DEPARTMENT OF
AERONAUTICS AND ASTRONAUTICS

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WRITTEN QUALIFYING EXAMINATION
FOR
DOCTORAL CANDIDATES

Wed., Jan. 19, 1994 37-212 9am - 1pm

• CLOSED BOOK AND NOTES •

Answer a total of five (5) questions (no more or less).

You must answer at least two (2) questions from Column A.
Please answer each question on a separate sheet (or sheets). *Do not put the answers to different questions on the same sheet of paper!*

Be sure that your name appears on *every* sheet of paper you turn in.

Oral examinations will be held Tuesday, January 25, 1994.

Results will be available Wednesday, January 26, 1994, after 2pm.

Column A	Column B
Mathematics	Instrumentation, Control and Estimation
Physics	Fluids
Dynamics	Structures
	Propulsion
	Systems
	Thermodynamics
	Avionics

1994

Mathematics

Written Exam Question

1. Find the point in the plane

$$Ax + By + Cz = D$$

which is nearest to the origin.

HINT: Minimize $x^2 + y^2 + z^2$ subject to the constraint that the point lies in the plane.

2. Second-order non-linear differential equations, not directly containing the independent variable, can be solved using the substitution

$$\frac{dy}{dx} = p \qquad \frac{d^2y}{dx^2} = \underbrace{\frac{dp}{dx}} = p\frac{dp}{dy}$$

a) Why is this true?

b) Use this idea to obtain the general solution of the equation

$$y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 0$$

3. Bernoulli numbers B_k are the coefficients in the power series expansion

$$\frac{t}{e^t - 1} = \sum_{k=0}^{\infty} \frac{B_k}{k!} t^k$$

a) Determine B_0 and B_1.

b) Prove that $B_3, B_5, \ldots, B_{2k-1}, \ldots$ are all zero.

HINT: Show that $\dfrac{t}{e^t - 1} - B_1 t$ is an even function of t.

4. Solve the following boundary-value problem for $U(x, y)$:

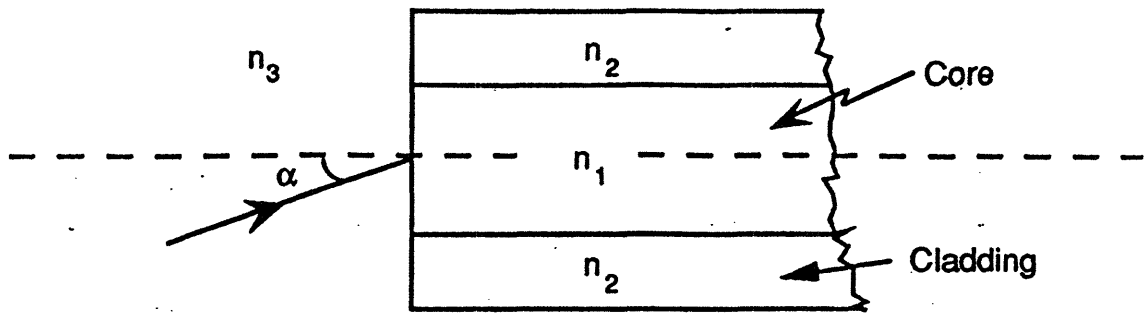
$$\frac{\partial U}{\partial x} = 2\frac{\partial U}{\partial y} + U$$

a) with boundary condition $U(x, 0) = 3e^{-5x}$.

b) What if the boundary condition were $U(x, 0) = 3e^{-5x} + 2e^{-3x}$?

HINT: Try the method of separation of variables.

Physics
Written Exam Question



The figure shows a 2-dimensional optical fiber or waveguide. The index of the core is n_1 and the index of the cladding is n_2, where $n_1 > n_2$. A ray of light entering the core from the outside at an angle of incidence, α , as shown, will propagate without loss via a series of total internal reflections at the core-cladding interface, as long as α is below a certain value.

Derive an expression for the maximum value of α, i.e., the maximum acceptance angle of the waveguide.

