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WRITTEN QUALIFYING EXAMINATION
FOR
DOCTORAL CANDIDATES

Wednesday, Jan. 18, 1995 | 37-212 | 9:00am-1:00pm

CLOSED BOOK AND NOTES

Answer a total of five (5) questions (no more or less).

You must answer at least two (2) questions from Column A.
Please answer each question on a separate sheet (or sheets). *Do not put the answers to different questions on the same sheet of paper!*

Be sure that your name appears on *every* sheet of paper you turn in.

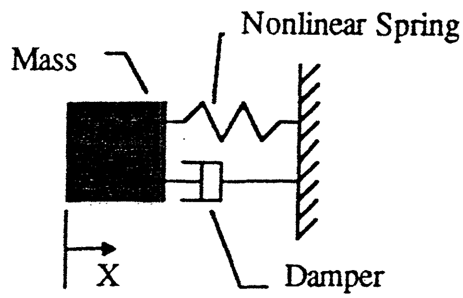
Oral examinations will be held on Tuesday, January 24, 1995.

Results will be available on Wednesday, January 25, 1995, after 2:00pm.

Column A	Column B
Mathematics	Instrumentation, Control and Estimation
Physics	Fluids
Dynamics	Structures
	Propulsion
	Systems
	Thermodynamics
	Avionics

1995

Mathematics
Written Exam Question



A nonlinear spring-mass-damper system is described by the following differential equation:

$$\ddot{x} + 3\dot{x} + 2\sin(x) = 0$$

$$x(t=0) = 0$$
$$\dot{x}(t=0) = \varepsilon$$

1) Find a solution for the behavour of the system, x(t), when $\varepsilon << 1$.

2) Sketch this solution.

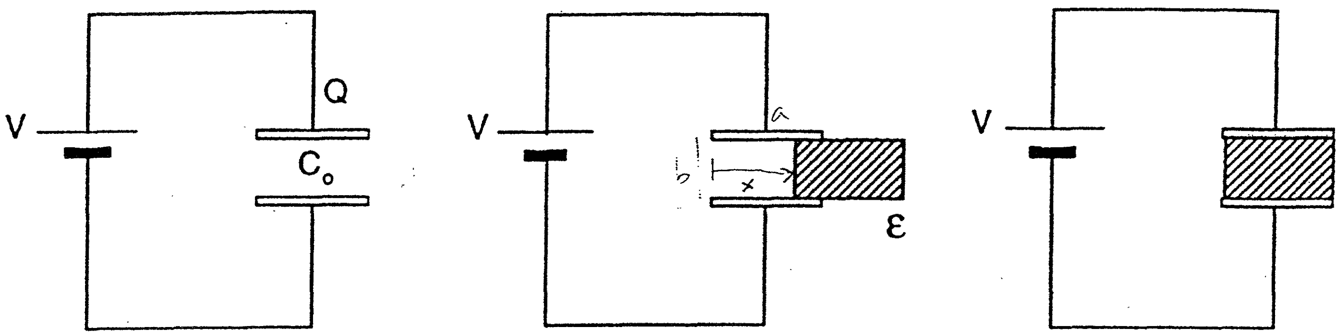
3) Find an expression for the initial motion of the mass accurate to $O(t^2)$

4) Estimate the time interval for which the expression derived in (3) is valid.

5) Estimate the maximum displacement of the mass.

1995

Physics
Written Exam Question



(a) (b) (c)

Figure (a) shows a capacitator C_0 connected to a battery of voltage V. The charge on the capacitor plate is Q. A slab of material of dielectric constant ε is inserted between the capacitor plates as shown in Fig. (b) and Fig. (c). (Assume that the capacitor plates are a square of side a, and separation b. Assume the slab is also a square of side a and thickness b.)

(a) Determine the stored energy in Fig. (a), Fig. (b) and Fig. (c).

(b) Is the stored energy in Fig. (c) greater than that in Fig. (a)?

