Energy Conversion

Here is a scheme which has been proposed to very efficiently produce electric power in an orbiting spacecraft by burning some on-board propellants.

(a) A very long (several km) conductive cable of length L is deployed vertically from the satellite. As it flies at speed $v_{orb.}$ across the Earth's magnetic field B, an electromotive force $V_{oc} = v_{orb.} BL$ is set up in the cable. One end of the cable is made to emit electrons, the other to capture electrons from the ionosphere, so that a current $I = V_{oc}/(R_L + R_C)$ circulates in series through the useful load (R_L) and the cable, of resistance R_C. The load power is $P_e = I^2 R_L = \frac{R_L}{(R_L + R_C)^2} V_{OC}^2 = \frac{R_L}{(R_L + R_C)^2} B_2^2 L^2 v_{orb}^2$,

(b) The passage of current produces a retarding force on the Satellite

$$F = I\,B\,L$$

To avoid losing orbital energy and eventually reentering, a chemical rocket is fired to produce a forward thrust with an average value of F. The rate of expenditure of chemical energy by this rocket is

$$P_{chem} = \frac{1}{2\eta_R} F\,c$$

where c is the effective jet velocity (c=Isp/g), and η_R is the thermodynamic efficiency of the rocket, $\eta_R = 1 - \left(\frac{P_{exit}}{P_{chamber}}\right)^{\frac{\gamma-1}{\gamma}}$. With a well-expanded nozzle, η_R can be 70-80%.

The system operates in steady state, as described. Calculate the overall conversion efficiency

$$\eta = \frac{P_e}{P_{chem}}$$

For a numerical application, assume $R_c = 0.5R_L$,
$v_{ORB} = 7600 m/s$, c = 4,000m/s, $\eta_R = 0.75$. If the result looks odd, explain what (if anything) is wrong with the argument.

Structures
Written Exam Question

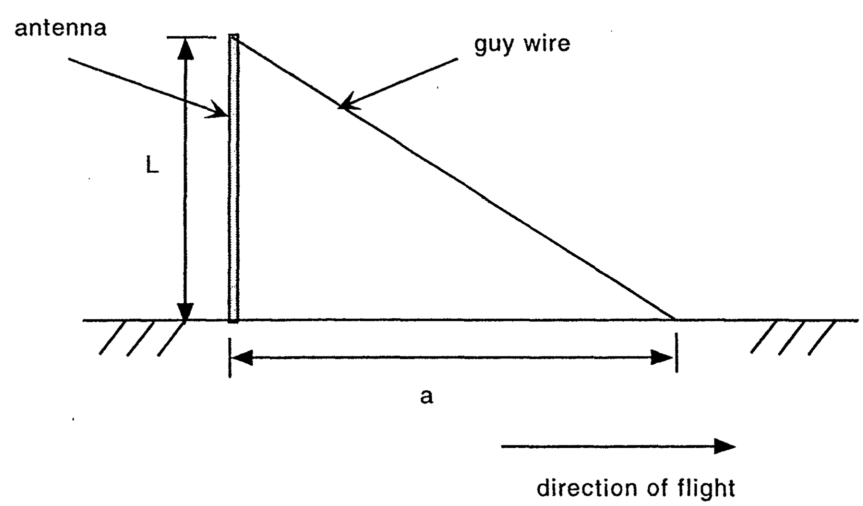
A radio antenna mounted on the fuselage of an airplane is described in the Figure below. A guy wire is attached to the end of the antenna to strength it against drag forces generated by the airplane in motion. Assuming that the drag force is uniformly distributed and has a total resultant *D*, find:

1. The force that the guy wire should exert in order to minimize the maximum bending moment in the antenna (consider that the antenna has a constant bending stiffness *EI*).

2. The lateral displacement at the tip of the antenna under the wire force determined in (1).

3. The wire axial stiffness (that will be used to dimension the cross-sectional area of the wire for a chosen material) needed in order to achieve the force determined in (1).

4. Considering the need to reduce the distance (represented by *a* in the figure) between the antenna and the attachment point of the wire on the fuselage, discuss what the failure mode, if any, of the system "antenna + wire" would be that restricts that reduction beyond a minimum distance.



Figure—Antenna mounted on an airplane fuselage and being reinforced with a guy wire

Reminder: *Please state the assumptions/rationale you are using throughout the solution of this problem*

Systems Qualifying Exam - Written Question

A customer has approached your company with a need that can be satisfied by the equivalent of an "electro-mechanical carrier pigeon." That is to say - the device to-be-designed must transport a parcel or object in a city environment (like Boston) that can not be sent by digital or telecommunication means. To keep the system small, simple, and reduce costs, this device will not be piloted. No system like this currently exists, but your company knows that the system must perform many functions that include: 1) parcel stowage, 2) fueling, 3) trajectory input/updating, 4) departure/arrival, 5) path navigation, 6) continuous base communications, 7) cruise, 8) obstacle avoidance, etc.

1) Develop a baseline functional flow diagram that effectively represents a model of how this system might operate. This diagram should include the functions that you believe will be most important for a successful design. Clearly show the functional relationships with functions that must be performed sequentially and functions that must, or can be, performed concurrently.

2) Prepare a sketch of a concept that can serve as a baseline design for trade study down-select purposes. Label the most subsystems.

3) In view of the system behavior captured in "1" and the architectural implementation characterized in "2," identify the top technical challenge that you think your company must meet.

Thermodynamics

1. A two-phase Carnot cycle is operated as shown in the figure below. The letters A, B, C, D, E, and F, refer to different states that occur during the cycle. The pressures and temperatures are such that the vapor behaves as a perfect gas. The specific volume at C is twice that at B. The specific volume of the liquid is much less than the specific volume of the vapor.

a) What is the ratio of the pressure at B to that at A?
b) What is the ratio of the pressure at C to that at A?
c) Is the specific volume at E greater than that at F?
d) Is the specific volume at D greater than that at E?
e) What is the difference in enthalpy between states B and A? (give the answer in terms of T and S)
f) What is the ratio of the overall heat received to the heat received in the regime in which the system is in a two phase condition?
g) What is the thermal efficiency of the Carnot cycle?