DEPARTMENT OF
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WRITTEN QUALIFYING EXAMINATION FOR DOCTORAL CANDIDATES

Wednesday, January 22, 1997 37-212 9:00 a.m. - 1:00 p.m.

CLOSED BOOK AND NOTES

Answer a total of five (5) questions (no more or less).

You must answer at least two (2) questions from Column A.
Please answer each question on a separate sheet.

Be sure that your name appears on **EVERY** sheet of paper you turn in.

Oral examinations will be held on Tuesday, January 28, 1997. Pick up your schedule on Monday, January 27, 1997 in the afternoon from Liz in 33-208.

Results will be available on Wednesday, January 29, 1997 after 2:00 p.m.
Please contact your advisor.

Column A	Column B
Mathematics	Avionics
Physics	Fluids
Dynamics	Humans and Automation
	Instrumentation, Control and Estimation
	Propulsion
	Structures
	Systems
	Thermodynamics

Written Doctoral Qualifying Exam

Math

Consider the following differential equation.

$$x^2y'' + xy' - y = 2x \ln x$$

1a) Find the homogeneous solution. [Hint: Try $y = x^p$].

1b) Find the particular solution.

1c) Is it possible to specify boundary condition(s) which would make $y(x)$ non-singular at $x = 0$? Similarly, is it possible to make $y'(x)$ non-singular at $x = 0$?

A point is randomly chosen from the interval $0 \ldots 1$, and is denoted by the random variable x_1 (i.e. x_1 is uniformly distributed between 0 and 1).

2a) Find the variance of x_1.

2b) Find the probability density function of the random variable y, defined from the chosen x_1 by $y = 3x_1 + 2$.

We now select, completely independently of the first point, another point x_2, also from the $0 \ldots 1$ interval.

Let the random variable w be defined as the distance between x_1 and x_2:

$$w = |x_1 - x_2|$$

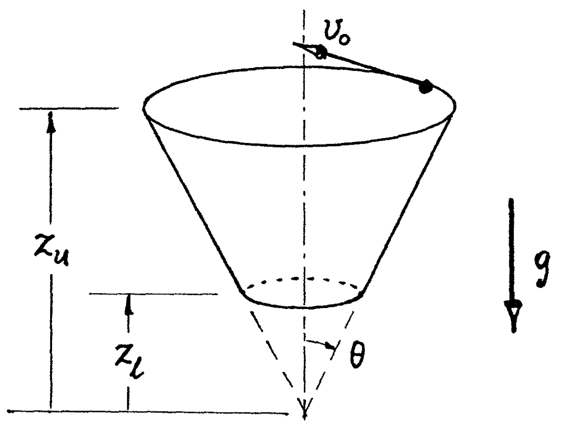
2c) Find the expected value of w.

2d) Find the probability density function of w.

Written Doctoral Qualifying Exam

Dynamics

A small ball is released at a height z_u at the rim of a conical funnel, and rolls without friction inside the funnel under the influence of gravity. The funnel has a half-angle θ, and has the lower part below $z = z_\ell$ removed. The initial velocity of the ball is purely tangential, with magnitude v_o.



a) What is the maximum v_o which allow the ball to fall out through the bottom hole?

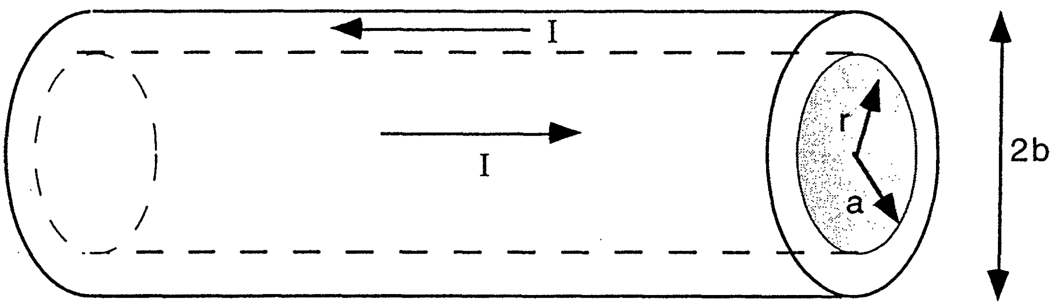
b) For what values of v_o will the ball "spill" over top rim?

c) For any v_o between these two limits, describe the motion of the ball after release. Is it periodic? Between what heights does the ball remain confined?

d) Consider now a shallow funnel of general shape $z(r)$. Assuming the ball travels only where $dz/dr \ll 1$, what must the shape $z(r)$ be in order to make the ball behave like planet orbiting a massive sun at $r = 0$? If the ball is released at some specific location $r = r_o$, what value of v_o corresponds to the "escape velocity"?

