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DEPARTMENT OF
AERONAUTICS AND ASTRONAUTICS

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WRITTEN QUALIFYING EXAMINATION
FOR
DOCTORAL CANDIDATES

Tuesday, January 23, 1990 37-232 9:00 a.m. - 1:00 p.m

• CLOSED BOOK AND NOTES •

Answer a total of five (5) questions (no more or less).

You must answer at least two (2) questions from Column A.
Please answer each question on a separate sheet (or sheets). *Do not put the answers to different questions on the same sheet of paper!*

Be sure that your name appears on *every* sheet of paper you turn in.

Oral examinations will be held Tuesday, January 30, 1990.

Results will be available Wednesday, January 31, 1990.

Column A	Column B
Mathematics	Instrumentation, Guidance and Control
Physics	Fluids
Dynamics	Structures
	Propulsion
	Systems
	Thermodynamics

Mathematics

Written Exam Question

Find the function y(x) that minimizes the integral

$$I = \int_0^1 (y'(x))^2 \, dx$$

subject to the conditions

$$\int_0^1 y^2(x) \, dx = 1$$

$$y(0) = 0$$

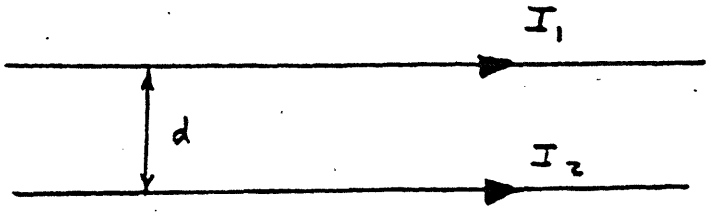
$$y(1) = 0$$

Is the answer unique? What is the minimum value of the integral? Explain fully.

Physics

Written Exam Question

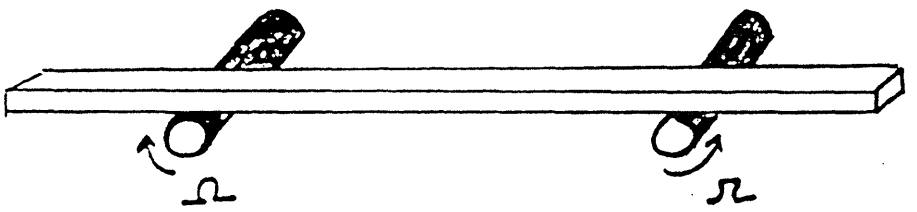
Current flows through two long, straight conductors, separated by distance d, as shown below. The current flowing through one conductor is I_1; the current in the other conductor is I_2. What is the force per unit length on each conductor? Be sure to specify both the magnitude and direction of the force for each conductor.



Dynamics
Written Exam Question

Two identical rollers of radius r, with fixed axes separated by distance L, are rotating in opposite directions, as shown below, with angular speed Ω. A uniform plank of mass M and length P, is placed across the rollers. The coefficient of friction between the plank and the rollers is μ.

Determine the motion of the plank.



Instrumentation, Guidance, and Control
Written Exam Question

Consider the feedback control system shown below.

(a) Find a compensator K(s) to meet the following specifications:

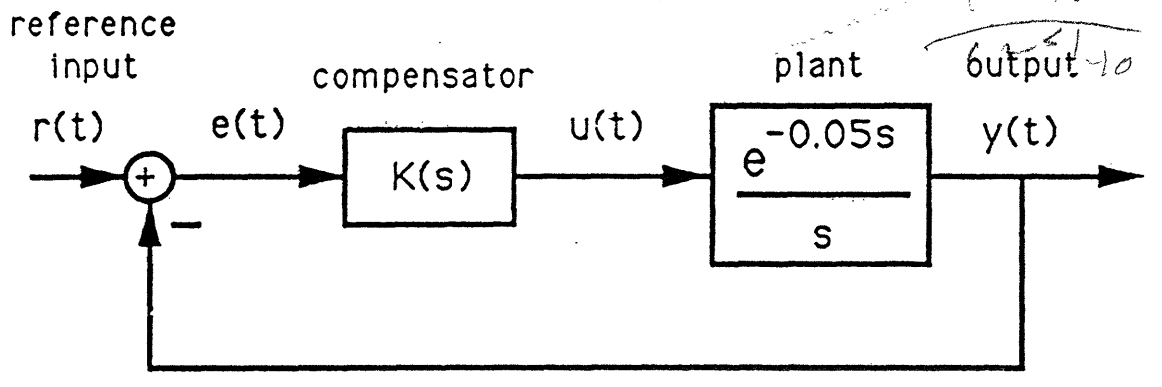
(1) The peak overshoot of the closed-loop step response should be less than 20%.