Assume the index of air is n_3, and you may find the following formulas useful:

$$\frac{n_{glass}}{n_{air}} = \frac{\sin \theta_{incidence}}{\sin \theta_{refraction}}$$

$$\frac{n_{air}}{n_{glass}} = \sin \theta_{critical}$$

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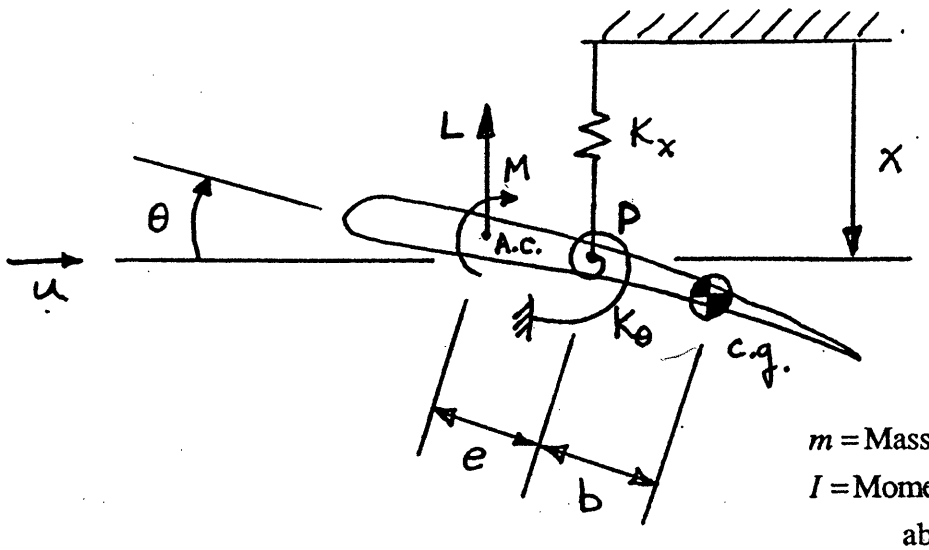
Dynamics

Written Exam Question



m = Mass

I = Moment of inertia about point P

u = Free stream velocity

α = Angle of attack

The straight wing section above is placed into a wind tunnel to study its flutter behavior. It is free to move vertically (x) and in pitch (θ) about point P, with springs attached to each degree of freedom $(K_x \& K_\theta)$. In addition, the aerodynamic forces can be approximated as:

$$L = Z_\alpha \alpha$$

$$M = M_{\dot{\alpha}} \dot{\alpha}$$

Lift acts at the aerodynamic center (A.C.), and the weight, aerodynamic, and elastic forces balance in steady state at a small value of θ.

(a) Write the differential equations governing the linearized perturbation dynamics of x and θ.

(b) Write an expression for the characteristic equation of the system (do not expand).

(c) Determine the system stability for the following parameter values:

$e = 25$ cm

$b = 0$

$m = 4$ kg

$I = 1 \text{ kg} \cdot \text{m}^2$

$u = 25$ m/s

$M_{\dot{\alpha}} = 0$

$Z_\alpha = 1000$ N/rad

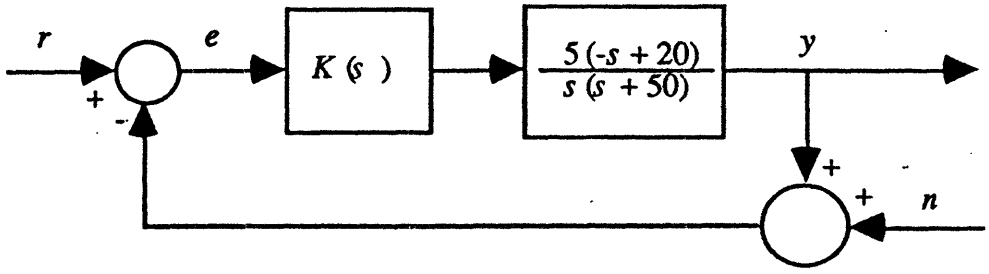
$K_x = 100$ N/m

$K_\theta = 200$ Nm/rad

(d) If e is reduced in part (c), will the system become more stable or less stable? Why?

Instrumentation, Control and Estimation
Written Exam Question

You are given the following system configuration.



Suggest a compensator, $K(s)$, such that this system meets the following specifications.

(1) Phase margin ≥ 30 deg.