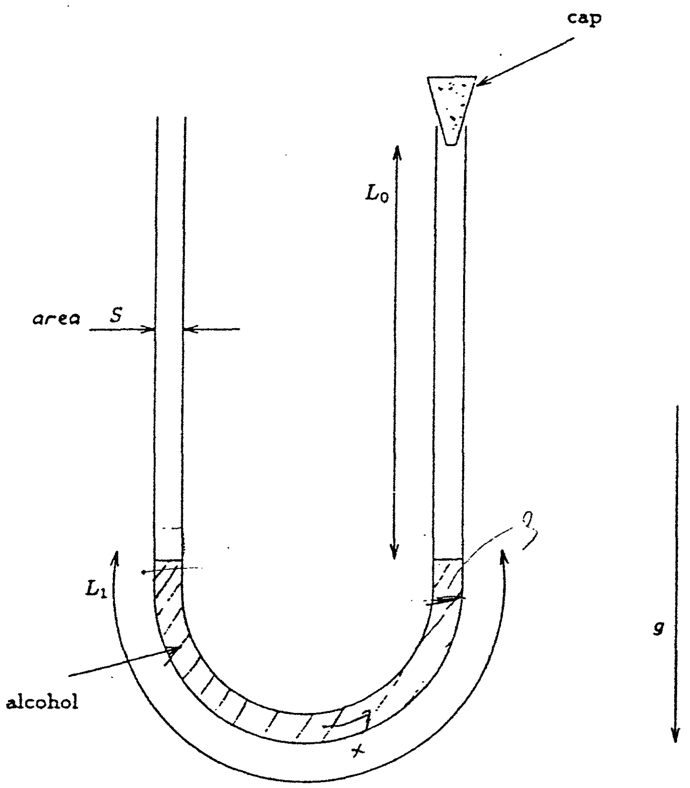
(c) Calculate the energy drawn from battery.

(d) Compare your answers in (c) and in (b).

(e) Explain the discrepancy you may have found in (d).

Dynamics
Written Exam Question

The following experimental setup is provided to you:



A U-shaped glass pipe (assumed to insulate heat perfectly) is filled with alcohol (density ρ). Its cross-section is S. It is assumed that the liquid inside the pipe is connected (*i.e.* no bubbles can form in it and it cannot divide itself into many segments) and that the total length of liquid is L_1. The right end of the pipe may or may not be tightly capped, and, when the liquid is at rest, the total length from the surface of the liquid to the right end of the pipe is L_0. We assume that the gas in the pipe and surrounding it is air (a perfect gas); we also assume normal nominal temperature and pressure conditions. When the system is at rest, the air pressure in the right half of the pipe is the atmospheric pressure. The whole setup is subject to gravity g.

1. Assuming the cap is removed, that there are no frictions (no viscosity) and the air inertia is negligible, derive the period of the oscillations of the alcohol in the pipe.

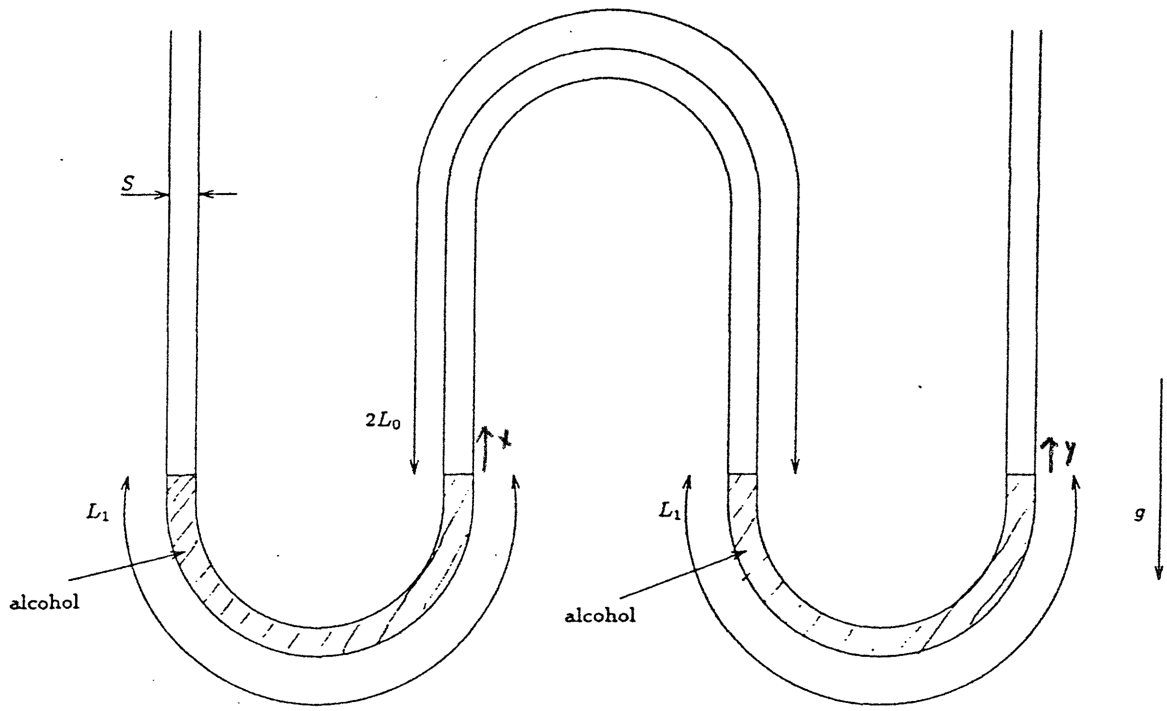
2. Solve the previous question when the cap is in place and holds tight. Assume small perturbations only. (Hint: for perfect gas and adiabatic conditions, the relation

