DEPARTMENT OF
AERONAUTICS AND ASTRONAUTICS

CAMBRIDGE, MASSACHUSETTS 02139
ROOM (617)253-
TELEX 92-1473 CABLE MIT CAM

WRITTEN QUALIFYING EXAMINATION
FOR
DOCTORAL CANDIDATES

Thursday, February 25, 1988 33-319 1:00 p.m. - 5:00 p.m.

• CLOSED BOOKS AND NOTES •

Answer six (6) but no more (and no less) of the following 18 questions.*

Please answer each question on a separate sheet (or sheets). Do not put the answers to different questions on the same sheet of paper!

Be sure that your name appears on every sheet of paper you turn in.

An announcement will be made when the results are available. Results should be available within 2 weeks.

Oral Examinations, if required, will be arranged individually.

---

*The questions are variously in the disciplines of

guidance and control	propulsion	structures
fluids	thermodynamics	bio-engineering
systems	dynamics	physics
airplane mechanics	kinetic theory	

Question 1.

a) Evaluate A^{-1} where $A = \begin{pmatrix} 1 & 0 & 2 \\ 1 & 2 & 0 \\ 0 & 3 & -2 \end{pmatrix}$.

b) Find the Taylor Series expansion of tan(x) for $x \sim 0$, including terms up to $O(x^2)$

(i.e., $\tan(x) = a + bx + cx^2 + O(x^3)$. Find a, b, c`

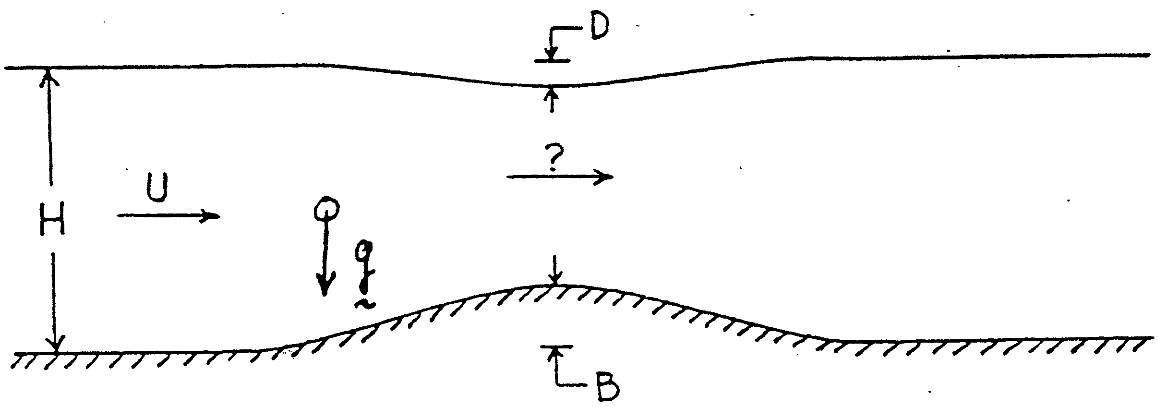
c) Let $\phi(x,y,z) = x + y^2 + z^3$. Evaluate $\nabla\phi$ and $\nabla^2\phi$. 31b

d) Integrate $\int_{0} x \cos(x)\, dx$.

Question 2.

Water, of depth H and velocity U, is flowing over a bump of height B as shown below. It is observed that the surface is depressed by an amount D over the bump. Relate the depression D to the velocity of water at that position, i.e., below the depression. Acceleration of gravity $\underset{\sim}{g}$ acts as shown.

State clearly any assumptions made.



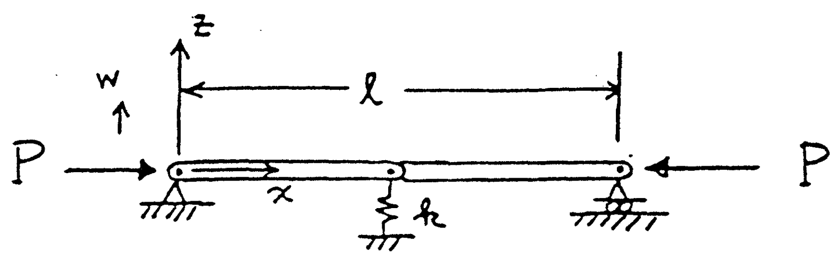
Question 3.

