DEPARTMENT OF
AERONAUTICS AND ASTRONAUTICS

CAMBRIDGE, MASSACHUSETTS 02139
ROOM 33-208 (617) 253-2260

WRITTEN QUALIFYING EXAMINATION
FOR
DOCTORAL CANDIDATES

Wed., Jan. 20, 1993 37-212 9am - 1pm

• CLOSED BOOK AND NOTES •

Answer a total of five (5) questions (no more or less).

You must answer at least two (2) questions from Column A.
Please answer each question on a separate sheet (or sheets). *Do not put the answers to different questions on the same sheet of paper!*

Be sure that your name appears on *every* sheet of paper you turn in.

Oral examinations will be held Tuesday, January 26, 1993.

Results will be available Wednesday, January 27, 1993, after 2pm.

Column A	Column B
Mathematics	Instrumentation, Guidance and Control
Physics	Fluids
Dynamics	Structures
	Propulsion
	Systems
	Thermodynamics
	Avionics

Mathematics

Written Exam Question

a. Use the Calculus of Residues to obtain a value for the definite integral

$$\int_0^{2\pi} \frac{dt}{5 + 3\cos t}$$

HINT: Make the substitution $z = e^{it} = \cos t + i \sin t$ where $i = \sqrt{-1}$.

b. The steady-state temperature distribution in a circular plate whose faces are insulated and whose circumference is kept at prescribed temperatures is governed by Laplace's equation, which, in polar coordinates, has the form

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2} + \frac{1}{r}\frac{\partial T}{\partial r} = 0$$

Find the general solution of this form of Laplace's equation and show how to find the internal temperature distribution from the boundary conditions.

c. Euler's Gamma function is defined as

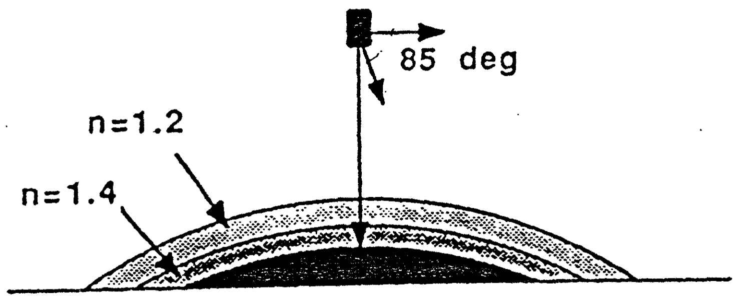
$$\Gamma(x) = \int_0^{\infty} t^{x-1} e^{-t}\, dt$$

Calculate $\dfrac{d\Gamma(x)}{dx}$ and $\dfrac{d^n\Gamma(x)}{dx^n}$.

1993

Physics
Written Exam Question

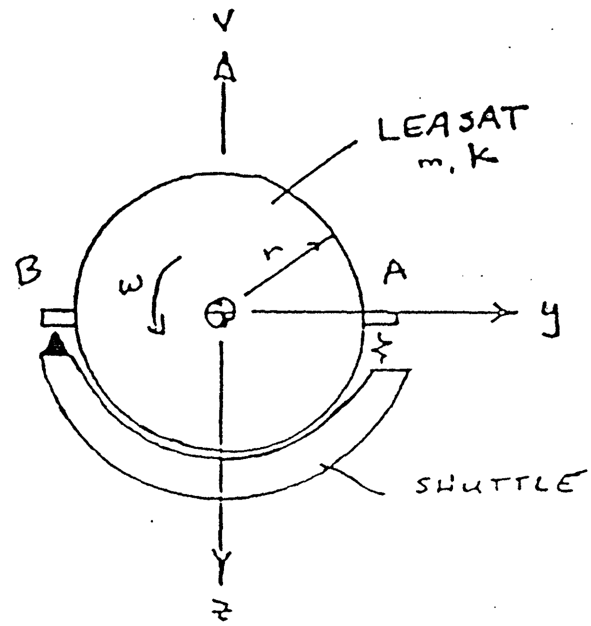
A satellite orbiting 200 km above a planet observes a surface feature at an apparent angle of 5° from nadir (85° from the horizon). The planet has a very dense atmosphere which can be crudely modeled as a 40 km lower layer with an index of refraction of 1.4 and a 40 km upper layer with an index of refraction of 1.2. If the satellite were to send a radar pulse to the feature, how long would it take for the return pulse to be received? (Note: you may assume that the satellite does not move much while the pulse is enroute.)



