MASSACHUSETTS INSTITUTE OF TECHNOLOGY
1861

# DEPARTMENT OF AERONAUTICS AND ASTRONAUTICS

# WRITTEN QUALIFYING EXAMINATION
# FOR
# DOCTORAL CANDIDATES

Wednesday, January 21, 1998 37-212 9:00 a.m. - 1:00 p.m.

CLOSED BOOK AND NOTES

Answer a total of five (5) questions (no more or less).

You must answer at least two (2) questions from Column A.
Please answer each question on a separate sheet.

Be sure that your name appears on **EVERY** sheet of paper you turn in.

Oral examinations will be held on Tuesday, January 27, 1998. Pick up your schedule on Monday, January 26, 1998 after 1:00 p.m. from Liz in 33-208.

Results will be available on Wednesday, January 28, 1998 after 2:00 p.m.
Please contact your advisor.

| Column A | Column B |
|---|---|
| Mathematics | Avionics |
| Physics | Fluids |
| Dynamics | Humans and Automation |
| | Instrumentation, Control and Estimation |
| | Propulsion |
| | Structures |
| | Systems |
| | Thermodynamics |

# Math
# Written Exam Question

a) A plane passes through the points with position vectors $\begin{pmatrix}1\\1\\2\end{pmatrix}, \begin{pmatrix}0\\3\\0\end{pmatrix}$ and $\begin{pmatrix}3\\1\\1\end{pmatrix}$. Give an equation describing this plane. Also give the equation of the unit normal to the plane.

What is the angle between the plane and the y axis?. What is the closest point on the plane to the origin and how far is it from the origin?.

b) An even function f(t) is periodic with period T=2, and $f(t) = cosh(t-1)$ for $0 \leq t \leq 1$. Sketch f(t) in the range $-2 \leq t \leq 4$' Find a Fourier series representation for f(t).

Hence or otherwise demonstrate that:

$$\sum_{n=1}^{\infty} \frac{1}{1+n^2\pi^2} = \frac{1}{e^2-1}$$

# Physics

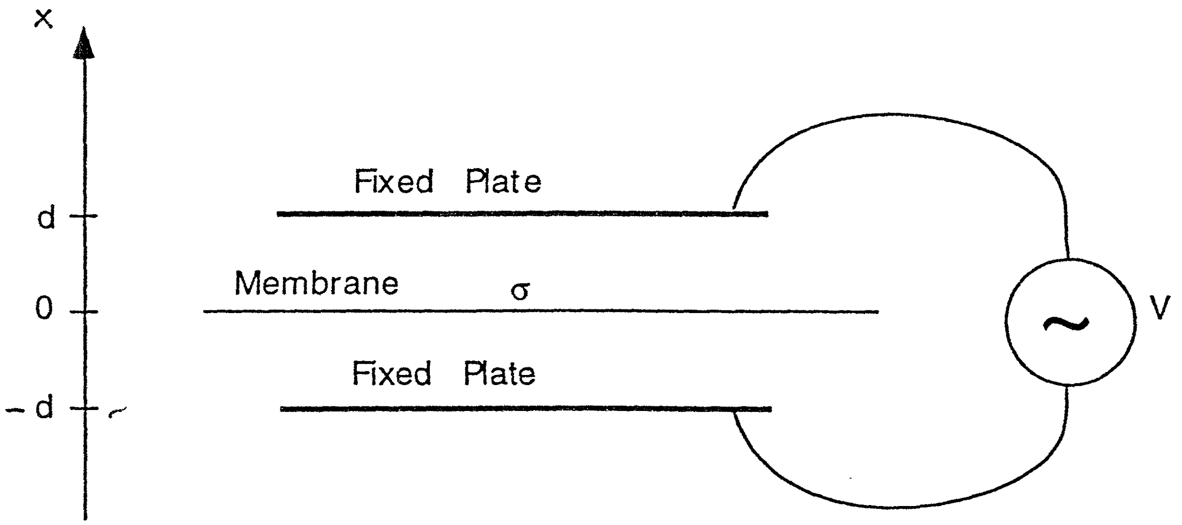
(a) Derive an expression for the electrostatic energy stored in a capacitor. Then show that the electrostatic force on a capacitor plate is F = 0.5 QE where Q is the charge and E is the electric field (3).