Given two beams pinned together, and supported against lateral motion by a linear spring k as shown. The beams are uniform and have equal stiffnesses EI.

a) Determine the buckling load P_c, assuming the beam stiffness EI is much greater than the spring stiffness k. (Beams are essentially rigid).

b) Determine P_c when the beam stiffness as well as the spring stiffness is also considered. What nondimensional spring-beam stiffness ratio parameter governs the problem here?

c) If the restraining spring force is nonlinear, i.e., one has $F_s = k_L w + k_N w^2$, determine the post-buckling behavior assuming the beams are essentially rigid compared to the spring (as in Part (a) above). Sketch the load versus center deflection relation, P versus w_c.

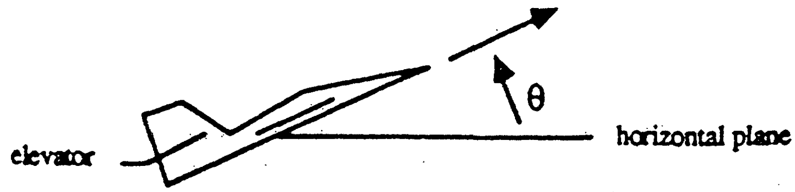


Question 4.

Under what conditions does the radiant energy transfer from a gas to a surface become comparable to the convected energy transfer? to the conducted energy transfer? Express your answer in terms of non-dimensional parameters and give numerical examples to describe the actual circumstances at which those parameters would be of order unity.

Question 5.

The pitch attitude of an aircraft is generally controlled by the pilot through the use of the elevator as shown below.



For the purposes of this problem, the dynamic relationship between the elevator deflection, δ, and the pitch attitude, θ, can be modeled as follows:

$$\dot{\theta} = K_a \delta$$

where K_a is the gain of the effect of elevator deflection upon aircraft pitch rate.

Part A: Simple Linear Pilot Model

The control response of a human pilot to error in pitch attitute can be modeled as:

$$\tau_n \dot{\delta} + \delta = K_h e$$

where e is the pitch error, τ_n is a neuromuscular lag time, and K_h is the gain of the pilot response.

Construct a block diagram of the closed-loop pilot/aircraft system and reduce the block diagram to obtain the closed-loop transfer function. Clearly label the inputs and outputs of each element of the block diagram.

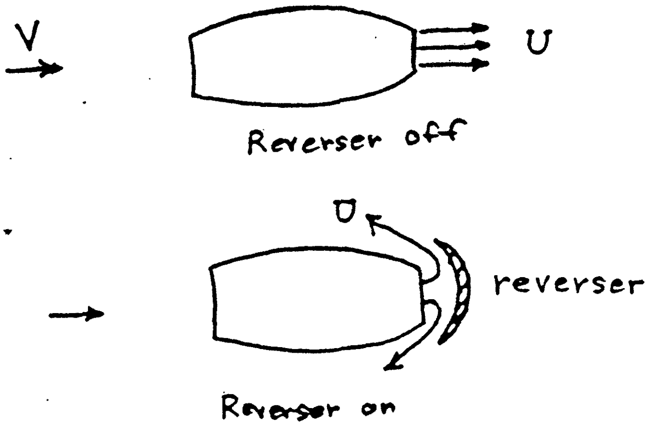
In flight, an atmospheric disturbance causes the aircraft to have an error in pitch attitude. Describe the response of the pilot/aircraft system in terms of system parameters. How might the pilot adjust the parameters of his response to the pitch error to achieve desired aircraft response? How might this depend upon the type of aircraft and mission that he/she is flying? What are the limits of the gain, K_h, that can be used by the pilot and how are they defined in terms of system response?