$$PV^\gamma = \text{Constant}$$

holds. (P is the pressure, V is the volume.))

Dynamics
Written Exam Question

3. Consider now the following setup:



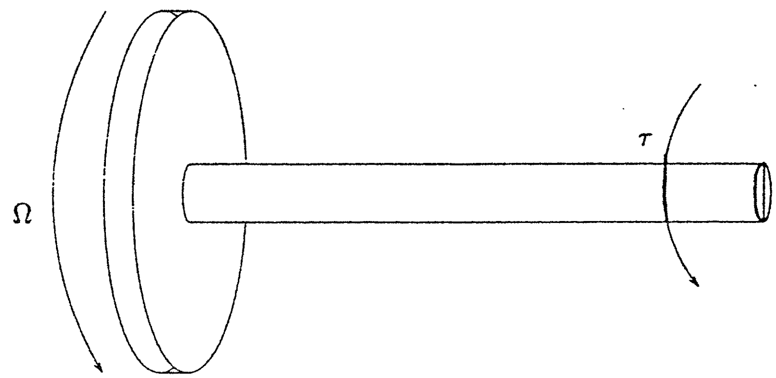
Assuming small perturbations, derive the characteristic frequencies of this system.

4. **Numerical application:** The following values are given to you:
   - density of alcohol: $\rho = 0.7$ kg/liter
   - nominal atmospheric air pressure: 1.0133×10^5 Pa
   - $\gamma = 1.4$, $g = 9.8\text{m/s}^2$
   - $L_0 = 1.5\text{m}$, $L_1 = 0.3\text{m}$

   Compute the characteristic frequencies for question 1, 2 and 3.

Instrumentation, Control and Estimation
Written Exam Question

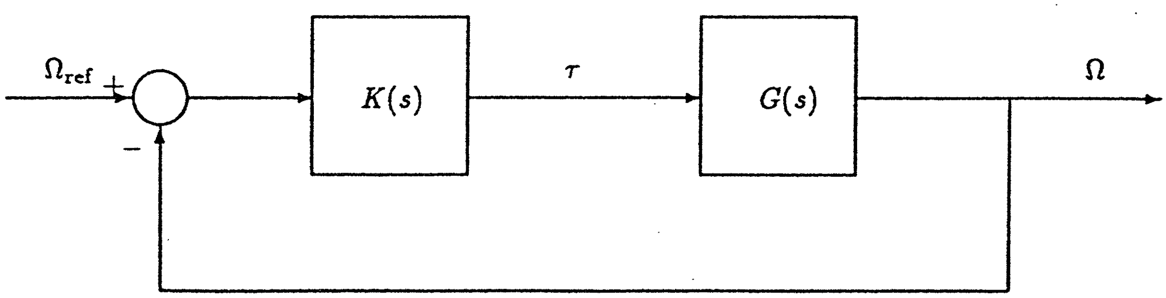
We consider the following flexible shaft-disc system:



We wish to regulate the rotational speed Ω of the disc around some nominal value using the torque τ. The shaft is flexible, such that the transfer function from τ to Ω may in principle be modeled as

$$G(s) = \frac{\Omega(s)}{\tau(s)} = \frac{1}{s(s^2+1)}.$$

The following control diagram is considered:



Assume we want to control this system with proportional feedback $K(s) = K \in (-\infty, \infty)$.