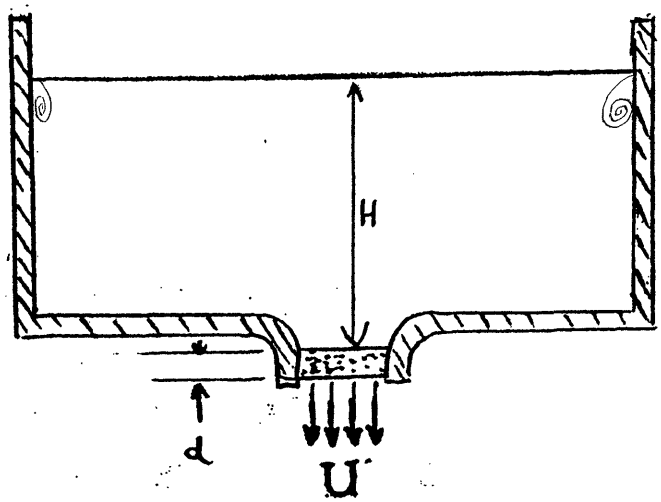
(2) $\left|\frac{e(j\omega)}{r(j\omega)}\right| \leq 0.05$ for $\omega \leq 1$ rad/sec.

(3) $\left|\frac{y(j\omega)}{n(j\omega)}\right| \leq 0.10$ for $\omega \geq 100$ rad/sec.

Describe the strategy that you intend to follow in designing the compensator, and carry out the design as well as you can in the available time.

Fluids

Written Exam Question



A container filled with water to a level H is fitted with a spout containing a porous plug through which the water empties. The resistance of the plug may be expressed by the formula

$$\Delta p = KUd$$

where Δp is the pressure drop across the plug.
U the flow velocity,
d is the thickness of the plug, and
K a coefficient of resistance.

A) The pores have a characteristic small pore size of (a), which is much smaller than the thickness, d, of the plug. Determine how K varies with a.

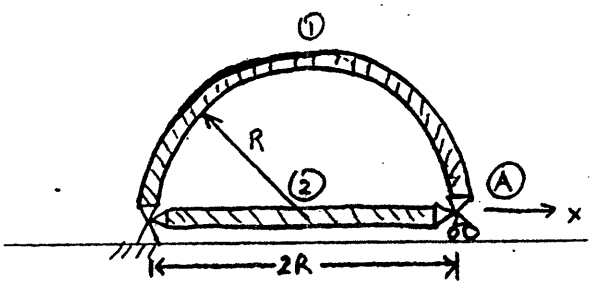
B) Determine the velocity through the plug, assuming H>>d.

C) Discuss what effects you will need to consider when H is comparable to d.

Structures

Written Exam Question

Given the system shown below consisting of a half-circular arch spanned by a beam-column.



beam 1 A_1 EI_1 α_1
beam 2 A_2 EI_2 α_2

Determine the thermally induced deflections at point A as a function of temperature change ($\Delta T > 0$, $\alpha_1 < \alpha_2$). In addition, in cases of high CTE (Thermal Expansion Coefficient) mismatch what failure modes would you consider?

Propulsion

Written Exam Question

A turbojet engine has a compressor pressure ratio of 10 and turbine inlet temperature of 1500 K. Its compressor airflow is 100 kg/s.

Estimate its thrust, in Newtons, at sea-level static conditions. The air temperature is 300 K.

Systems
Written Exam Question

1) You have been asked to design the following for some particular aircraft.

- An external tension cable (exposed to the airflow)
- A landing gear compression strut

What considerations influence the designs of these components?

2) You have the choice of two materials for your components:

	modulus (psi)	max stress (psi)	density (g/cc)	raw cost/lb
Steel	30 million	150 000	8	$1
Gr/Epoxy	20 million	100 000	2	$50

Discuss the relevant material selection criteria for each component.

