DEPARTMENT OF
AERONAUTICS AND ASTRONAUTICS

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WRITTEN QUALIFYING EXAMINATION FOR DOCTORAL CANDIDATES

Wednesday, January 17, 1996 37-212 9:00 a.m.-1:00 p.m.

CLOSED BOOK AND NOTES

Answer a total of five (5) questions (no more or less).

You must answer at least two (2) questions from Column A.
Please answer each question on a separate sheet.

Be sure that your name appears on *every* sheet of paper you turn in.

Oral examinations will be held on Tuesday, January 23, 1996.

Results will be available on Wednesday, January 24, 1996, after 2:00 p.m.
Please contact your advisor.

Column A	Column B
Mathematics	Instrumentation, Control and Estimation
Physics	Fluids
Dynamics	Structures
	Propulsion
	Systems
	Thermodynamics

MATHEMATICS

1. Discuss the convergence of the power series
$$x + 2^2x^2 + 3^3x^3 + \cdots + n^n x^n + \cdots$$
i.e., for what values of x will the series converge?

2. Find the first four terms in the Taylor series for e^{e^x}, i.e., c_0, c_1, c_2, c_3 in
$$e^{e^x} = c_0 + c_1x + c_2x^2 + c_3x^3 + \cdots$$

3. From the integral form of the Bessel function
$$J_n(x) = \frac{1}{\pi}\int_0^\pi \cos(n\theta - x\sin\theta)\,d\theta$$
find a bound L for which $|J_n(x)| \le L$ for all n and x.

4. For $y = y(x)$, the first and second derivatives are $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. If we regard x as a function of y, i.e., $x = x(y)$, we know that
$$\frac{dx}{dy} = \frac{1}{\frac{dy}{dx}}$$
But how do you express $\frac{d^2x}{dy^2}$ in terms of $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$?

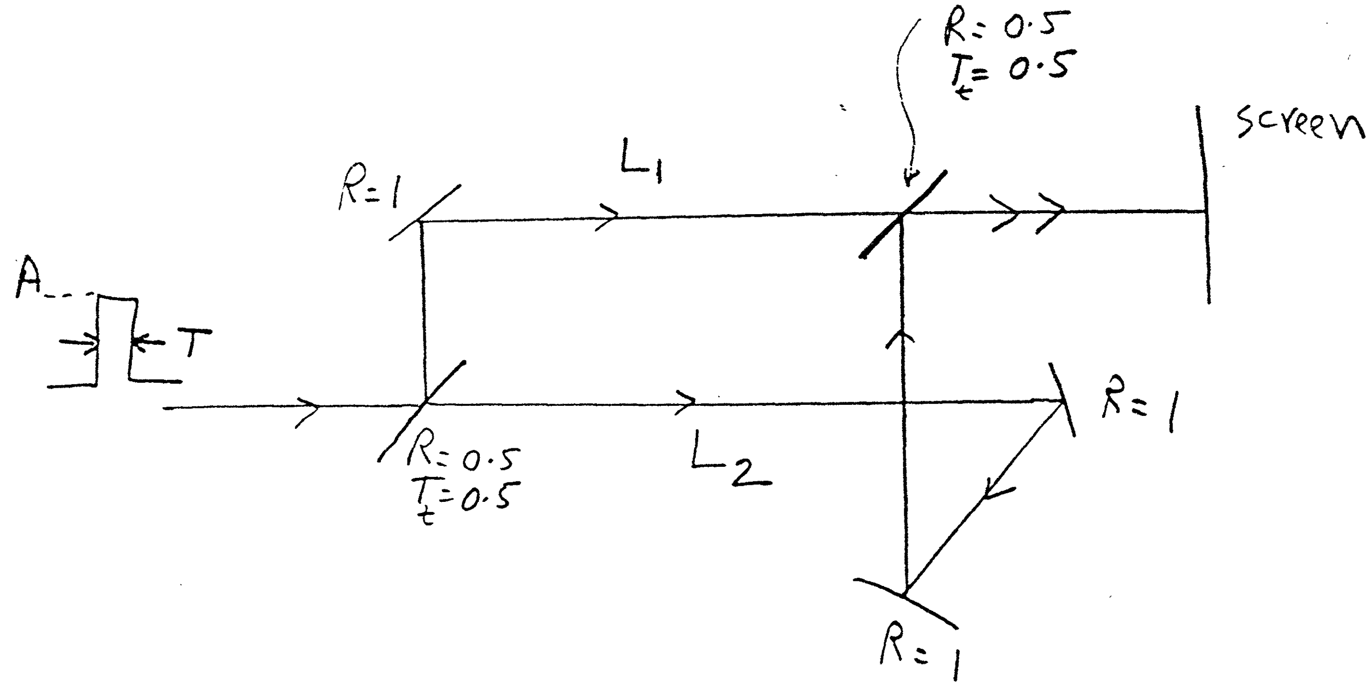
5. Solve the differential equation
$$(1+t)^2\frac{d^2x}{dt^2} + x = 0$$
Discuss the stability behaviour of the solution for $t \to \infty$.