1. Draw the corresponding root-locus.

2. Are there values of $K \in (-\infty, \infty)$ for which the system is stable?

In order to overcome the troubles generated by the shaft flexibility, control engineers sometimes make use of a so-called "notch" compensator. A typical notch compensator for our system is given

Instrumentation, Control and Estimation
Written Exam Question

by

$$K(s) = K\frac{s^2 + 0.1s + 0.8}{(s+10)^2}, \qquad K \geq 0.$$

3. Draw the corresponding root-locus.

4. Choose the gain K such that one of the closed-loop poles is -5.

The locations of the closed-loop system poles for the choice of K in part (4) turn out to be

$$\begin{array}{c} -13.111 \\ -5.0 \\ -1.679 \\ -0.105 \pm 0.962j. \end{array}$$

Assume now that $G(s)$ is not exactly known due to thermal and mechanical variations. $G(s)$ can then be written:

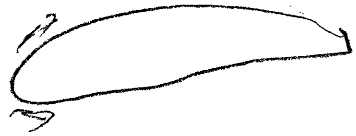
$$G(s) = \frac{1}{s(s^2 + 1 + \delta)},$$

where δ is unknown (thus 0 corresponds to the original, nominal system).

5. Using the same notch compensator, sketch the locus of the closed-loop poles for small variations of δ (it can be positive or negative. Hint: This should be set up so δ appears as a gain in the characteristic equation.)

6. Assume that δ varies between -0.3 and 0.3. How would you modify the notch compensator to ensure good (that is: stable) behavior of the closed-loop system whatever the value of δ is?

1995



Fluids
Written Exam Question

1. List four critical assumptions that lead to the small perturbation velocity potential (φ) equation in two dimensions:

$$(1 - M_\infty^2)\,\varphi_{xx} + \varphi_{yy} = 0$$

The coordinates x, y are fixed to the body and M_∞ is the freestream Mach number.

2. Define U_∞ as the freestream speed, assumed parallel to the x axis. The upper and lower surfaces of the body are defined as follows:

$$\begin{aligned} y_u &= g_u(x) - \alpha x \\ y_l &= g_l(x) - \alpha x \end{aligned}$$

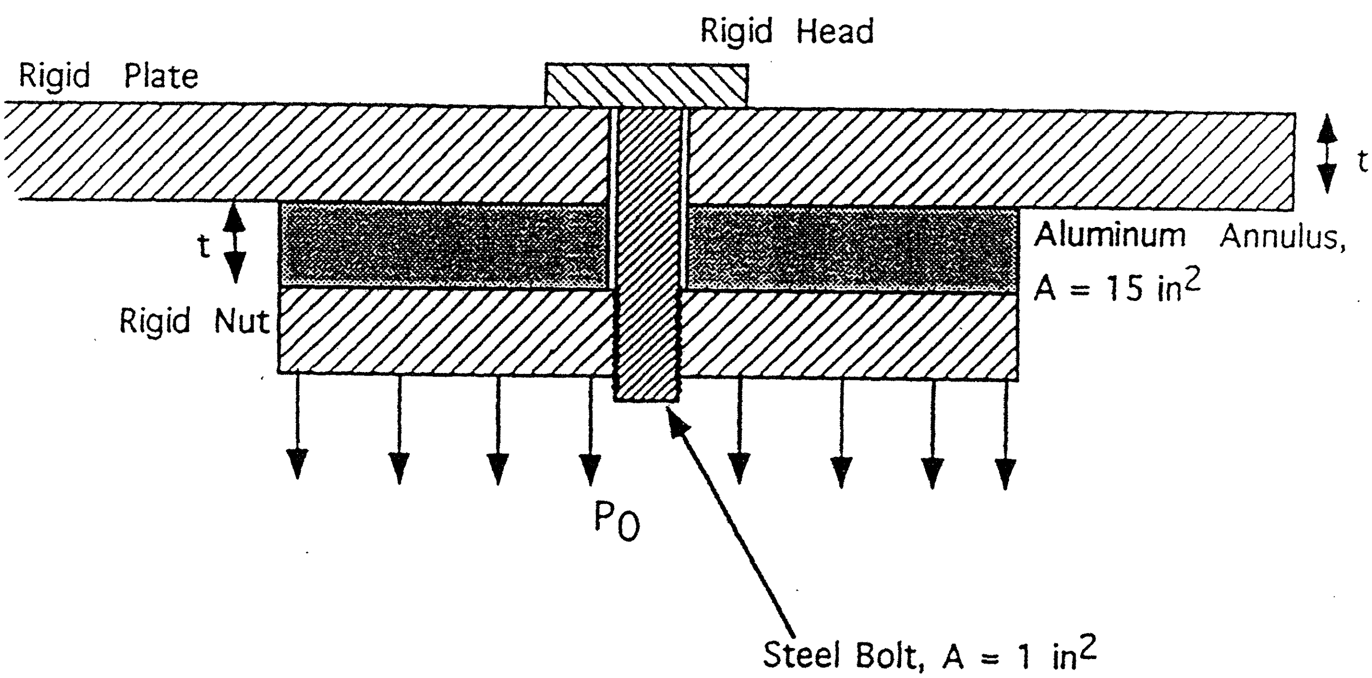
If the body has solid walls, i.e. no flow though the surface, give the inner boundary condition, and state where it is applied in the context of small-perturbation theory. Explain your answer.