Thermodynamics

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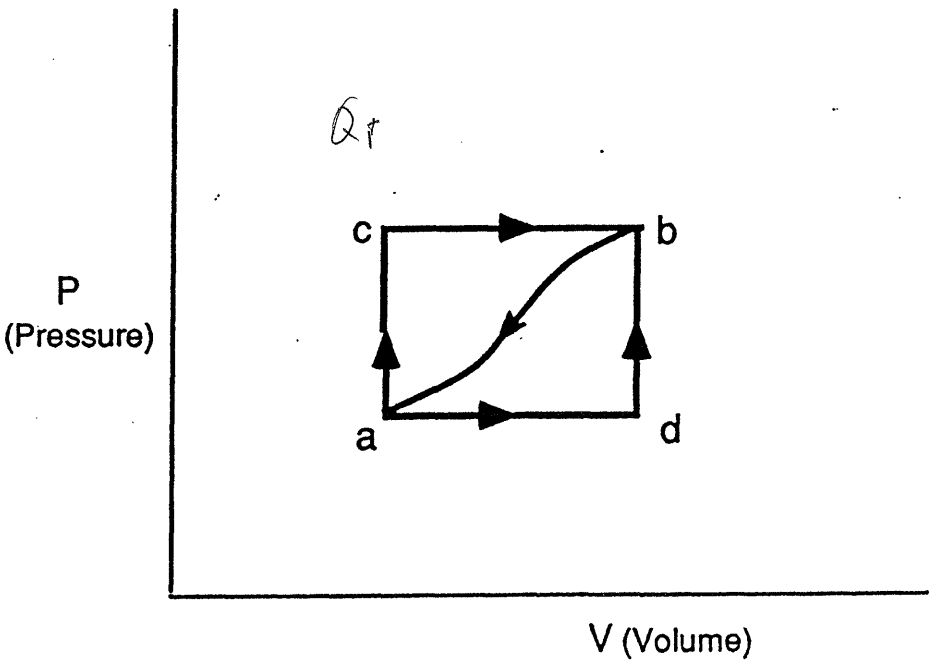
Written Exam Question

When a system consisting of a simple compressible substance is taken from state *a* to state *b*, in the Figure below, along the path *acb*, 80 J of heat flow into the system and the system does 30 J of work.

a) How much heat flows into the system along path *adb*, if the work done by the system is 10 J?

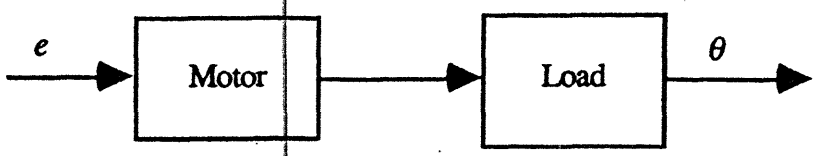
b) The system is returned from state *b* to state *a* along the curved path. The work done on the system is 20 J. Does the system absorb or liberate heat and how much?

c) If $U_a = 0$, $U_d = 40$ J, what is the heat absorbed by the system in processes *ad* and *db*?



Avionics
Written Exam Question

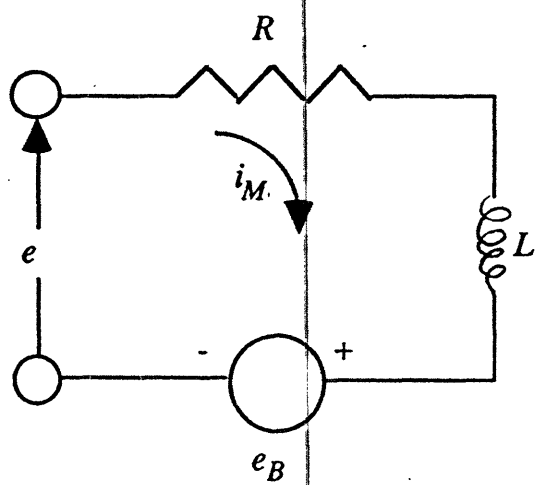
You are given the following servo drive system and load.



The load is connected directly to the motor. Their combined moment of inertia is 4 slug ft^2.

The motor torque is $K_T i_M$ where i_M is the current through the motor armature. $(K_T = 4\,\text{lb ft/amp})$

The motor current is governed by the following circuit.



$R = 0.1$ ohms

$L = 0.01$ henries

e_B is the motor back $emf = K_B\dot{\theta}$ $(K_B = 1$ v/rad/sec$)$.

(a) What is the transfer function from e to θ?

(b) If this were enclosed in a unity feedback loop to form a positioning servomechanism, what compensation and gain could be used so that the closed loop system would have a dominant response to a step function command which has a 5 percent overshoot and response time to 5 percent error of 0.5 sec? If you cannot find a complete design, suggest the form of compensation that you think would work.