(b) Consider an electrostatic loudspeaker the cross section of which is sketched in the figure below where the light polymer membrane A carries a charge $\sigma$ per unit area. The signal V is applied between the plates at x = d and at x = -d to displace the membrane from its rest position which is initially x = 0.

Derive an expression for the total electrostatic force on the membrane when it has been displaced to position x and a signal voltage V is applied to the loudspeaker, stating any assumptions or approximations that you have made. (5)

c) Comment on the form of this expression and the requirements for a high quality loudspeaker. (2)

Calculate the value of $\sigma$ required if the equlibrium displacement of the membrane is 100 μm for V = 1V.



# Qualifier Written Question (Dynamics)

Consider the small gyroscope of mass $m$ and angular momentum magnitude $h$, supported on a massless rod of length $\ell$ which is free to pivot at the origin as sketched in the figure. Gravitational acceleration $g$ is present. The $x, y, z$ location of the gyroscope is completely defined by $\psi$ and $\theta$, where $\psi$ is the azimuth angle about the $z$-axis, and $\theta$ is the angle down from the $x$-$y$ plane.

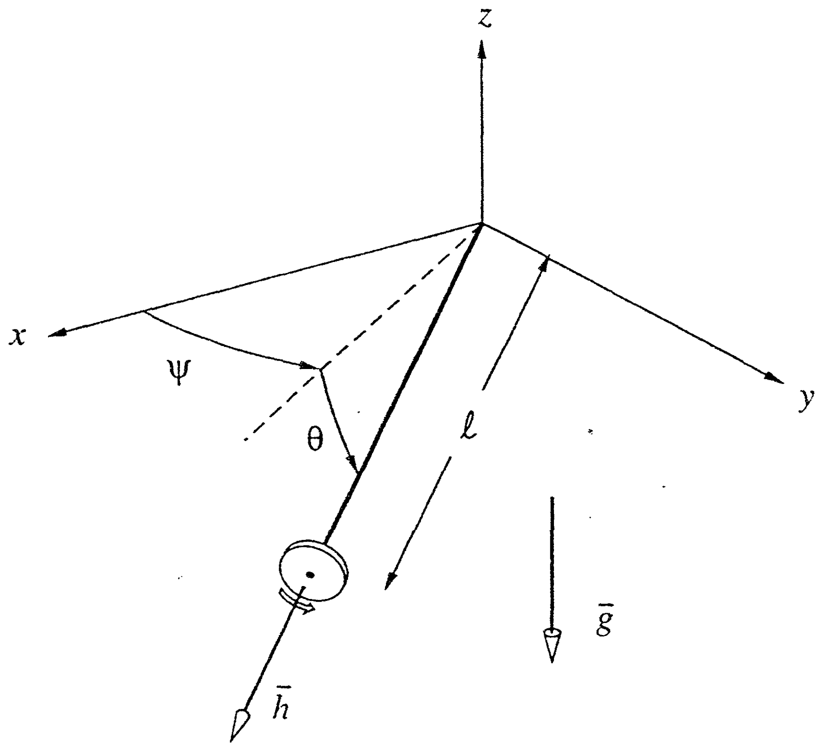
$$\left\{ \begin{array}{c} x \\ y \\ z \end{array} \right\} = \ell \left\{ \begin{array}{c} \cos\psi \cos\theta \\ \sin\psi \cos\theta \\ -\sin\theta \end{array} \right\}$$

1) [25%] If the rod is swinging at some rates $\dot{\psi}, \dot{\theta}$, what is the total angular momentum $\vec{H}$ of the gyroscope about the $x, y, z$ origin? Specify all three components in terms of $\psi, \theta$ and their rates.

2) [20%] With the rod horizontal ($\theta = 0$), what is the steady precession rate $\dot{\psi}$?

3) [25%] What is the precession rate when the rod is at some angle $\theta \neq 0$?

4) [30%] The rod is held horizontal at $\psi = \theta = 0$ against gravity, and then released at time $t = 0$. Determine the initial motion. Specifically, what are $\psi(t)$ and $\theta(t)$ for small $t$?



