DEPARTMENT OF
AERONAUTICS AND ASTRONAUTICS

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WRITTEN QUALIFYING EXAMINATION

FOR

DOCTORAL CANDIDATES

Tuesday, January 24, 1989 37-212 9:00 a.m. - 1:00 p.m.

• CLOSED BOOKS AND NOTES •

Answer a total of five (5) questions (no more or less).

You must answer at least two (2) questions from Column A.

Please answer each question on a separate sheet (or sheets). Do not put the answers to different questions on the same sheet of paper!

Be sure that your name appears on every sheet of paper you turn in.

An announcement will be made when the results are available. Results should be available within 2-3 weeks.

Oral Examinations will be held Tuesday, January 31, 1989.

Column A	Column B
Mathematics	Instrumentation, Guidance, and Control
Physics	Fluids
Dynamics	Structures
	Propulsion
	Systems
	Thermodynamics

Mathematics

Written Exam Question

a) Find the limiting form of the two roots of the quadratic equation

$$\varepsilon x^2 + bx + c = 0 ,$$

in the limit as $\varepsilon \to 0$.

b) Solve the o.d.e.

$$\frac{d^2u}{dy^2} = k^2u, \qquad \text{k is real, positive}$$

subject to the boundary conditions

$$u(0) = 1 , \qquad U(\infty) = 0.$$

How would your answer change if k were complex?

c) Given that $\psi(x,y,z) = x + y^2 + z^3$ evaluate $\nabla\psi$ and $\nabla \times (\psi\vec{j})$ (where $\vec{j}$ is the unit vector in the y-direction).

d) Given that

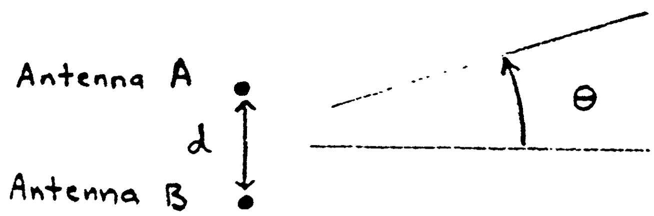
$$f(x) = x \sin x + x^2 \cos x$$

evaluate the first three terms of the Taylor series expansion of $f(x)$ about $x = \frac{\pi}{4}$.

Physics

Written Exam Question

Two omnidirectional antennas separated by a distance d radiate similarly polarized electromagnetic waves. Antenna A lags antenna B by 180° of phase and the separation distance d is one wavelength λ. Say as much as you can about the antenna pattern in the far field, particularly the location of the nodes and anti-nodes.



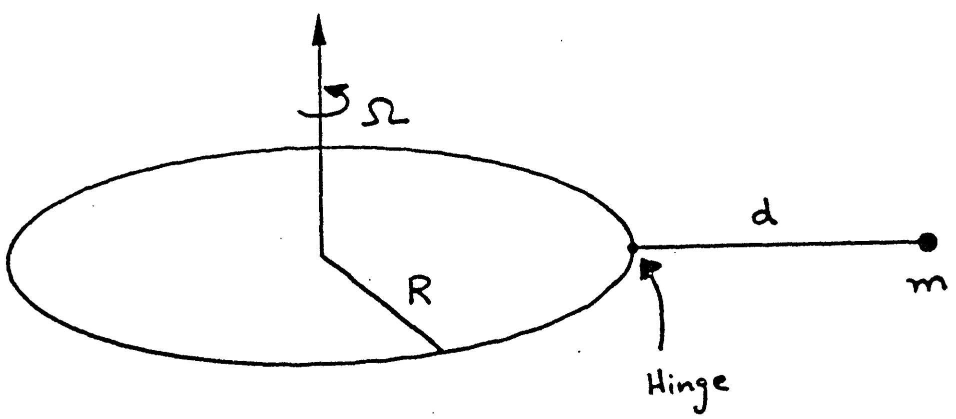
Dynamics

Written Exam Question

A pendulum is attached to the edge of a spinning disk, as shown in the sketch below. The disk spins at a constant angular rate, Ω, about its center. For the two cases described below, find the (nonlinear) equations of motion of the pendulum, and the natural frequency of the pendulum for small amplitude motion. (Assume there is no gravity.)

<u>Case 1</u>: The pendulum is hinged so that its motion lies in the plane of the disk.

<u>Case 2</u>: The pendulum is hinged so that its motion is perpendicular to the plane of the disk.



Instrumentation, Guidance, and Control

Written Exam Question

A plant to be controlled has the experimentally determined frequency response $G(j\omega)$ shown on the attached Bode plot. Find a compensator $K(s)$ for the unity feedback control system (Figure 1) that will meet the following specifications:

e_0	$= 0$	(zero steady state error to a step input)
P_0	$= 15\%$	(peak overshoot)
t_r	$= 0.025$ sec	(rise time)

Of course, you will not be able to test whether your compensator meets specs, so you should treat your design as a first cut. Be sure to explain your approach in as much detail as time permits.