(2) The steady-state error to a unit ramp input, r(t) = t, should be less than 0.01.

(3) The rise time should be as short as practicable, subject to the above constraints.

(Of course, you will not be able to test whether your compensator meets the specifications. Therefore, you should find a "first cut" for the desired compensator.)

(b) Will the closed-loop system exhibit any undesirable characteristics? Explain.



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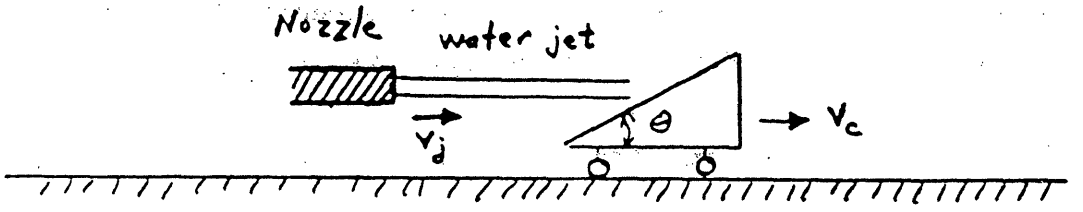
Fluids

Written Exam Question

A water jet acts on the slant side of a triangular carriage, which is rolling along the floor, as shown in the figure. What are the forces on the carriage? You may assume that the water is inviscid.



Structures

Written Exam Question

Consider a simply supported wood beam loaded as shown below. To increase the strength, thin straps of graphite/epoxy composite material are glued to the top and bottom surface of the beam. Determine,

(a) The largest normal stress σ_{xx} in the wood beam without the straps. $z = \frac{h}{2}$

(b) The largest normal stress σ_{xx} in the wood beam with the straps. $z = \frac{h}{2}$

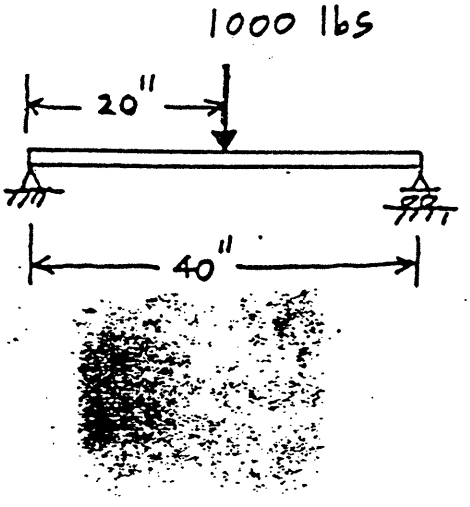
(c) The largest normal stress σ_{xx} in the composite straps. $z = \frac{h}{2} + t$

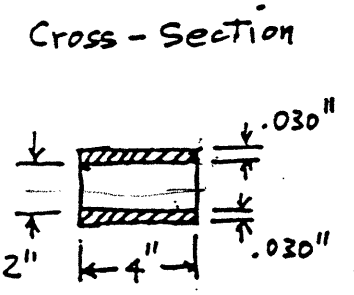
(d) The shear stress σ_{zx} at the composite-wood interface away from the edges.

Use the following material properties:

Wood	$E_w = 2 \times 10^6$ lbs/in^2
Composite	$E_c = 20 \times 10^6$ lbs/in^2

$$\sigma_{xx} = -\frac{Mz}{I}$$

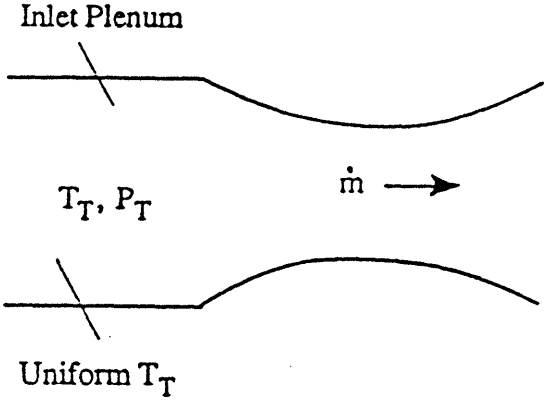


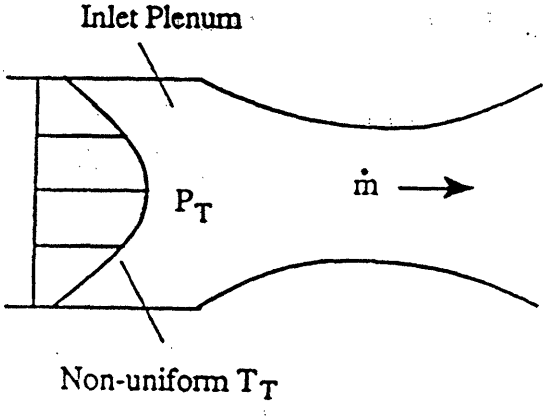


Propulsion
Written Exam Question

The gas leaving the combustion chamber of a propulsive device has a non-uniform temperature distribution, such that streamtubes near the walls are colder than those near the centerline. Nozzle expansion proceeds without mixing to some common exit pressure (equal to ambient pressure).

Comparing to a hypothetical device with the same flow rate, the same total enthalpy flux, and the same pressure ratio, but with uniform inlet plenum temperature, which will have a higher static thrust? By what ratio?





Systems
Written Exam Question

Liquid hydrogen has been proposed as an alternative fuel for flight vehicles. Examine the design implications of this fuel choice in terms of the Breguet equation:

(a) Derive the Brequet equation for jet aircraft in standard form.

(b) Show that

$$\frac{Vg}{SFC} = \eta_0 H$$

where

g = acceleration due to gravity
V = flight vehicle speed
SFC = engine specific fuel consumption
η_0 = overall efficiency of the engine
$\eta_0 = \eta_{propulsive} \times \eta_{thermal}$
H = heating value of fuel

(c) Discuss the possible effects of this fuel choice on flight vehicle design, given the following fuel properties:

	Jet Fuel	Liquid Hydrogen
heating value (J/kg)	55,000	154,000
density (kg/m^3)	781	71.3

(d) Discuss the advantages and disadvantages of liquid hydrogen as a fuel in the context of a total air transportation system.

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Thermodynamics
Written Exam Question

An ideal refrigerator is used to produce liquid oxygen (LOX) from room temperature air (initial state, p=1 atm, T=288.15K). To extract the oxygen, the air (assume 79% N_2 and 21% O_2 by volume) is cooled to the normal boiling point of oxygen at which temperature the oxygen is completely liquefied and the nitrogen remains a gas. The nitrogen is then separated from the LOX and brought back to room temperature and p=1 atm.

Assume that all processes are reversible, that the components of air behave ideally when in the gaseous state, and that the condenser of the refrigerator is at room temperature.

A. Determine the following for the production of 1 kg of LOX:
   a. The entropy change of the oxygen.
   b. The entropy change of the surroundings.

B. What would be the entropy change of the entire system (nitrogen-oxygen) measured from the initial state, if the LOX were also warmed to room temperature and maintained at one atmosphere in a container separate from that of nitrogen?

Table A.4
Thermodynamic Properties of Oxygen[a]
Table A.4.1
Saturated Oxygen

		Specific Volume m³/kg			Enthalpy kJ/kg		
Temp. K	Press. MPa P	Sat. Liquid v_f	Evap. v_{fg}	Sat. Vapor v_g	Sat. Liquid h_f	Evap. h_{fg}	Sat. Vapor h_g
54.3507	0.00015	0.000 765	92.9658	92.9666	−193.432	242.553	49.121
60	0.00073	0.000 780	21.3461	21.3469	−184.029	238.265	54.236
70	0.00623	0.000 808	2.9085	2.9093	−167.372	230.527	63.155
80	0.03006	0.000 840	0.681 04	0.681 88	−150.646	222.289	71.643
90	0.09943	0.000 876	0.226 49	0.227 36	−133.758	213.070	79.312
100	0.25425	0.000 917	0.094 645	0.095 562	−116.557	202.291	85.734
110	0.54339	0.000 966	0.045 855	0.046 821	−98.829	189.320	90.491
120	1.0215	0.001 027	0.024 536	0.025 563	−80.219	173.310	93.091
130	1.7478	0.001 108	0.013 488	0.014 596	−60.093	152.887	92.794
140	2.7866	0.001 230	0.007 339	0.008 569	−37.045	125.051	88.006
150	4.2190	0.001 480	0.003 180	0.004 660	−7.038	79.459	72.421
154.576	5.0427	0.002 293	0.000 000	0.002 293	32.257	0.000	32.257