# WRITTEN QUALIFYING EXAM QUESTION
# AVIONICS

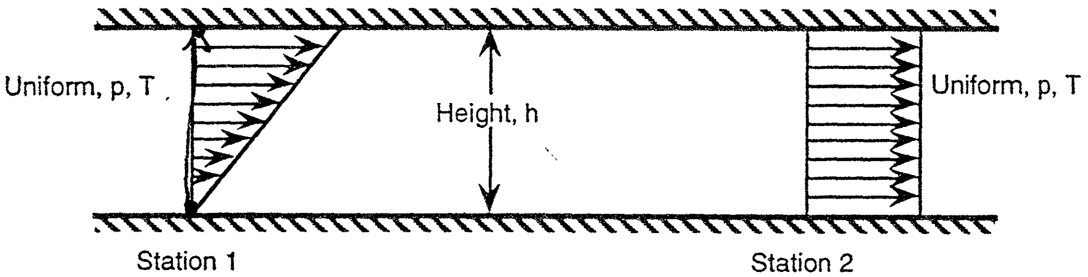
The conventional artificial horizon in an aircraft cockpit consists of a two degree of freedom gyro for spatial stabilization and a pendulum to track the apparent local vertical and to correct for gyro drift. A new configuration for the artificial horizon is proposed which would eliminate the pendulum and replace its function using a single accelerometer and a Global Positioning System (GPS) receiver. The accelerometer is fixed to the aircraft frame and senses specific force in the x body axis. The GPS receiver produces position information (latitude and longitude), and velocity and acceleration information in local north, east and down coordinates. This information is very accurate because selective availability is turned off.

Can such a configuration work to produce consistent information for the artificial horizon function? If your answer is no please explain in terms of physical arguments. If your answer is yes please provide a preliminary design of the system.

# Written Doctoral Qualifying Exam

## Fluids

An inviscid incompressible fluid flows through a two-dimensional channel with inlet and exit velocity profiles as shown below. The walls of the channel are adiabatic. Although the inlet velocity profile is nonuniform, through mixing, the exit velocity profile is uniform.



a) Given that the maximum velocity at Station 1 is $U_{max}$, what is the velocity of the flow at Station 2?

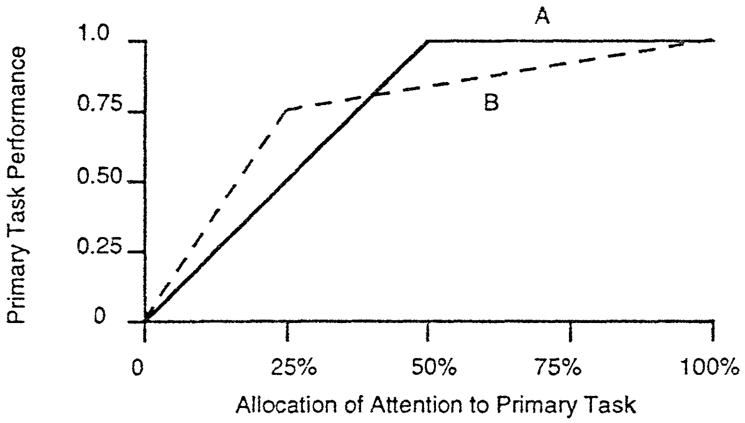
b) Given that the pressure is uniform and equal to $p_1$ at Station 1, and that the pressure is uniform at Station 2, what is the pressure at Station 2?

c) Given that the temperature of the fluid is uniform at Station 1 and again uniform at Station 2, what is the change in temperature from Station 1 to Station 2?

d) Is this process reversible? (Prove your answer.)

# 1998 Humans & Automation Qualifier Question

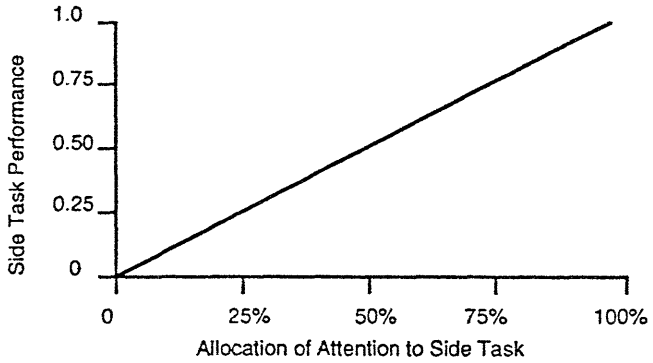
Consider an experiment in which pilots are instructed to fly an airplane along a given course. The pilots perform this task under two conditions: A and B. As the pilots vary the amount of attention they allocate to the primary task, their performance (0=worst, 1=best) varies according to the following curves:



1. Describe the implications of the shapes of the curves (i.e., what is fundamentally different between conditions A and B?).