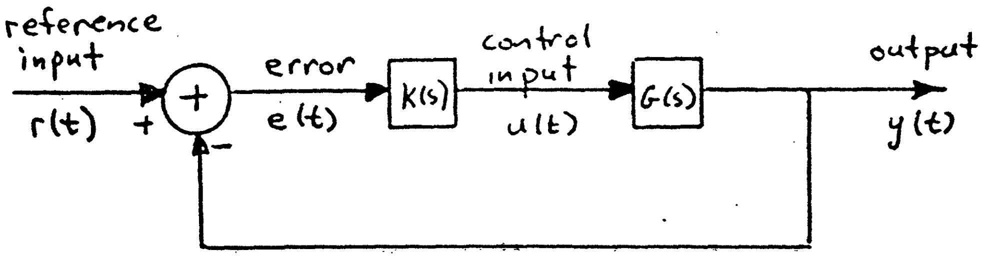
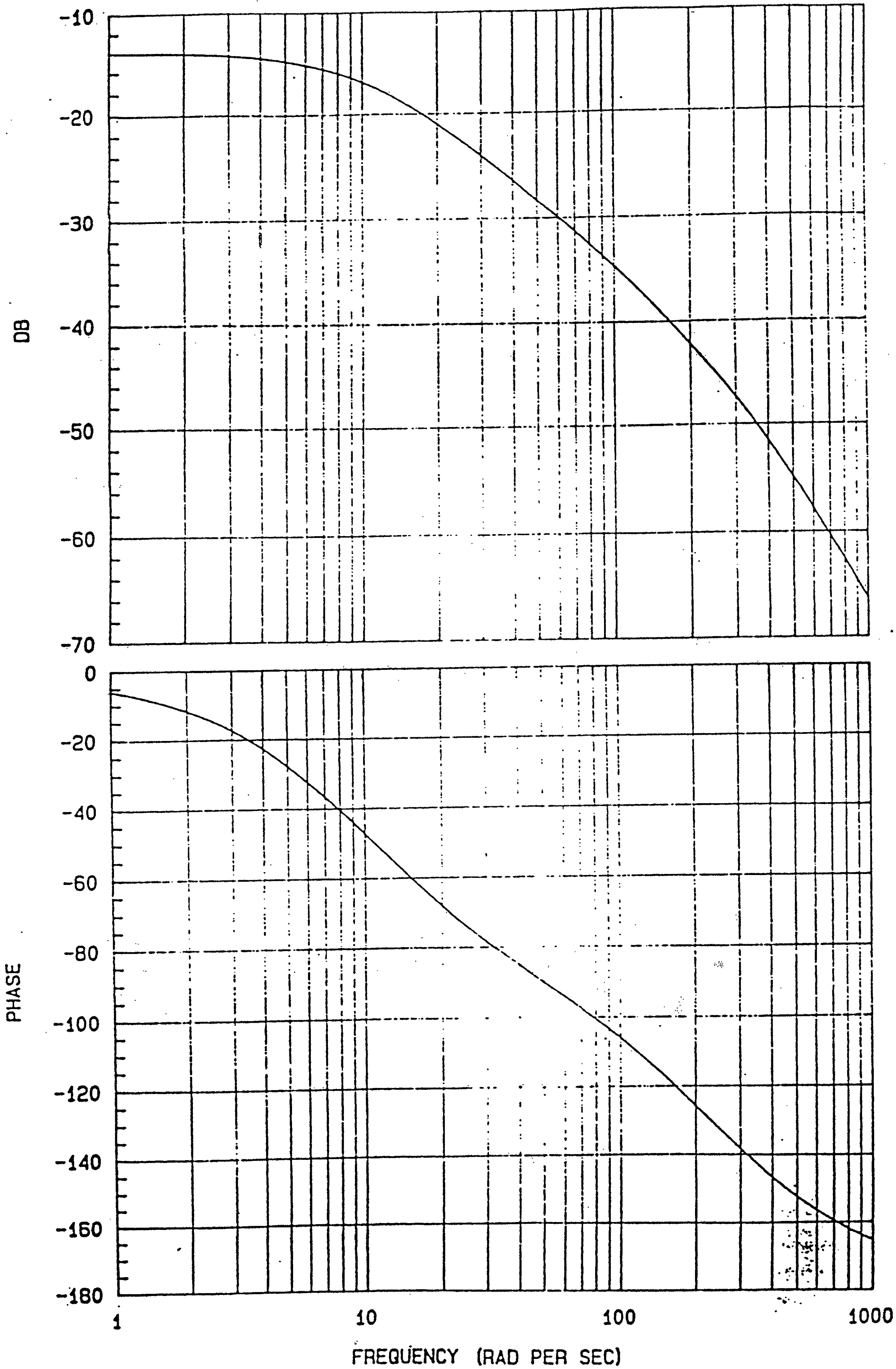


Figure 1.