note: sketch is not to scale

1993

Dynamics
Written Exam Question



LEASAT was ejected from the shuttle using a "Frisbee" Launch which gave it a translational velocity, v, of 0.4 m/s and a rotational angular velocity, ω, of 0.2 rad/s by releasing a spring which pushed on the mount at point A. LEASAT pivoted on its mount at point B on its way out of the shuttle cargo bay. You may assume the shuttle was inertially "fixed".

a) Find the impulse in newton-sec on LEASAT at A due to the action of the spring.

b) Find the impulse in newton-sec on LEASAT at B due to the reaction of the support at B.

c) Find the energy in Joules in the spring before release assuming it all goes into kinetic energy of LEASAT.

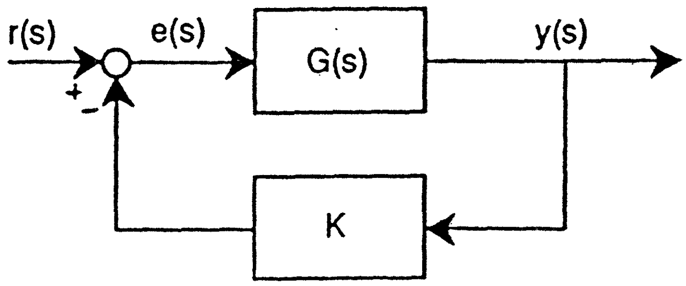
d) Find the velocity of the shuttle relative to the body-fixed, rotating, y-z axes fixed in LEASAT.

Assume for LEASAT:

- Mass = m = 7000 kgm
- physical radius = r = 2m
- radius of gyration about c.m. = $k = \frac{\sqrt{2}}{2} r$

Instrumentation, Guidance and Control
Written Exam Question

The attached frequency response is for the open loop transfer function, G(s), of a system with negative feedback:

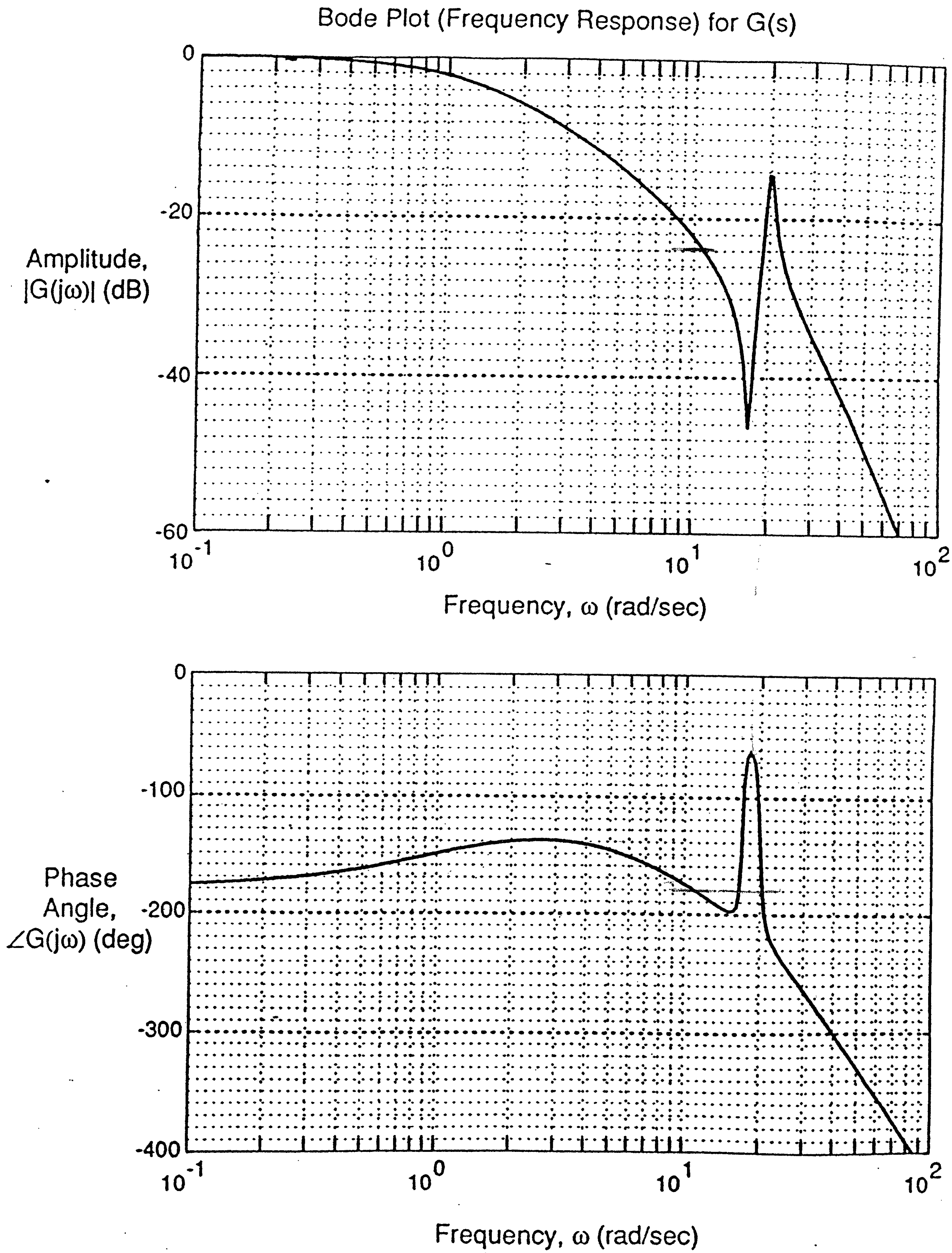


You are to answer some questions based on the information provided by this Bode plot. The only additional information that you need is that all poles and zeros of G(s) have negative real parts except one pole, which is real and unstable, at s = 1.25 rad/sec.

1. Make a rough sketch of the Nyquist Plot [$G(j\omega)$ plotted in the complex plane] of G(s) and determine whether the system can be stabilized using a constant-gain feedback K, as shown in the above drawing. What is the minimum gain to stabilize the system?