HINT: You might want to consider something of the form $(1+t)^m$.

6. Let the columns of a three-dimensional matrix **M** be the three column vectors **a**, **b** and **c**, i.e., $\mathbf{M} = [\mathbf{a}\ \ \mathbf{b}\ \ \mathbf{c}]$. Let the vector products (or cross products) of any pair of these vectors be considered as row vectors.

Explain how the matrix
$$\mathbf{Q} = \frac{1}{(\mathbf{a}\cdot\mathbf{b}\times\mathbf{c})}\begin{bmatrix}\mathbf{b}\times\mathbf{c}\\ \mathbf{c}\times\mathbf{a}\\ \mathbf{a}\times\mathbf{b}\end{bmatrix}$$
is related to the matrix **M**.

PHYSICS

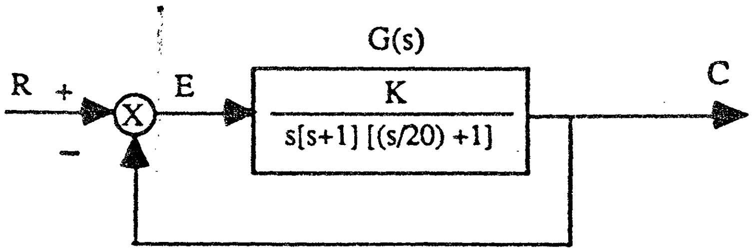


If the input to the interferometer shown in the figure is a pulse of amplitude A= 1 watt, and width T= 1 nanosecond, sketch the intensity you would observe on the screen as a function of pathlength difference (L_2- L_1) from 0 to 50 cm. Comment on your observations.

Assume the two beam splitters are identical with reflectivity = transmission = 0.5; the reflectivity of the the two mirrors is 1; the wavelength of the light is 1 micrometer; and the speed of light is 3x 10^8m/s. State any other assumptions you need to make.

INSTRUMENTATION CONTROL & ESTIMATION

You are given the following system:



a) Numerically, what are the ranges of K (including all + and - values) for which the closed loop system is stable.

b) Examine the answer in a) by sketching a Nyquist diagram and a root locus diagram. Here we are interested in the topology of the sketches, not exact values. In each diagram, show that the regions of instability agree with the answers in a).

c) For those values of K for which stable operation occurs, what is the steady state ($t \to \infty$) error.

   i) with a unit step input $r(t) = 1$

   ii) with a unit ramp input $r(t) = t$.

d) By any method of your choice, demonstrate the existence of a dominant mode or modes of the closed loop step response. Hint: It is not necessary to evaluate the closed loop poles.

e) A certain value of K is specified with the given plant, G(s). The transient response of the closed loop system is satisfactory. Unfortunately, there is insufficient dc gain (by a factor of 10) for control of steady state errors.