3. If M_∞ is supersonic, is any function $f\left(x \pm \sqrt{M_\infty^2 - 1}\, y\right)$ a solution to the small perturbation equation in 1.? If so, explain which $\pm$ sign you would choose for a solution on the upper surface.

4. Find an expression for the pressure coefficient on the upper and lower surface, and discuss the validity of this expression.

Structures
Written Exam Question

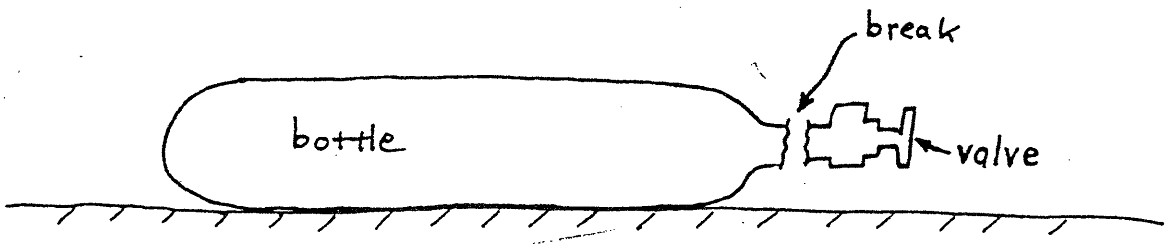
An annulus of an aluminum alloy is attached to a perfectly rigid infinite plate by a steel bolt with a perfectly rigid nut. The shank of the bolt is smooth where it passes through the rigid plate, but is threaded to accept the nut. The plate and aluminum annulus are of equal thickness, t. The bolt has a cross-sectional area of 1 in^2., the annulus has a cross sectional area of 15 in^2. The bolt is pretensioned to 11000 lbs and then a load, P_0 is applied, distributed over the nut. If the maximum load the bolt can take in tension is 12000 lbs, what distributed load P_0, must be applied to cause the bolt to fracture?. How does this change if the annulus is made of steel rather than aluminum?. Consider elastic deformations only, the modulus of the aluminum alloy is 10 Msi, the modulus of steel is 30 Msi.



Cross Sectional View of Assembly

Propulsion
Written Exam Question

A gas cylinder, initially filled with 30 kg of air (molecular weight 29 and specific heat ratio 1.4) at 3000 psi (20 MPa), is dropped and the valve cleanly broken off leaving a hole of area 10^{-4} m^2 with its axis aligned with the axis of the bottle. Find the force acting to accelerate the bottle as a function of time, beginning with the severing of the valve.



Systems
Written Exam Question

Design an efficient "low altitude terrain alerting" system for aircraft. Efficient means being able to allow reliable alerts over irregular terrain for a range of descent rates and forward speeds without requiring unnecessary early or nuisance alerts and excessive margins. The only measurements available for the system are:

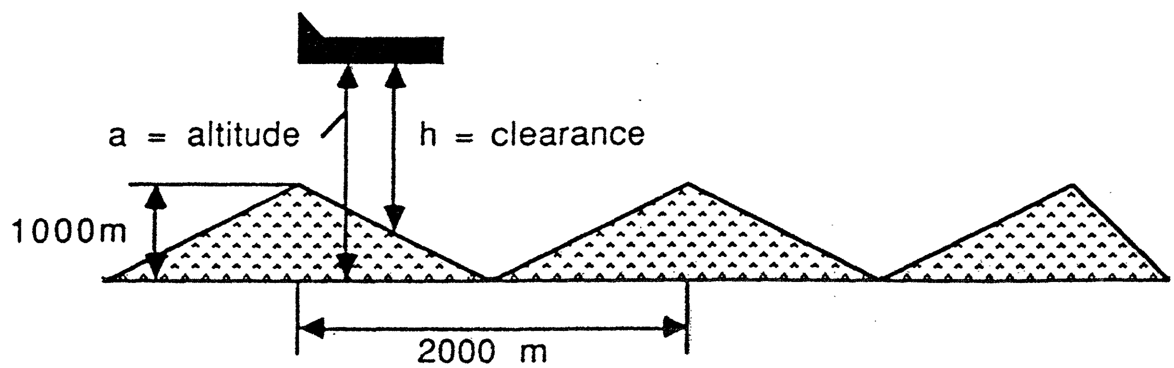
1) pressure altitude, a, and its rate, $\dot{a}$ ($\dot{a}$ has a 6 second lag)

2) ground clearance, h, and its rate, $\dot{h}$ ($\dot{h}$ is averaged over the last 12 seconds). This measurement is vertical from the aircraft irrespective of its attitude

3) horizontal ground speed, v

The escape maneuver is pull-up limited to 0.25g to a path angle of 15° above horizontal, with a delay of 3 seconds after the alert. The critical terrain profile is "saw-tooth" triangular mountains of height 1000 meters and wavelength of 2000 meters. It is desired to ensure a miss-distance of 100 meters above peak terrain.

What is the minimum level flight altitude for the aircraft not to receive any nuisance alerts at 400 meters/second ground speed?



Thermodynamics
Written Exam Question

1995

An engine uses a series of processes as follows: Air is compressed adiabatically from 1 atm and 300 K to 30 atm and 900 K. It is then heated at constant pressure to 1800 K, adiabatically expanded back to 1 atm and 800 K, and cooled at constant pressure to 300 K.

Assume the air is a thermally perfect gas with constant specific heat C_p = 1000 Joule/kg K.

a) Calculate the entropy changes, if any, in the compression and expansion processes.

b) Sketch the four processes on temperature-entropy coordinates.

c) Find the thermal efficiency and power per kg/s of airflow of an engine operating on this cycle.

d) Find the maximum efficiency possible for an engine operating on this type of cycle, between pressures of 1 atm and 30 atm.

e) For the compressor, find the adiabatic efficiency, defined as the reversible adiabatic work divided by the actual work for the same pressure ratio.

f) Supposing that the peak temperature is fixed at 1800 K (and for simplicity that the compressor and turbine efficiencies are both unity), what compressor temperature ratio will yield the maximum power per unit of airflow?

Avionics
Written Exam Question

This problem addresses the navigation of an aircraft using the Global Positioning System (GPS). GPS satellites, whose orbits are accurately known by users, transmit precisely timed encoded messages which are received and decoded by user aircraft. By precisely measuring the arrival times of these messages the relative distances to the satellites can be determined, and thus, knowing satellite orbits, the location of the user aircraft can be inferred.

Relative to our user aircraft the azimuths from true north, and elevations above the horizon, of three GPS satellites, are:

Satellite#	Azimuth	Elevation
1	45 deg	45 deg
2	135 deg	45 deg
3	225 deg	45 deg

Knowing the positions of the satellites, the timing of the signals they send, and using aircraft position as indicated by its onboard inertial navigation system, arrival times of signals from the three satellites are predicted. Using a precision time reference the aircraft system measures the time differences between when the actual signals arrive and the predicted times of arrival. The differences are as follows:

Satellite#	Arrival time minus predicted arrival time
1	1.0 microseconds
2	5.0 microseconds
3	0.2 microseconds

1. Assuming perfect timing of signals from the satellites and using the speed of light at 300,000 km/second, determine the errors in the onboard inertial navigation system in the north, east, and down directions.

2. Now assume that the aircraft has been flying for a long period of time so its time reference has drifted and knowledge of absolute time is uncertain. Set up the problem for determining time as well as position corrections using a fourth satellite.

Satellite#	Azimuth	Elevation
4	0 deg	45 deg