DEPARTMENT OF
AERONAUTICS AND ASTRONAUTICS

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WRITTEN QUALIFYING EXAMINATION
FOR
DOCTORAL CANDIDATES

Wednesday, January 22, 1992 37-212 9:00 a.m. - 1:00 p.m

• CLOSED BOOK AND NOTES •

Answer a total of five (5) questions (no more or less).

You must answer at least two (2) questions from Column A.
Please answer each question on a separate sheet (or sheets). *Do not put the answers to different questions on the same sheet of paper!*

Be sure that your name appears on *every* sheet of paper you turn in.

Oral examinations will be held Tuesday, January 28, 1992.

Results will be available Wednesday, January 29, 1992, after 2:00pm.

Column A	Column B
Mathematics	Instrumentation, Guidance and Control
Physics	Fluids
Dynamics	Structures
	Propulsion
	Systems
	Thermodynamics
	Avionics

Systems
Written Exam Question

The thrust T of a jet aircraft is related to its fuel weight flow $\dot{w}$ by

$$T = \frac{\dot{w}}{SFC_T}$$

where SFC_T is the thrust-specific fuel consumption. Similarly, the propulsive power of a piston-engined aircraft is

$$TV = \eta_p P = \eta_p \frac{\dot{w}}{SFC_P}$$

where SFC_P is the power-specific fuel consumption, V is the flight speed, and η_p is the propulsive efficiency.

1) Use the above relations to derive the Breguet range equation for each type of aircraft. How does the flight altitude impact the range of each type?

2) There is typically some uncertainty in the dry weight of a commercial aircraft prior to flight, this requiring some conservatism in the takeoff fuel load chosen. Determine the sensitivity of fuel burn to the dry weight. Is this sensitivity greater for short- or long-range aircraft?

Thermodynamics

Written Exam Question

Some of the responses required below require explanations. Please limit each explanation to two clear and concise sentences.

A. i. Typically, heat engine and refrigeration cycles are designed with at least four "legs". The ideal cycle is precisely four legged. Explain why.

ii. What is the *minimum* number of "legs" that can form a heat engine cycle? Explain.

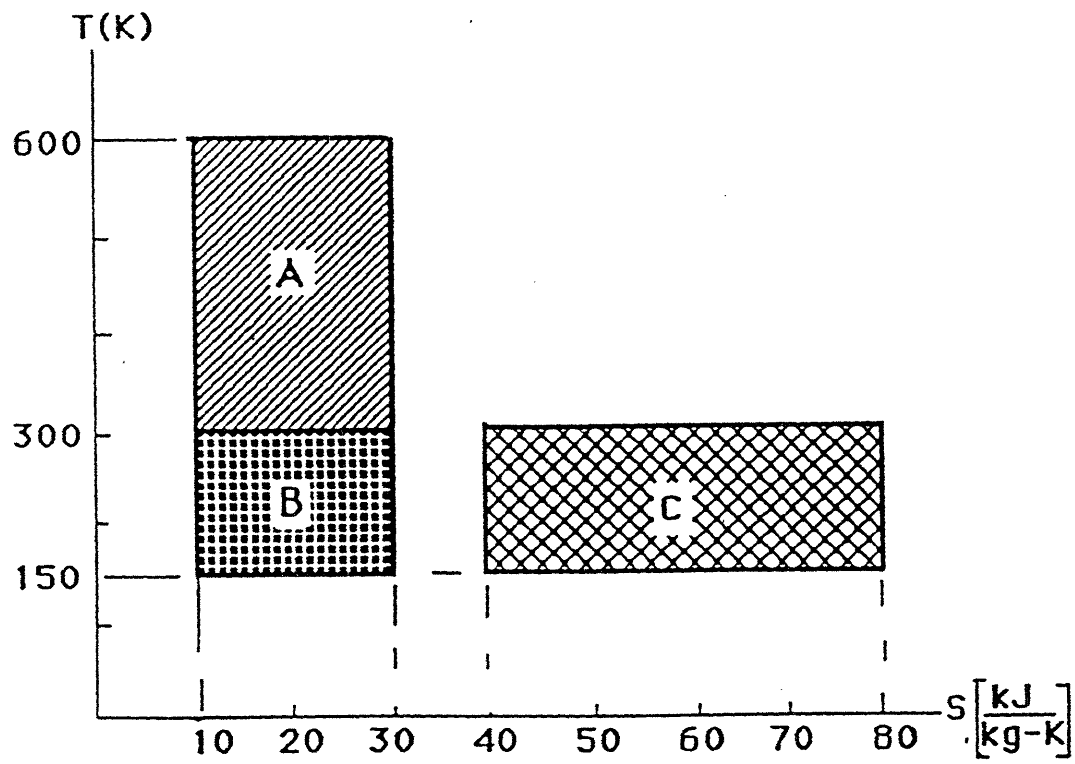
iii. On any appropriate thermodynamic plane sketch an example of a cycle pattern that would operate with this minimum number of legs.

iv. Explain how the performance characteristics of a practical device operating on the cycle you have proposed would differ from those of an ideal cycle.