Part B: The Effect of Reaction Time

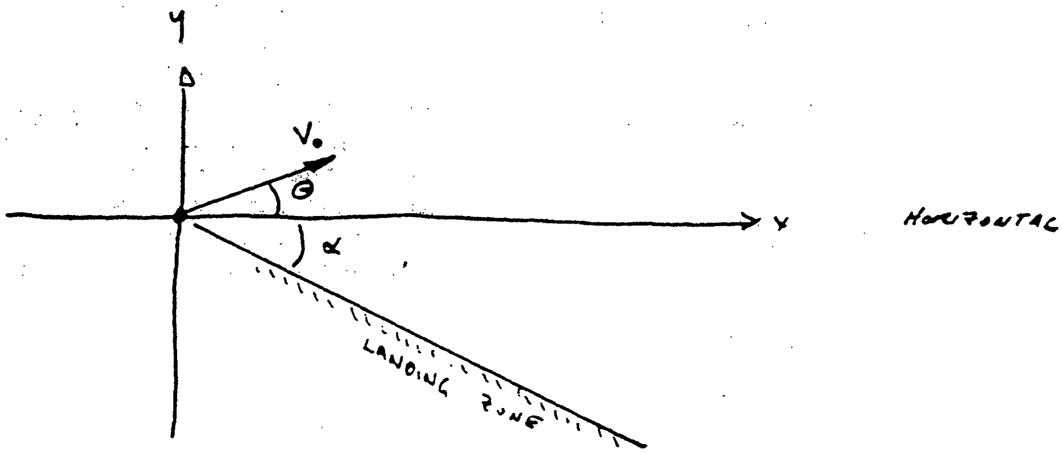
If, in addition to the neuromuscular lag, we add a pilot reaction time, τ_r, to the pilot model where τ_r is a simple time delay between the observation of the error and the initiation of a response, how does this change the pilot's transfer function? What are the effects of this reaction time upon system response and stability? If the reaction time causes instability, how might the pilot alter his/her gain to achieve stability?

Question 6.

A thrust reverser for a jet engine is shown shematically at the left. If the reverser turns the flow through an angle of 135° without appreciably altering the <u>relative</u> exit speed flow, U, if the forward speed of the engine is V and the engine massflow is $\dot{m}$, what is the change in thrust between operation with reverser off and reverser on?



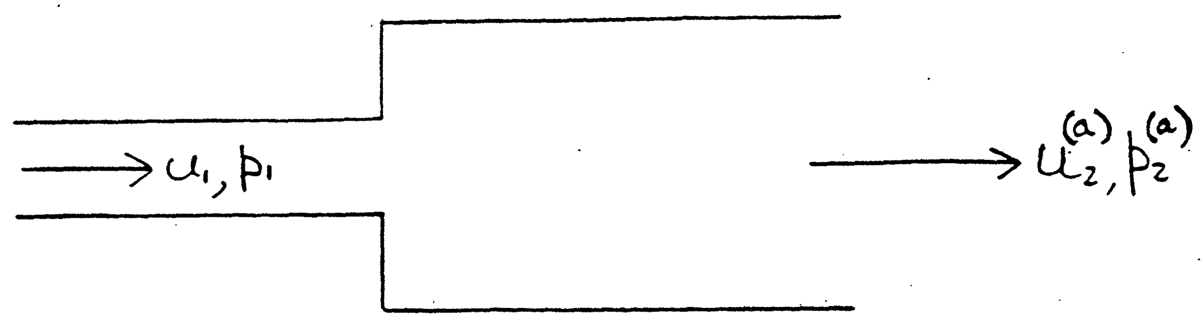
Question 7.