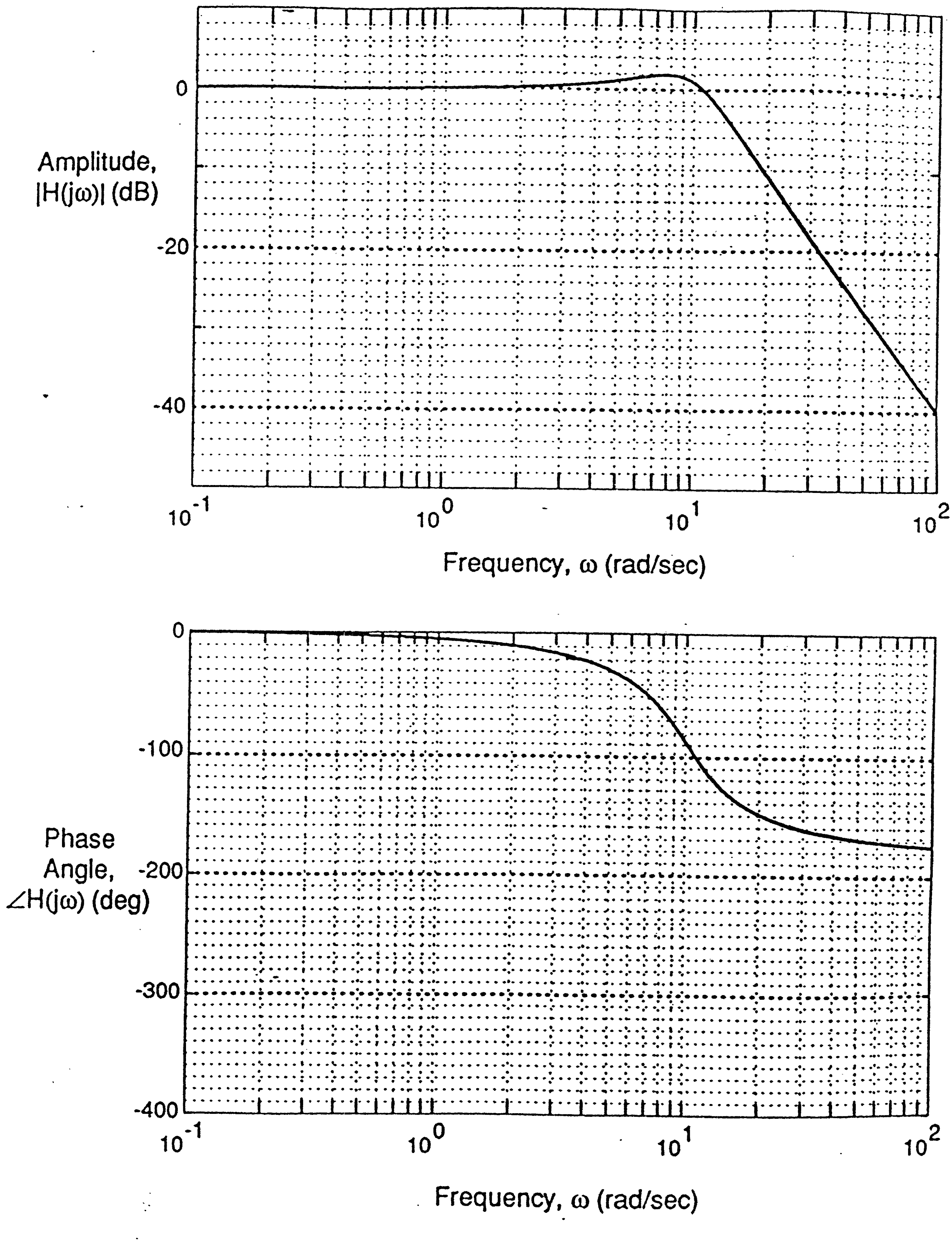
2. Using the attached Bode plot of G(s), estimate the following:
   a) the value of K which stabilizes the system with maximum phase margin;
   b) the phase margin achieved with this value of K;
   c) the gain margin(s) (all of them) for this value of K; and
   d) the steady-state error [e(t) as t approaches ∞] for the following step input::
      $r(t) = 0$ for all $t < 0$
      $r(t) = 1$ for all $t \geq 0$.

3. Now replace K in the feedback path with K·H(s), where H(s) is the second-order low-pass filter shown in the second attached Bode plot. Estimate and briefly comment on the benefits and drawbacks of this compensator, for the system characteristics you estimated in Question 2. For this question, assume that K is adjusted to give the maximum phase margin, as in Question 2.



Bode Plot (Frequency Response) for G(s)
Amplitude, |G(jω)| (dB)
0
-20
-40
-60
10^{-1}
10^{0}
10^{1}
10^{2}
Frequency, ω (rad/sec)
Phase Angle, ∠G(jω) (deg)
0
-100
-200
-300
-400
10^{-1}
10^{0}
10^{1}
10^{2}
Frequency, ω (rad/sec)

Bode Plot (Frequency Response) for H(s)
(For Use in Question 3)