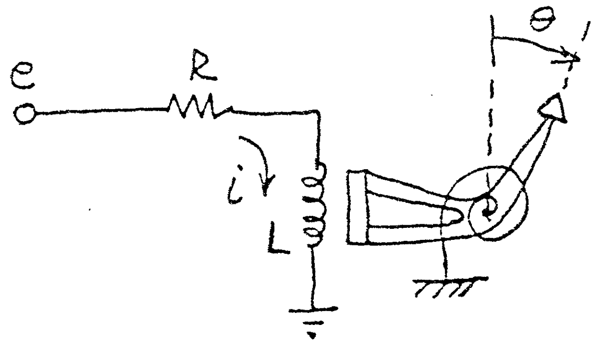
B. i. Compare the important practical performance characteristics of engines operated on the three ideal cycles shown in the figure below.

ii. List and justify specific practical considerations that would dictate adopting or rejecting A or B or C, for (a) engines, (b) heat pumps, and (c) refrigerators.

(Limit the justification of each item on the list to two clear and concise sentences.)



An electric meter movement is shown schematically below.



The input voltage e drives a current through the coil which has resistance and inductance. The magnetic field of the coil interacts with a permanent magnet to produce a torque on the indicator movement which is restrained by a spring and fluid damping. Numerical values are:

R = coil resistance = 5 ohms

L = coil inductance = 0.1 henry

S = torquer scale factor = 1 lb ft/amp (Torque = S i)

I = indicator moment of inertia = 10^{-3} slug ft^2

C = indicator viscous damping coefficient = 0.1 lb ft/rad/sec

K = indicator spring constant = 0.9 lb ft/rad

1. What do you estimate the response time of this indicator to be?

2. If the indicator were provided with a signal generator to give an electrical indication of θ, you could then operate the indicator in a closed loop manner by producing the voltage e as the amplified difference between the input signal and the fed back indication of θ. Without further compensation, what response time do you estimate you could achieve with this configuration?

Mathematics

Written Exam Question

a. Derive the following series expansion by any convenient method

$$\int_0^x e^{-x^2}\,dx = x - \frac{x^3}{3} + \frac{1}{2!}\frac{x^5}{5} - \frac{1}{3!}\frac{x^7}{7} + \cdots$$

b. The curvature of a plane curve is

$$\frac{1}{\rho} = \frac{\dfrac{d^2y}{dx^2}}{\left[1+\left(\dfrac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}$$

Derive the corresponding expression for polar coordinates where

$$x = r\cos\theta \quad \text{and} \quad y = r\sin\theta$$

c. Find the least and greatest distance from the origin to the curve

$$4x^2 + 9y^2 + 16z^2 = 16$$

and give a geometric interpretation.

d. Solve the boundary-value problem

$$\frac{\partial U}{\partial x} = 4\frac{\partial U}{\partial y}$$

where $U(0, y) = 8e^{-3y}$.

Physics

Written Exam Question

A collimated beam of electromagnetic radiation is passed through a region of space containing only free electrons. It is noted that some radiation can be detected at angles $\vartheta \neq 0$ to the direction of the original beam.

(a) Will this be at the same wavelength as the incident beam? Why?

(b) For simplicity, suppose $\vartheta = 180°$ and assume the electrons are initially at rest. Calculate the wavelength shift.

(c) Textbooks usually associate this effect with X-rays. Why would it not be apparent for, say, visible light? Hint: Consider how the effect could be measured, given its magnitude.