-10
-20
-30
-40
-50
-60
-70
DB
0
-20
-40
-60
-80
-100
-120
-140
-160
-180
PHASE
1
10
100
1000
FREQUENCY (RAD PER SEC)

Fluids

Written Exam Question

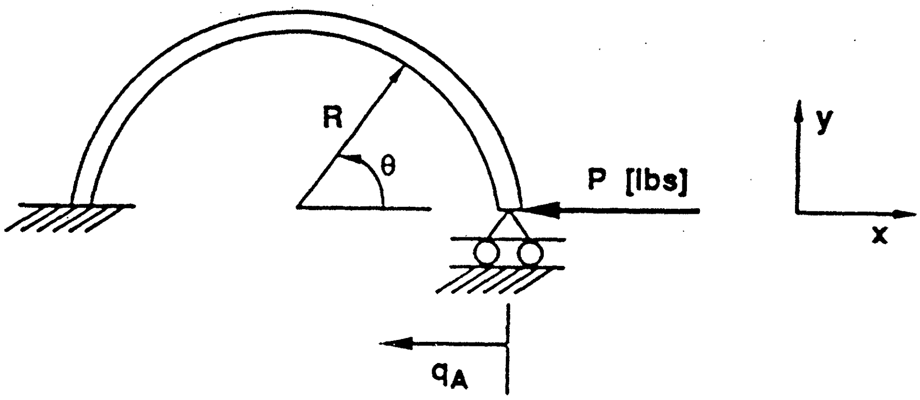
An incompressible fluid flows downward from a small hole in the bottom of a deep tank. Assume the effective area of the hole is "A" and the velocity, assumed uniform, is "V_0". Calculate the shape of the bounding streamline as a function of the distance z, below the location of the hole.

Discuss the key limitations to your solution (include the determination of the effective area). Be sure to discuss all the implicit assumptions that underlie your calculations.

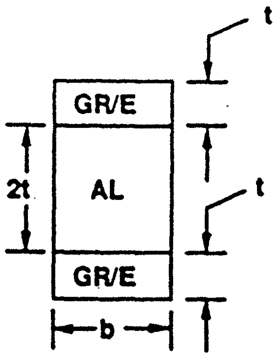
Structures

Written Exam Question

Determine the deflection q_A of the arch illustrated below under the action of the force P assuming that only bending deflections are significant. The radius of the arch and the cross-sectional property EI are constant. Assume b,t << R.



Cross Section of Arch



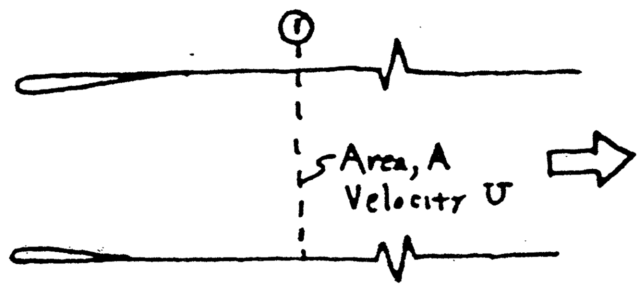
Material Properties

GR/E	E=20 Msi	v=.34
AL	E=10 Msi	v=.30

Propulsion

Written Exam Question

An inlet has a flow area A (see below). The velocity far from the inlet (i.e., outside it) is zero and the velocity at A is U. If the fluid can be considered to have constant density, ρ, what is the force exerted on the inlet? Indicate in which direction this force points.



Systems

Written Exam Question

Discuss briefly but quantitatively how the following factors might influence the choice of design wing loading of an aircraft (only choose four of the five factors for your answer):

a) Takeoff distance

b) Cruise altitude

c) Critical Mach number

d) Gust response

e) Noise

Thermodynamics

Written Exam Question

A long vertical adiabatic cylinder is sealed at its top end, and its bottom end is open to the atmosphere. The cylinder is fitted with a 1 kg frictionless adiabatic piston of negligible length. Initially the piston is pinned, as shown in the sketch, and nitrogen gas is trapped above the piston at a temperature of 300K and a pressure of 1 atm, which is the same as the external pressure. When the pin is removed the piston is allowed to fall until it stops; then the system comes to a new equilibrium state. The cylinder is sufficiently long so that when the piston stops it is still well within the cylinder.

a) Compute the work done by the nitrogen alone, and by the entire system (nitrogen + piston) on the atmosphere in this process.

b) Compute the entropy change of the system (nitrogen + piston) and that of the atmosphere in this process.