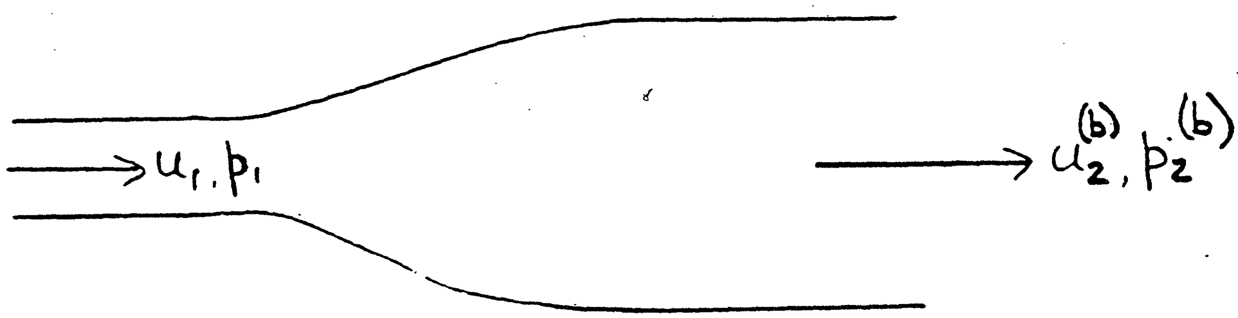


An olympic ski jumper leaves the origin of coordinates at time zero with a velocity magnitude, V_0, and free-falls to a landing on a flat surface that is inclined to the horizontal by an angle α. What angle, θ, should his initial velocity make with the horizontal to achieve the maximum length jump? What horizontal distance will he cover if he uses the optimum θ?

Neglect aerodynamic forces.

Question 8.

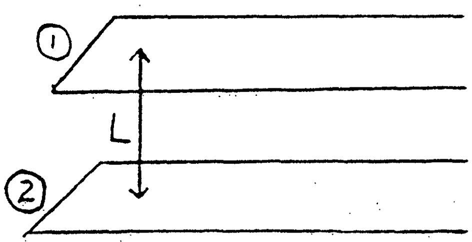




Consider the flow of an incompressible fluid in the junction between two pipes of cross-sectional area A_1 and A_2. Assuming that the Reynolds number is large, discuss the different qualitative nature of the flow in an abrupt and a smooth junction (pictured above). Explaining any approximations clearly, derive equations for the outflow variables u_2, p_2 in the two cases.

Question 9.

A low density gas is contained between two parallel infinite flat plates separated by a distance L. The upper plate is kept at temperature T_1 and the lower plate is kept at temperature T_2. The upper plate has an energy accommodation coefficient $\alpha_1 = 1$ and the lower plate α_2 may vary $0 < \alpha_2 < 1$. The mean free path of the gas is λ. Find the gas temperature distribution T(y) between the plates for $\alpha_2 = 0$, $\alpha_2 = 0.5$ and $\alpha_2 = 1.0$ for $Kn \equiv \lambda/L \ll 1$ and $Kn \gg 1$. What can you say if $Kn = O(1)$?



$Kn \equiv$ Knudsen number

Question 10.