Physics
Written Exam Question

Current flows in an infinite straight conductor with radius a and returns through a thin outer conductor of radius b as shown. The current density J in the center conductor is not uniform, but varies with radius r according to $J(r) = J_0 \frac{a}{r}$ where J_0 is a constant.



(a) Find the total current I in tems of J_0 and any other given parameters.

(b) The current flow in the outer conductor is uniform. Find the magnitude of the magnetic field B(r): A) within the center conductor, B) between the conductors, and C) outside the outer conductor.

(c) The current is slowly increased from zero. Neglecting resistance losses what is the energy expended?.

(d) By equating the total stored magnetic energy due to current I with the energy in an equivalent inductor L carrying current I, find the inductance of a length l of the system.

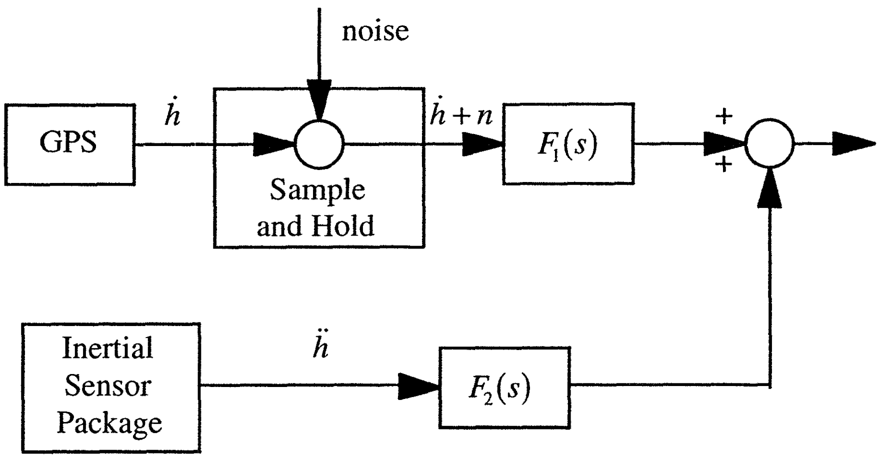
Avionics Qualifying Exam Question

January 22, 1997

An upgrade of the avionics system in a commuter aircraft is under development. It is proposed that the new global positioning system (GPS) receiver, included as part of the upgrade, be used to create a voltage proportional to altitude rate ($\dot{h}$) as input to drive an electronic display in the cockpit . This receiver provides a vertical velocity output, in digital form, at a sample rate of 5 hz. Since the cockpit display requires continuous voltage inputs the GPS receiver is fitted with a sample and hold circuit, transforming the digital altitude rate output into a voltage.

When tested the modified GPS system is found to provide very good altitude rate information at frequencies below 5 Hz, but the sample and hold circuit introduces excessive noise in the output at 5 Hz and higher frequencies. If the display is to operate satisfactorily the amplitude of the noise at 5 Hz must be reduced by a factor of 10.

Searching for a solution to this problem the developers find that a voltage signal indicating vertical acceleration ($\ddot{h}$) is available from an inertial sensor package contained within the vertical situation indicator (VSI) system. This source provides good information at higher frequencies but its response to inputs at frequencies below .05 Hz is decreased. The developers propose the following complimentary filter architecture to combine the two sources of information.



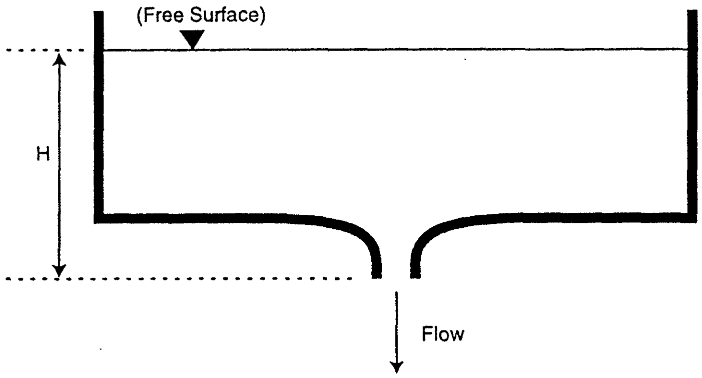
1. Develop transfer functions for the filters $F_1(s)$ and $F_2(s)$ so that the system output provides accurate altitude rate information in the frequency range from .05 Hz to 5 Hz, and that the noise from the GPS sample and hold circuit is reduced by at least a factor of 10.