<u>NOTES:</u> Photon energy $= h\nu = \frac{hc}{\lambda}$

Photon momentum $= \frac{h\nu}{c} = \frac{h}{\lambda}$

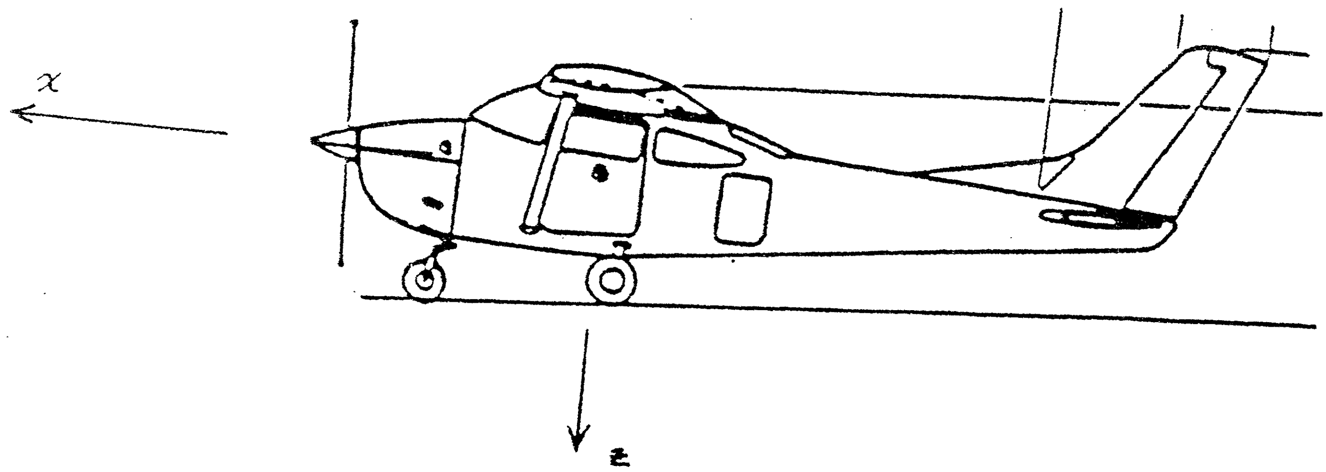
Electron momentum $= p_e$

Electron energy $= \sqrt{m_o^2 c^4 + p_e^2 c^2}$ (relativistic)

where m_o = electron ______________ rest mass = 0.91×10^{-30} Kg

$c = 3 \times 10^8$ m/sec $h = 6.62 \times 10^{-34}$ Joule x sec

DYNAMICS
WRITTEN EXAM QUESTION



A C-182 has a weight of 2800 lbs. and principal moments of inertia in body-fixed axes of:

$I_x = 948$ slug ft^2

$I_y = 1346$ slug ft^2

$I_z = 1967$ slug ft^2

The aircraft is in a steady spin with

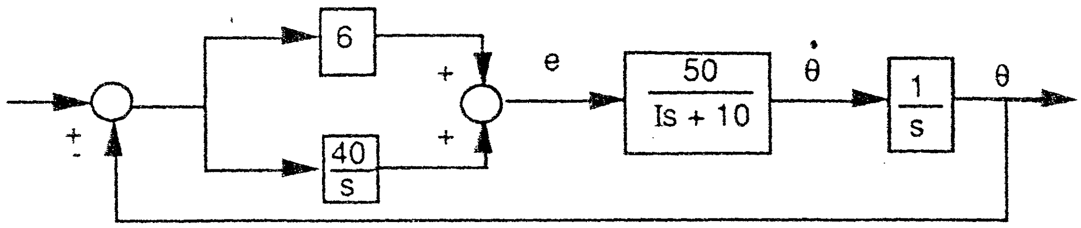
Elevation angle	$= \theta = -55°$	(i.e. nose down)
Heading rate	$= \dot{\psi} = 360°/\text{sec}$	(clockwise from above)
Roll angle	$= \phi = 0$	(i.e. y axis level)

a) What moment acts on the aircraft in body coordinates?
b) What is the kinetic energy due to rotation?
c) Compare the kinetic energy of rotation with the kinetic energy due to translation at a velocity of 50 kts.
d) What is the physical source of the moment you found in a) and why doesn't it cause the kinetic energy due to rotation to vary with time?

Note: 1 kt = 1.689 ft/sec.

WRITTEN QUALIFYING EXAM QUESTION - CONTROL

The following is a block diagram of a one-arm manipulator with a proportional plus integral compensator.



The moment of inertia, I, is variable due to the different payloads that the manipulator grasps.

Sketch a locus of the closed loop roots as a function of the moment of inertia, I. The sketch must be qualitatively correct but it need not be quantitatively accurate.

You need not calibrate the loci in terms of values of I. But show clearly the asymptotic behavior for large and small values, and determine the range of I for stable operation.

Qualifying Exam - Written Question in Fluids

In solving boundary value problems the boundary conditions are, strictly speaking, applied on the boundary surface. In the linearized approximation, however, the boundary conditions are applied on a more convenient locus. Thus in two dimensional airfoil theory for an airfoil whose thickness is

$$y=\pm\tau\ f(x) \qquad 0\leq x\leq c$$

the tangential flow condition is applied on the surface $y=0$ for $0\leq x\leq c$. τ is the thickness ratio and $f(x)$ defines the chordwise thickness profile.