   You are told to add a series compensator $G_c(s)$ (in the forward path) to G(s). The compensator is constrained to include one pole and one zero, and must be designed to have a minimal effect on the stability of the uncompensated system.

   Using either a root locus sketch or a Bode diagram, show explicitly where the pole and zero of the compensator should be placed. Write out the transfer function for your compensator.

f) Now, G(s) contains a right half-plane zero.

$$G(s) = \frac{K(-s+1)}{s[s+1][(s/20)+1]}$$

   How does the presence of this right half-plane zero affect the performance that can be achieved with the closed loop system?

FLUIDS

A strong axisymmetric and two-dimensional vortex in an incompressible fluid is often modeled as a so called Rankine vortex, in which the core of radius a is assumed to rotate as a solid body and the flow outside the core is irrotational. Determine:

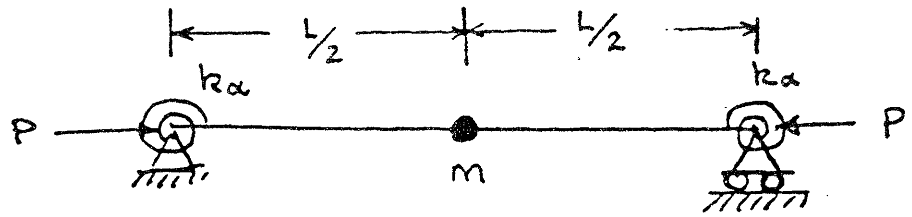
a) The pressure distribution in the vortex.

b) The stagnation pressure distribution and explain its shape.

c) Does this flow satisfy the Navier Stokes equations?

STRUCTURES

Consider the discrete system shown below, made up of two massless rigid pinned bars, two torsional springs, and a concentrated mass at the center hinge.



1. Can this system undergo elastic instability? If no, prove so. If yes, derive the instability condition. Compare and contrast with the stability of a continuous beam of equal length.

2. In the presence of the load P, calculate the natural frequency of vibration.

3. Correlate your answers in parts 1 & 2 and comment.

PROPULSION

A jet engine has a mass flow m and a flight velocity U_o. It discharges flow at the nozzle exit at a pressure equal to atmospheric and with a given velocity U_e.

a) If the fuel/air ratio is f, what is the fuel specific impulse?

b) If the engine is operated so that the exhaust velocity, U_e, and the mass flow rate, m, are constant over the flight regime (with the nozzle area varied to keep the exit pressure equal to atmospheric), at what flight velocity, U_o, is the thrust power maximized?

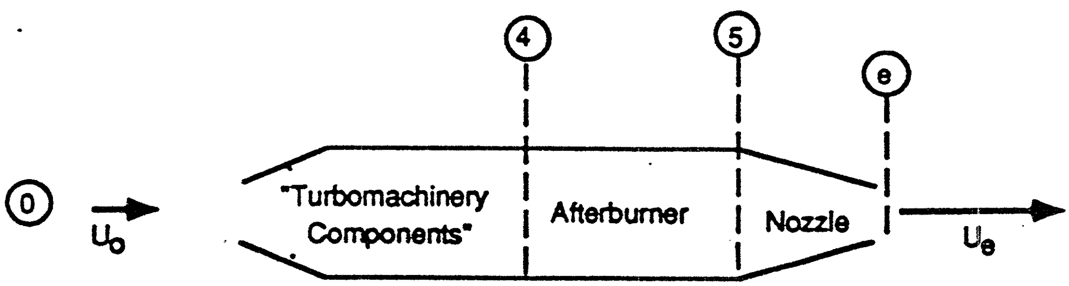
c) The engine can be operated with or without its afterburner on. With the afterburner off, at a flight velocity U_o, the exit stagnation temperature is T_{t_e}. With the afterburner on, the exit stagnation temperature is twice th value with the afterburner off, i.e., $T'_{t_e} = 2T_{t_e}$, where the prime (') denotes conditions with the afterburner on. The mass flow through the engine, the flight velocity, and the exit stagnation pressure remain unchange when the afterburner is lit.