1993

Fluids

Written Exam Question

Given a wavy wall $y = \varepsilon \sin \lambda x$, where $\varepsilon << 1$. The region above the wall is filled with an inviscid, perfect gas. Far from the wall the fluid has a velocity in the plus x direction U_∞. The governing equation for the velocity potential is

$$\left(1 - M_\infty^2\right) \varphi_{xx} + \varphi_{yy} = 0$$

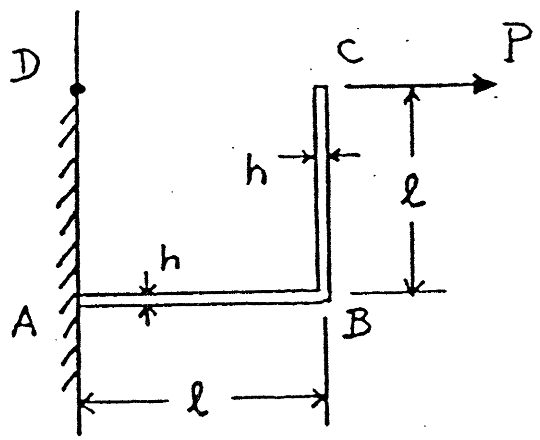
Find the pressure coefficient as a function of the wall parameters and the Mach number for both subsonic & supersonic flow. Explain the difference in force between the two cases.

Structures

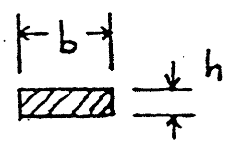
Written Exam Question

Given an L-shaped frame as shown below subjected to a load P at point C. The frame is made of two beams joined together at point B.

(a) Determine the maximum stress in the structure and its location.
(b) Determine the horizontal and vertical deflection at point C.
(c) If a spring of magnitude k (lb/in) is inserted between points C and D, determine the horizontal deflection of point C as a function of $k\ell^3/EI$.



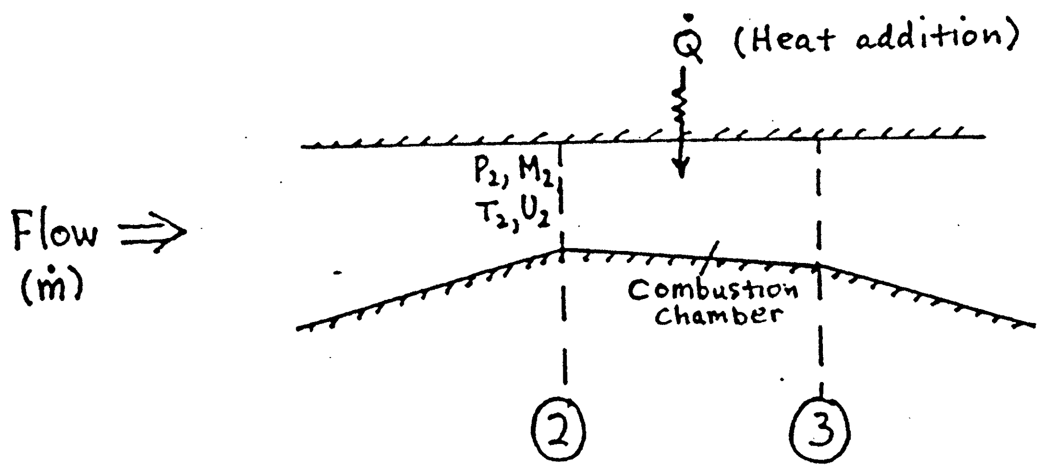
Cross-section
of each member



PROPULSION QUESTION FOR WRITTEN QUALIFYING EXAM

As a simple model of the combustion process in a supersonic combustion ramjet (SCRamjet) consider a **constant pressure inviscid flow with heat addition.** As shown below, the fluid enters the combustion chamber at station 2 at M_2, with static pressure p_2, static temperature T_2, and velocity U_2. The mass flow through the SCRamjet is $\dot{m}$. In the combustion chamber the total rate of heat addition between stations 2 and 3 is $\dot{Q}$. The flow through the device can be taken to be described as a (quasi one-dimensional) channel flow and as an ideal gas with constant specific heats.

a) In such a channel flow, how are differential changes in velocity related to differential changes in static pressure.
b) Using the expression arrived at in part (a) and the "givens" of the problem, find the velocity, U_3, at the combustion chamber exit.
c) What is the static temperature at the combustion chamber exit? [The answer should be expressed in a form which includes the parameter $q = \dot{Q}/\dot{m}$.]
d) What is the area ratio, exit area/inlet area, of the combustion chamber?
e) **Qualitative only** - Describe the differences in Mach number and in stagnation pressure at stations 2 and 3. In particular, are the Mach number and the stagnation pressure at station 3 greater, less than, or equal to the values at station 2? This does not have to be quantitative, but you should present some brief reasoned arguments that support the differences you describe.