A. Derive this condition analytically.

B. Explain the implications of your approximation.

C. By extension of your results discuss the case of a two dimensional cambered airfoil at an angle of attack α_0.

Structures

Written Exam Question

In all cases, give a justified qualitative answer before trying to derive a quantitative one.

A beam supported as shown in Figure 1 must support its own weight.

Beam material stiffness = E
Beam material strength = σ_{ult}
Beam material mass density = ρ
Beam cross-sectional properties =
A (area)
I (moment of inertia)
Acceleration due to gravity = g

a) What are the loads on the beam?

b) What are the reactions at points B and C?

c) What are the deflections? (assume they are small, at least for this question)

d) How would it fail if
   i) it is stiff (E large) but not strong (σ_{ult} small)
   ii) it is strong (σ_{ult} large) but not stiff (E small)

e) Figure 2 shows an additional support member. It is designed to reduce the deflections of the beam, and increase its strength. How does it accomplish these objectives? What fundamental difficulty in analysis is introduced by the additional member? (The support member has the same properties as the beam, but is massless).

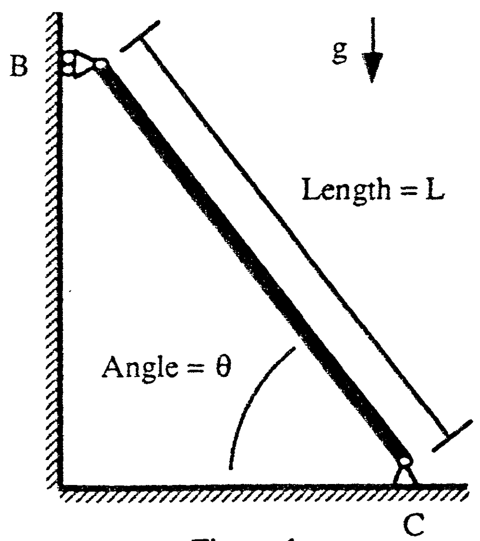


Figure 1

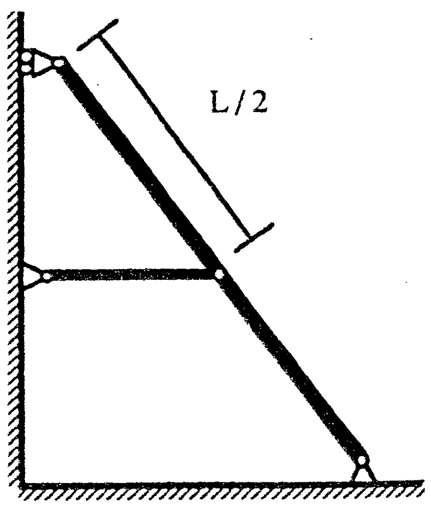
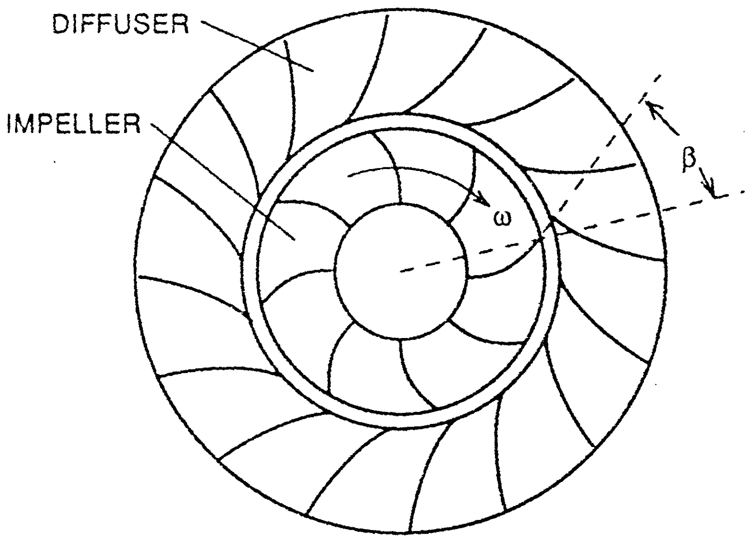


Figure 2

Propulsion

Written Exam Question

A centrifugal pump (or low-speed centrifugal compressor), has impeller vanes which are inclined backward from the radial direction at the tip, at an angle β. Suppose the flow in the impeller and diffuser is loss-free, that the flow exits the impeller parallel to the blades, and that the diffuser reduces the velocity to zero. Also assume that the flow has no swirl at entance to the impeller.



a) Write an expression for the overall stagnation pressure rise of the pump in terms of the rotational speed ω, impeller tip radius r and the angle β.

b) Write an expression for the ratio of the static pressure rise in the diffuser, to the overall pressure rise found in a), and sketch its variation with β.