2. Simplify your design as much as you can.

3. Calculate the response of your system to a step in vertical aircraft acceleration of $1 \text{ ft} / \text{sec}^2$.

Written Qualifiers, Jan 1997.

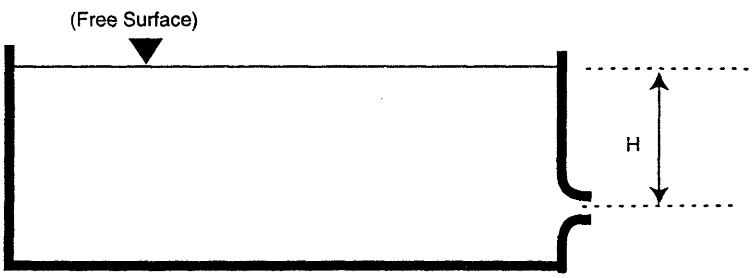
Fluids Question



A large bucket is filled with water and has a narrow hole at the bottom, as shown in the figure above (the bucket and hole are both two-dimensional, extending into the page to infinity).

a) What is the velocity of the water as it leaves the hole?

b) Derive an equation to describe the shape of the column of water as a function of the distance measured from the bottom of the bucket.



Now consider a second bucket, shown in the figure above in which the hole is on the side.

c) Derive an equation to describe the trajectory of the water as it leaves the bucket and the thickness of the water jet as a function of vertical distance measured from the hole

d) Describe, qualitatively, the effects (if any) of viscosity and surface tension

Humans and Automation Question

You've been hired as a consultant to evaluate the workload of tasks performed by pilots of an aerospace vehicle. What workload measurement technique would you suggest for this purpose? Describe how your data would be obtained and the scale used to determine the amount of workload. Provide a detailed argument of the advantages of your proposal and also discuss its limitations and disadvantages.

Controls Written Qualifier Question

January 1997

You are to design a controller for a high performance fighter aircraft that is longitudinally unstable. The short period dynamics consist of one stable and one unstable pole. In addition, there is significant delay in the flight control computer, which is modelled as a nonminimum phase zero. Ignoring the phugoid, the dynamics are

$$G(s) = \frac{\theta}{\delta_e}(s) = \frac{1.5\left(1 - \frac{s}{10}\right)}{(1+s)\left(1 - \frac{s}{0.1}\right)}$$

where θ is the pitch attitude angle and δ_e is the elevator deflection. The controller should meet the following specifications:

- The phase margin should be at least 45 degrees.

- The gain margins should be greater than 6 dB.

1) If a constant gain controller is used, approximate the maximum gain which would satisfy the phase margin requirement. Is the gain margin requirement met by this controller? What is the approximate bandwidth of the resulting closed loop system?

2) Using sketches, describe a control law that would improve, as much as possible, the bandwidth of the system, while satisfying the gain and phase margin specifications. Be as quantitative as possible in the time available.

3) What are the fundamental performance limitations associated with control of systems containing nonminimum phase zeros? Explain briefly why these limitations exist.

PROPULSION
Written Qualifying Exam Question

1/16/97

Consider a simplified model of the operation of a liquid rocket combustion chamber, in which gas is generated in the chamber a time τ_v after the corresponding liquid has been injected:

$$\dot{m}_{gas}(t) = \dot{m}_{inj}(t - \tau_v)$$

The liquid is supplied to the injection orifices, of total area A_i, at a constant pressure P_o, greater than the chamber pressure P.