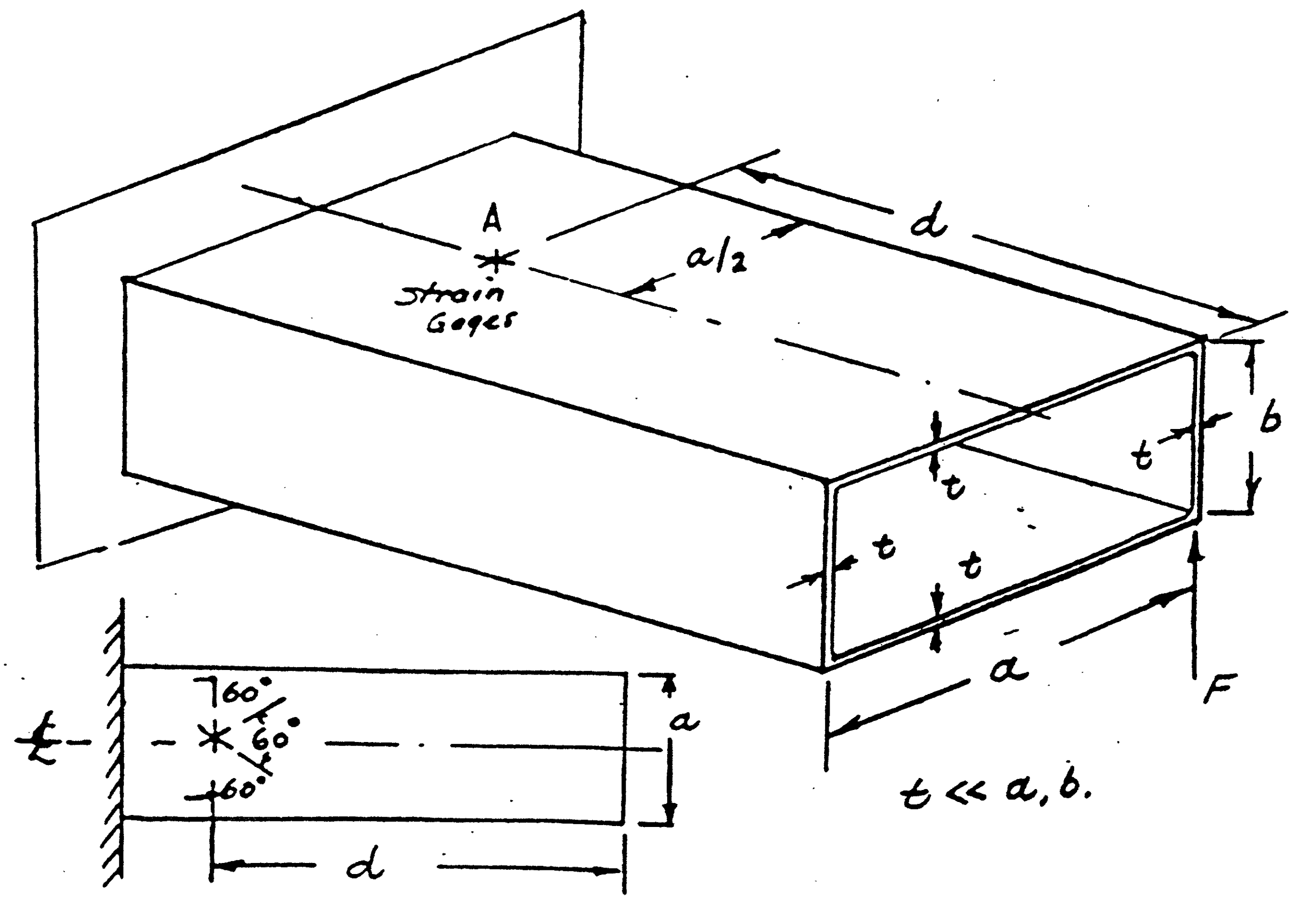
The arresting gear on an aircraft carrier consists of a steel cable strung between two shafts which turn paddles in two large water tanks (capacity 50,000 liters each). The tanks are baffled to suppress rotation of the water. Calculate the temperature rise in the water after recovery of an F-14 at a landing gross weight of 23,000 kg and a carrier approach speed of 250 km/hr. State any assumptions clearly. (Note that, for water C = 1 cal/gm deg C, and 1 cal = 4.186 joules)

Question 11.

A large insulated tank has a small valve closing it from the atmosphere. The tank is initially evacuated. By opening the valve, air is bled into the tank until the pressure in the tank rises to atmospheric. What is the temperature of the air in the tank? How much was the entropy of the universe increased by this process?

Question 12.

A method has been suggested for the determination of elastic properties of a given material. The set-up consists of a thin wall box beam with parallel webs and parallel top and bottom surfaces made of the specific isotropic material. It is cantilevered at one end and loaded by a concentrated force F along the vertical web at the other end. Three strain gages at 60° angle apart are mounted at point "A" which is on the top of the surface and at a distance "d" from the loaded edge. Based on the three measured strain gage readings determine the Young's modulus E, the shear modulus G and the Poisson's ratio ν. [Write, in details, the various steps in your solution.] If the material is orthotropic with structural axes coincide with the axes of the beam, can you suggest a method to determine the 4 independent elastic constants for this plane stress state using a similar set up?

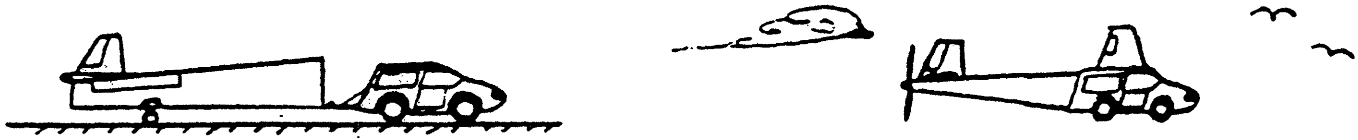


Question 13.

A particle of mass m is attracted toward the origin with a force proportional to the distance. If it starts from the point $x = a$, $y = 0$ with a velocity v_0 perpendicular to the x-axis, find the path described. Furthermore, if $v_0 = 0$, how long will it take to reach the origin?

Question 14.