2. In a typical experiment, the shapes of these curves above are not known, but the primary task performance is measured. Is the use of primary task performance as a measure of workload appropriate in this case? Why or why not?

Now consider an experiment in which the pilots are given a side task in addition to the primary task above. Assume that the pilots allocate any and all spare attention from the primary task to the side task. The performance of the side task as a function of the attention allocated to it is shown in the curve below:



3. Plot the pilots' performance on the primary task (for both conditions A and B) as a function of the corresponding performance on the side task (using the entire range of attention allocation from 0% - 100%). Mark a point on each curve where the pilots would operate if they were told to perform equally well on the primary and side tasks.

4. Is the use of side task performance as a metric of workload appropriate in this case? Why or why not? What is the value-added of measuring primary task performance at the same time as side task performance?

# Qualifier Written Question (Control)

Respond to the following three bullets:

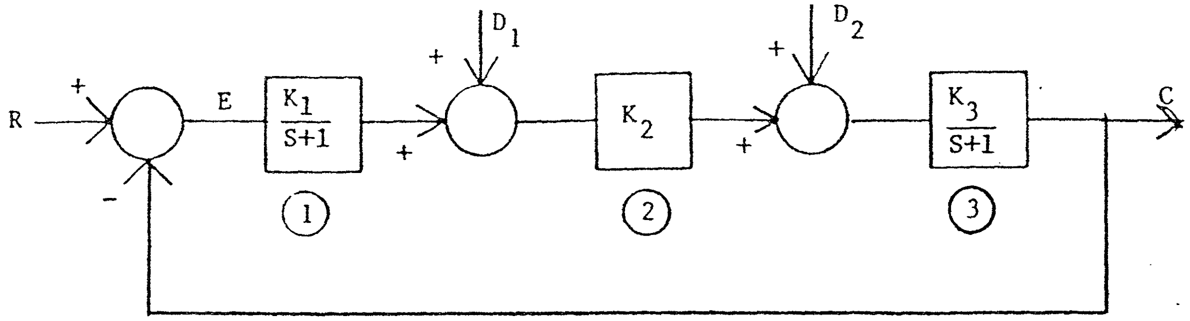
- Problem 1-C
- Problem 2-C
- Problem 3-C or Problem 4-C (but not both)

Notation for closed loop systems:

| | |
|---|---|
| $R$ | reference input |
| $C$ | controlled-variable output |
| $E$ | error ( $= R - C$ for unity-feedback systems) |
| $G(s)$ | forward transfer function |
| $H(s)$ | feedback transfer function |

# Problem 1-C

The system shown below is a regulator, and is supposed to have a small steady-state error, $E_{SS}$ when $R = 0$ and $D_1$ and/or $D_2$ are constant.



You may add a factor of $\frac{1}{s}$, a simple integrator, in any one of the three blocks in the forward path. Assume that the resultant system is stable.

a. In which block should the integrator be placed so that the steady-state error for conditions defined above is zero?

b. Assume the integrator is placed in one of the blocks so that the product of the forward path transfer function is $\frac{K_1K_2K_3}{s(s+1)^2}$. The input is made a unit ramp, $R(t) = t, t > 0$, and $D_1 = D_2 = 0$. What is the steady-state value of $E(t)$ for this system?

c. For the system described in part b, it is found that for some value of $K_1, K_2, K_3$, the response of the system $C(t)$ to unit step for $R(t)$ with $D_1 = D_2 = 0$ includes a component $Acos(\omega_1 t + \phi)$. What is the value of the $K_1K_2K_3$ product in this case? What is the value of $\omega_1$? Explain your reasoning.