Systems
Written Exam Question

A transport aircraft cruising at 35,000 ft. burns fuel at a rate given by:

$$FBR = 10{,}000 + (333 - V)^2 \qquad \text{lbs. per hour}$$
$$(V \text{ in knots, true airspeed})$$

a) Assuming zero wind, what is the least fuel required to fly a trip of 1,000 nautical miles?

b) If the airplane is late, it can fly faster to make an on-time arrival. If this means the 1,000 nm trip must be done in 2.5 hours, what is the least fuel required for the trip?

c) What is the speed for maximum range at zero wind?

d) If there is a headwind of 50 knots, what is the airspeed for maximum range?

1993

Thermodynamics
Written Exam Question

Energy transfer as heat from any space vehicle must take place by radiation. The weight of a space radiator is directly proportional to its area, which is determined by the rate at which the energy must be radiated as heat. The rate of energy radiation is proportional to the product of the area and the fourth power of the absolute temperature of the radiator, i.e.:

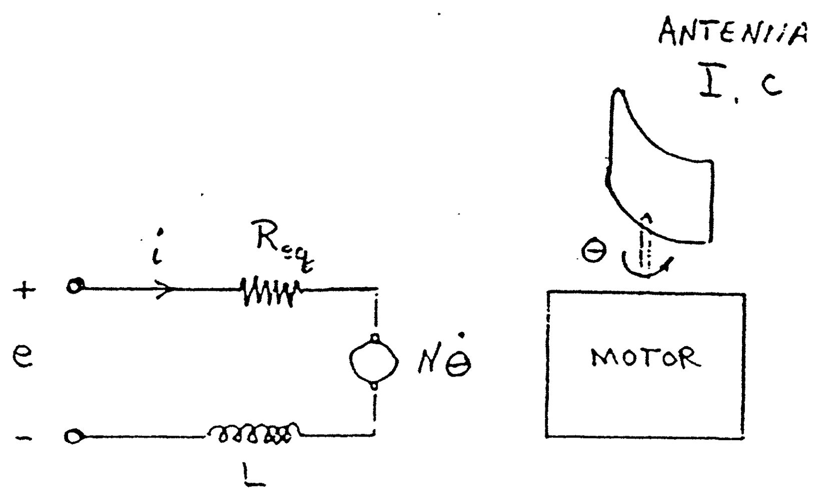
$$\text{Rate of energy radiated away as heat} = (\text{Area}) \cdot (\sigma T^4) \quad ; \quad \text{units are J/s}$$

where σ is a constant.

Suppose that a spacecraft is powered by a reversible Carnot cycle working between a fixed temperature source at T_H (the high temperature reservoir), and the radiator (which acts as the low temperature reservoir). The work of the cycle per unit time (the power output) is fixed. What is the radiator temperature that minimizes radiator weight? [The answer is to be expressed as some fraction of T_H.]

Avionics

Written Exam Question



An electric motor is used to position an antenna. The moment applied to the antenna is given by Ki. The antenna is modeled by its moment of inertia, I, and viscous damping, c. The electric motor has a back EMF given by $N\dot{\theta}$, and inductance, L. Electric losses are accounted for by an equivalent resistance, R_{eq}.

1. Find the transfer function, θ/e, from input control voltage to antenna position.

2. Design a closed-loop controller of θ by making the voltage, e, proportional to the difference between a commanded position, θ_c, and a measured position, θ_m. Draw the signal flow diagram. Is it possible to stabilize the system with a pure gain?

3. How will the values of R_{eq}/L and c/I affect your design?