The <u>Aerocar</u> is an automobile which can be converted to an aircraft on the road (the wings and tail are towed as a trailer behind the "car" when the vehicle is driven on the ground).



The vehicle characteristics are:

Airframe + pilot mass: $m_e = 500$ kg   Max fuel mass: $m_f = 500$ kg

<u>Ground</u>: best mileage: $\mu = 10$ km/kg fuel   (independent of weight)

<u>Air</u>: best $L/D = 20$
specific fuel consumption: $\sigma = .00025$ kg/(s · kW)
Propulsive efficiency: $\mu = 0.8$
acceleration due to gravity: $g = 10$ m/s^2

1. The driver/pilot sets off on a long journey with a full fuel load. How far will he get if

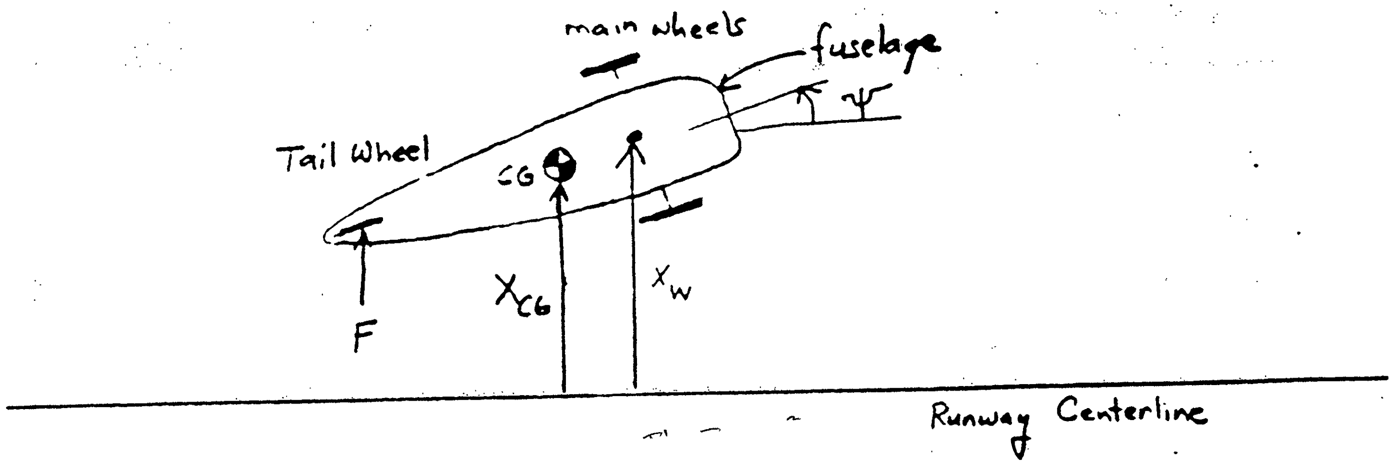
1 pt. a) He only travels on the road?
3 pts. b) He only travels by air?

6 pts. 2. What should he do to get the farthest with his initial fuel load?

Question 15.

The problem is to understand why it is difficult to steer an aircraft with a tail wheel rather than a nose wheel.

A model for the aircraft is shown below:



where

V = forward velocity
Ψ = yaw angle
L_1 = distance from main wheels to c.g.
L_2 = distance from tail wheel to c.g.
X_w = distance of main wheels to centerline
X_{cg} = distance of c.g. to centerline

The tail wheel pivots freely, whereas the main wheels cannot pivot. Assume that the aircraft has mass m and moment of inertia about the yaw axis I. The application of rudder control by the pilot essentially causes a lateral force F to be applied to the tail wheel.

Find the transfer function from the force F to position X_{cg}. You may assume that the angle Ψ is small. Why is the aircraft difficult to control?

[Hint: To find the equations of motion, you will have to use the constraint that the main wheels roll in the direction in which they are pointed, i.e., cannot slip sideways].