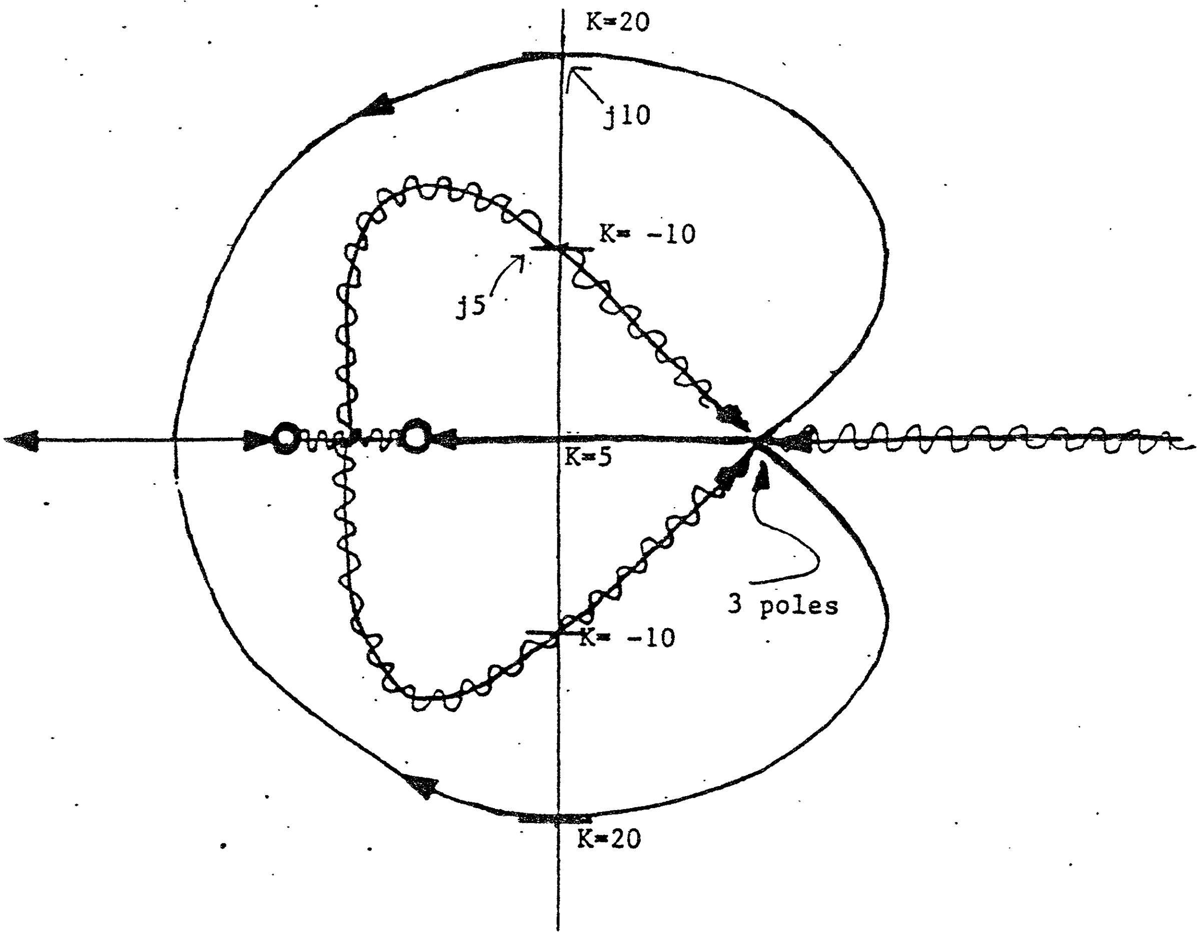
# Problem 2 - C

The root locus diagram for a system open-loop transfer function $KG(s)H(s)$ is shown below for both positive and negative values of $K$. Sketch a Nyquist plot for the function $G(s)H(s)$. Your plot should clearly indicate the magnitude of $GH$ at all points where the plot crosses the real axis. Also indicate the value of $\omega$ at the real axis crossings where you have sufficient information to do so.

Note:

→ are locii for $0 < K < +\infty$

⇝ are loci for $-\infty < K < 0$



# Problem 3 - C

Determine the forward transfer function to satisfy the following requirements. Pay particular attention to limitations on poles and zeros.

1. The steady state value of $E$ with $R(t) = \frac{t^2}{2}$, $t > 0$, is 0.004.

2. The system crossover frequency must be 20 radians per second to meet frequency response objectives.

3. The magnitude of the input to error transfer function is less than 0.1 at any frequency less than 5 radians per second ($|\frac{E(j\omega)}{R(j\omega)}| \leq 0.1$ for any $\omega \leq 5$).