The gas temperature can be taken as a constant. Gas leaves the chamber through its choked throat, of area A_t, at a rate

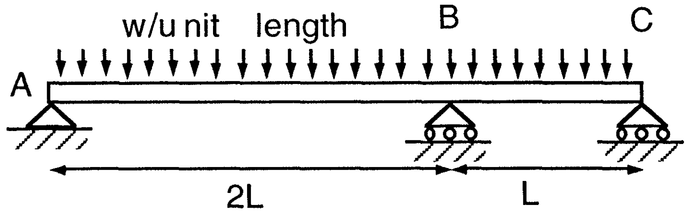
$$\dot{m}_{out}(t) = \frac{P(t)A_t}{c^*}$$

where P is the chamber pressure and c^* is the "characteristic velocity" (constant if $T = const.$). The chamber volume is V, and, at steady state, $P = \overline{P}$.

(30%)	(a) Is there a reason to suspect that this chamber may become unstable under some conditions? Discuss this qualitatively, and point to factors which may drive towards instability.
(20%)	(b) Formulate the differential equation governing $P(t)$.
(20%)	(c) Linearize this equation, and assume solutions of exponential form, with a complex exponent.
(20%)	(d) Derive the condition for instability, if it exists. This should be a certain relationship among the parameters of the problem, preferably in non-dimensional form.
(10%)	(e) Discuss your result in (d) and compare to the qualitative arguments in (a).

Materials and Structures
Written Exam Question

A uniform horizontal beam of rectangular cross-section with flexural rigidity EI and length 3L carries a uniformly distributed load, w (units of force/length), and rests on three simple supports, at its ends A and C, and at B where AB is of length 2L.



(i) Find the *change* in the reaction at B caused when the support at B sinks through a small distance d such that the beam remains in contact with the support. (7/10)

Assume that the beam remains elastic and that the supports can provide either upward or downward reactions.

(ii) The beam is made of a ductile metal with an elastic-perfectly plastic behavior (i.e there is no work hardening) with modulus E and yield stress σ_y. The support at B is fixed in its initial position, describe *qualitatively* what the overall behavior of the beam would be as the distributed load is increased. (Hint: sketches of the stress distribution through the depth of the beam at key locations at critical load levels may help your explanation). (3/10)

Systems Qualifying Exam Question for Jan 97

Written

You are asked to design a space based radar system. The Air Force gives you the following A specification:
Design a space based radar system which provides global coverage, can detect aircraft of 1 square meter radar cross section and is affordable.

A) The radar range equation is (Received Signal/Noise is equal to transmitted power *receiver gain**2*wavelength**2*radar cross section/range**4/(noise*bandwidth)/(4*pi)**3). An effective system requires a signal to noise ratio of 15 dB. For reasonable choices of wavelength, system temperature(T) (noise=kT W/Hz) and gain, estimate the power required on orbit as a function of range. What conclusion do you arrive at from the numbers that you obtained?

B) For global coverage and using simple geometry, estimate the number of satellites that you will need as a function of altitude. Take the radius of the Earth as 6400 km.

C) Using the rule of thumb that current technology allows us to build systems with spacecraft specific power of 2-4 W/(kg dry mass) estimate the dry mass of each satellite. For reasonable choices of orbits and orbit transfer technologies and assuming that the satellites are launched on a system which deposits them at 200 km altitude, estimate the total mass of each satellite as a function of range.

D) Using the rules of thumb that satellite construction costs are \$77K/(kg dry mass) and that launch costs are approximately \$10K/kg to LEO and up to \$62K/kg to GEO, estimate the total cost of the system as a function of range.

Using the numbers that you have obtained, formulate and discuss the trade offs that you would make to come up with a candidate design.

Written Doctoral Qualifying Exam

Thermodynamics

200 ml of tea at 95°C is mixed with 100 ml of milk at 5°C. Both liquids can be assumed to have the thermodynamic properties of plain water:

$$\rho = 1\,\mathrm{g/cm^3} \qquad\qquad c_p = 4.2\,\mathrm{J/gC^\circ}$$

1) What is the temperature of the mixture after mixing? What is the overall increase in entropy that results from the mixing?

2) After mixing, the mixture is then allowed to reach room temperature, at 20°C. What is the additional increase in entropy of the universe that results from this?

3) In a second serving, both the tea and the milk are first allowed to reach the room temperature before they are mixed. What is the total entropy increase of the universe in this case?

4) In a third serving, a small Carnot-cycle engine has its hot/cold heat exchangers dipped in the tea/milk when they are at their temperature extremes, and 100J of electrical work is thereby generated and stored in a perfect battery. The rest of the serving execise proceeds as in 3). What is the entropy increase of the universe during this overall exercise? (You may assume that the temperatures of the tea/milk barely change as a result of the small work generation).