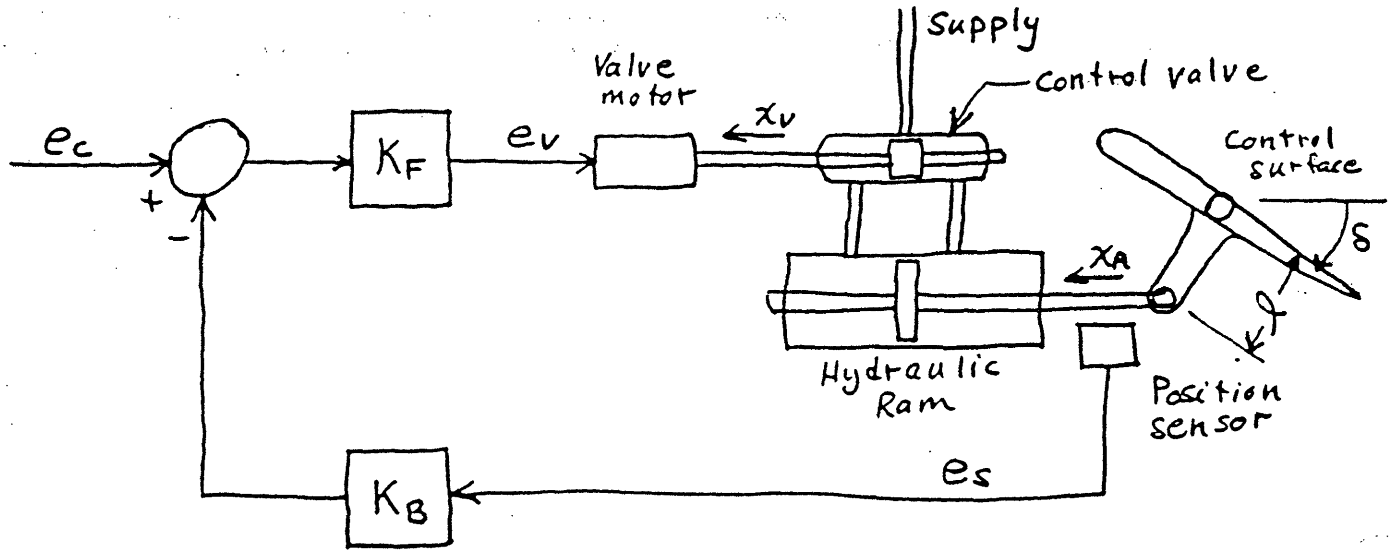
Question 16.

1. Derive the rocket equation.

2. Derive the velocity change (ΔV) requirement for a Hohmann (co-elliptical, co-tangent) transfer between an initial circular orbit at r_1 and a final circular orbit at r_2 ($> r_1$).

Question 17.

The most common method of driving the control surfaces of aircraft is with a hydraulic ram. The configuration in simplified form is shown below:



The control valve can be represented as follows:

$$\frac{x_v}{e_v} = \frac{S}{\frac{1}{\omega_n^2} s^2 + \frac{2\zeta}{\omega_n} s + 1}$$

$S = 0.2$ in/volt
$\omega_n = 120$ rad/sec
$\zeta = 0.5$

$K_v = 8 \ \frac{\text{in}^3/\text{sec}}{\text{in}}$

$Q = K_v x_v$

Q is the fluid flow rate considered positive into the right hand chamber of the ram. Neglect fluid compressibility effects.

Position sensor: $e_s = K_s X_A$

$K_s = 2$ volts/inch

Hydraulic piston face area = 0.8 in^2

Control horn length = ℓ = 5 inches

What values of forward gain, K_F, and feedback gain, K_B, are required to meet the following specifications?

a) Static sensitivity: $\delta / e_c = 0.1$ rad/volt

b) Response time approximately 0.1 sec.

Question 18.

Consider a nuclear-electric propulsion system with a payload of 1,000 kg and which is designed to produce a $\Delta v = 10{,}000$ m/sec in gravity free space. If the power to mass ratio of the power supply is 1.5 kW/kg and the initial acceleration is 10^{-3} m/s^2 find the optimum exhaust velocity. If the propellant is Cs^+ with $q/m = 7.25 \times 10^5$ Coulomb/kg find the ion engine accelerating voltage. If the electrodes in the engine are 1 mm apart, calculate the space change limited current density. Show that the fact that the current density is limited implies that the thrust per unit area of the ion engine is limited. Using this fact work out the area of the thruster.

[$\varepsilon_0 = 8.8542 \times 10^{-12}$ Farad/m]