4. To limit the effect of high frequency noise,

$$\left|\frac{C(j\omega)}{R(j\omega)}\right| \leq 0.05$$

   for any $\omega \geq 200$ radians per second.

5. The phase margin is greater than 45°. You should use only the asymptotic approximation for the magnitude of $G(j\omega)$. You may use no more than a total of four poles and zeros for your choice of $G(s)$.

# Problem 4-C

a. A unity feedback system has an open-loop transfer function

$$G(s) = \frac{K_C e^{-\frac{\pi}{2}s}}{s+1}$$

Find $K_C$ for a phase margin of 45°.

b. A unity feedback system has an open-loop transfer function

$$G(s) = \frac{K}{s}\left(\frac{1+s}{1-s}\right)^2.$$

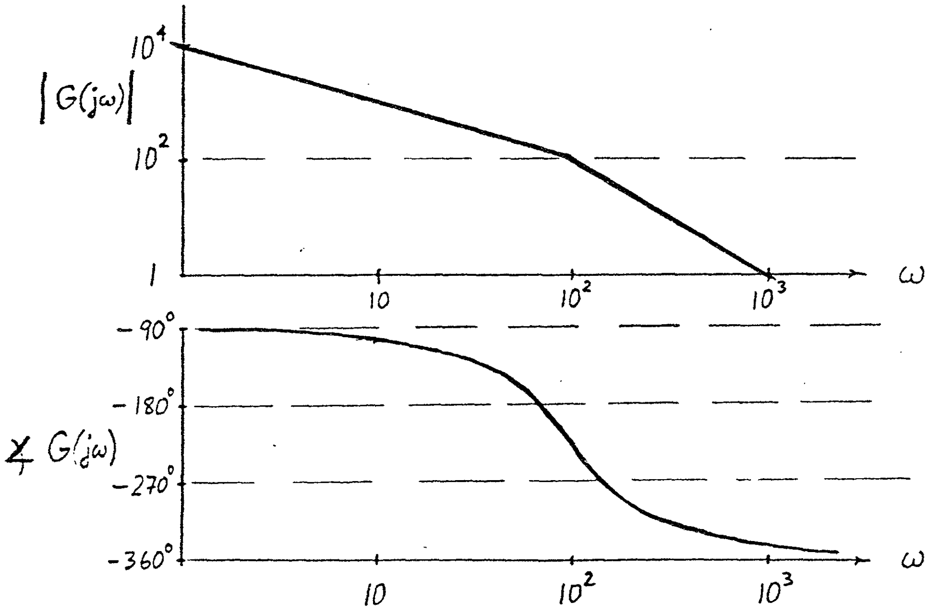
Sketch the Nyquist diagram and indicate locations of the $-1/K$ point for stability.

c. A unity feedback system has an open-loop transfer function

$$G(s) = \frac{K}{s}\left(\frac{1-s}{1+s}\right)^2.$$

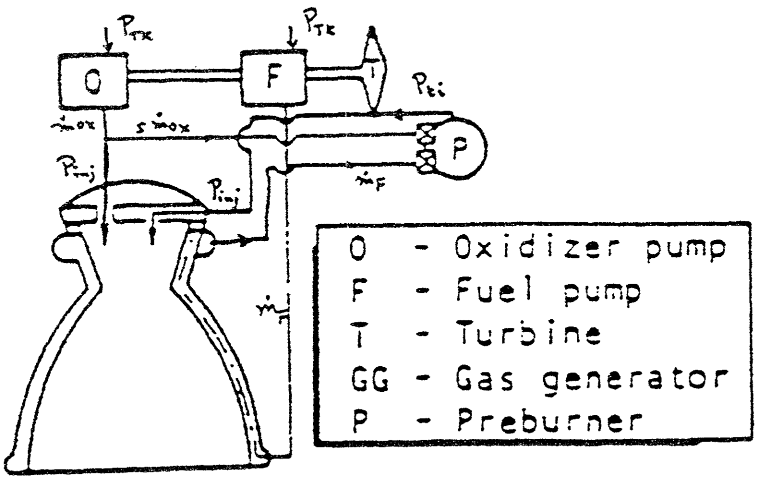
Sketch the root locus for $-\infty < K < +\infty$.

d. The asymptotic Bode diagram of a rational transfer function $G(s)$ is shown below. Determine an expression for $G(s)$ that is consistent with this diagram.