What is the ratio of thrust with afterburner on to thrust with afterburner off? (This should be an expression involving only the given quantities U_e and U_o.)

d) Suppose that the aircraft climbs to another altitude where the atmospheric pressure is half of what it was in sections (a), (b), and (c), but the atmospheric temperature is unchanged.

How are (i) the mass flow through the engine, and (ii) the thrust, altered?

Make any assumptions that you think are warranted, but be sure to include justification of the reasoning behind the assumptions.



Turbojet can be run with afterburner off (exit temperature = T_{t_e}),
or on (exit temperature = $T'_{t_e} = 2T_{t_e}$)

SYSTEMS

Argue for the relative merits of either a geosynchronous constellation or a LEO (200 km) constellation of communication satellites for communicating with aircraft in flight. Assume that the bandwidth necessary to support the required communications rate is .25 MHz. Consider both the aircraft and space elements. You are free to choose frequency and transmission power as you see fit.

Some potentially but not necessarily useful numbers:

Radius of the Earth: 6.37×10^3 km

Mass of the Earth: 5.98×10^{24} kg

Boltzman's Constant: $k = -228.4$ dBW $= 1.38 \times 10^{-23}$ Wsec/°K

Assumed atmosphere/space background temperature: 150 °K

Assumed earth temperature: 270 °K

Speed of light: 3×10^8 m/sec

THERMODYNAMICS

An electric current of 10 Amperes (10 A) is maintained for 1 second in a resistor of 25 Ohms (25 Ω), while the temperature of the resistor is kept constant at 300 K, which, for this problem, is the temperature of the surroundings, and the resistor is exposed to atmospheric pressure. The rate of power dissipation in the resistor is given by: Power = $(\text{Current})^2$ x Resistance.

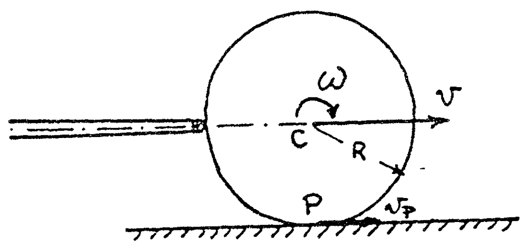
a) What is the entropy change of the resistor?
b) What is the entropy change of the surroundings?
c) What is ΔS_{total} (the total entropy change of surroundings plus resistor)?
d) What is the loss of available work?

The same current is maintained for the same time in the same resistor, but now thermally insulated. The initial temperature is 300 K. If the resistor has a mass of 0.01 kg and a specific heat of 10^3 J/kg-K:

e) What is the entropy change of the resistor?
f) What is the entropy change of the surroundings?
g) What is ΔS_{total} (the total entropy change of surroundings plus resistor)?
h) What is the loss of available work?

DYNAMICS

A billiards player strikes the cue ball (of mass M) horizontally at exactly its mid-height. Since the instantaneous force from the strike is large, the wall friction can be ignored



during it, but afterwards, any slip motion υ_p of the contact point P is resisted by a dry friction force

$-\mu Mg\left(\frac{\vec{\upsilon}_p}{\upsilon_p}\right)$ ($\mu = 0.5$ is a friction coefficient).

(a) If the ball starts with a forward velocity $\upsilon_o = 2\ m/\sec$ calculate the initial rotation rate ω_o and slip υ_{po}.

(b) Find the time and distance it will take for the ball to stop slipping, and the velocity υ and rotation rate ω it will have after that time.

(c) What would happen if the strike is off-center (but still horizontal)? Is there a particular striking height for which the ball will have a pure rolling motion from the beginning ?