# Written Qualifying Exam
## Propulsion Question

The schematic shows a staged combustion cycle for pressurization of a liquid propellant rocket. The fuel is routed from the fuel pump discharge through the nozzle cooling circuit (where it picks up a total amount of heat $Q_{cool}$) to a preburner (P). A fraction s of the oxidizer is similarly sent to P, the rest being directly injected into the main combustor. The preburner exhaust gas (temperature $T_{ti}$, molecular mass M', specific heat ratio $\gamma'$) is used as the working gas for the turbine, which drives both pumps, and is then injected into the main combustor. The temperature $T_{ti}$ is low enough to allow uncooled turbine operation, which means the preburner is run very fuel rich. The following quantities are assumed known or estimated:



Staged combustion

$P_{TK}, P_{inj}$

$\Delta P_p, \Delta P_{cool}$ (pressure drops in the preburner injector and in the cooling passages, respectively.

Turbine and pump efficiencies

$\gamma', M', Q_{cool} / \dot{m}_F, T_{ti}$

Enthalpies of F and OX in tanks

Fuel heat volume h (heat release per unit mass of fuel if burnt stoichiometrically)

$O/F = \dfrac{\dot{m}_{OX}}{\dot{m}_F}$ and $(O/F)_{\text{stoichiometric}}$

Imposing (a) A mechanical work balance, and (b) a thermal balance of the preburner, derive a model that will allow calculation of the oxidizer fraction s to the precombustor, and of the turbine inlet pressure $P_{ti}$. Explain your assumptions.

# Materials and Structures
# Written Exam Question

A unidirectional composite is made from carbon fibers in an epoxy matrix. The volume fraction of fibers is 65%. The bond between the fiber and matrix is made at a temperature of 175°C. Estimate the following, explaining any assumptions that you make in your calculations:

a) The density of the composite (1 point)

b) The Young's modulus in the fiber direction. (1)

c) The Young's modulus in the transverse direction. (1)

d) The thermal expansion coefficient in the fiber direction. (2)

e) The longitudinal residual stress in the fibers and matrix at room temperature (291 K). (3)

f) Comment on the accuracy of each of these estimates. What would be the relative sensitivity of each of the properties you have calculated to voids in the matrix? (2)

The relevant properties of the constituents are as follows:

| | Density $kgm^{-3}$ | Young's Modulus GPa | Coefficient of Thermal Expansion ($K^{-1} \times 10^{-6}$) |
|---|---|---|---|
| Graphite Fibers | 1600 | 140 | -1 |
| Epoxy | 1200 | 2.41 | 100 |

# Qualifiers Systems Question
# Written
# January 1998

The objective of this question is to see how you organize your thoughts with respect to the initial design of a large scale system, even though you may not be familiar with the technologies specific to the system. To this end, consider a replacement to the Air Force's AWACS (Airborne Warning and Control System) which is used to search for and then track enemy aircraft via Radar. The replacement system will consist of a combination of space-based platforms (SBP) and unmanned aerial vehicles (UAV's). The goal is to cost effectively provide a replacement to AWACS which provides at least the same level of performance as well as timely information transfer to the warfighter in the field. For this problem, please answer the following:

1) Please provide a few sentences about your technical background (e.g., research, classwork, etc.) and expertise so that your answer can be placed in proper context.

2) Assume that you are leading the first meeting of your company's team which is responding to a Request for Proposal (RFP) from the Air Force for a design of this replacement.

a) Provide an outline of the proposal.

b) List and describe the focus of the various teams which will be working on the proposal.

c) Identify and describe proposal selection criteria by which you think the Air Force will evaluate your company's proposal.

3) In general terms, perform the following steps. Depending upon your background, you may focus the bulk of your answer on SBP's or UAV's. However, be sure to provide some detail on the other area as well as on the **interfaces** between these areas.

a) List what you think are the driving mission requirements.

b) Describe two mission architectures and explain why you think that they are capable of meeting the requirements.

c) Identify some key trades that need to be conducted in order to discriminate between these two architectures and the metrics you would use to evaluate these trades.